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The Higgs mass coincidence problem.

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Abstract

We present a phenomenological evaluation of the ratio $\rho_t = m_Z m_t/m_H^2$, from the LHC combined m_H value, we get $((1\sigma))$

$$\rho_t^{(exp)} = 0.9956 \pm 0.0081.$$

This value is close to one with a precision of the order $\sim 1\%$. Similarly we evaluate the ratio $\rho_{Wt} = (m_W + m_t)/(2m_H)$. From the up-to-date mass values we get $\rho_{Wt}^{(exp)} = 1.0066 \pm 0.0035$ (1σ). The Higgs mass is numerically close (at the 1% level) to the $m_H \sim (m_W + m_t)/2$. From these relations we can write any two mass ratios as a function of, exclusively, the Weinberg angle (with a precision of the order of 1% or better):

$$\frac{m_i}{m_j} \simeq f_{ij}(\theta_W), \quad i, j = W, Z, H, t. \qquad (1)$$

For example: $m_H/m_Z \simeq 1 + \sqrt{2}s^2_{\theta_W/2}$, $m_H/m_t c_{\theta_W} \simeq 1 - \sqrt{2}s^2_{\theta_W/2}$. In the limit $\cos\theta_W \to 1$ all the masses would become equal $m_Z = m_W = m_t = m_H$.

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1. The ratio $\rho_t = m_Z m_t/m_H$. In the light of the recent results from the LHC coming from the experiments ATLAS and CMS, the parameter defined by the relation

$$\rho_t = \frac{m_Z m_t}{m_H^2} \qquad (2)$$

where m_Z, m_t are the masses of the Z^0 gauge boson and the top quark and m_H is the Higgs mass has become experimentally measurable. We estimate its current value to be

$$\rho_t^{(exp)} = 0.9956 \pm 0.0081 \qquad (3)$$

where we have used the current values for [2] and the combined value of the boson masses presented by ATLAS and CMS [3, 4]

$$m_H = 125.9 \pm 0.4 \pm 0.4 \quad \text{GeV/c}^2. \qquad (4)$$

The combined value of the boson mass is obtained by standard statistic techniques, we neglect correlations among the systematic component of the errors. The value (3) is obtained by a MC simulation (see [1]). The conclusion is that the experimental value of the ratio ρ_t is close to one with a precision of the order or less than 1%. This precision is not far from the precision at which the well known ratio

$$\rho = m_W^2/m_Z^2 \cos^2\theta_W$$

is presentely measured,

$$\rho = 1.0008 \pm 0.001$$

[2] with θ_W the Weinberg angle and m_W the charged electroweak gauge boson mass. The closeness of this parameter ρ_t to one might be merely a coincidence which will dissapear with any new measurement or might be not.

It is also interesting to consider an alternative way to express the closeness of the ratio ρ_t to one. If we consider the individual mass rations $m_Z/m_H, m_H/m_t$, their current experimental values are [1].

$$\frac{m_Z}{m_H} = 0.725 \pm 0.003, \quad (5)$$

$$\frac{m_H}{m_t} = 0.727 \pm 0.005 \quad (6)$$

where we have taken the LHC combined value of m_H. and PDG m_Z, m_t masses. Both ratios are the same at the level of 1% (and totally compatible at even higher precision according to present error bars). Very similar results are obtained if we use any of the ATLAS or CMS individual measurements

Similarly to ρ_t we define now another ratio of masses involving the Higgs, vector bosons and top quarks, whose experimental value is also seen to be close to one. Let us take

$$\rho_{Wt} = \frac{m_W + m_t}{2m_H} \quad (7)$$

where m_W is the mass of the W boson. We estimate the current value of this ratio (using a similar MC technique as explained above, see fig.(**??**)(down)) to be

$$\rho_{Wt}^{(exp)} = 1.0066 \pm 0.0035 \quad (8)$$

where we have used the current value for M_W [2] $M_W = 80.385 \pm 0.015\mathrm{GeV/c}^2$ and the combined value for the Higgs mass, Eq.(4). If the individual values for each of the experiments are used instead, we get $\rho_{Wt}^{(exp)} = 1.0082 \pm 0.0036$ ($m_{h,ATLAS}$), $\rho_{Wt}^{(exp)} = 1.0056 \pm 0.0036$ ($m_{h,CMS}$).

