



## P-odd effects in heavy ion collisions at NICA

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### Abstract

Experimental manifestation of P-odd effects related to the vorticity and hydrodynamic helicity in non-central heavy ion collisions at MPD@NICA and BM@N detectors is discussed. For the NICA and FAIR energy range characterized by the large baryonic charge of the forming medium the effect should manifest itself in the specific neutron asymmetries. The polarization of strange particles probing the vorticity and helicity is also discussed.

*Keywords:* vorticity, helicity, handedness

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### 1. Introduction

A search for local violations of discrete (C and P) symmetries from the discovery of the famous Chiral Magnetic Effects (CME) [1] is a subject of intense theoretical and experimental studies. Here we concentrate on the effects related to the P-odd "mechanical" effects, like the orbital angular momentum, vorticity and helicity.

In particular, we address to the neutron angular asymmetries [2], effects of the helicity separation [3, 4] and baryon polarization [2, 3] as well as the emergence of momentum correlations (handedness) [4]. We discuss the manifestation of these effects at the NICA and FAIR energy range as well as new theoretical ideas on the role of maximal helicity (Beltrami) relativistic flows.

### 2. Generalized Chiral Vortical Effect (CVE) and neutron asymmetries [2]

It was shown [5] that CVE provides another source for the observed consequences of CME. On the other hand, CVE leads also to the separation of charges different from the electric one [2].

In particular, the large baryonic chemical potential appearing in the collisions at comparatively low energies at NICA and FAIR (and possibly SPS and RHIC at

low energy scan mode) may result in the separation of the baryonic charge. A special interest is manifestation of this separation in *neutron* asymmetries with respect to the production plane, as soon as neutrons, from the theoretical side, are not affected by CME and, from the experimental side, there is a unique opportunity to study neutron production and asymmetries at MPD@NICA.

### 3. Hydrodynamical helicity separation [3, 4]

The helicities  $\vec{v} \cdot \text{rot}\vec{v}$  in gold-gold collisions at different impact parameters were evaluated in different domains. The helicity calculated with inclusion of the all particles is zero (see Fig.1, blue online). For the particles with the definite sign of the velocity components, which are orthogonal to the reaction plane (and may be selected also experimentally), the helicity is nonzero and changes a sign for the different signs of these components (red and green lines, respectively). The effect is growing with the growing impact parameter and it represents a sort of the saturation in time. This effect holds in both QGSM [3] and HSD [4] models.

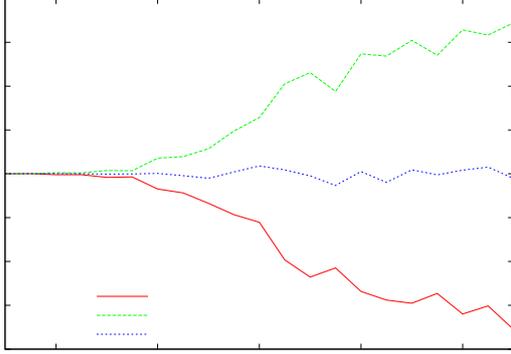


Figure 1: (color online) Time dependence of helicity

#### 4. Baryon polarization [2, 3]

Such a polarization can emerge due to the anomalous coupling of vorticity to the (strange) axial quark current via the respective chemical potential, being very small at RHIC but substantial at NICA and FAIR energies. In that case the  $\Lambda$  polarization at NICA due to the triangle anomaly can be explored in the framework of the experimental program of polarization studies at NICA performed in the both collision points.

#### 5. Momentum correlations [4]

One may introduce the following "handedness" pseudoscalars:

$$\sum (\vec{p}_3, \vec{p}_2, \vec{p}_1),$$

where  $(\vec{p}_3, \vec{p}_2, \vec{p}_1)$  - triple product

$$(\vec{p}_3, [\vec{p}_2, \vec{p}_1]),$$

$\vec{p}_1, \vec{p}_2, \vec{p}_3$  - momenta of pion in the final state. Momenta in each triple product were sorted in the following way:

$$|p_3|^2 < |p_2|^2 < |p_1|^2.$$

The following quantity was computed:

$$\eta = \frac{\sum (\vec{p}_3, \vec{p}_2, \vec{p}_1)}{\sum |(\vec{p}_3, \vec{p}_2, \vec{p}_1)|},$$

where the sums are over all such triplets. As the result, the indications of small but nonzero correlations at NICA energy range were obtained.

