



## Five jet production at next-to-leading order QCD

S. D. Badger<sup>a</sup>, B. Biedermann<sup>b,1</sup>, P. Uwer<sup>c</sup>, V. Yundin<sup>d</sup><sup>a</sup>Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland<sup>b</sup>Universität Würzburg, Institut für Physik, Emil-Hilb-Weg 22, 97074 Würzburg, Germany<sup>c</sup>Humboldt-Universität zu Berlin, Institut für Physik, Newtonstraße 15, 12489 Berlin, Germany<sup>d</sup>Max-Planck-Institut für Physik, Föringer Ring 6, 80805 München, Germany

---

### Abstract

We discuss theoretical predictions for the production of five hard jets at next-to-leading order accuracy QCD at the Large Hadron Collider. Results are shown for both the total cross section as well as for differential distributions of the jet transverse momenta and rapidities. As a general pattern, we find moderate corrections of the order of 10% with respect to the LO result and a significant reduction of the scale dependence. Furthermore, ratios of different jet-multiplicity are studied. Our results are compared with data from the ATLAS collaboration.

*Keywords:* massless QCD, jet physics, hadronic collisions, unitarity method, next-to-leading order corrections

---

### 1. Introduction

The experiments at the Large Hadron Collider (LHC) represent an excellent opportunity to test perturbative quantum chromo-dynamics (QCD) with jets as its main observables. A phenomenologically and theoretically important jet production mode stems from pure QCD reactions with every observed jet originating from the same hard scattering matrix element involving exclusively QCD interactions. Such processes are of great interest to constrain the parton distribution functions (PDF) and the value of the strong coupling constant  $\alpha_s$  itself. Furthermore, they may contribute as important backgrounds for new physics searches. Since the occurring processes at the LHC may involve many particles in the final state a precise understanding of multi-jet production is necessary. Multi-jet computations at leading order (LO) are by now well established. However, they suffer in general from large theoretical uncertainties like the residual dependence of the unphysical factorisation and renormalisation scales. Such uncertainties are in general reduced when including higher order corrections.

For di-jet production this has been achieved at next-to-leading order (NLO) more than 20 years ago [1] and the full next-to-next-to-leading order (NNLO) prediction is almost completed [2]. Three-jet production at NLO was completed in 2002 [3], and implemented in the public code NLOJET++, though pure gluonic contributions were known previously [4]. A notable breakthrough has been the computation of four-jet production at NLO [5, 6], with results generally in good agreement with the experimental data [7].

NLO computations have meanwhile achieved a very high level of automation in sophisticated public computer programs such as [8, 9, 10, 11, 12]. Important contributions from on-shell unitarity methods [13, 14] and integrand reduction [15] as well as alternative new algorithms [16, 17, 18] have lead to a variety of high-multiplicity phenomenological applications in perturbative QCD (recent examples can be found in [19, 20, 21, 22, 23, 24, 22, 25], see also [26] for a more complete overview). In these proceedings, we discuss the production of five hard jets at NLO accuracy in QCD.

The paper is organised as follows: we first outline the methods and tools used for the computation in section 2 and describe the detailed numerical setup in section 3.

---

<sup>1</sup>Speaker

In section 4, we present results for the total cross section and differential distributions for the jet transverse momenta and rapidity, as well as jet ratios of different multiplicity jet production. Where possible, the results are compared with data from the ATLAS measurements. We finally present our conclusions. More results can further be found in [20].

## 2. Outline of the computation

The computation is done in the five-flavour scheme in massless QCD. In fixed-order perturbation theory, the  $n$ -jet differential cross section is expanded in the coupling  $\alpha_s$ . At NLO, this reads  $d\sigma_n = d\sigma_n^{\text{LO}} + d\delta\sigma_n^{\text{NLO}} + \mathcal{O}(\alpha_s^{n+2})$  with  $d\sigma_n^{\text{LO}} \sim \alpha_s^n$  and  $d\delta\sigma_n^{\text{NLO}} \sim \alpha_s^{n+1}$ . The partonic processes that occur at leading order may be derived from the four basic channels

$$\begin{aligned} 0 &\rightarrow gggggg, & 0 &\rightarrow q\bar{q}gggg, \\ 0 &\rightarrow q\bar{q}q'\bar{q}'ggg, & 0 &\rightarrow q\bar{q}q'\bar{q}'q''\bar{q}''g \end{aligned}$$

