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Theory of ionizing neutrino-atom collisions: The role of atomic recoil

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Abstract

We consider theoretically ionization of an atom by neutrino impact taking into account electromagnetic interactions predicted for massive neutrinos by theories beyond the Standard Model. The effects of atomic recoil in this process are estimated using the one-electron and semiclassical approximations and are found to be unimportant unless the energy transfer is very close to the ionization threshold. We show that the energy scale where these effects become important is insignificant for current experiments searching for magnetic moments of reactor antineutrinos.

Keywords: neutrino-impact atomic ionization, atomic recoil, neutrino magnetic moments, BSM physics

1. Introduction

Neutrinos are very intriguing objects in particle physics. They interact very weakly and their masses are much smaller than those of the other fundamental fermions (charged leptons and quarks). In the Standard Model (SM), neutrinos are massless and have only weak interactions. However, the observation of neutrino oscillations by many experiments implies that neutrinos are massive and mixed. Therefore, the SM must be extended to account for neutrino masses. In many extensions of the SM, neutrinos also acquire electromagnetic properties through quantum loop effects (see Refs. [1–3] for detail). Hence, the theoretical and experimental study of neutrino electromagnetic interactions is a promising tool to search for the fundamental theory beyond the SM (BSM).

The most theoretically studied electromagnetic properties of neutrinos are the dipole magnetic and electric moments. The neutrino magnetic moments expected in the minimally extended SM are very small and proportional to the neutrino masses:

$$\mu_\nu = 3 \times 10^{-19} \mu_B \left(\frac{m_\nu}{1\,\text{eV}} \right),$$

with $\mu_B = e/(2m_e)$ being the electron Bohr magneton (in units $\hbar = c = 1$), and m_e is the electron mass. Any larger value of μ_ν can arise only from the BSM physics [1–3]. Current direct experimental searches for a magnetic moment of the electron antineutrinos from reactors have lowered the upper limit on its value down to $\mu_\nu < 2.9 \times 10^{-11} \mu_B$ [4]. These ultra low background experiments use germanium crystal detectors exposed to the neutrino flux from a reactor and search for scattering events by measuring the energy deposited by the neutrino scattering in the detector. Their sensitivity to μ_ν crucially depends on lowering the threshold for the energy transfer T. This is because the electromagnetic contribution to the inclusive differential cross

section for the neutrino scattering on a free electron (FE) at small energy transfer ($T \ll E_\nu$) behaves as $d\sigma^{\mathrm{FE}}_{(\mu)}/dT \propto 1/T$ [5], while that induced by weak interaction, $d\sigma^{\mathrm{FE}}_{(w)}/dT$, is practically constant in T [5].

The current experiments using germanium detectors have reached threshold values of T as low as few keV, where one can expect modifications of the FE formulas due to the binding of electrons in the germanium atoms. Our theoretical analysis (see Ref. [6] and references therein), involving the WKB and Thomas-Fermi models, has shown that the so-called stepping approximation, introduced in [7] from an interpretation of numerical data, works with a very good accuracy. The stepping approximation treats the process as scattering on independent electrons occupying atomic orbitals and suggests that the cross sections $d\sigma_{(\mu)}/dT$ and $d\sigma_{(w)}/dT$ follow the FE behaviors down to the ionization threshold for the orbital; and below that energy the electron on the corresponding orbital is "inactive", thus producing a sharp "step" in the T dependence of the cross section.

To the best of our knowledge, the issue of the center-of-mass atomic motion has remained practically unadressed so far in theoretical studies devoted to the ionization channel of neutrino-atom collisions. Usually the recoil effects are neglected under the assumption that the atomic nucleus due to its large mass stays at rest during the ionization process. At the same time, it appears that in the case of reactor antineutrinos the recoil energy can be comparable to atomic binding energies. Thus, in this contribution, we analyze the role that can play the center-of-mass atomic motion in the discussed processes. In particular, we inspect how it can affect the validity of the stepping approximation.

2. Theoretical estimate of atomic-recoil effects

We specify the incident neutrino energy and momentum by E_ν and $\mathbf{p}_\nu$, respectively. The atomic target is supposed to be initially at rest, unpolarized and in its ground state $|0\rangle$ with the corresponding energy E_0. We treat the initial and final electronic systems nonrelativistically under conditions $T \ll m_e$ and $\alpha Z \ll 1$, where Z is the atomic number and α is the fine-structure constant. The incident and final neutrino states are described by the Dirac spinors assuming $m_\nu \approx 0$.

First we consider the experimentally measured single-differential inclusive cross section $d\sigma/dT$ of atomic ionization by neutrino impact, without accounting for atomic recoil in this process. The standard electroweak contribution to the cross section is given by (see

e.g. in [6])

$$
\frac{d\sigma_{(w)}}{dT} = \frac{G_F^2}{4\pi} \left(1 + 4\sin^2\theta_W + 8\sin^4\theta_W \right)
$$
$$
\times \int_{T^2}^{(2E_\nu - T)^2} S(T, q^2)\, dq^2, \qquad (1)
$$

where G_F is the Fermi constant, θ_W is the Weinberg angle, and $S(T, q^2)$ is the dynamical structure factor that can be presented as follows ($q = |\mathbf{q}|$):

$$
S(T, q^2) = \sum_n |\langle n|\rho(\mathbf{q})|0\rangle|^2\, \delta(T - E_n + E_0), \qquad (2)
$$

with $\rho(\mathbf{q})$ being the Fourier transform of the electron density operator and the n sum running over all the atomic states $|n\rangle$ with energies E_n of the electron system. The μ_ν contribution to the cross section can be similarly expressed in terms of the same factor (2) as [6]

$$
\frac{d\sigma_{(\mu)}}{dT} = 4\pi\alpha\mu_\nu^2 \int_{T^2}^{(2E_\nu - T)^2} S(T, q^2)\, \frac{dq^2}{q^2}, \qquad (3)
$$

where the function $S(T, q^2)$ is integrated over q^2 with q^{-2}, rather than a unit weight as in Eq. (1).

