High Precision Prediction for the lightest CP-even MSSM Higgs-Boson Mass

Sven Heinemeyer, IFCA (CSIC, Santander)

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based on collaboration with
T. Hahn, W. Hollik, H. Rzehak, G. Weiglein

1. Introduction

2. The best of both worlds

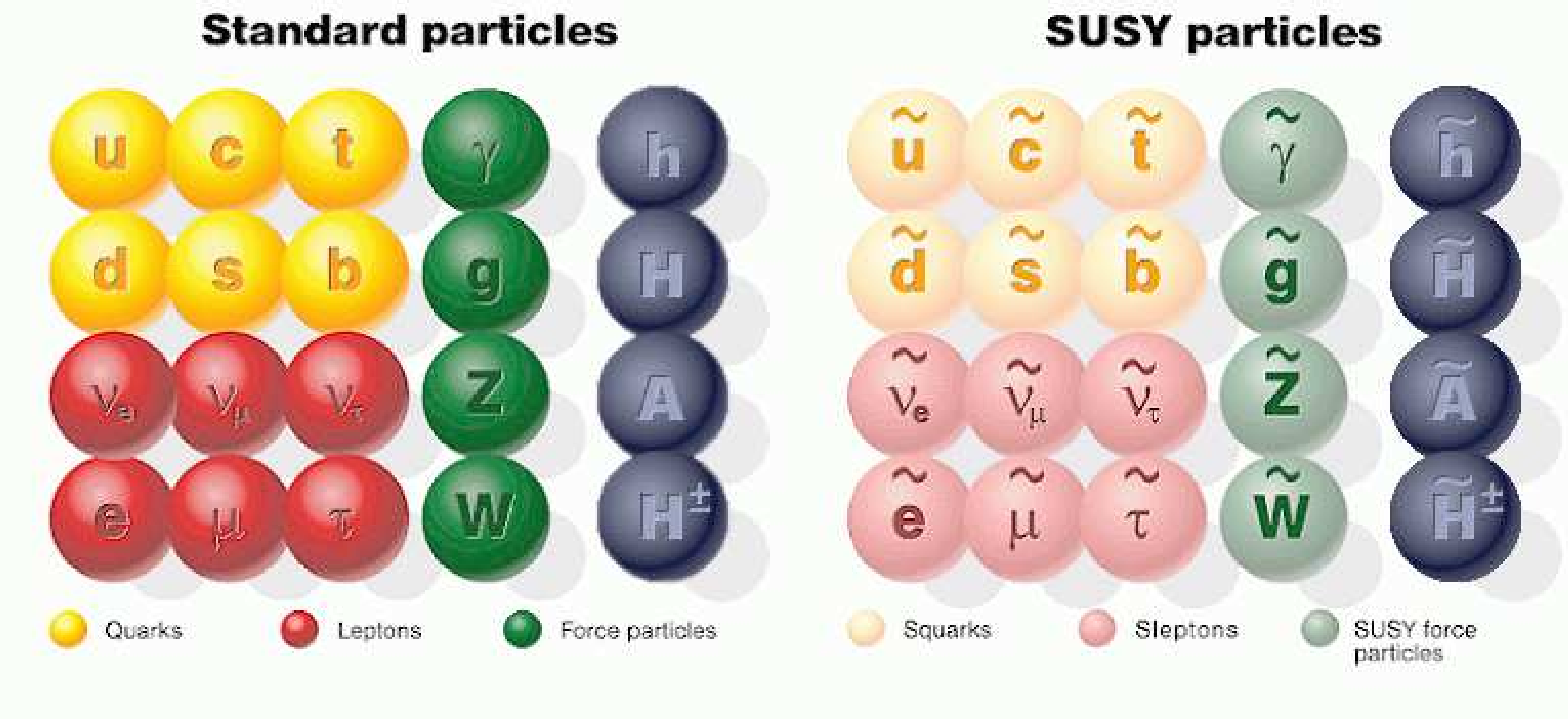
3. Results

4. Conclusions

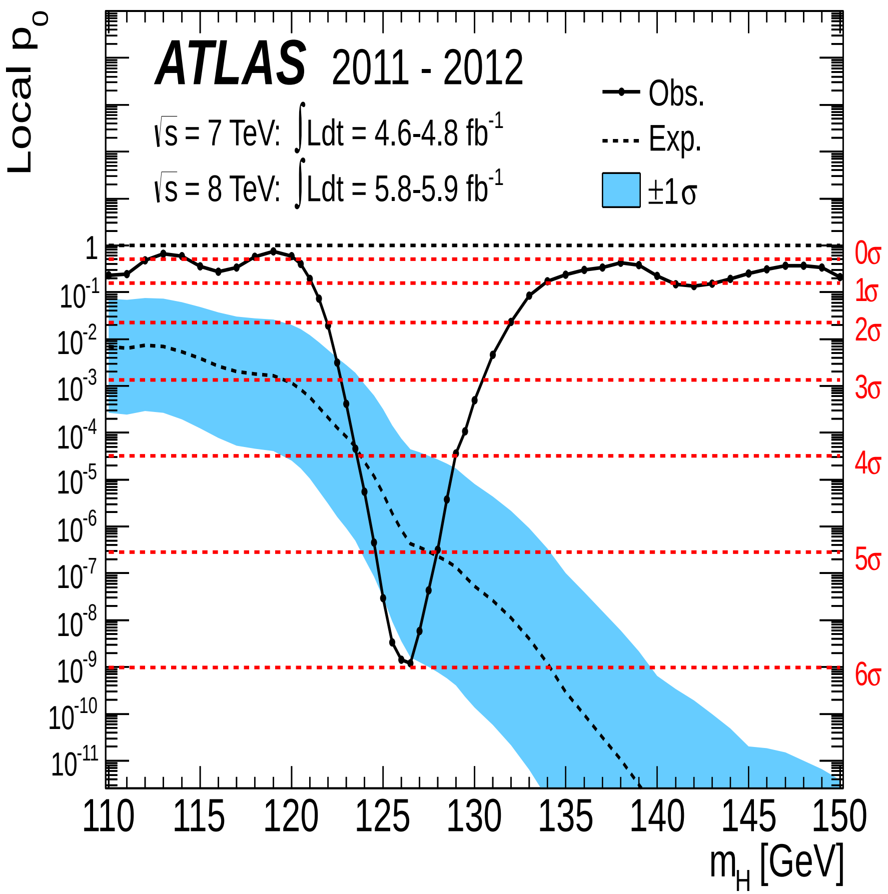
1. Introduction

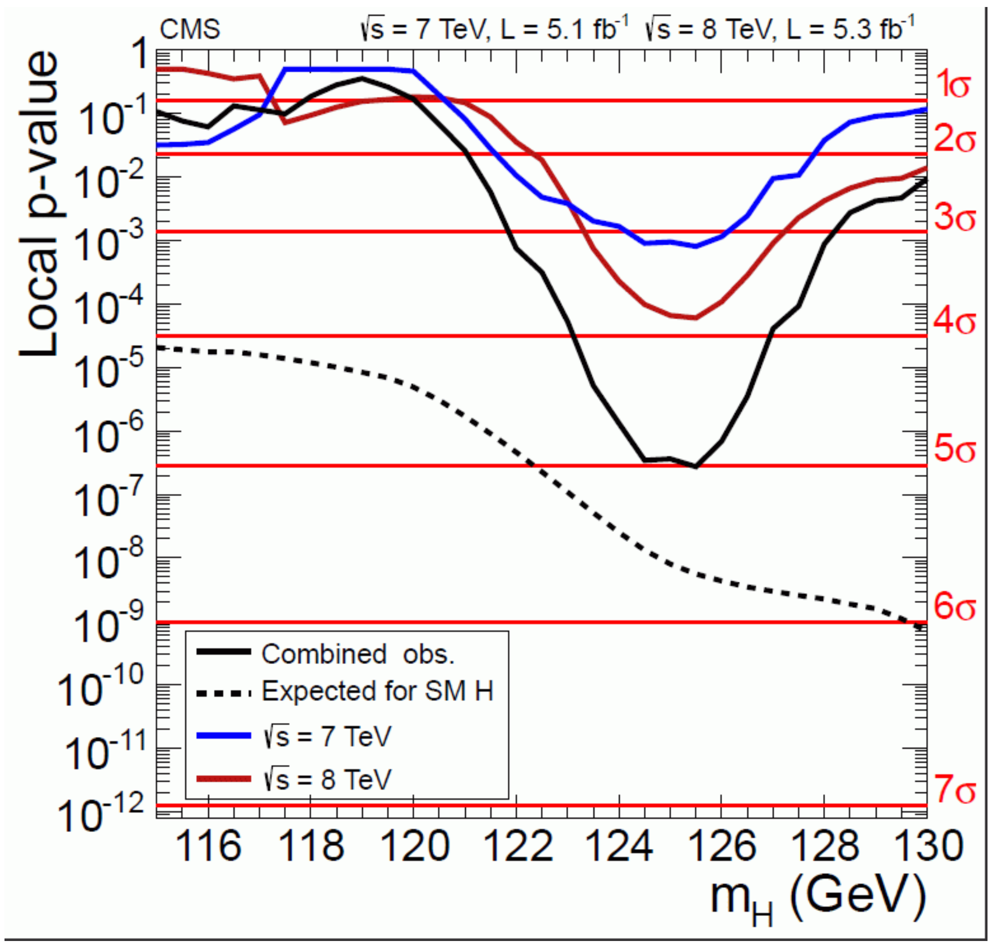
The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles



We have a discovery!





→ MSSM always predicted $M_h \lesssim 135$ GeV

→ MSSM predicts (over large parts of the parameter space) that the lightest Higgs is SM-like

⇒ discovery can be identified with the lightest MSSM Higgs boson!

MSSM: Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1\bar{H}_1 + m_2^2 H_2\bar{H}_2 - m_{12}^2(\epsilon_{ab}H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}\,|H_1\bar{H}_2|^2$$