via crossing symmetry where  $q$ ,  $q'$  and  $q''$  denote generic quark flavours. The phase space integration for the leading-order cross section is performed with the Sherpa Monte-Carlo event generator [27], the necessary tree-level amplitudes being evaluated with the built-in matrix element generator Comix [28]. The NLO corrections include both the one-loop virtual corrections with born kinematics, as well as the real radiation with extended kinematics due to an additionally emitted parton in the final state. The two contributions are separately collinear and infrared divergent. The divergencies cancel only after integration over phase space and factorisation of the initial-state singularities into renormalized parton distributions. Within the Catani-Seymour subtraction scheme [29]—a technical prescription how to cancel the singularities analytically beforehand—both virtual and real corrections may be integrated separately with a Monte-Carlo program. We use again the Sherpa Monte-Carlo event generator with its own implementation of the Catani-Seymour formalism for the phase space integration. Comix [28] is employed as tree-level matrix element generator for the real corrections. The virtual corrections are computed with the publicly available program NJET<sup>2</sup> [11] and linked via the Binoth-Les Houches accord [30, 31] to Sherpa. Based on NGLUON [8], NJET is a one-loop matrix-element generator for high multiplicity processes in massless QCD, in its initial version being able to evaluate all matrix elements for

two-, three-, four- and five-jet production. Meanwhile, there is an extended version of NJET available allowing the computation of processes with one vector boson ( $W, Z, \gamma$ ) and up to five jets, and for diphoton production with up to four jets. NJET uses methods from generalised unitarity [32, 33, 34, 35],  $D$ -dimensional generalised unitarity [36, 37] and the integrand-reduction procedure of OPP [15] to construct multi-parton one-loop primitive amplitudes from tree-level amplitudes. The tree-level input for the unitarity cuts is evaluated with efficient Berends-Giele recursion [38]. The scalar loop integrals are obtained from the QCDLoop/FF Package [39, 40]. For a detailed study of the implemented algorithms, we refer to Refs. [8, 11, 41]. We note that we use the full colour and full helicity matrix elements for our computation, although the new version of NJET allows a separation of leading-colour and subleading-colour contributions.

## 3. Numerical setup

For the recombination of the partons into jets, we employ the anti-kt jet algorithm [42]. In particular, we use the implementation of FASTJET [43] linked to the Sherpa Monte-Carlo event generator. In order to compare our predictions with data of multi-jet cross sections from the ATLAS collaboration [7], we apply asymmetric cuts on the jets ordered in transverse momenta

$$p_T^{j_1} > 80, \text{ GeV} \quad p_T^{j_{\geq 2}} > 60 \text{ GeV}, \quad R = 0.4$$

with  $R$  being the jet resolution parameter. NNPDF2.3 with  $\alpha_s(M_Z = 0.118)$  is used as the standard PDF in the NLO setup, and NNPDF2.1 with  $\alpha_s(M_Z = 0.119)$  for the LO analysis. The analysis is done for centre-of-mass energies of 7 TeV and 8 TeV. Both for the renormalisation and factorisation scale, we use a single dynamical scale  $\mu = \mu_r = \mu_f$ , based on the sum of the transverse momenta of the final state partons

$$\widehat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}}.$$

We choose  $\mu = \widehat{H}_T/2$  to be the central scale.

## 4. Numerical results

With the setup described in the previous section, we get the following results for the 5-jet cross section at 7 and 8 TeV:

<sup>2</sup>The NJET code is available at <https://bitbucket.org/njet/njet/>.

$\mu$	$\sigma_5^{7\text{TeV-LO}}$ [nb]	$\sigma_5^{7\text{TeV-NLO}}$ [nb]
$\widehat{H}_T/2$	$0.699 \pm 0.004$	$0.544 \pm 0.016$
$\widehat{H}_T$	$0.419 \pm 0.002$	$0.479 \pm 0.008$
$\widehat{H}_T/4$	$1.228 \pm 0.006$	$0.367 \pm 0.032$

$\mu$	$\sigma_5^{8\text{TeV-LO}}$ [nb]	$\sigma_5^{8\text{TeV-NLO}}$ [nb]
$\widehat{H}_T/2$	1.044(0.006)	0.790(0.021)
$\widehat{H}_T$	0.631(0.004)	0.723(0.011)
$\widehat{H}_T/4$	1.814(0.010)	0.477(0.042)

