

An alternative scenario for critical scalar field collapse in AdS_3

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Abstract

In the context of gravitational collapse and black hole formation, we reconsider the problem to describe, analytically, the critical collapse of a massless and minimally coupled scalar field in $2 + 1$ gravity.

Keywords: Critical collapse, Black Holes, 2+1 gravity

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1. Introduction

The first exact solution of Einstein's field equations, discovered by Schwarzschild in 1916,

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2 \quad (1)$$

describes an uncharged and non-rotating black hole of mass M . As shown by Birkhoff in 1923, this solution is the unique static and spherically symmetric vacuum solution of General Relativity. Its properties, i.e. the existence of a trapped region ($r < 2M$) bounded by an event horizon ($r = 2M$) and with a (central) singularity ($r = 0$), are generic properties of black holes even beyond spherical symmetry.

Black holes form from the gravitational collapse of massive stars. By the no-hair theorems [1, 2, 3, 4] time evolution proceeds towards a static (stationary) black

hole solution uniquely characterized by its conserved charges (mass, charge and angular momentum). At the threshold of black hole formation, given by the Chandrasekhar limit, we have a static (stationary) black hole solution with a finite mass.

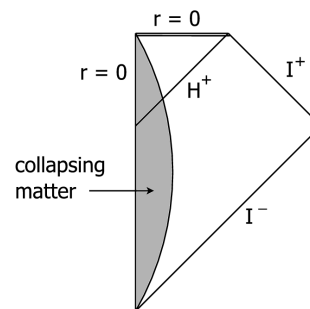


Figure 1: Conformal (Carter-Penrose) diagram of gravitational collapse and black hole formation.

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There is, however, also a critical threshold for black hole formation, discovered by Choptuik in 1993 [5]. He showed that by a fine-tuning of the initial data one can make arbitrarily small black holes with universal power-law scaling of the mass and a (continuously or discretely) scale invariant threshold (critical) solution. There is a striking similarity between the latter and critical phase transitions in statistical mechanics [6].

2. Critical phenomena in gravitational collapse

Choptuik [5] studied the collapse of a spherically symmetric massless and minimally coupled scalar field coupled to 4D Einstein gravity. He considered 1-parameter (p) families of regular initial data and found, empirically, the critical value p_* discriminating between strong data ($p > p_*$), in which a black hole forms, and weak data ($p < p_*$), which disperse. He showed that around the critical point $p = p_*$ there is a power-law scaling of the black hole mass

$$M \simeq C(p - p_*)^\gamma \quad (2)$$

with a universal exponent $\gamma \simeq 0.374$. Moreover, for a finite time in a finite region of space, near-critical ($p \sim p_*$) data approach the same universal solution.

Choptuik found that the critical solution is discretely self-similar (DSS), i.e. it is the same provided we rescale space and time according to

$$(r, t) = (e^\Delta r, e^\Delta t), \quad (3)$$

where $\Delta \sim 3.44$ (this phenomena is called scale-echoing). For perfect fluids the critical solution is continuously self-similar (CSS), i.e. it is invariant under (infinitesimal and finite) rescaling of space and time.

Generically, the critical solution has a strong (naked) singularity, and is characterized by having only one growing perturbation mode. Analytical approaches consider (CSS or DSS) solutions regular at the center and at the past light cone of the (naked) singularity (see [6] and references therein).

3. Critical scalar field collapse in 2+1 dimensions

In 2+1 dimensions, matter affects space-time only globally and not locally by producing conical singularities [7]. A black hole solution to the vacuum Einstein equations was found provided we include a negative cosmological constant ($\Lambda = -\frac{1}{l^2}$) [8]

$$ds^2 = -(-M + \frac{r^2}{l^2})dt^2 + \frac{dr^2}{(-M + \frac{r^2}{l^2})} + r^2 d\theta^2, \quad (4)$$

which is not asymptotically ($r \rightarrow \infty$) Minkowski, as in (1), but Anti-de Sitter (AdS). The solution (4) is a black hole (the BTZ black hole) for $M > 0$, while for $M < 0$ ($M \neq -1$) it describes a naked conical singularity and for $M = -1$ it is regular AdS space.

