lattice QCD studies of the leading order hadronic contribution to the muon $g-2$

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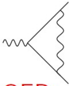
$g_\mu - 2$

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{S}$$

- gyromagnetic ratio : g_μ connects magnetic moment $\vec{\mu}$ to the spin $\vec{S}$
- anomalous magnetic moment of the muon : $a_\mu \equiv \frac{g_\mu - 2}{2}$
- elementary fermion : Dirac theory $g_\mu = 2 \rightsquigarrow a_\mu = 0$
- Schwinger : anomaly $a_\mu = \frac{\alpha}{2\pi} \approx 0.00116 \approx 11\,614\,097.3 \times 10^{10}$



QED: $11\,658\,471.8 \quad (0.0)$ [Aoyama, Hayakawa, Kinoshita, Nio '12]

$\rightsquigarrow$ 5-loop

- contributions: Weak, hadronic, ...
- a_μ : precesion of muon spin around the momentum in a homogeneous magnetic field

$g_\mu - 2$ experiments : CERN, BNL

$g_\mu - 2$

- anomalous magnetic moment of the muon : $a_\mu \equiv \frac{g_\mu - 2}{2}$

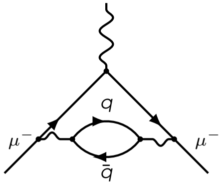
$$a_\mu^{\text{exp}} = 116\,592\,091\,(54)\,(33) \cdot 10^{-11} \qquad [0.5\,\text{ppm}]$$
$$a_\mu^{\text{th}} = 116\,591\,803\,(42)\,(26)\,(01) \cdot 10^{-11} \qquad [0.4\,\text{ppm}]$$

- theory vs. experiment: $\sim 3.6\sigma$ [PDG, 2013]
- uncertainty on a_μ : hadronic effects :

1. LO: $O(\alpha^2)$: $42 \cdot 10^{-11}$ [85% of total error]
2. NLO: $O(\alpha^3)$: $26 \cdot 10^{-11}$

[talk by Eigo Shintani]

3. Weak: $1 \cdot 10^{-11}$ [$153(1) \cdot 10^{-11}$]
4. QED: up to (α^5): $0.08 \cdot 10^{-11}$



new $g_\mu - 2$ experiments: FNAL E989 in 2017 and JPARC

hadronic vacuum polarisation : $\Pi(Q^2)$

- at the quantum level, the photon receives vacuum polarisation corrections
- charge screening $\rightsquigarrow$ running of QED coupling: $\Delta\alpha_{\mathrm{QED}}(Q^2)$
- leading order (LO) contribution

$$\int \mathrm{d}^4x\, e^{iQx} \langle J_\mu(x)\, J_\nu(0)\rangle = (Q_\mu Q_\nu - Q^2\,\delta_{\mu\nu})\,\Pi(Q^2)$$

$$J_\mu(x) = \sum_{f=1}^{N_\mathrm{f}} Q_f\, \overline{\psi}_f(x)\gamma_\mu \psi_f(x)$$

$$Q_f \in \{-1/3, 2/3\}$$

- $\Pi(Q^2)$: photon vacuum polarization function (VPF)

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- $\Pi(Q^2)$: photon vacuum polarization function (VPF)

$$\Delta\alpha_{\mathrm{QED}}(Q^2) = 4\pi\alpha\,\left(\Pi(Q^2) - \Pi(0)\right)$$

- Adler function $D(Q^2)$:

$$D(Q^2) = -12\pi^2\,\frac{d\,\Pi\,(q^2)}{d\log(q^2)}$$

$$= \frac{3\pi}{\alpha}\,\frac{d}{d\log(q^2)}\,\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(q^2)$$

$Q^2 = -q^2$

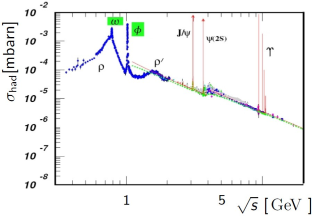
pheno. evaluation of $a_\mu^{\rm had}[{\rm LO}]$

- phenomenological approach :

  combine analyticity and unitarity to experimental data

  dispersion relation + optical theorem + $(e^+e^- \to$ hadrons$)$ cross section

$$a_\mu^{\rm had}[{\rm LO}] = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{R_{\rm had}(s)\, K(s)}{s^2}$$