While the statistical error of the Monte-Carlo Integration (given in parenthesis) is at permille level for the LO computation, it is at percent level at NLO showing the complexity of the computation. Besides the central scale  $\mu = \widehat{H}_T/2$ , we quote also the cross section at an upper scale  $\mu = \widehat{H}_T$  and at a lower scale  $\mu = \widehat{H}_T/4$ . As expected, we observe a significant reduction of the residual scale dependence when going from LO to NLO: For the given scale choices, the uncertainty is reduced by about 70-80%. This is also illustrated in

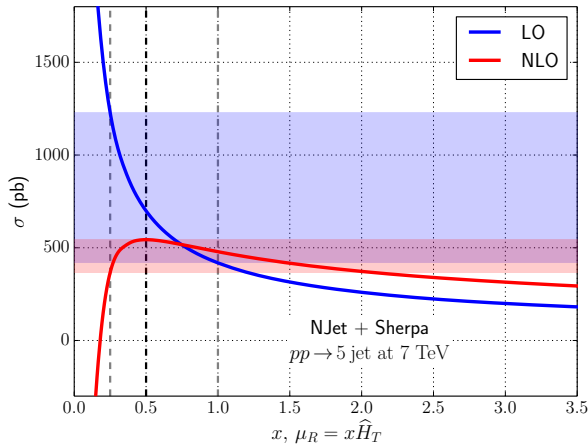


Figure 1: Residual scale dependence of the 5-jet cross section in leading and next-to-leading order.

Fig. 1 where the LO and NLO cross section is plotted as a function of  $\mu$ . The black dashed vertical line indicates the cross section at the central scale  $\mu = \widehat{H}_T/2$ , while the blue and the red horizontal band shows the scale variation limited by the upper and lower scales. Furthermore, the variation of the NLO cross section is rather flat with a maximum at the central scale which is a hint for the goodness of the dynamical scale. In Fig. 2, we show again the dependence of the total cross section on the

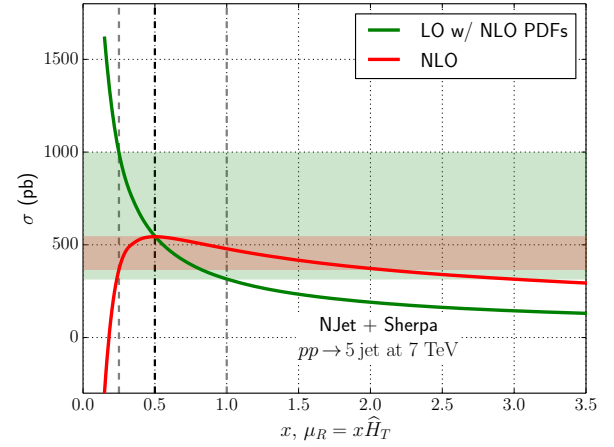


Figure 2: Same as Fig. 1 but using the NLO setup in LO.

scale  $\mu$  but this time using the NLO PDFs also for the LO cross section. While the qualitative behaviour with respect to reduction of the residual scale dependence is the same as in the LO setup, we observe in addition that the NLO corrections at the central scale become very small which is remarkable for an NLO-QCD computation. A similar pattern has been observed already studying the production of four and three hard jets with a similar setup [6] where it is shown that the main reduction of the corrections' size is due to the change of  $\alpha_s$  within the two different PDF sets.

For the comparison with experimental data, we state besides the five-jet cross sections also the NLO values for two-, three- and four-jet production at 7 TeV that have been computed with the same numerical setup:

$\sigma_2^{7\text{TeV-NLO}}$	$1175(3)_{1295(-)}^{1046(+)} \text{ nb}$
$\sigma_3^{7\text{TeV-NLO}}$	$52.5(0.3)_{33.2(-)}^{54.4(+)} \text{ nb}$
$\sigma_4^{7\text{TeV-NLO}}$	$5.65(0.07)_{3.72(-)}^{5.36(+)} \text{ nb}$
$\sigma_5^{7\text{TeV-NLO}}$	$0.544(0.016)_{0.367(-)}^{0.479(+)} \text{ nb}$
$\sigma_6^{7\text{TeV-LO}}$	$0.0496(0.0005)_{0.0992(-)}^{0.0263(+)} \text{ nb}$

The values in superscript are the cross sections at the upper scale and those in subscript the ones at the lower scale. The fact that five-jet production at next-to-leading order for the real radiation requires the computation of matrix elements with six partons in the final state allows in a rather easy way the evaluation of the six-jet rate, however, consequently only at leading order.