BTZ black hole formation was analysed by [9] in the collision of point-particles and by [10] in the gravitational collapse of a dust ring. No critical solution is involved in these cases.

Pretorius and Choptuik [11] considered the circularly symmetric collapse of a massless and minimally coupled scalar field ϕ in 2+1 gravity. They considered families of initial data with length scale $r_0 \sim 0.32l$ (so that the effects of the cosmological constant are suppressed by a factor 0.1) and tuned to the threshold of black hole formation on the initial implosion. They find CSS critical behaviour and power-law scaling (of the maximum value of the Ricci scalar and of the mass from the apparent horizon) $(p - p_*)^{2\gamma}$, with $\gamma \sim 1.20 \pm 0.05$. Independently, Husain and Oliver [12] found $\gamma \sim 0.81$.

4. Analytical approach to critical scalar field collapse in AdS_3

Garfinkle [13] found a 1-parameter (n) family of CSS solutions to the $\Lambda = 0$ equations of motion, regular at the center, to reproduce the observed critical solution near the singularity. In appropriate double-null coordinates the metric reads

$$ds^2 = -A(v^n + (-u)^n)^{4-2/n} dudv - \frac{1}{4}(v^{2n} + (-u)^{2n})^2 d\theta^2. \quad (5)$$

Regularity of the solutions at the past light-cone of the singularity requires that n is a positive integer. He found that the solution $n = 4$ agrees well with the numerical data.

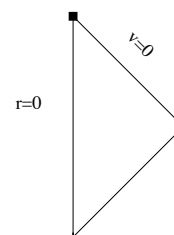


Figure 2: Past light-cone of the singularity of the critical solution ($r = g_{\theta\theta}$).

Garfinkle and Gundlach [14] carried out a linear perturbation analysis of the Garfinkle solutions. The relevant time parameter being $\tau = -\ln(-u)^{2n}$ ($\tau = +\infty$ corresponds to the singularity), the perturbations are

expanded in modes $e^{k\tau} = (-u)^{-2nk}$ which grow when $Re(k) > 0$. k is related to scaling in near-critical collapse: a quantity Q with dimension (length)^s will scale as $|p - p_*|^{s/k}$, i.e. $\gamma = \frac{1}{k}$.

They imposed regularity condition at the center ($g_{\theta\theta} = 0$) and smoothness at $v = 0$: they found that the $n = 4$ solution has 3 unstable modes, while $n = 2$ has only one unstable mode with $k = \frac{3}{4}$, giving $\gamma = \frac{4}{3}$. This analysis was extended to $O(\Lambda)$ in [15].

5. An alternative scenario

A point probably overlooked in the Choptuik and Pretorius analysis is that the introduction of a point particle (a conical singularity) left the critical solution unchanged (up to a phase shift in proper time, related to the mass of the particle). This suggests that the critical solution, instead of having a regular center, might have no center at all (as the $M = 0$ BTZ vacuum).

Moreover, in the Garfinkle solutions $v = 0$ is an apparent horizon: the critical solution must be something else outside the past light-cone of the singularity, and it is not clear what are the correct boundary conditions to be imposed on the perturbations along this surface.

Recently, Baier, Stricker and Taanila [16] (BST) derived, from a self-similar ansatz, a class of solutions conformal to the 3D Minkowski cylinder (i.e. with no center). Such solutions belong to a class of separable solutions [17]

$$ds^2 = F^2(T)[-dT^2 + dR^2 + G^2(R)d\theta^2], \quad \phi = \phi(T), \quad (6)$$

characterized by two parameters α and b . When b (the scalar field strength) vanishes we recover the BTZ solutions with $\alpha = -M$, while the BST solutions correspond to $b \neq 0$, $\alpha = 0$. BST incorrectly suggested that $b = \alpha = 0$ leads to the critical solution: this is not possible since the observed critical solution has a strong singularity [18].