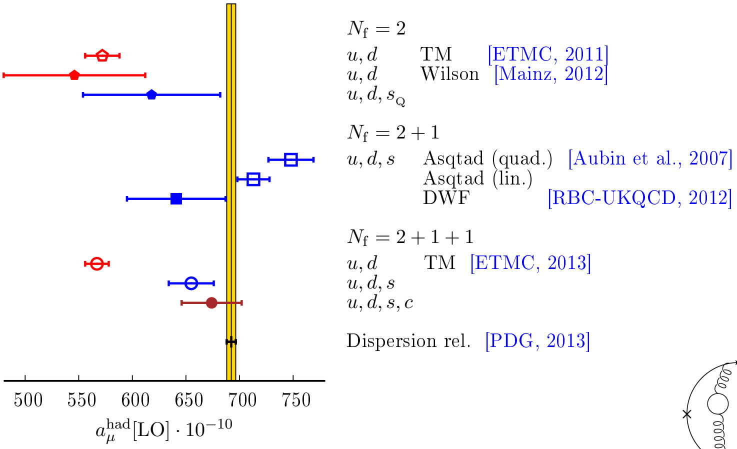
  low-energy regions contributes significantly

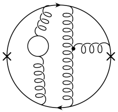
  compatibility among experiments

  theoretical prediction that relies on experimental data

- lattice QCD

$a_\mu^{\mathrm{had}}[\mathrm{LO}]$





$a_\mu^{\mathrm{had}}[\mathrm{LO}] = 692(4) \cdot 10^{-10}$ [0.6%] [PDG, 2013]

$N_f = 1 + 1 + 1 + 1$ [BMW, 1311.4446]

s and c contributions [HPQCD, 1403.1778]

[talk by Jonna Koponen]

see also updates from “lattice 2014” conference [BMW, ETMC, HPQCD, Mainz, RBC-UKQCD]

Why is it difficult?

$$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\, 4\pi^2 \left[\Pi(Q^2) - \Pi(0)\right]$$

[T. Blum, hep-lat/0212018]

- signal is dominated by $Q^2 \sim m_\mu^2$: $Q_\nu = 2\pi n_\nu / L \quad \rightsquigarrow \quad L \gtrsim 12\,\mathrm{fm}$
- $\Pi(Q^2 = 0)$ is not directly accessible:

$$\Pi_{\mu\nu}(Q) = (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu)\, \Pi(Q^2)$$

noise increases as $Q^2 \to 0$

- quark-disconnected contributions 
- $u,\ d,\ s,\ c, \ldots$ contribute
- resonances : ρ
- more effects emerge when increasing precision . . .

recent advances in lattice studies of $a_\mu^{\mathrm{had}}[\mathrm{LO}]$

- Q^2 behaviour near $Q^2 = 0$
  - twisted boundary conditions $Q_\nu = \theta_\nu / L + 2\pi n_\nu / L$
    [Mainz, 1112.2894; C. Aubin et al., 1307.4701; BMW, 1311.4446]
  - Padé approximants model-independent & systematically improvable
    [Mainz, 1112.2894; C. Aubin et al., 1205.3695; M. Golterman et al., 1309.2153, 1405.2389]
    [talk by Santiago Peris]
  - derivatives of vacuum polarisation : $\rightsquigarrow$ $\Pi(Q^2 = 0)$, Adler function, moments, ...
    [G. de Divitiis et al., 1208.5914; ETMC, 1301.2607; Mainz, 1311.6975; HPQCD, 1403.1778]
    [E. de Rafael, 1406.4671]
  - new representations of the VPF
    [A. Francis et al., 1306.2532; X. Feng et al., 1305.5878]

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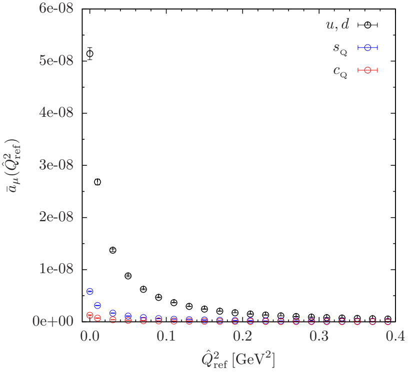
- quark-disconnected contributions
  [M. Della Morte & A. Jüttner, 1103.4818; ETMC, 1311.3885; A. Francis et al., 1306.2532]
- chiral extrapolation
  - rescaling of the kernel [ETMC, 1103.4818]
  - simulations at the physical point [ETMC, 1311.3885; BMW, 1311.4446; HPQCD, 1403.1778]
- charm quark contribution [ETMC, 1308.4327; BMW, 1311.4446; HPQCD, 1403.1778]
- variance reduction techniques [T. Blum et al., 1208.4349]