In Fig. 3, we compare our theoretical NLO predictions for multi-jet production with the actual measurements of the total jet cross sections performed by the

ATLAS collaboration [7] at 7 TeV. Apart from the two-

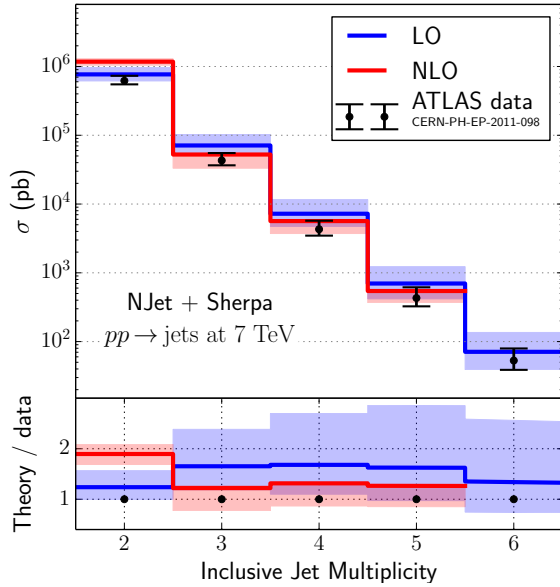


Figure 3: Cross sections for 2-, 3-, 4-, 5- and 6-jet production in leading and next-to-leading order as calculated with NJet as well as results from ATLAS measurements [7]. The LO quantities use NNPDF2.1 while the NLO quantities use NNPDF2.3. The 6-jet cross section is only available in LO accuracy.

jet cross section, we see a remarkably good agreement within the uncertainty of the ATLAS data. One reason for the mismatch with the two-jet data could arise from the soft gluon regime which is known to give rise to large corrections that cannot be treated within fixed order perturbation theory. Another reason are the asymmetric cuts that we applied: At leading order, the two jets are back-to-back, i.e., the asymmetric cut  $p_T^{j_1} > 80$  GeV and  $p_T^{j_2} > 60$  GeV is in fact a symmetric cut with both jets fulfilling  $p_T > 80$  GeV. At NLO the real radiation can employ some of the asymmetric phase space which explains qualitatively why the NLO cross section is so much larger than the LO value, in contrast to the three-, four- and five-jet case where the corrections are negative.

It is instructive to study the ratios of different multiplicity cross sections since in such observables various uncertainties (e.g. stemming from luminosity, scale dependence, PDF dependence etc.) may cancel. To this end we define a jet ratio  $\mathcal{R}_n$  as

$$\mathcal{R}_n = \frac{\sigma_{(n+1)\text{-jet}}}{\sigma_{n\text{-jet}}}.$$

This is a phenomenologically interesting quantity since it is proportional to  $\alpha_s$  at leading order. The ratios

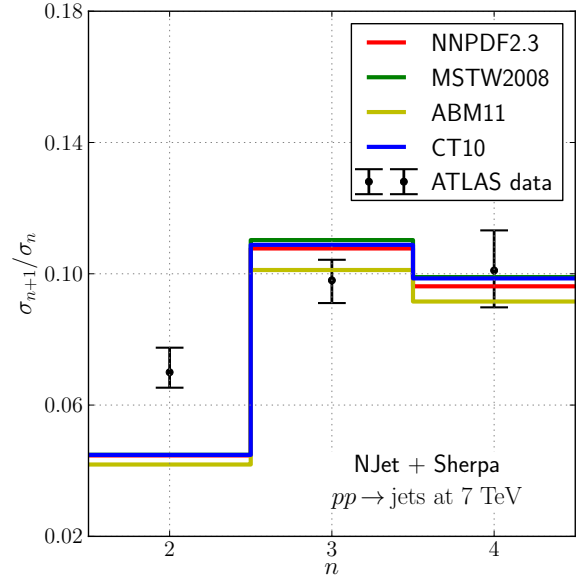


Figure 4: Theoretical predictions for the jet ratios  $\mathcal{R}_n$  compared with ATLAS measurements [7]. Theoretical predictions are made with the central values of the 4 listed PDF sets with NLO  $\alpha_s$  running.  $\alpha_s(m_Z) = 0.118$  for NNPDF2.3, CT10 and ABM11 and  $\alpha_s(m_Z) = 0.120$  for MSTW2008

$\mathcal{R}_2$ ,  $\mathcal{R}_3$  and  $\mathcal{R}_4$  are shown in Fig. 4 where the ATLAS data are compared with the NLO computation using, besides our standard PDF set NNPDF2.3 [44], also MSTW2008 [45], ABM11 [46] and CT10 [47]. While the predictions based on NNPDF2.3, MSTW2008 and CT10 give compatible results, the values from ABM11 are slightly smaller. The explicit values are shown in Tab. 1 for the NNPDF2.3 PDF set. We observe for  $\mathcal{R}_3$