gauge couplings, in contrast to SM $\Rightarrow m_h \leq M_Z$

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

The lightest MSSM Higgs boson

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}, \frac{e\, m_t^2}{M_W s_W}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

> The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Upper bound predicted:

$$M_h \lesssim 135 \text{ GeV}$$

[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. Weiglein '02*]

$\tilde{t}$ sector of the MSSM:

Stop mass matrices

$$\mathbf{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + DT_{t_1} & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + DT_{t_2} \end{pmatrix} \xrightarrow{\theta_{\tilde{t}}} \begin{pmatrix} m^2_{\tilde{t}_1} & 0 \\ 0 & m^2_{\tilde{t}_2} \end{pmatrix}$$

with

$$X_t = A_t - \mu / \tan\beta$$

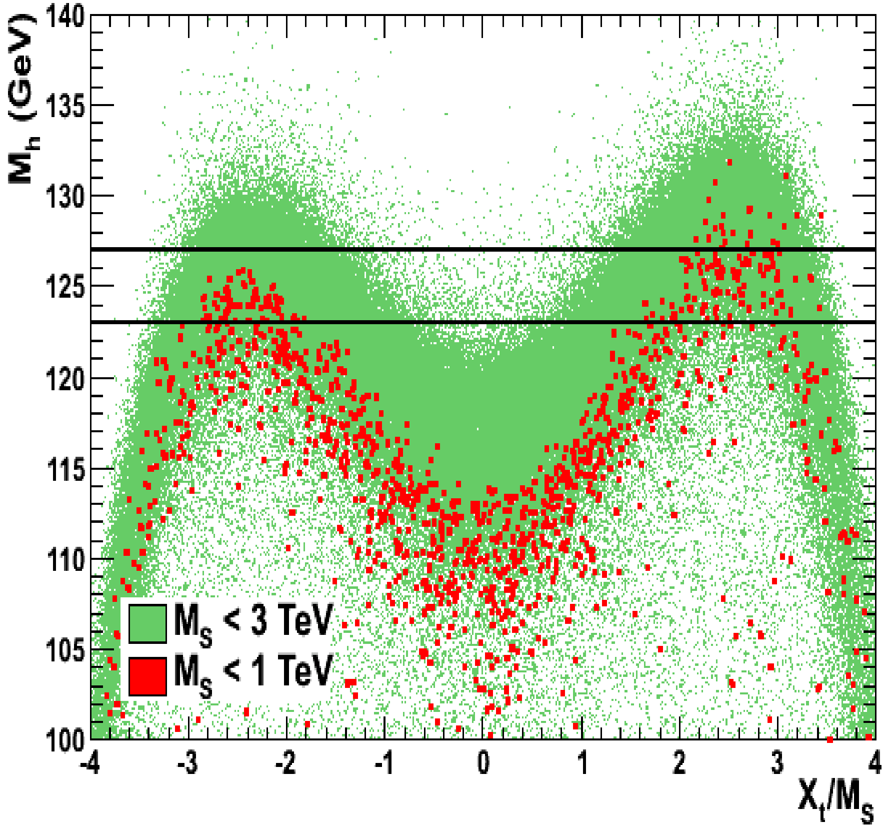
$\Rightarrow$ mixing important in stop sector!