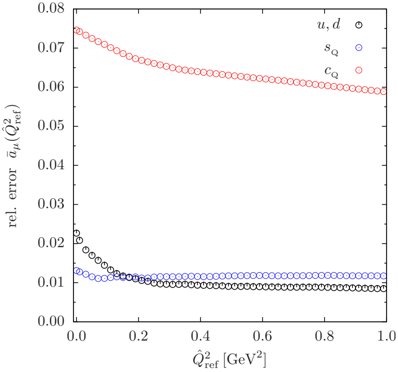
$\bar{a}_\mu^{\rm HLO}(Q^2_{\rm ref})$

$a_\mu^{\rm HLO}$ is dominated by the low Q^2 region : noisy and long-distance contributions

$$\bar{a}_\mu^{\rm HLO}(Q^2_{\rm ref}) \equiv 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{\rm ref}}^{\infty} dQ^2\, f\left(Q^2, m_\mu^2\right) \left[\Pi(Q^2) - \Pi(Q^2_{\rm ref})\right] \xrightarrow{Q^2_{\rm ref}\to 0} a_\mu^{\rm HLO}$$

integrand is peaked at $Q^2 \sim m_\mu^2$ $\qquad$ $m_\mu^2 \sim 0.01\,{\rm GeV}^2$





Mainz : $g_\mu - 2$ lattice project

[Mainz, 1112.2894]

- Q^2 behaviour near $Q^2 = 0$
  - twisted boundary conditions
  - Padé approximants
  - derivatives of VP tensor : Adler function
  - new "mixed" representation of the VPF

use/combine different methods to estimate systematic effects

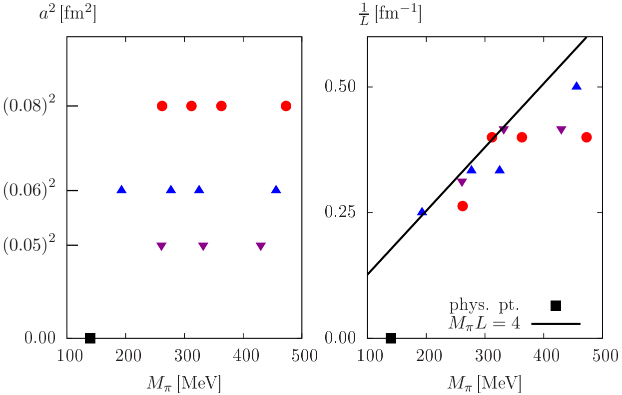
- quark-disconnected contributions

provide an upper-bound for their size

- charm quark contribution
- variance reduction techniques

improve statistical precision at low Q^2

lattice setup



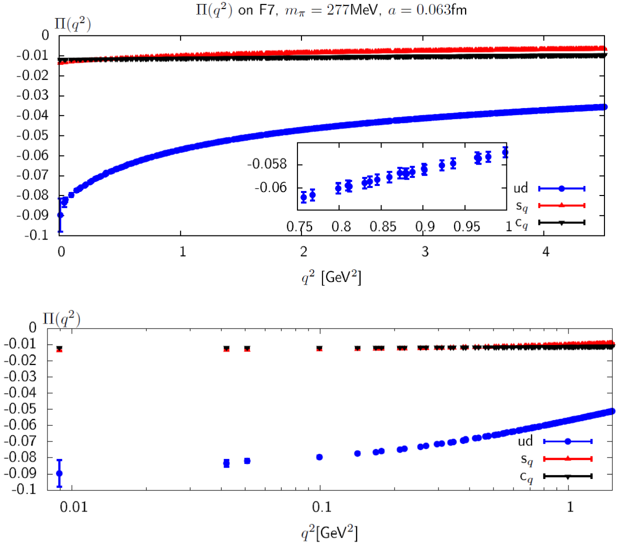
$N_\mathrm{f} = 2$ $\mathcal{O}(a)$ improved Wilson fermions [CLS]