#### 6. Relativistic Beltrami flows

Chaotic Beltrami flows producing Lagrangian turbulence of streamlines play an essential role in the theory of dynamical systems, astrophysics, plasma physics, etc. (see, e.g. [6] and references therein). Quite recently, Beltrami equation was studied in the context of Chiral Magnetic Effect in heavy ion collisions (HIC) [7, 8]. As Beltrami flows obviously possess the maximal hydrodynamic helicity, their relevance for HIC also emerges due to the recently discovered phenomena of helicity separation and baryon polarization [2, 3]. Also, the chaotic streamlines of such flows [6] may provide an interesting mechanism of fast thermalization.

As the fluid velocities are essentially relativistic, a generalization of Beltrami flows to the relativistic case is required.

Let us rewrite the expression for isentropic flows in the following way:

$$\widehat{u}^\mu \left( \partial_\mu \widehat{u}_\nu - \partial_\nu \widehat{u}_\mu \right) = 0 \quad (1)$$

where  $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$ ,  $u^\mu = \gamma(1, \frac{v}{c})$  is the fluid four-velocity and the effective velocity

$$\widehat{u}_\mu \equiv \frac{\gamma w u_\mu}{\rho}. \quad (2)$$

Here,  $c$  is the velocity of light,  $\gamma = (1 - v_j^2/c^2)^{-\frac{1}{2}}$ , the Lorentz scalar  $n$  is the number density of the fluid particle of the mass  $m$ , so that  $\rho = nm\gamma$ . Note that the relativistic enthalpy (heat function)  $w = e + p$ , where the energy density  $e$  includes the rest energy  $\frac{\rho c^2}{\gamma}$ .

The relativistic-covariant generalization of the steadiness condition is

$$\eta^\mu \partial_\mu f = 0 \quad (3)$$

where  $\eta$  is an arbitrary constant four-vector which characterizes the fluid and  $f$  is an arbitrary function. In case when  $\eta$  is a time-like four-vector, in particular  $\eta = (1, 0, 0, 0)$ , this condition reproduces the standard steadiness condition.

The relativistic-covariant generalization of the Bernoulli equation

$$u^\mu \partial_\mu \widehat{u}_\nu \eta^\nu = 0 \quad (4)$$

can be derived by contracting eq. (1) with four-vector  $\eta$  and using eq. (3). It is satisfied in the whole volume occupied by the fluid

$$\partial_\mu \widehat{u}_\nu \eta^\nu = 0 \quad (5)$$

if the following relativistic-covariant generalization of the non-relativistic Beltrami condition  $\vec{v} \times \text{rot } \vec{v} = 0$  is valid

$$\epsilon_{\mu\nu\alpha\beta} \eta^\nu \widehat{u}^\alpha w^\beta = 0 \quad (6)$$

where the dual velocity strength-tensor  $\omega^{\nu\mu}$  and the generalized relativistic vorticity  $\omega^\nu$  are

$$\omega^{\nu\mu} \equiv \epsilon^{\nu\mu\alpha\beta} \partial_\alpha \widehat{u}_\beta, \quad \omega^\nu \equiv \omega^{\nu\mu} \eta_\mu. \quad (7)$$

This condition is fulfilled by the following relativistic-covariant generalization of the non-relativistic Beltrami equation:

$$\omega^{\nu\mu} = \phi(x) \left( \eta^\mu \widehat{u}^\nu - \eta^\nu \widehat{u}^\mu \right), \quad (8)$$

which leads to the following equation

$$\omega^\nu = \phi(x) \eta^2 \left( g^{\nu\mu} - \frac{\eta^\nu \eta^\mu}{\eta^2} \right) \widehat{u}_\mu. \quad (9)$$