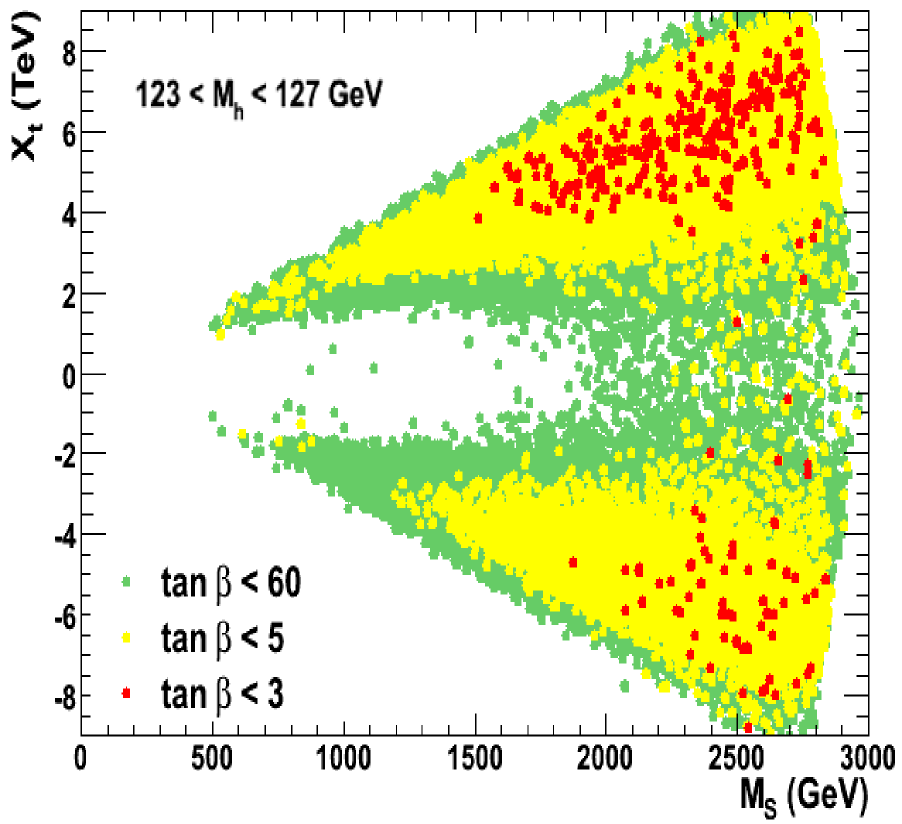
Simplifying abbreviation:

$$M_{\text{SUSY}} := M_{\tilde{t}_L} = M_{\tilde{t}_R}$$

Stop masses for $M_h = 125.5$ GeV

[*A. Arbey et al., '11*]





$\Rightarrow M_h \sim 125.5$ GeV requires large X_t and/or large M_{SUSY}

The embarrasing situation:

Experiment:

ATLAS: $M_H^{\mathrm{exp}} = 125.7 \pm 0.4 \pm 0.2$ GeV

CMS: $M_H^{\mathrm{exp}} = 125.7 \pm 0.3 \pm 0.3$ GeV

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Theory:

$$\delta M_h^{\text{theo}} \sim 3 \text{ GeV}$$

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Theory:

$$\delta M_h^{\text{theo}} \sim 3 \text{ GeV}$$

$\Rightarrow$ Theory prediction must be improved
to match the experimental accuracy!

2. The best of both worlds

Method I:

Higher-order corrections in the Feynman diagrammatic method:

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_H^2 & 0 \\ 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
($\to$ Feynman-diagrammatic approach):

$$M_{hH}^2(q^2) = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H)$: renormalized Higgs self-energies

$\mathcal{CP}$-even fields can mix

$\Rightarrow$ complex roots of $\det(M_{hH}^2(q^2))$: $\mathcal{M}_{h_i}^2 (i = 1, 2)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

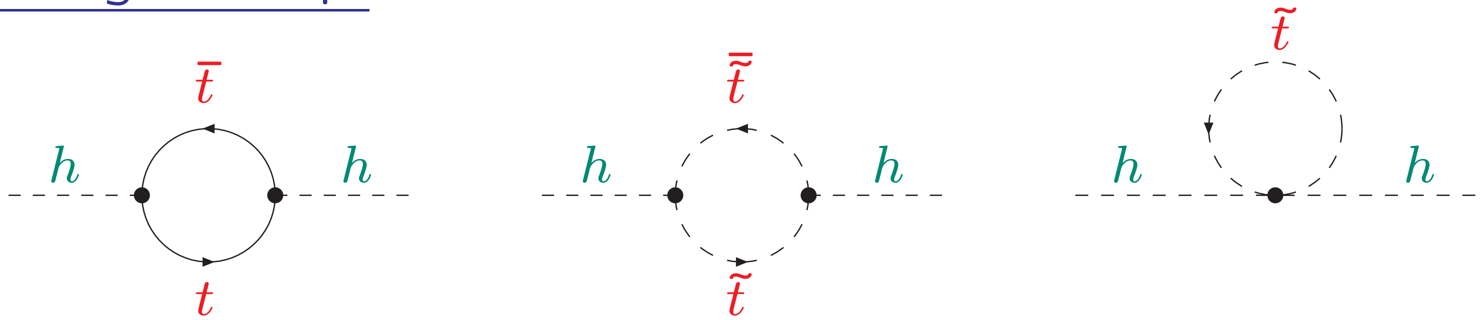
Calculation of renormalized Higgs boson self-energies:

$\hat{\Sigma}(q^2) = \hat{\Sigma}^{(1)}(q^2) + \hat{\Sigma}^{(2)}(q^2) + \ldots$

all MSSM particles contribute

main contribution: $t/\tilde{t}$ sector ($\tilde{t}$: scalar top, SUSY partner of the t)

Very leading 1-Loop:



2-Loop:

To avoid large corrections:

On-shell renormalization of the scalar top sector $\Rightarrow X_t^{\mathrm{OS}}$

$$\sim m_t^4 \left[\log^2 \left(\frac{m_{\tilde{t}}}{m_t} \right) + \log \left(\frac{m_{\tilde{t}}}{m_t} \right) \right]$$

Structure of higher-order corrections:

One-loop:

$$\Delta M_h^2 \sim m_t^2 \alpha_t \left[L + L^0\right] \; , \qquad L := \log\left(\frac{m_{\tilde{t}}}{m_t}\right)$$

Two-loop:

$$\Delta M_h^2 \sim m_t^2 \left\{\alpha_t \alpha_s \left[L^2 + L + L^0\right] + \alpha_t^2 \left[L^2 + L + L^0\right]\right\}$$

Three-loop:

$$\begin{aligned}\Delta M_h^2 \sim m_t^2 \Big\{ \quad & \alpha_t \alpha_s^2 \left[L^3 + L^2 + L + L^0\right] \\ + & \alpha_t^2 \alpha_s \left[L^3 + L^2 + L + L^0\right] \\ + & \alpha_t^3 \left[L^3 + L^2 + L + L^0\right] \Big\}\end{aligned}$$

Partial results: [*S. Martin '07*]