$\mathcal{R}_n$	ATLAS[7]	LO	NLO
2	$0.070^{+0.007}_{-0.005}$	0.0925(0.0002)	0.0447(0.0003)
3	$0.098^{+0.006}_{-0.007}$	0.102(0.000)	0.108(0.002)
4	$0.101^{+0.012}_{-0.011}$	0.097(0.001)	0.096(0.003)
5	$0.123^{+0.028}_{-0.027}$	0.102(0.001)	--

Table 1: Results for the jet ratios  $\mathcal{R}_n$  for the central scale of  $\widehat{H}_T/2$  and NNPDF2.3 PDF set.

and  $\mathcal{R}_4$  remarkably stable results with deviation from the LO result by less than 10%. Also the agreement with the experimental data is good within the estimated uncertainty. As expected,  $\mathcal{R}_2$  does not agree well with the data. The NLO corrections amount to -50%. The

reason can again be found in the estimation of the total two-jet cross section with asymmetric cuts and the missing soft gluon contributions.

In Fig. 5, we show  $\mathcal{R}_n$  as a function of the transverse momentum  $p_T$  of the leading jet. The dashed lines rep-

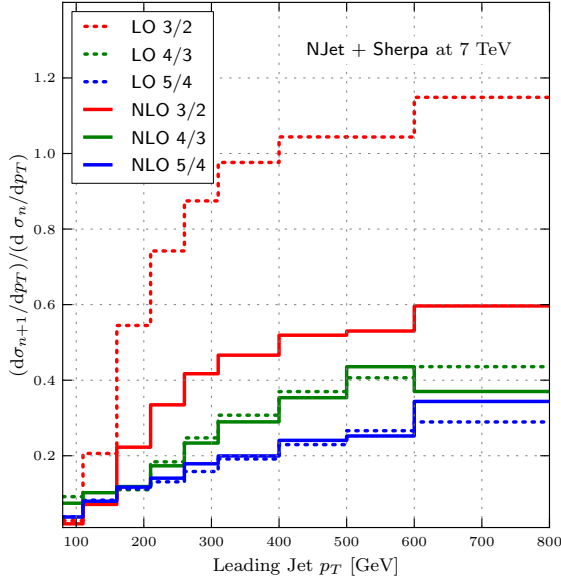


Figure 5: The  $\mathcal{R}_n$  ratio as a function of the  $p_T$  of the leading jet.

resent the LO ratio while the solid lines denote the corresponding NLO result. For the 3/2 ratio, we see also at the differential level large negative corrections at NLO for all values of  $p_T$ , as expected from the discussion above. The corrections for the 4/3 and the 5/4 ratios, however, are moderate in size for all values of  $p_T$ . This stable result makes  $\mathcal{R}_3$  and  $\mathcal{R}_4$  good candidates for future  $\alpha_s$  measurements.

The transverse momentum distribution for the leading jet without any ratio is shown for the 7 TeV case in Fig. 6. Note that we have used the NLO pdf set NNPDF2.3 both for the LO and NLO distribution. The NLO corrections in this setup are rather small, for most of the spectrum, the LO values are modified by less than 10%. The horizontal bands denote again the scale variation in the same way as described for the total cross section in Figs. 1 and 2. Over the whole  $p_T$ -range we observe a significant reduction of the residual scale dependence. A remarkable feature is the nearly constant K-factor over almost the whole shown distribution. The reason for this is most likely the dynamical scale choice which may re-sum possibly large logarithms in the high  $p_T$  range and thus helps to improve the convergence of the perturbative expansion. Note, however, that a dy-

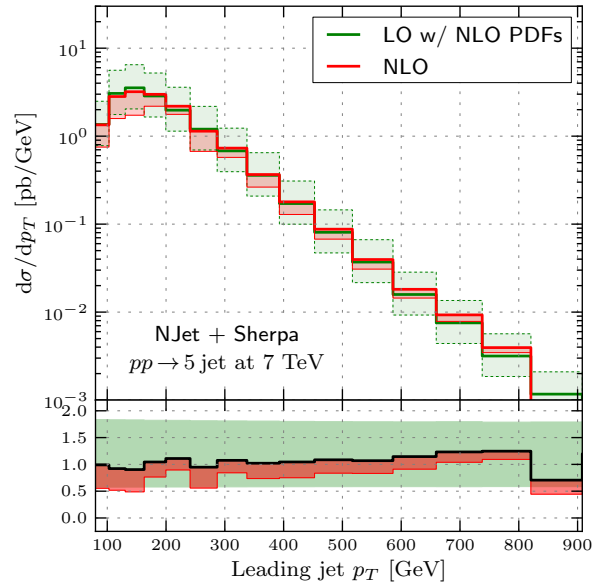


Figure 6: The  $p_T$  distribution of the leading jet. Both LO and NLO use the NNPDF2.3 pdf set with  $\alpha_s(M_Z) = 0.118$