increased statistics

strange and charm are quenched : s_Q , c_Q

only quark-connected contributions

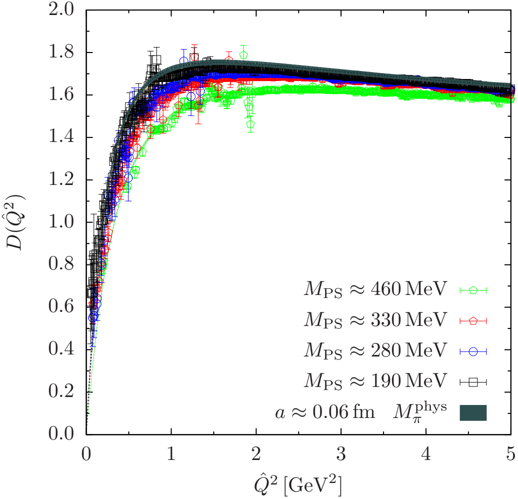
vacuum polarisation function: $\Pi(Q^2 = 0)$



$m_\mu^2 \sim 0.01\,\text{GeV}^2$

Adler function : light-quark mass dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$



u, d

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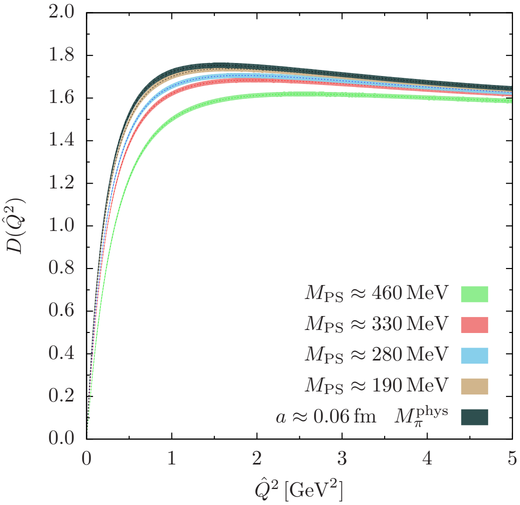

u, *d*

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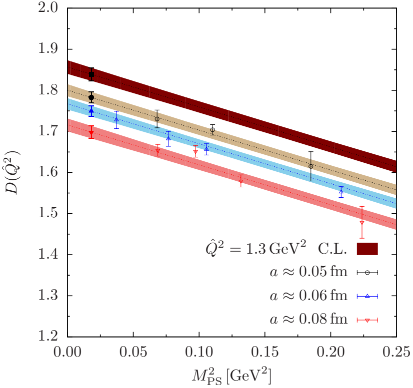

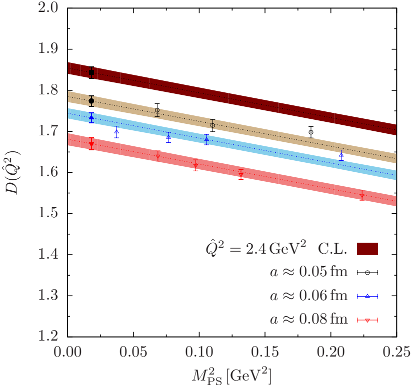

Adler function : strange quark

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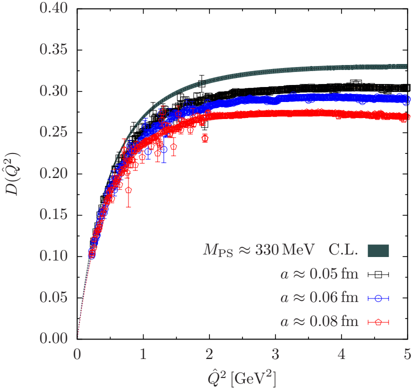
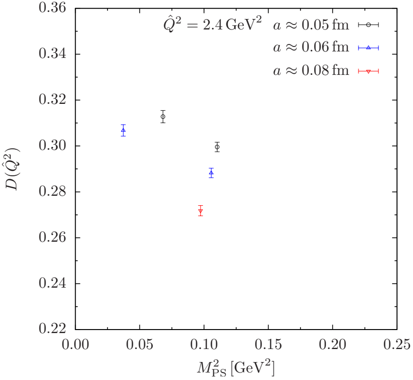
s_Q





lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,\text{GeV}^2$

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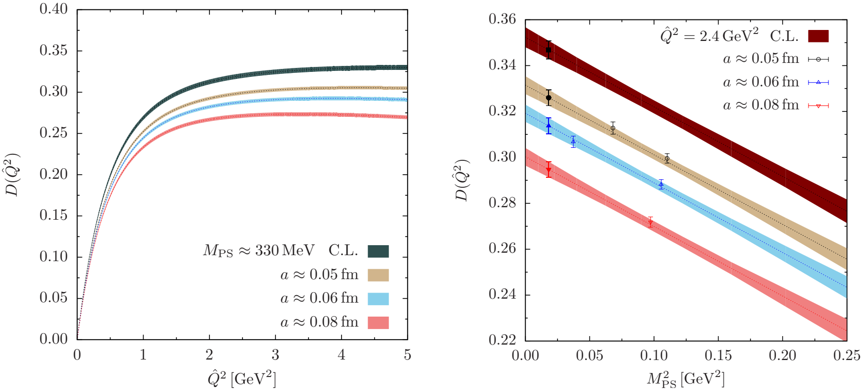