[*R. Harlander, P. Kant, L. Mihaila, M. Steinhauser '08*] $\Rightarrow$ *H3m*

H3m adds $\mathcal{O}\left(\alpha_t \alpha_s^2\right)$ corrections to *FeynHiggs*

Large $m_{\tilde{t}}$ $\Rightarrow$ large L $\Rightarrow$ resummation of logs necessary $\Rightarrow$ Method II

Advantages of Feynman-diagrammatic method:

– all contributions at fixed order are captured

– trivial to include many SUSY scales

– full control over Higgs boson self-energies
  $\rightarrow$ needed for other quantities (production and decay)

Problems of Feynman-diagrammatic method:

– always only fixed order

– large logs not captured beyond the calculated order

Method II: Log resummation via RGE's:

Simple example for log resummation:

SUSY mass scale: $M_{\mathrm{SUSY}} = M_{\mathrm{S}} \sim m_{\tilde{t}}$

Above M_{SUSY}: MSSM
Below M_{SUSY}: SM

Relevant SM parameters:
- quartic coupling λ
- top Yukawa coupling h_t ($\alpha_t = h_t^2/(4\,\pi)$)
- strong coupling constant g_s ($\alpha_s = g_s^2/(4\,\pi)$)

Procedure:

1. Take: $h_t(m_t), g_s(m_t)$

   SM RGEs for h_t, g_s: $h_t, g_s(m_t) \to h_t, g_s(M_{\mathrm{S}})$

2. Take $\lambda(M_{\mathrm{S}}), h_t(M_{\mathrm{S}}), g_s(M_{\mathrm{S}})$

   SM RGEs for λ, h_t, g_s: $\lambda, h_t, g_s(M_{\mathrm{S}}) \to \lambda, h_t, g_s(m_t)$

3. Evaluate M_h^2

$$M_h^2 \sim 2\lambda(m_t)v^2$$

Advantages of RGE log resummation:

– large logs taken into account to all orders

– calculation can easily be extended to very large scales

Problems of RGE log resummation:

– not all contributions at fixed order are captured

$\to$ sub-leading logs more difficult

$\to$ momentum dependence

– difficult (impossible?): include many different SUSY scales

– difficult (impossible?): control over Higgs boson self-energies

$\to$ needed for other quantities (production and decay)

Our resummation procedure:

- SM two-loop RGEs
- one-loop threshold correction for $\lambda(M_{\mathsf{SUSY}})$:
  ($g_1 = g_2 = 0 \Rightarrow$ pure loop correction)

$$\lambda(M_S) = \frac{3\,h_t^4}{8\,\pi^2} x_t^2 \left[1 - 1/12\,x_t^2\right] \ , \quad x_t = X_t^{\overline{\mathsf{MS}}}/M_S$$

$\Rightarrow$ at n-loop order: $L^n + L^{n-1}$

1. Take: $h_t(m_t), g_s(m_t)$

   2L SM RGEs for h_t, g_s: $h_t, g_s(m_t) \to h_t, g_s(M_S)$ (neglect λ contribution)

2. Take $\lambda(M_S), h_t(M_S), g_s(M_S)$

   2L SM RGEs for λ, h_t, g_s: $\lambda, h_t, g_s(M_S) \to \lambda, h_t, g_s(m_t)$

3. Run up and down till convergence is reached

4. Evaluate $\Delta M_h^2 \sim 2\lambda(m_t)v^2$

Combination of FD and RGE result:

$\Rightarrow$ to avoid double counting:
subtract leading and subleading logs at one- and two-loop

Problem:

– FD result with $X_t^{\text{OS}}, M_S^{\text{OS}}, \overline{m}_t$

– RGE result with $X_t^{\overline{\text{MS}}}, M_S^{\overline{\text{MS}}}, \overline{m}_t$

$$\overline{m}_t = \frac{m_t^{\text{pole}}}{1 + \frac{4}{3\pi}\alpha_s(m_t^{\text{pole}}) - \frac{1}{2\pi}\alpha_t(m_t^{\text{pole}})}$$

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}}\left[1 + 2L\left(\frac{\alpha_s}{\pi} - \frac{3\,\alpha_t}{16\,\pi}\right)\right]$$

$$M_S^{\overline{\text{MS}}} \sim M_S^{\text{OS}} \quad : \text{ no log differences!}$$

Combination of FD and RGE result:

$$\Delta M_h^2 = (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}}, M_S^{\overline{\text{MS}}}, \overline{m}_t) - (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\text{OS}}, M_S^{\text{OS}}, \overline{m}_t)$$

$$M_h^2 = (M_h^2)^{\text{FD}} + \Delta M_h^2$$

Technical aspect:

$$(\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\text{OS}}, M_S^{\text{OS}}, \overline{m}_t)$$

$$:= (\Delta M_h^2)^{\text{FD,LL1,LL2}}(X_t^{\overline{\text{MS}}}, M_S^{\overline{\text{MS}}}, \overline{m}_t)\Big|_{X_t^{\overline{\text{MS}}} \to X_t^{\text{OS}}, M_S^{\overline{\text{MS}}} = M_S^{\text{OS}}}$$

⇒ combination of best FD result with
resummed LL, NLL corrections for large $m_{\tilde{t}}$