namical scale goes, strictly speaking, beyond fixed order perturbation theory. Similar results hold also for the subleading jets and for the rapidity distributions as described in [20].

## 5. Conclusion

In these proceedings, the production of five hard jets at next-to-leading order in massless QCD is being discussed. The process has been computed with the publicly available one-loop matrix element generator NJET linked to the Sherpa Monte-Carlo event generator. As a general pattern we find that the inclusion of the NLO QCD correction reduces the residual scale dependence. The corrections are in general small, of the order of 10% when using NLO PDF sets both for the LO and NLO cross section. Using a dynamical scale based on the sum of the transverse momenta of the final state partons, we find a remarkably constant K-factor in transverse momentum and rapidity distributions. Comparing our results with ATLAS data, we find good agreement between theory and data for observables with more than two hard jets. In particular the 4/3 and the 5/4 jet ratios seem to be good candidates for future  $\alpha_s$  measurements.

## References

- [1] W. Giele, E. N. Glover, D. A. Kosower, Higher order corrections to jet cross-sections in hadron colliders, Nucl.Phys. B403

- (1993) 633–670. arXiv:hep-ph/9302225, doi:10.1016/0550-3213(93)90365-V.
- [2] A. G.-D. Ridder, T. Gehrmann, E. Glover, J. Pires, Second order QCD corrections to jet production at hadron colliders: the all-gluon contribution, *Phys.Rev.Lett.* 110 (2013) 162003. arXiv:1301.7310, doi:10.1103/PhysRevLett.110.162003.
- [3] Z. Nagy, Three jet cross-sections in hadron hadron collisions at next-to-leading order, *Phys.Rev.Lett.* 88 (2002) 122003. arXiv:hep-ph/0110315, doi:10.1103/PhysRevLett.88.122003.
- [4] W. B. Kilgore, W. Giele, Next-to-leading order gluonic three jet production at hadron colliders, *Phys.Rev.D* 55 (1997) 7183–7190. arXiv:hep-ph/9610433, doi:10.1103/PhysRevD.55.7183.
- [5] Z. Bern, G. Diana, L. Dixon, F. Febres Cordero, S. Hoeche, et al., Four-Jet Production at the Large Hadron Collider at Next-to-Leading Order in QCD, *Phys.Rev.Lett.* 109 (2012) 042001. arXiv:1112.3940, doi:10.1103/PhysRevLett.109.042001.
- [6] S. Badger, B. Biedermann, P. Uwer, V. Yundin, NLO QCD corrections to multi-jet production at the LHC with a centre-of-mass energy of  $\sqrt{s} = 8$  TeV, *Phys.Lett.* B718 (2013) 965–978. arXiv:1209.0098, doi:10.1016/j.physletb.2012.11.029.
- [7] G. Aad, et al., Measurement of multi-jet cross sections in proton-proton collisions at a 7 TeV center-of-mass energy, *Eur.Phys.J.* C71 (2011) 1763. arXiv:1107.2092, doi:10.1140/epjc/s10052-011-1763-6.
- [8] S. Badger, B. Biedermann, P. Uwer, NGLuon: A Package to Calculate One-loop Multi-gluon Amplitudes, *Comput.Phys.Commun.* 182 (2011) 1674–1692. arXiv:1011.2900, doi:10.1016/j.cpc.2011.04.008.
- [9] G. Bevilacqua, M. Czakon, M. Garzelli, A. van Hameren, A. Kardos, et al., HELAC-NLO arXiv:1110.1499.
- [10] G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, et al., Automated One-Loop Calculations with GoSam, *Eur.Phys.J.* C72 (2012) 1889. arXiv:1111.2034, doi:10.1140/epjc/s10052-012-1889-1.
- [11] S. Badger, B. Biedermann, P. Uwer, V. Yundin, Numerical evaluation of virtual corrections to multi-jet production in massless QCD, *Comput.Phys.Commun.* 184 (2013) 1981–1998. arXiv:1209.0100, doi:10.1016/j.cpc.2013.03.018.
- [12] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, et al., The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations, *JHEP* 1407 (2014) 079. arXiv:1405.0301, doi:10.1007/JHEP07(2014)079.
- [13] Z. Bern, L. J. Dixon, D. C. Dunbar, D. A. Kosower, One loop n point gauge theory amplitudes, unitarity and collinear limits, *Nucl.Phys.* B425 (1994) 217–260. arXiv:hep-ph/9403226, doi:10.1016/0550-3213(94)90179-1.
- [14] Z. Bern, L. J. Dixon, D. C. Dunbar, D. A. Kosower, Fusing gauge theory tree amplitudes into loop amplitudes, *Nucl.Phys.* B435 (1995) 59–101. arXiv:hep-ph/9409265, doi:10.1016/0550-3213(94)00488-Z.