Unlike the case of the Garfinkle solutions we have exact solutions for $\Lambda \neq 0$ [19]. The $\alpha < 0$ solutions are black-hole like, while those for $\alpha > 0$ have a center (regular if $\alpha = 1$). Therefore $\alpha = 0$ is a candidate threshold (critical) solution. In the limit $\Lambda = 0$ our solutions take the form

$$\begin{aligned} ds^2 &= b^2 \sinh^2(T)[-dT^2 + dR^2 + (e^R - \frac{\alpha}{4}e^{-R})^2 d\theta^2] \\ \phi &= \sqrt{2} \ln \tanh(-\frac{T}{2}) \end{aligned} \quad (7)$$

and we see that the center ($\alpha > 0$) is sent to $R \rightarrow -\infty$ when $\alpha \rightarrow 0$. They have a singularity at $T = 0$, and

when $\alpha = 0$ (unlike the Garfinkle solutions) the apparent horizon is at infinite geodesic distance.

Moreover, the subcritical ($\alpha > 0$) solutions near the singularity are in qualitative agreement with numerical data, and for a regular center ($\alpha = 1$) approximate the $n = 1$ Garfinkle solution.

6. Open questions

The linear perturbation analysis of our solutions indicates that there is only one unstable growing mode with $k = 2$, giving $\gamma = \frac{1}{2}$ which disagrees with the value $\gamma = O(1)$ from the numerical analysis. Also, our subcritical solution agrees for $\alpha = 1$ with the $n = 1$ Garfinkle solution, but the numerical data are best fit for $n = 4$. It will be interesting to see if, along these lines, one could find a family of solutions which approximate, near the singularity, the $n = 4$ solution, while leading to a critical exponent $\gamma = O(1)$.

References

- [1] W. Israel, *Commun. Math. Phys.* **8** (1968), 245
- [2] B. Carter, *Phys. Rev. Lett.* **26** (1971), 331
- [3] S.W. Hawking, *Commun. Math. Phys.* **25** (1972), 152
- [4] D.C. Robinson, *Phys. Rev. Lett.* **34** (1975), 905
- [5] M.W. Choptuik, *Phys. Rev. Lett.* **70** (1993), 9
- [6] C. Gundlach, *Phys. Rept.* **376** (2003), 339
- [7] S. Deser, R. Jackiw and G. 't Hooft, *Annals of Phys.* **152** (1984), 220
- [8] M. Bañados, C. Teitelboim and J. Zanelli, *Phys. Rev. Lett.* **69** (1992), 1849
- [9] D. Birmingham and S. Sen, *Phys. Rev. Lett.* **84** (2000), 1074
- [10] Y. Peleg and A. Steif, *Phys. Rev.* **D51** (1995), 3992
- [11] F. Pretorius and M.W. Choptuik, *Phys. Rev.* **D62** (2000), 124012
- [12] V. Husain and M. Olivier, *Class. Quant. Grav.* **18** (2001), L1
- [13] D. Garfinkle, *Phys. Rev.* **D63** (2001), 044007
- [14] D. Garfinkle and C. Gundlach, *Phys. Rev.* **D66** (2002), 044015
- [15] M. Cavaglià, G. Clément and A. Fabbri, *Phys. Rev.* **D70** (2004), 044010
- [16] R. Baier, S.A. Stricker and O. Taanila, *Class. Quant. Grav.* **31** (2014), 025007
- [17] G. Clément and A. Fabbri, *Nucl. Phys.* **B630** (2002), 269
- [18] G. Clément and A. Fabbri, *Class. Quant. Grav.* **31** (2014), 098001
- [19] G. Clément and A. Fabbri, "A scenario for critical scalar field collapse in AdS_3 ", arXiv:1404.0589 [gr-qc]