⇒ most precise M_h prediction for large $m_{\tilde{t}}$ ⇒ FeynHiggs 2.10.0

3. Results

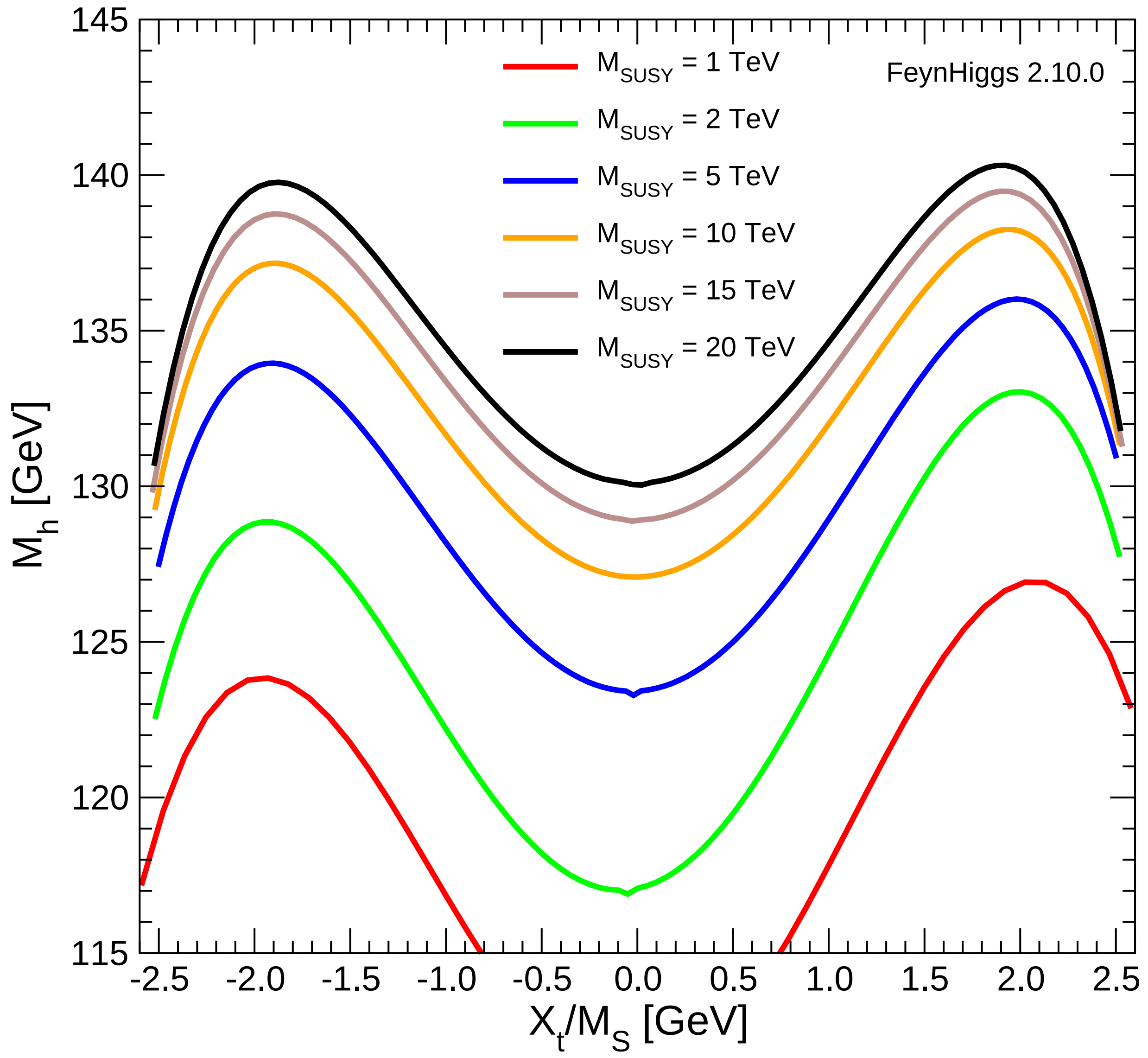
[FeynHiggs 2.10.0]

Parameters:

$$
\begin{aligned}
M_S &= \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \\
M_A &= 1000 \text{ GeV} \\
\mu &= 1000 \text{ GeV} \\
M_2 &= 1000 \text{ GeV} \\
m_{\tilde{g}} &= 1600 \text{ GeV} \\
\tan\beta &= 10
\end{aligned}
$$

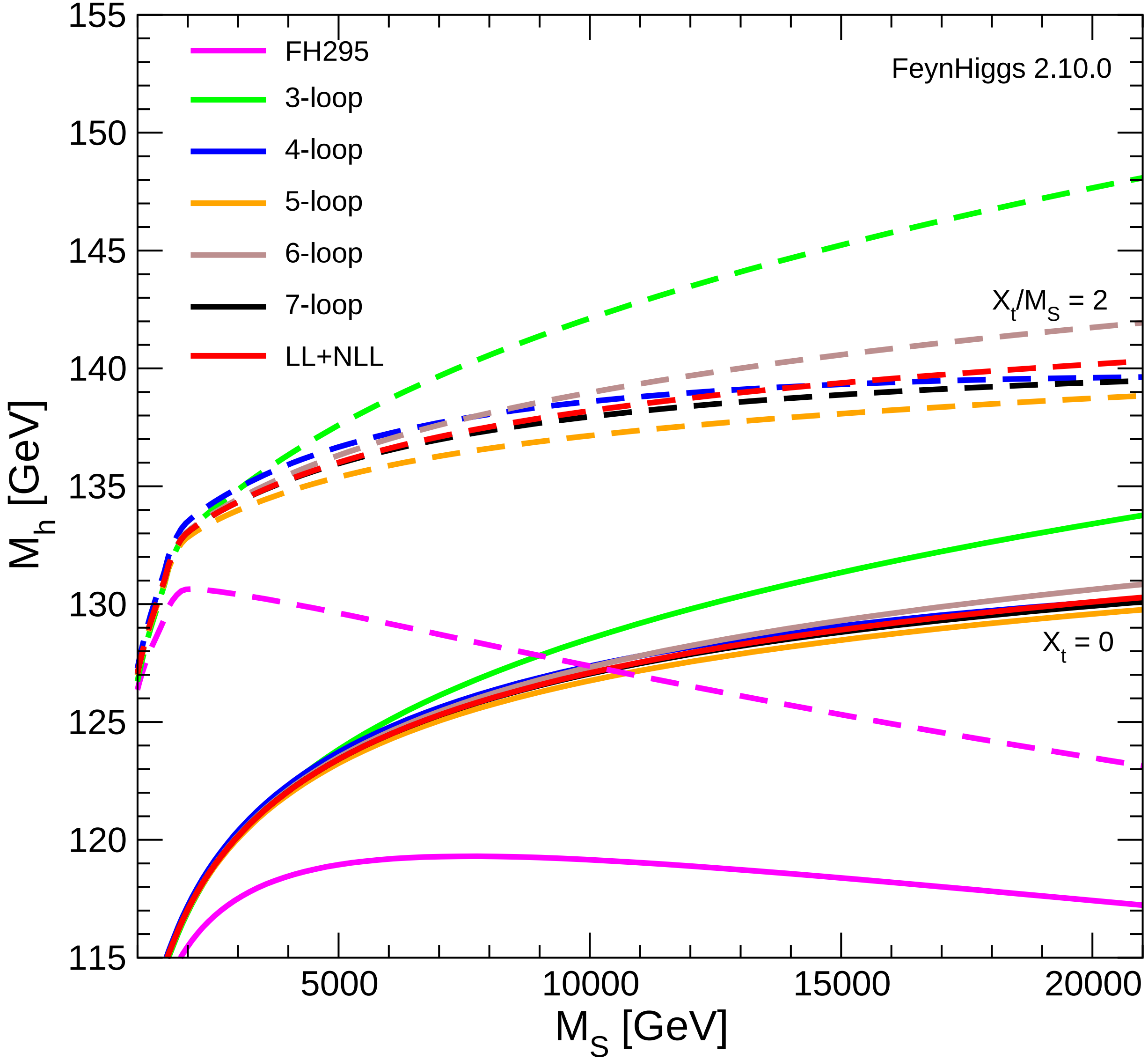
Vary M_S, X_t to analyze effects

$M_h(X_t/M_S)$: [FeynHiggs 2.10.0]