The relations $\rho_t \simeq \rho_{Wt} \simeq 1$ imply that any two of the quantities m_H, m_W, m_Z, m_t can be written in terms of the other two. Taking into account also the relation $\rho \simeq 1$ we can write any two mass ratios as a function of, exclusively, the Weinberg angle (with a precision of the order of 1% or better):

$$\frac{m_i}{m_j} \simeq f_{ij}(\theta_W), \quad i, j = W, Z, H, t. \quad (9)$$

Examples of these relations are:

$$\frac{m_W}{m_Z} \simeq \cos\theta_W, \quad (10)$$

$$\frac{m_H}{m_Z} \simeq 1 + \sqrt{2}\sin^2\frac{\theta_W}{2}, \quad (11)$$

$$\frac{m_H}{m_t}\cos\theta_W \simeq 1 - \sqrt{2}\sin^2\frac{\theta_W}{2}. \quad (12)$$

In the limit $\cos\theta_W \to 1$ all the masses would become equal $m_Z = m_W = m_t = m_H$.

[1] "God" or "golden" particle?. The difference between any of the m_H/m_Z, m_Z/m_H ratios and the Golden Ratio $(\sqrt{5}+1)/2$ is a "mere" 15%. Equality would be exact if $m_t = m_H + m_Z$

2. In the SM. In a model independent way, the quantity ρ_t can be viewed as the ratio of the highest massive representatives of the spin $(0, 1/2, 1)$ particles of the Standard Model and, to a very good precision the experimental evidence tell us that

$$\rho_t^{(exp)} \sim \frac{m_{s=1} m_{s=1/2}}{m_{s=0}^2} \simeq 1. \quad (13)$$

Somehow the mass of the "lowest" scalar particle mass is numerically the geometric mean of the highest spin 1 and spin 1/2 masses. In the Standard Model (SM) with a standard Higgs sector consistent of one Higgs doublet Φ the tree level top, gauge and Higgs boson masses are given in terms of v and their respective Yukawa couplings $m_W = g\frac{v}{2}$, $m_Z = \sqrt{g^2 + g'^2}\frac{v}{2}$, $m_t = g_t\frac{v}{2}$, $m_H^2 = -2\mu^2 = 2\lambda v^2$. Moreover we have $g' = g\tan\theta_W$ or $\sqrt{g^2 + g'^2} = g/\cos\theta_W$,$G_F m_W^2/\sqrt{2} = g^2/8$. In terms of these quantities the tree level ratio

$$\rho_t^{0(SM)} = \frac{1}{4\sqrt{2}}\frac{g g_t}{\cos\theta_W \lambda}. \quad (14)$$

Similarly, the tree level SM ρ_{Wt} ratio is given by:

$$\rho_{Wt}^{0(SM)} = \frac{m_W + m_t}{2m_H} = \frac{g + g_t}{4\sqrt{(2\lambda)}}. \quad (15)$$

In the SM, the Higgs selfcoupling λ is non-determined. At tree level any two of the quantities λ, g, g', g_t can be written in terms the two others using the expressions:

$$\lambda \simeq c\sqrt{g^2 + g'^2} g_t, \quad (16)$$

$$\lambda \simeq c^2 (g + g_t)^2 \quad (17)$$

where $c \sim o(1)$. Let us take into account both expressions. For $g_t >> g$ the second equation becomes $\lambda \simeq c^2 g_t^2$, inserting it in the first one we arrive to ($\kappa \simeq 1 + o(g/g_t)$)

$$\lambda \simeq \kappa\left(g^2 + g'^2\right). \quad (18)$$

Including one loop corrections, the three level relations above should be replaced, in particular by (where μ_0 the renormalization scale, $\mu_0 \sim m_Z - m_t$)

$$g_t(\mu_0) = \frac{\sqrt{2}m_t}{v}\left(1 + \delta_t(\mu_0)\right), \quad (19)$$

$$\lambda(\mu_0) = \frac{\sqrt{m_H^2}}{2v^2}\left(1 + \delta_\lambda(\mu_0)\right), \quad (20)$$

we consider negligible the running of the gauging couplings $g_i(\mu_0)$. The first order corrected ratio ρ_t is then, using expresions (19,20),

$$\rho_t^{SM} \simeq \rho_t^0\left(1 + \delta_\lambda - \delta_t\right). \quad (21)$$