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[PRELIMINARY]

Adler function : charm quark

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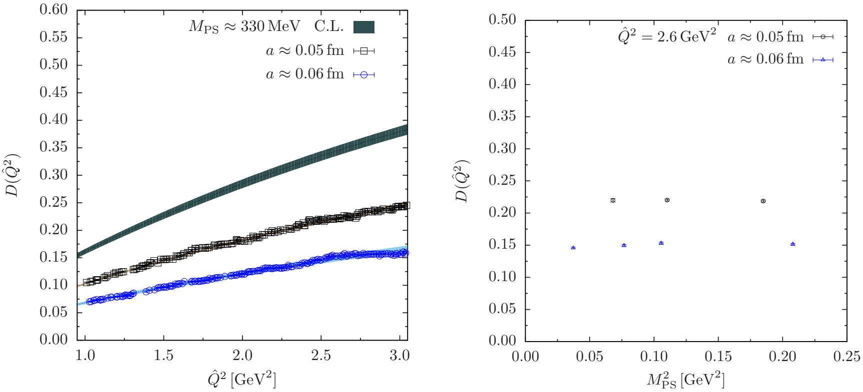
c_Q



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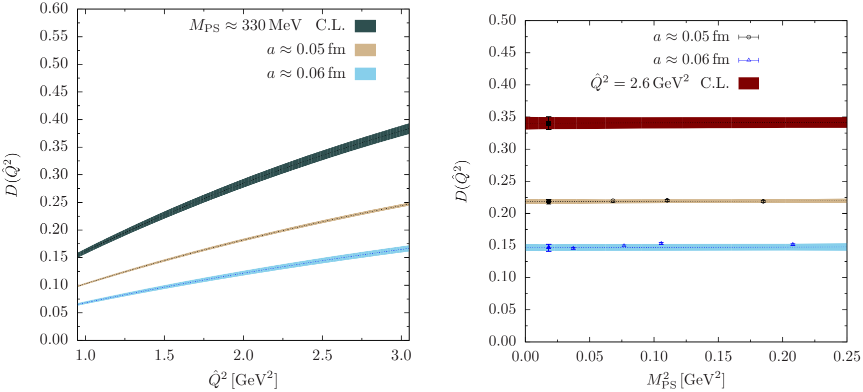

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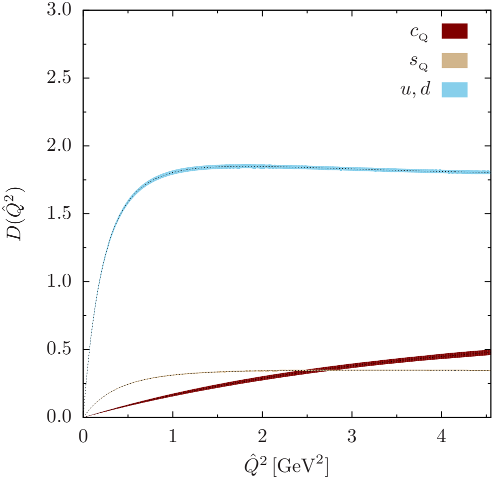
[PRELIMINARY]

Adler function : flavour contributions

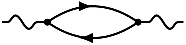
$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