$\Rightarrow$ increase with M_S, maxima at $X_t/M_S = \pm 2$

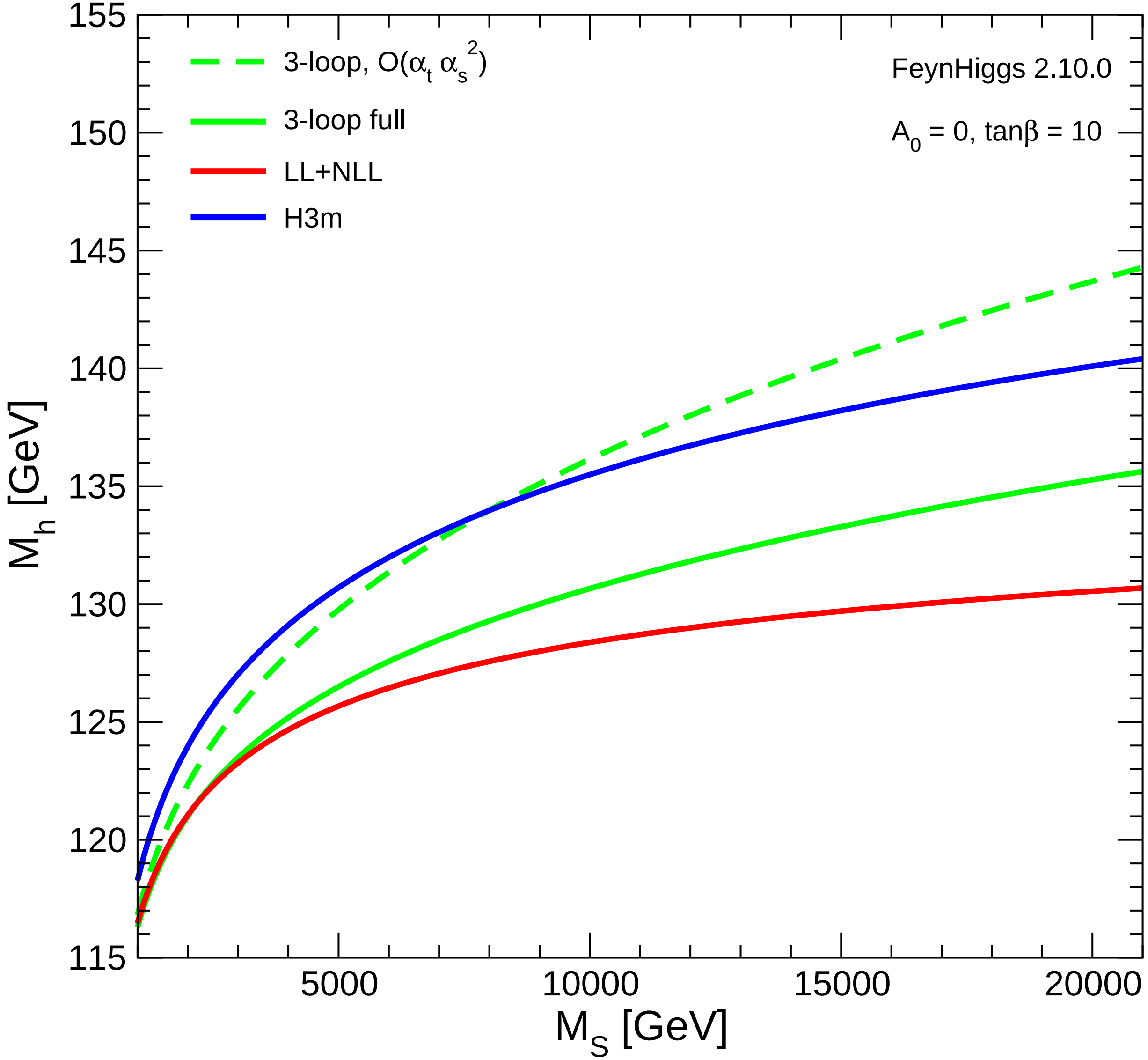
$M_h(M_S)$ for various approximations: [*FeynHiggs 2.10.0*]



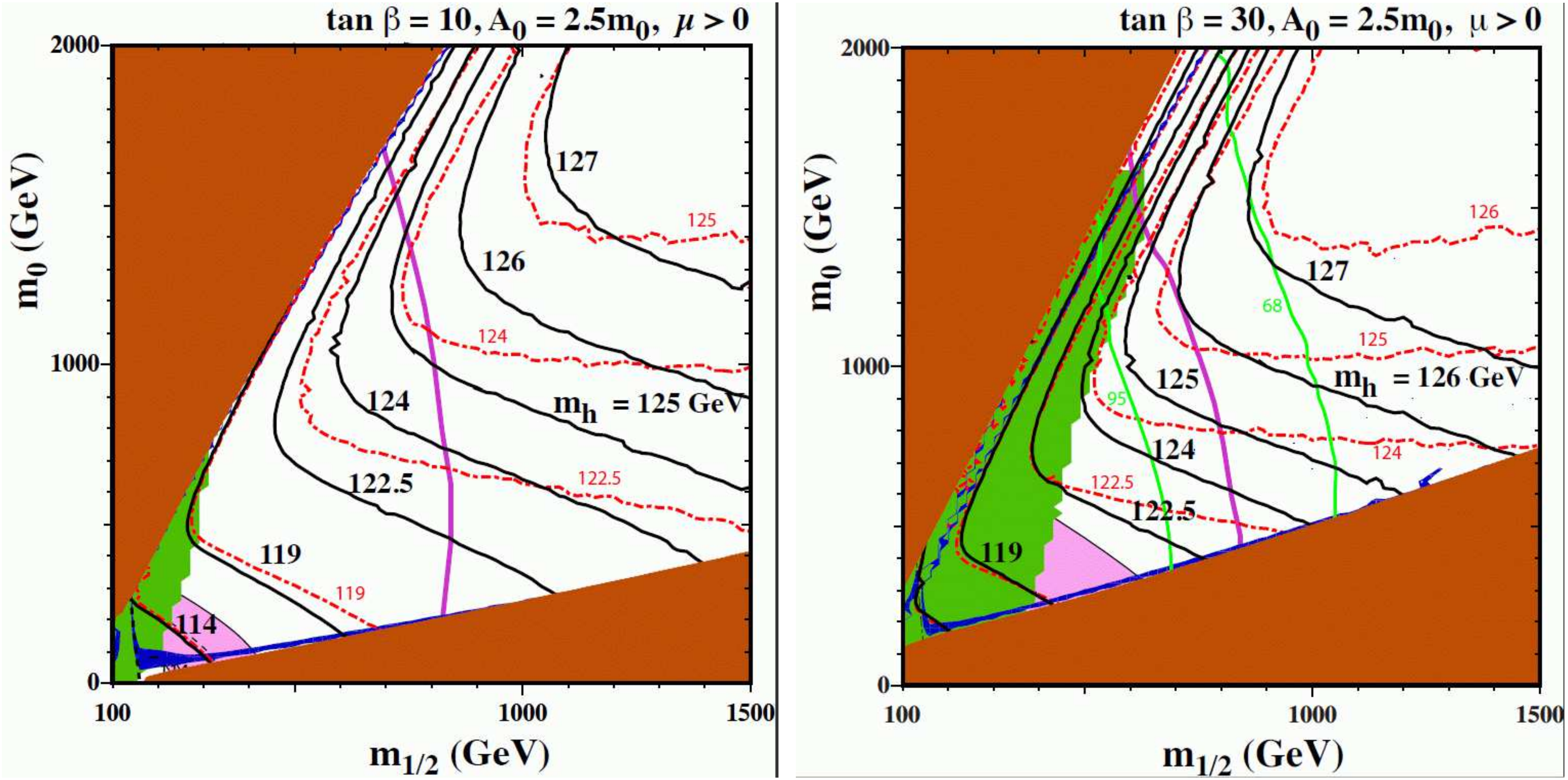
$\Rightarrow$ 3-loop good for $M_S \lesssim 2$ TeV, 7-loop: $\Delta \sim 1$ GeV for $M_S = 20$ TeV

