Towards a quantitative understanding of the $D \rightarrow \pi$ and $B \rightarrow \pi$ semileptonic decays

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Outline

• Motivation

• D→π and B→π form factors:
  - Pionic couplings and decay constants
    → New values of $f_{D^*/D}$ and $f_{B^*/B}$ from Lattice QCD
  - The three pole model
    → New (updated) description of the form factor

• Fitting the experimental data
  Belle, Cleo-c + new data from BaBar

• Results and conclusions
Motivation

- For a semileptonic decay $P \rightarrow P' \ell \nu$ ($P = D^0$ or $B^0$ and $P' = \pi^-$):

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{qq_2}|^2 P'_{\pi}^3}{24\pi^3} |f_+(q^2)|^2$$

$$q^2 = (P_\ell + P_\nu)^2 = (P_\ell - P_{\pi})^2$$

- $|V_{ub}|$ extraction:

$$\frac{d\Gamma(B \rightarrow \pi e \bar{\nu}_e)/dE_{\pi}}{d\Gamma(D \rightarrow \pi e \bar{\nu}_e)/dE_{\pi}} = \left| \frac{V_{ub}}{V_{cd}} \right|^2 \frac{M_B}{M_D} \left| \frac{f_{B \rightarrow \pi}(q^2)}{f_{D \rightarrow \pi}(q^2)} \right|^2$$

Same $E_{\pi}$

Common range in the energy of the ejected pion: $E_{\pi} \lesssim 1$ GeV

(in the rest frame of the heavy-light meson)
**D→π and B→π form factors**

- **Form factor model:**
  
  Based on “polology”, particularly:
  

  The form factor satisfies the dispersion relation:

  \[
  f_+(q^2) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\text{Im} f_+(t)}{t - q^2 - i\varepsilon} \, dt
  \]

  with the residues of the vector resonances:

  \[
  f_{+H}(q^2) \approx \sum_{i}^{\infty} \frac{\text{Res}(f_{+H})_{H^i}}{m_{H^i}^2 - q^2}
  \]

  and satisfying:

  \[
  \text{Res}(f_{+H})_{H^i} = \frac{1}{2} m_{H^i} f_{H^i} g_{H^i H\pi}
  \]

  Sum of an infinite series of poles:
  
  vector resonances with \(J^P=1^{-}\) \(H^*, H^{**}, H^{***} \ldots\)

  Superconvergence condition:

  (from ff behaviour at large \(q^2\))
B and D spectroscopy

- From Godfrey and Isgur [PRD32 (85)189]

**Measurement (GeV)**

- **JP =1⁻ states**
  - **Prediction: D mesons**
    - \( m_0 = 2.037 \text{ GeV} (L=0) \)
    - \( m_1 = 2.645 \text{ GeV} (L=0) \)
    - \( m_2 = 2.816 \text{ GeV} (L=2) \)
    - \( m_3 = 3.11 \text{ GeV} (L=0) \)

- **Measurement (GeV)**
  - **Prediction: B mesons**
    - \( m_0 = 5.37 \text{ GeV} (L=0) \)
    - \( m_1 = 5.93 \text{ GeV} (L=0) \)
    - \( m_2 = 6.11 \text{ GeV} (L=2) \)
    - \( m_3 = 6.355 \text{ GeV} (L=0) \)

- **Measurement (PDG)**
  - \( 2.0103(1) \)
- **Measurement (LHCb)**
  - \( 2.609(4) \)
  - \( 2.649(5) \)

- **Measurement (CDF)**
  - \( 5.325(1) \)
  - \( 5.970(13) \)

→ Lowlying state: D*, B*

→ Radially excited states: observed by BaBar and LHCb (D*'), and CDF (B*')

![BaBar](image1.png)

![CDF](image2.png)

[PRD82(10)111101]

[arXiv:1309.5961 [hep-ex]]
D$\to\pi$ and B$\to\pi$ form factors

\[ f_{+,H}^\pi(q^2) \approx \sum_i \frac{\text{Res}(f_{+,H}^\pi)_{H^*_i}}{m_{H^*_i}^2 - q^2} \]