The function  $\phi(x)$  is constrained by the fluid particle-number conservation. Calculating the divergence of both sides of eq. (8) and using eq. (3) one gets

$$\partial_\mu (\phi u^\mu) = 0. \quad (10)$$

Comparing it with the fluid particle-number conservation

$$\partial_\mu \frac{\rho u^\mu}{\gamma} = 0, \quad (11)$$

where  $n = \frac{\rho}{\gamma}$  is the density, one can derive expression for  $\phi$ :

$$\phi = k \frac{\rho}{\gamma}, \quad (12)$$

where  $k$  is an arbitrary real number. This implies (recall that  $\rho = nm\gamma$ ) the following form of a relativistic-covariant generalization of the Beltrami equation which for constant particle density is more close in the form to the non-relativistic case:

$$\omega^{\nu\mu} \equiv \epsilon^{\nu\mu\alpha\beta} \partial_\alpha \widehat{u}_\beta = m \left( \eta^\mu \widehat{u}^\nu - \eta^\nu \widehat{u}^\mu \right), \quad (13)$$

$$\begin{aligned} \omega^\nu \equiv \omega^{\nu\mu} \eta_\mu &= \epsilon^{\nu\mu\alpha\beta} \eta_\mu \partial_\alpha \widehat{u}_\beta \\ &= m \eta^2 \left( g^{\nu\mu} - \frac{\eta^\nu \eta^\mu}{\eta^2} \right) \widehat{u}_\mu, \end{aligned} \quad (14)$$

where for brevity we changed the notation  $knm \rightarrow m$ . A wide class of solutions to equation (13) for the relativistic generalized Beltrami fields is:

$$\begin{aligned} \widehat{u}_\mu(x) &= n^\nu \left( \delta_\mu^\alpha - \frac{\eta_\mu \eta^\alpha}{\eta^2} \right) \\ &\times \left( m \eta^\beta \epsilon_{\alpha\nu\beta\rho} \partial^\rho + g_{\alpha\nu} \square - \partial_\alpha \partial_\nu \right) \Phi(x) + c \eta_\mu, \end{aligned} \quad (16)$$

where the function  $\Phi(x)$  is constrained by

$$(\square - m^2 \eta^2) \Phi(x) = 0, \quad \eta^\mu \partial_\mu \Phi(x) = 0, \quad (17)$$

$n_\mu$  is an arbitrary constant four-vector and  $\square \equiv g^{\mu\nu} \partial_\mu \partial_\nu$ .

## 7. Conclusions

The P-odd effects related to angular momentum provide an interesting field of the investigation at NICA.

The relativistic Beltrami flows open an interesting opportunity of theoretical studies of P-odd effects. They may explain the emergence of hydrodynamic helicity and, what is especially challenging, generate the fast thermalization due to chaos of streamlines.

## References

- [1] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A **803** (2008) 227 [arXiv:0711.0950 [hep-ph]].
- [2] O. Rogachevsky, A. Sorin and O. Teryaev, Phys. Rev. C **82** (2010) 054910 [arXiv:1006.1331 [hep-ph]].
- [3] M. Baznat, K. Gudima, A. Sorin and O. Teryaev, Phys. Rev. C **88** (2013) 061901 [arXiv:1301.7003 [nucl-th]].
- [4] O. Teryaev and R. Usubov, arXiv:1406.4451 [nucl-th].
- [5] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A **797** (2007) 67 [arXiv:0706.1026 [hep-ph]].
- [6] G.M. Zaslavsky, Hamiltonian chaos and fractional dynamics, Oxford University Press, 2006.
- [7] M. N. Chernodub, arXiv:1002.1473 [nucl-th].
- [8] Z. V. Khaidukov, V. P. Kirilin, A. V. Sadofyev and V. I. Zakharov, arXiv:1307.0138 [hep-th].