$M_h(M_S)$ compared with H3m:

[*FeynHiggs 2.10.0*]



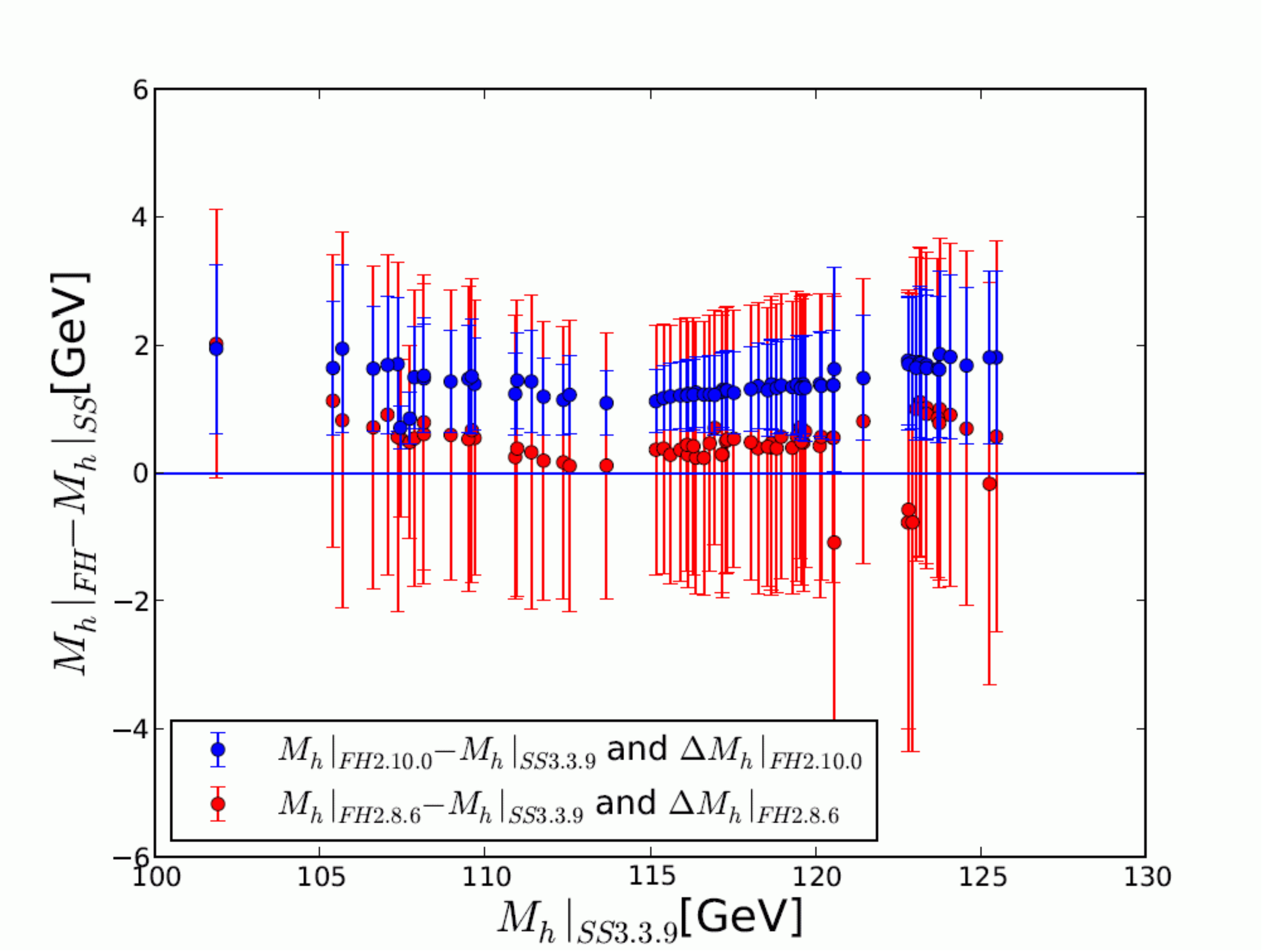
$\Rightarrow$ 3-loop $\mathcal{O}\left(\alpha_t^2\alpha_s, \alpha_t^3\right) \oplus$ beyond 3-loop important for precise M_h prediction!



red-dashed: FeynHiggs 2.8.5 black: FeynHiggs 2.10.0

⇒ shift to larger masses, $M_h \sim 125.5$ GeV "easier"

Effects in the CMSSM: *FeynHiggs* vs. *SoftSusy* [*O. Buchmüller et al. '13*]



red: FeynHiggs 2.8.5 (incl. unc.) blue: FeynHiggs 2.10.0 (incl. updated unc.)