The kinematical limits for q^2 are explicitly indicated in Eqs. (1) and (3). At large E_ν, typical for the reactor antineutrinos ($E_\nu \sim 1\,\mathrm{MeV}$), the upper limit can in fact be extended to infinity, since in the discussed here nonrelativistic limit the range of momenta $q \sim E_\nu$ is indistinguishable from infinity on the atomic scale. The lower limit can be shifted to $q^2 = 0$, since the contribution from the region of $q^2 < T^2$ can be expressed in terms of the photoelectric cross section [8] and is negligibly small (at the level of below one percent in the considered range of T). For this reason one can discuss, without loss of accuracy, the momentum-transfer integrals in Eqs. (1) and (3) running from $q^2 = 0$ to $q^2 = \infty$.

In the one-electron approximation and in the semiclassical limit, the dynamical structure factor (2) acquires the form [9]

$$
S(T, q^2) = \frac{m_e}{2pq}\, \theta(T - I)\, \theta(q_+ - q)\, \theta(q - q_-), \qquad (4)
$$

where p is an average momentum of the electron bound in the atomic orbital from which ionization takes place, I is the ionization potential for this orbital, and

$$
q_\pm = \sqrt{p^2 + 2m_e T} \pm p.
$$

It is straightforward to show (see Ref. [9] for detail) that using the semiclassical approximation (4) in Eqs. (1)

and (3) one arrives at the FE expressions [5]

$$\frac{d\sigma^{\text{FE}}_{(w)}}{dT} = \frac{G_F^2 m_e}{2\pi}\left(1 + 4\sin^2\theta_W + 8\sin^4\theta_W\right), \quad (5)$$

$$\frac{d\sigma^{\text{FE}}_{(\mu)}}{dT} = \frac{4\pi\alpha\mu_\nu^2}{T}. \quad (6)$$

Let us take into account the atomic recoil. This means that the energy transfer T is partly deposited in the center-of-mass atomic motion. The energy of this motion, that is, the recoil energy, is $T_R = q^2/(2M_a)$, where M_a is the atomic mass. Hence, the first argument of the function $S(T, q^2)$ in (1) and (3) must be replaced with

$$T_q = T - T_R = T - \frac{q^2}{2M_a}.$$

Substituting $T \to T_q$ in Eq. (4), we obtain

$$S(T_q, q^2) = \frac{m_e}{2pq}\,\theta(q_a - q)\,\theta(q_+ - q)\,\theta(q - q_-), \quad (7)$$

where $q_a = \sqrt{2M_a(T - I)}$. From Eq. (7) it follows that the atomic recoil has no effect if $q_a > q_+$. Indeed, in the latter case the FE results (5) and (6) for the cross sections (1) and (3), respectively, remain unaltered. In contrast, when $q_a < q_+$ in Eq. (7), the cross sections become suppressed relative to the respective FE values and even vanish at $q_a = q_-$.

Clearly, the dynamical structure factor of a real atom exhibits more complex dependencies on T and q as compared to its semiclassical approximation (7). Nevertheless, this approximation mimics the main qualitative features pertinent to the exact function $S(T_q, q^2)$, for example, such as "spread and shift" of the free-electron δ peak [9]. Thus, the criterion for the atomic-recoil effects to come into play can be formulated for a real atom as $q_a \lesssim q_+$ or, accordingly, $T - I \lesssim 2p^2/M_a$. At such energy-transfer values the cross sections (1) and (3) become suppressed and vanish when $T \to I$.

General numerical estimates can be obtained within the Thomas-Fermi model of manyelectron atoms. The scale for the average electron momentum p in this model is determined by $\alpha Z^{2/3}m_e$. It gives the following estimate for the energy range where the atomic-reoil effects are important:

$$T - I \lesssim 2Z^{4/3}E_h(m_e/M_a),$$

where $E_h = \alpha^2 m_e = 27.2\,\text{eV}$ is the Hartree energy. For germanium ($Z = 32$) we obtain $T - I \lesssim 0.04\,\text{eV}$. This energy scale is insignificant for the experiments searching for magnetic moments of reactor antineutrinos [4]. As mentioned in the introduction, these experiments have reached threshold values of T as low as few keV. Such values are already below the ionization threshold for K electrons in germanium ($I_K \approx 11\,\text{keV}$). These atomic electrons are most strongly bound, and the Thomas-Fermi average p value substantially underestimates their average momentum. The latter can be evaluated using the virial theorem as $p = \sqrt{2m_e I_K}$. The corresponding energy scale is $T - I \lesssim 0.3\,\text{eV}$. It appears to be about an order of magnitude larger than that given by the Thomas-Fermi model, but it is still insignificant for the discussed experiments.

3. Conclusions

In summary, we have inspected how the center-of-mass atomic motion can affect ionization of an atom by neutrino impact. Employing the semiclassical approximation, we have derived the criterion that defines the energy-transfer range where the atomic-recoil effects are important. Our numerical estimates have shown that these effects play no appreciable role in current experiments searching for neutrino magnetic moments of electron antineutrinos from reactors with Ge detectors.

Acknowledgments

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