- The residue can be written in terms of ratios of decay constant: (systematics cancel in Lattice computations)

\[ \text{Res}(f_{+,H}^\pi)_{H^*} = \frac{1}{2} m_{H^*} \frac{f_{H^*}}{f_H} f_H g_{H^*H\pi} \]

\[ f_{H^*}/f_H \text{ computed by Lattice} \]
\[ f_H \text{ measured in } H \to \ell\nu \text{ (for D), or computed by Lattice (for B)} \]
\[ g_{H^*H\pi} \text{ obtained from experimental measurement of the } H^* \text{ width (D) or by Lattice (B)} \]

Pionic couplings for D* and B*:

- From the BaBar measurement of the D*+ width:

\[ g_{D^*D\pi} = 16.92(19) \quad \text{[BaBar, PRD88(13)5]} \]

\[ g_c = \frac{f_\pi}{2\sqrt{m_Dm_{D^*}}} g_{D^*D\pi} = 0.570(6) \]

In agreement with LQCD


- From Lattice (RBC/UKQCD):

\[ g_b = 0.569(76) \quad \Rightarrow \quad g_{B^*B\pi} = 46(6) \quad \text{[arXiv: 1311.2251]} \]
D→π and B→π form factors

Decay constants $f_{D^*}$ and $f_{B^*}$: New

Using the ETMC Lattice field configuration

Regularization:
Twisted mass QCD with $N_f = 2$ dynamical light quarks

Continuum limit:
Working at 4 different fine lattice spacings:
a \in [0.054, 0.100] \text{ fm}

Chiral limit:
m_\pi \in [280,500] \text{ MeV}

Fully unquenched setup:
Working directly with $m_\pi$ in $m_{\text{valence}} = m_{\text{sea}}$

Renormalization:
Non perturbative (RI-MOM)
D→π and B→π form factors

**Decay constants**

\[
\langle 0 | P | H_q (\vec{0}) \rangle = f_{H_q}^2 m_{H_q}^2 \\
\langle 0 | V_i | H_q^* (\vec{0}, \lambda) \rangle = f_{H_q^*} m_{H_q^*} \epsilon_i^\lambda
\]

**Ratio of decay constants:**

- Systematics due to chiral extrapolation cancel out
- No conversion from lattice units
- Known to be 1 for \( m_h \to \infty \) (HQS)

Chiral and continuum extrapolation for different lattice spacing →

\[
\frac{f_{D^*}}{f_D} = 1.197 \pm 0.014
\]

(without \( m_c \) uncertainty)

with:

→ Larger statistics than previous works
→ Smearing procedure to isolate the lowest lying state
→ Computed for several quark masses (MS) allowing the interpolation to \( 1/m_b \)

\[
F(m_h, m_q) = \frac{f_{H_q^*}}{f_{H_q}} \\
= F(m_h, m_{ud})^{\text{cont.}} \left[ 1 + b_h \frac{m_q}{m_s} + c_h \frac{a^2}{(0.086 \text{ fm})^2} \right]
\]
D→π and B→π form factors

**Decay constants**

Interpolation of the form factor ratio to 1/m_b:

\[
R(m_h) = 1 + \frac{\alpha_1}{m_h} + \frac{\alpha_2}{m_h^2} + \ldots
\]

\[
\alpha_1 = 0.579(57) \text{ GeV}
\]

\[
\alpha_2 = -0.117(78) \text{ GeV}^2
\]

In the heavy quark mass limit:

\[
\lim_{m_q \to \infty} \frac{f_{H_q^*}}{f_{H_q}} = 1
\]

\[
R(m_h) = \frac{F(m_h, m_{ud})^{\text{cont.}}}{C(m_h)}
\]

With \( C(m_h) \) accounting for the matching between perturbative QCD and HQET

[Broadhurst et al, PRD52(95)4082; Campanario et al, NPB663(2003)280]

then

\[
\lim_{m_h \to \infty} R(m_h, m_{ud}) = 1
\]

\[
\frac{f_{B^*}}{f_B} = 1.042 \pm 0.014
\]

\[
\frac{f_{D^*}}{f_D} = 1.197 \pm 0.024
\]

\[
m_{b\overline{MS}}(2 \text{ GeV}) = 4.93(16) \text{ GeV}, \quad m_{c\overline{MS}}(2 \text{ GeV}) = 1.14(4) \text{ GeV}
\]
**D→π and B→π form factors**