- [15] G. Ossola, C. G. Papadopoulos, R. Pittau, Reducing full one-loop amplitudes to scalar integrals at the integrand level, *Nucl.Phys.* B763 (2007) 147–169. arXiv:hep-ph/0609007, doi:10.1016/j.nuclphysb.2006.11.012.
- [16] F. Cascioli, P. Maierhofer, S. Pozzorini, Scattering Amplitudes with Open Loops, *Phys.Rev.Lett.* 108 (2012) 111601. arXiv:1111.5206, doi:10.1103/PhysRevLett.108.111601.
- [17] S. Becker, D. Goetz, C. Reuschle, C. Schwan, S. Weinzierl, NLO results for five, six and seven jets in electron-positron annihilation, *Phys.Rev.Lett.* 108 (2012) 032005. arXiv:1111.1733, doi:10.1103/PhysRevLett.108.032005.
- [18] S. Actis, A. Denner, L. Hofer, A. Scharf, S. Uccirati, Recursive generation of one-loop amplitudes in the Standard Model, *JHEP* 1304 (2013) 037. arXiv:1211.6316, doi:10.1007/JHEP04(2013)037.
- [19] S. Badger, A. Guffanti, V. Yundin, Next-to-leading order QCD corrections to di-photon production in association with up to three jets at the Large Hadron Collider, *JHEP* 1403 (2014) 122. arXiv:1312.5927, doi:10.1007/JHEP03(2014)122.
- [20] S. Badger, B. Biedermann, P. Uwer, V. Yundin, Next-to-leading order QCD corrections to five jet production at the LHC, *Phys.Rev.* D89 (2014) 034019. arXiv:1309.6585, doi:10.1103/PhysRevD.89.034019.
- [21] G. Bevilacqua, M. Worek, Constraining BSM Physics at the LHC: Four top final states with NLO accuracy in perturbative QCD, *JHEP* 1207 (2012) 111. arXiv:1206.3064.
- [22] Z. Bern, L. Dixon, F. Febres Cordero, S. Hoeche, H. Ita, et al., Next-to-Leading Order W + 5-Jet Production at the LHC, *Phys.Rev.* D88 (2013) 014025. arXiv:1304.1253, doi:10.1103/PhysRevD.88.014025.
- [23] G. Cullen, H. van Deurzen, N. Greiner, G. Luisoni, P. Mastrolia, et al., NLO QCD corrections to Higgs boson production plus three jets in gluon fusion arXiv:1307.4737.
- [24] H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, et al., NLO QCD corrections to Higgs boson production in association with a top quark pair and a jet arXiv:1307.8437.
- [25] T. Gehrmann, N. Greiner, G. Heinrich, Precise QCD predictions for the production of a photon pair in association with two jets arXiv:1308.3660.
- [26] J. Alcaraz Maestre, et al., The SM and NLO Multileg and SM MC Working Groups: Summary Report arXiv:1203.6803.
- [27] T. Gleisberg, F. Krauss, Automating dipole subtraction for QCD NLO calculations, *Eur.Phys.J.* C53 (2008) 501–523. arXiv:0709.2881, doi:10.1140/epjc/s10052-007-0495-0.
- [28] T. Gleisberg, S. Hoeche, Comix, a new matrix element generator, *JHEP* 0812 (2008) 039. arXiv:0808.3674, doi:10.1088/1126-6708/2008/12/039.
- [29] S. Catani, M. Seymour, The Dipole formalism for the calculation of QCD jet cross-sections at next-to-leading order, *Phys.Lett.* B378 (1996) 287–301. arXiv:hep-ph/9602277, doi:10.1016/0370-2693(96)00425-X.
- [30] T. Binoth, F. Boudjema, G. Dissertori, A. Lazopoulos, A. Denner, et al., A Proposal for a standard interface between Monte Carlo tools and one-loop programs, *Comput.Phys.Commun.* 181 (2010) 1612–1622, dedicated to the memory of, and in tribute to, Thomas Binoth, who led the effort to develop this proposal for Les Houches 2009. arXiv:1001.1307, doi:10.1016/j.cpc.2010.05.016.
- [31] S. Alioli, S. Badger, J. Bellm, B. Biedermann, F. Boudjema, et al., Update of the Binoth Les Houches Accord for a standard interface between Monte Carlo tools and one-loop programs arXiv:1308.3462.
- [32] R. Britto, F. Cachazo, B. Feng, Generalized unitarity and one-loop amplitudes in N=4 super-Yang-Mills, *Nucl.Phys.* B725 (2005) 275–305. arXiv:hep-th/0412103, doi:10.1016/j.nuclphysb.2005.07.014.
- [33] R. Ellis, W. Giele, Z. Kunszt, A Numerical Unitarity Formalism for Evaluating One-Loop Amplitudes, *JHEP* 0803 (2008) 003. arXiv:0708.2398, doi:10.1088/1126-6708/2008/03/003.
- [34] D. Forde, Direct extraction of one-loop integral coefficients, *Phys.Rev.* D75 (2007) 125019. arXiv:0704.1835, doi:10.1103/PhysRevD.75.125019.
- [35] C. Berger, Z. Bern, L. Dixon, F. Febres Cordero, D. Forde, et al., An Automated Implementation of On-Shell Methods for