⇒ shift to larger masses, not captured by other codes

4. Conclusinos

[arXiv:1312.4937]

FeynHiggs 2.10.0

`www.feynhiggs.de`

First and only code that provides:

Combination of

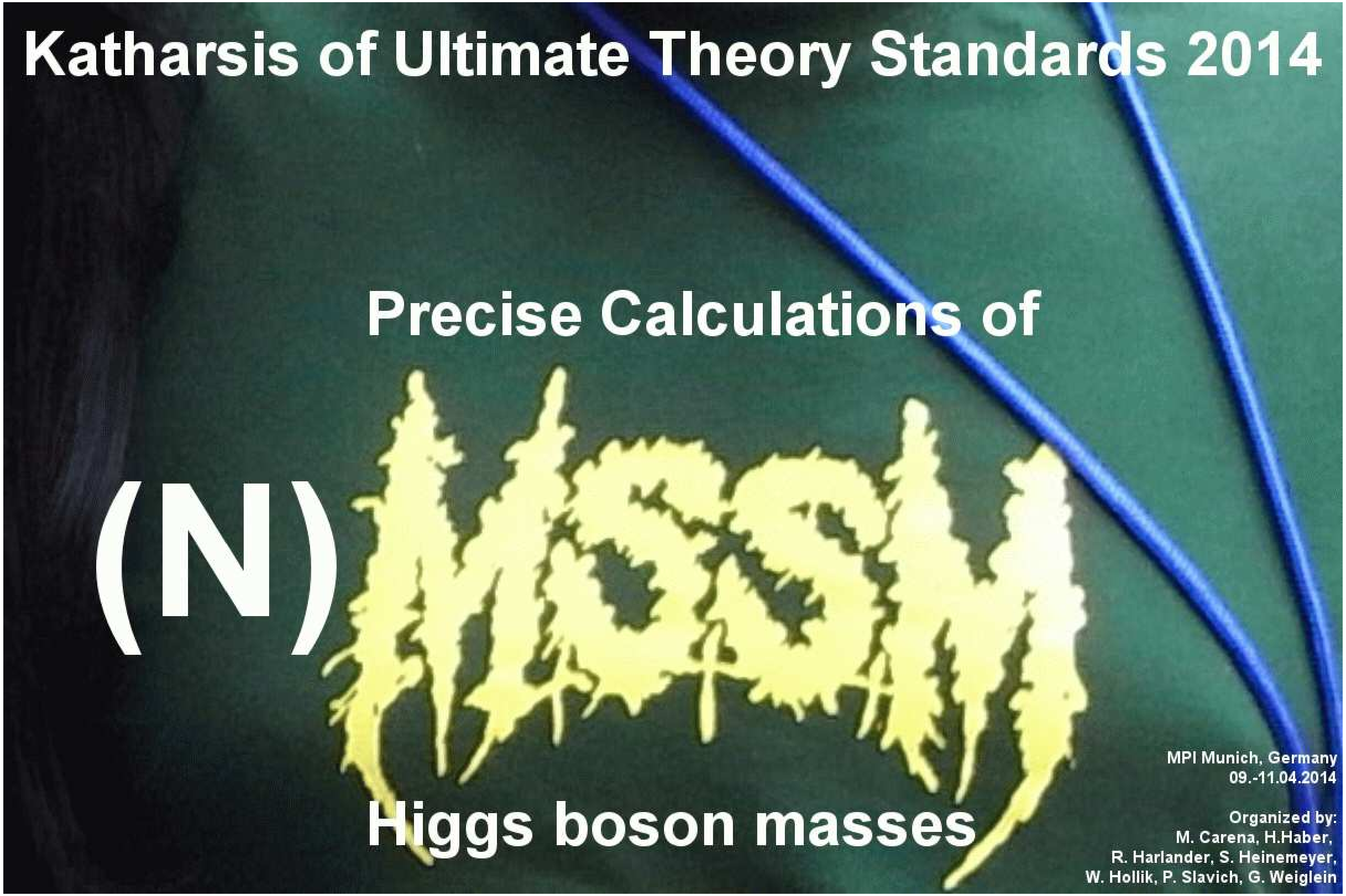
1.) Best available Feynman-diagrammatic result

and

2.) Leading and subleading logs from the top/stop sector

Supplemented by
Improved calulation of theory uncertainty: $\Delta M_h \lesssim 1.5$ GeV
(for the points analyzed so far)

Working group dedicated to SUSY Higgs mass calculations:



Next meeting: 20.-22.10.2014, DESY, Hamburg, Germany

Back-up

Perspectives

Can the theory precision meet the experimental precision?

A) Intrinsic uncertainty in the Feynman-diagrammatic method

B) Intrinsic uncertainty in the RGE method

C) Parametric uncertainties from SM input

A) Intrinsic uncertainty in the Feynman-diagrammatic method

$\mathcal{O}\left(\alpha_t\alpha_s^2\right)$ exists in *H3m*

$\rightarrow$ expansion in many mass scales necessary

$\Rightarrow$ progress possible, but difficult and slow

$\mathcal{O}\left(\alpha_t^2\alpha_s, \alpha_t^3\right)$ probably possible

Inclusion of $b/\tilde{b}$ very difficult (more and very different scales)

Corrections beyond 3-loop???

$\Rightarrow$ dedicated effort necessary!

B) Intrinsic uncertainty in the RGE method

Good recent overview paper: [*P. Draper, G. Lee, C. Wagner, arXiv:1312.5743*]

Missing in *FeynHiggs*:
– 3-loop RGE's
– 2-loop threshold corrections
– inclusion of more scales: EW scale, M_A

⇒ inclusion in *FeynHiggs* probably possible, but far from trivial
⇒ combination of FD and RGE method crucial!

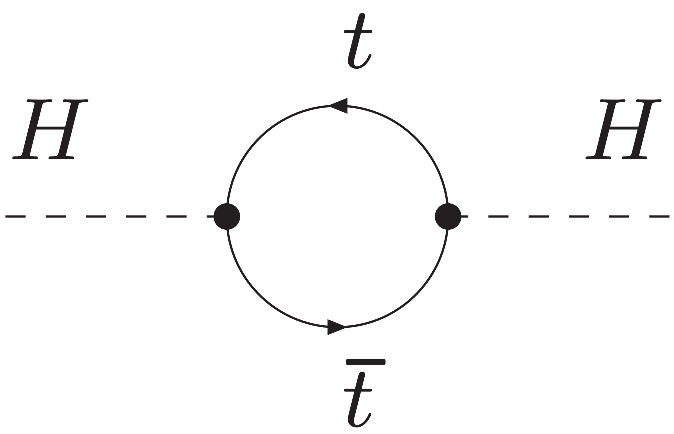
⇒ dedicated effort necessary!

Goal for future *FeynHiggs* version (5-10 years from now): $\Delta M_h^{\text{intr.}} \lesssim 500$ MeV

⇒ knowledge of SUSY mass scales would be extremely helpful . . .

C) Intrinsic uncertainty from m_t:

Nearly any model: large coupling of the Higgs to the top quark:



$\Rightarrow$ one-loop corrections $\Delta M_H^2 \sim G_\mu m_t^4$

$\Rightarrow$ M_H depends sensitively on m_t in all models where M_H can be predicted (SM: M_H is free parameter)

SUSY as an example: $\Delta m_t \approx \pm 1$ GeV $\Rightarrow$ $\Delta M_h \approx \pm 1$ GeV

$\Rightarrow$ Precision Higgs physics needs e^+e^- precision of $\Delta m_t \sim 100$ MeV
$\Rightarrow$ $\Delta M_h \sim 100$ MeV cannot be surpassed (soon)