### Value of the residues

\[
\text{Res}\left(f^\pi_{+,H}\right)_{H^*} = \frac{1}{2} m_{H^*} \frac{f_{H^*}}{f_H} f_H g_{H^* H \pi}
\]

- Having all ingredients to compute the first residues (from D* and B*):

\[
\text{Res}_{q^2=m^2_{D^*}} f^D_{+, \pi}(q^2) = \frac{1}{2} m_{D^*} \frac{f_{D^*}}{f_D} f_D g_{D^* D \pi} = 4.15(14) \text{ GeV}^2
\]

3% accuracy!

\[
\text{Res}_{q^2=m^2_{B^*}} f^B_{+, \pi}(q^2) = \frac{1}{2} m_{B^*} \frac{f_{B^*}}{f_B} f_B g_{B^* B \pi} = 24.1(3.3) \text{ GeV}^2
\]

13% accuracy \(g_{B^*B\pi}\)

- And constraining the second one (D*'*, B*'*):

\[
\text{Res}_{q^2=m^2_{D^*'}} f^D_{+, \pi}(q^2) = -(1.0 \pm 0.3) \text{ GeV}^2
\]

20-30% accuracy

\[
\text{Res}_{q^2=m^2_{B^*'}} f^B_{+, \pi}(q^2) = -(7.7 \pm 1.6) \text{ GeV}^2
\]

Using:

\[
\frac{f_{D^*'}}{f_{D^*}} = \frac{f_{D'}}{f_D} = 0.57(16)
\]  
[D.B. et al, NPB872(13)313]

\[
f_{B^*'} = 165(46)_{-12} \text{ MeV}
\]  

D*' width measured by BaBar

[PRD82(10)111101]
Fitting the experimental data

• Using charm semileptonic data:

CLEOc: tagged $D^0 \rightarrow \pi^- \ell^+ \nu$ and $D^+ \rightarrow \pi^0 \ell^+ \nu$ events (818pb$^{-1}$)  
[PRD80(09)032005]

CLEOc: untagged $D^0 \rightarrow \pi^- \ell^+ \nu$ and $D^+ \rightarrow \pi^0 \ell^+ \nu$ (281pb$^{-1}$)  
[PRD77(08)112005], [PRL100(08)251802]

Belle: tagged $D^0 \rightarrow \pi^- \ell^+ \nu$ ($\ell = e, \mu$) (282fb$^{-1}$)  
[PRL97(06)061804]

BaBar (preliminary): untagged $D^0 \rightarrow \pi^- \ell^+ \nu$ (347fb$^{-1}$)

$z$-expansion: (a general parameterization of the form factor)

$$ |z| < < 1 \quad t \equiv q^2 \quad F(t) = \frac{1}{P(t) \phi(t, t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(t, t_0)^k $$

But we know that

$$ f_+(q^2) = \sum_{n=0}^{\infty} \frac{\text{Res} f_+(q^2)}{\frac{m_{D_n^*}^2 - q^2}{m_{D_n^*}^2}} $$

The form factor cannot be explained by the nearest two poles alone ($D^*$ and $D^{*'}$) → need more than two poles if one looks for the physics description of the form factor behavior.
Superconvergence

The “three” pole model:

\[ f_+(q^2) = \frac{\gamma_0}{m_{D^*}^2 - q^2} + \frac{\gamma_1}{m_{D^{**}}^2 - q^2} + \frac{\gamma_{\text{eff}}}{m_{\text{eff}}^2 - q^2} \]

\( \gamma_n = \text{Res}_{q^2=m_{D_n^*}^2} f_+^{D\pi}(q^2) \)

Results:

\( \gamma_0 = 3.95 \pm 0.11 \text{ GeV}^2 \)
\( \gamma_1 = -1.13 \pm 0.30 \text{ GeV}^2 \)

\( m_{\text{eff}} = 3.9 \pm 0.4 \text{ GeV} \)

Larger than the mass of the 3rd pole \( (m_{D^*3} \sim 3.1 \text{ GeV}) \)

The form factor is not saturated by three poles: **the third pole is effective**

and includes contributions from higher mass states

\( \gamma_{\text{eff}} = -\gamma_0 - \gamma_1 \) it is verified in the fit
Fitting the experimental data

Results by experiment:

- **CLEO tag. π⁺**
  - 1374 events

- **CLEO tag. π⁰**
  - 838 events

- **CLEO untag. π⁺**
  - 6165 events

- **CLEO untag. π⁰**
  - 6104 events

- **Belle π⁺**
  - 232 events

- **BoBar π⁺**
  - 5303 events
  (Preliminary)
Fitting the experimental data

- Using B semileptonic data:
  
  BaBar: $B^0 \rightarrow \pi^- \ell^+ \nu$ and $B^+ \rightarrow \pi^0 \ell^+ \nu$ ($656 \times 10^6$ BB)
  
  Belle: $B^0 \rightarrow \pi^- \ell^+ \nu$ ($657 \times 10^6$ BB)

  Constrainig $\beta_0$ and $\beta_1$ to the residues of B and B$^*$
  
  (and verifying the superconvergence condition):

  $\beta_0 = 24.4 \pm 3.3 \text{ GeV}^2$
  
  $\beta_1 = -7.7 \pm 2.0 \text{ GeV}^2$
  
  $m_{\text{eff}} = 7.5 \pm 0.3 \text{ GeV}$

  $|V_{ub}| = (2.72 \pm 0.12 \pm 0.40|\beta_0 \pm 0.03|\beta_1) \times 10^{-3}$
Results and Conclusions

→ Data from BaBar (preliminary), Belle and CLEOc on B/D → πℓν
→ New accurate results from Lattice QCD on the ratio of the decay constants:

\[
\frac{f_{D^*}}{f_D} = 1.197 \pm 0.024 \quad \frac{f_{B^*}}{f_B} = 1.042 \pm 0.014
\]

→ and experimental measurements of the D* and D*’ resonances

• The D→ π form factor cannot be described by the D* and D*’ contributions, not even adding a D*”

• We have extended the Bečirević and Kaidalov [PLB(2000) 417] description of the form factor in a “three” pole model, the 3rd pole being effective

\[
f_+(q^2) = \frac{\gamma_0}{m_{D^*}^2 - q^2} + \frac{\gamma_1}{m_{D^*'}^2 - q^2} + \frac{\gamma_{\text{eff}}}{m_{\text{eff}}^2 - q^2}
\]

Constraining the first two residues and verifying superconvergence (\(\sum \text{Res}_{H^*)i} = 0\))

• Using a scaling law between D and B meson couplings, superconvergence, and theoretical and experimental inputs, we fit this model to B→πℓν data and obtain:

\[
|V_{ub}| = (2.72 \pm 0.12_{\text{exp.}} \pm 0.40_{\text{theo.}}) \times 10^{-3}
\]

where the theoretical uncertainty is dominated by the coupling \(g_b\)
Thank you!
| $m_2$ [GeV] | $\chi^2$  | $|V_{ub}| \times \beta_0/m_B^2 \times 10^2$ | $R_1$       | $R_2$       | $1 + \beta_1/\beta_0 + \beta_2/\beta_0$ |
|------------|----------|----------------------------------------|-------------|-------------|----------------------------------------|
| 6          | 31.0     | 7.92 ± 0.33                            | -0.26 ± 0.07| -0.40 ± 0.07| +0.17 ± 0.02                           |
| 7          | 24.9     | 6.84 ± 0.29                            | -0.24 ± 0.06| -0.37 ± 0.06| +0.05 ± 0.04                           |
| 8          | 22.5     | 6.51 ± 0.29                            | -0.24 ± 0.06| -0.36 ± 0.06| -0.11 ± 0.06                           |
| 10         | 20.7     | 6.26 ± 0.29                            | -0.24 ± 0.06| -0.34 ± 0.05| -0.52 ± 0.11                           |
| 12         | 20.0     | 6.17 ± 0.30                            | -0.25 ± 0.06| -0.34 ± 0.05| -1.0 ± 0.2                             |
| 15         | 19.6     | 6.11 ± 0.30                            | -0.25 ± 0.06| -0.33 ± 0.05| -2.0 ± 0.3                             |
| 30         | 19.2     | 6.04 ± 0.30                            | -0.25 ± 0.06| -0.33 ± 0.05| -9.7 ± 1.6                             |

Table 4: Results obtained with a 3-poles model for $f_2^{B\pi}(q^2)$ and for different values of the third pole mass. The total number of degrees of freedom is equal to 23. The superconvergence condition is not used. Untagged analyses by BaBar and Belle are used.