- One-Loop Amplitudes, Phys.Rev. D78 (2008) 036003. arXiv:0803.4180, doi:10.1103/PhysRevD.78.036003.
- [36] W. T. Giele, Z. Kunszt, K. Melnikov, Full one-loop amplitudes from tree amplitudes, JHEP 0804 (2008) 049. arXiv:0801.2237, doi:10.1088/1126-6708/2008/04/049.
- [37] S. Badger, Direct Extraction Of One Loop Rational Terms, JHEP 0901 (2009) 049. arXiv:0806.4600, doi:10.1088/1126-6708/2009/01/049.
- [38] F. A. Berends, W. T. Giele, Recursive Calculations for Processes with n Gluons, Nucl. Phys. B306 (1988) 759. doi:10.1016/0550-3213(88)90442-7.
- [39] G. van Oldenborgh, FF: A Package to evaluate one loop Feynman diagrams, Comput.Phys.Commun. 66 (1991) 1–15. doi:10.1016/0010-4655(91)90002-3.
- [40] R. K. Ellis, G. Zanderighi, Scalar one-loop integrals for QCD, JHEP 0802 (2008) 002. arXiv:0712.1851, doi:10.1088/1126-6708/2008/02/002.
- [41] B. G. Biedermann, New methods for evaluating one-loop corrections of multi-jet production at the Large Hadron Collider, Logos Verlag Berlin (2013).
- [42] M. Cacciari, G. P. Salam, G. Soyez, The Anti-k(t) jet clustering algorithm, JHEP 0804 (2008) 063. arXiv:0802.1189, doi:10.1088/1126-6708/2008/04/063.
- [43] M. Cacciari, G. P. Salam, G. Soyez, FastJet user manual, Eur.Phys.J. C72 (2012) 1896. arXiv:1111.6097, doi:10.1140/epjc/s10052-012-1896-2.
- [44] R. D. Ball, V. Bertone, S. Carrazza, C. S. Deans, L. Del Debbio, et al., Parton distributions with LHC data, Nucl.Phys. B867 (2013) 244–289. arXiv:1207.1303, doi:10.1016/j.nuclphysb.2012.10.003.
- [45] A. Martin, W. Stirling, R. Thorne, G. Watt, Parton distributions for the LHC, Eur.Phys.J. C63 (2009) 189–285. arXiv:0901.0002, doi:10.1140/epjc/s10052-009-1072-5.
- [46] S. Alekhin, J. Blumlein, S. Moch, Parton distribution functions and benchmark cross sections at NNLO arXiv:1202.2281.
- [47] H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. M. Nadolsky, et al., New parton distributions for collider physics, Phys.Rev. D82 (2010) 074024. arXiv:1007.2241, doi:10.1103/PhysRevD.82.074024.