The top yukawa δ_t can be written as $\delta_t = \delta_t^{QCD} + \delta_t^w$. The corrections are ([5] and references therein), ignoring logarithm terms, $\delta_\lambda = \frac{1}{16\pi^2} c_\lambda \lambda$, $\delta_t^w = \frac{1}{16\pi^2} \frac{c_t}{8} g_t^2$, $\delta_t^{QCD} = (-1/(3\pi^2)) g_s^2$, with the numerical coefficients $c_\lambda \simeq 25/2 - 9\pi/(2\sqrt{3}) \simeq 4.3$, $c_t \simeq 6.1$. Thus $\frac{\delta_\lambda}{\delta_t^w} \simeq \frac{c_\lambda}{c_t} \left(\frac{m_H}{m_t}\right)^2 \simeq 0.3$. Then $\rho_t = \rho_t^0 \left(1 + c_1 \lambda - c_2 g_t^2 - c_s g_s^2\right)$, The correction $\delta_t^{QCD} \sim 5\%$ is the most important one, acting to diminish slightly the ratio. Both corrections, $\delta_t^w, \delta_\lambda$, are of opposite sign and very small, of the order of 1%.

3. SM Renormalization group equations. The RGE equations for the individual couplings take the form (see for example [6, 7, 8, 9]) (with $t = \log(\mu/\Lambda)$, expression valid for high, but no so high, scales $\mu >> m_t, m_H$, or for $\Lambda \to \infty$):

$$\frac{dg_t^2}{dt} = \frac{9}{16\pi^2} g_t^4, \quad (22)$$

$$\frac{d\lambda}{dt} = \frac{6}{16\pi^2} \left(4\lambda^2 + 2\lambda g_t^2 - g_t^4\right). \quad (23)$$

If we introduce the variable $R = \frac{\lambda}{g_t^2}$ the RGE equations for g_t, R and $\rho_t(t)$ become decoupled with nested solutions, $g_t = g_t(\mu)$,$R = R(g_t)$,$\rho_t = \rho_t(R)$. In addition to Eq.(22), we have

$$g_t^2 \frac{dR}{dg_t^2} = \frac{1}{3} f(R), \quad (24)$$

$$\frac{d\rho_t}{dR} = -\frac{3\rho_t}{2f(R)} \left(1 + \frac{2f(R)}{3R}\right). \quad (25)$$

with $f(R) = 8R^2 + R - 2$. The equations (22,24,25) can be solved explicitly. For a light Higgs and large top mass the ratio R is small, at low scales $R^{exp} \sim 10^{-1}$, Eq.(**??**). For such a small R the solution of the differential equations is approximately: $R(g_t) = R_c - \frac{4}{3} \log g_t$, and $\rho_t \sim kR^2 \sim (R_c - \frac{4}{3} \log g_t)^2 \sim kR_c^2 \sim \rho_t^0$. At large energies ($\mu >> m_t$, as long as $R > 0$ or $\lambda > 0$), the ratio $\rho_t(\mu)$ keeps approximately constant, only sligtly decreasing with the logarithm of g_t.

If we consider a reduced Higgs-top-strong system where the λ, g_t, g_s are non-vanishing and allowed to run together with the ratios R, ρ_t. One ends with a similar system of equations where the evolution of ρ_t is of the type $g_t^2 d\rho_t / g_t^2 \sim \rho_t h(R, g_t^2)$ and similar results are obtained.

4. Conclusions and further discussion. We expect new physics that cuts off the divergent top, gauge and higgs loop contributions to the Higgs Mass at scales $\lesssim$ 10 TeV. Many different possibilities have been well explored, they usually include, more or less ad-hoc, new particles with properties tightly associated to those of the SM. Some of these possibilities are for example (and any combinations among them)[10, 11]: a) The new particles are just the, softly broken, SUSY, superpartners with couplings and Yukawas strongly dictated by supersymmetry and the soft breaking itself. b) The Higgs is a composite resonance, or c) The “Little” Higgs is a pseudo-Nambu-Goldstone boson with respect a “softly” broken approximate global symmetry. This scalar sector is accompanied by some new particles belonging to enlarged multiplets together with the SM particles. It is a general feature that, in all or most of these models, the quartic self coupling, and then the Higgs mass, is related to the gauge coupling constants and to the top yukawa in a more or less explicit way, reminding of the relation (**??**) suggested by the experimental evidence $\rho_t \simeq 1$. The reason is clear [10], the new one-loop which are proportional to the couplings of the SM gauge sector (or to a subsector of an enlarged gauge sector) have to match and cancel the top and the other cuadratic loops. We have briefly review the situation in the MSSM and Littlest Higgs scenarios in [1].

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