$a \to 0$

$M_\pi^{\rm phys}$



u, d, s_Q, c_Q



stat. error only

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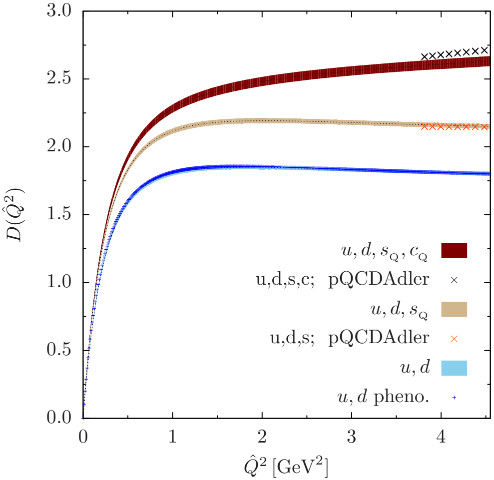
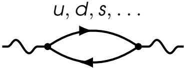


u, d, s_Q, c_Q



stat. error only

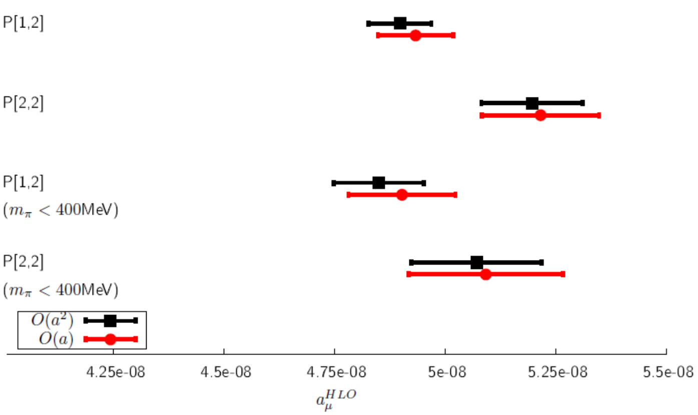
pheno. model u, d [D. Bernecker & H. Meyer, 1107.4388]

[pQCDAdler package, F. Jegerlehner]

Adler function : $a_\mu^{\mathrm{had}}[\mathrm{LO}]$

Adler function : allows to compute $a_\mu^{\mathrm{had}}[\mathrm{LO}]$ without determining $\Pi(Q^2 = 0)$

u, d

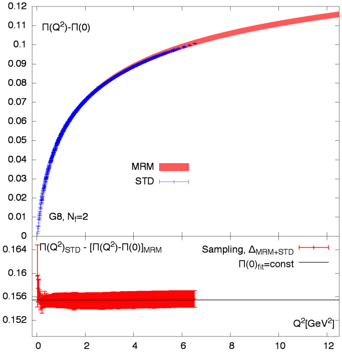


mixed (time-momentum) representation

$$\Pi(Q_0^2) - \Pi(0) = \int_0^\infty dx_0\, G(x_0) \left[x_0^2 - \frac{4}{Q_0^2} \sin^2\left(\frac{1}{2} Q_0 x_0\right) \right]$$

$$G(x_0)\delta_{k\ell} = -\int d^3\vec{x} \, \langle\, J_k(x)\, J_\ell(0)\, \rangle$$

- continuous function of Q_0^2 including $Q_0^2 = 0$
- dominated at long distance by the resonance nature of the ρ meson

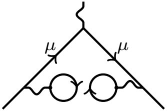


[A. Francis et al., 1306.2532]

see also [X. Feng et al., 1305.5878]

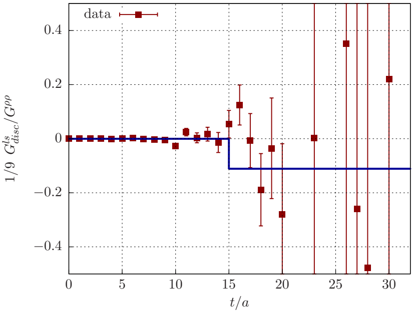
- possibility to combine different representations
- extension to include ρ meson decay

mixed representation: quark-disconnected contribution



- u, d, s : SU(3) breaking effect : noise reduction
- $t \to \infty$

$$\frac{1}{9}\,\frac{G^{\ell s}_{\rm disc}(t)}{G^{\rho\rho}(t)} \quad \longrightarrow \quad -\frac{1}{9}$$



4% : conservative upper-bound for systematic error from neglecting disconnected contribution

conclusions

- currently, lattice results for $a_\mu^{\rm had}$: [$\sim$ 6%]
- many recent studies from various groups
- implementation of new ways to compute $a_\mu^{\rm had}$

  Adler function, mixed representation
- quark-disconnected diagrams

  upper-bound : 4%
- related quantities : $\Delta\alpha_{\rm QED}^{\rm had}$, α_s

in view of future experimental results . . . or to address *e.g.* the $e^+e^- - \tau$ difference

$\rightarrow$ [1%] precision

- effect of the ρ meson decay
- variance reduction techniques
- . . .

- . . . hadronic light-by-light scattering contribution