$\alpha_s$ determination from C-parameter

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Outline

- Motivation & Introduction
- Factorization & log resummation at $N^3$LL
- Singular & Non-singular terms
- Power corrections and hadron mass effects
- Fits for $\alpha_s$
- Conclusions and Outlook
Motivation & Introduction
Event shape analyses with analytic power corrections get consistently low values for $\alpha_S$.

$\alpha_S(m_Z)$ determination from event shape fits

- **MC power corrections**
  - $\tau, \rho, Y_3, B$
- **Analytic power corrections**
  - $\tau$ tail fits
  - moments fits

### Resummation
- **N$$^3$$LL** (Dissertori et al.)
- **N$$^2$$LL** (Gehrmann et al.)
- **NLL** (Becher & Schwartz)

### References
- [Dissertori et al.]
- [Chien & Schwartz]
- [Davison Webber]
- [AFHMS]
- [Gehrmann et al.]
- [Becher & Schwartz]
The world average

Determinations are first “averaged” within a given process
The various averages are later combined together for the final average

Completely dominated by lattice results !!!

\[ \tau \text{-decays} \]
\[ \text{Lattice} \]
\[ \text{DIS} \]
\[ e^+e^- \text{ annihilation} \]
\[ Z \text{ pole fits} \]

\[ \alpha_s(M_Z) \]

Figures taken from PDG
many details in review
[arXiv:1110.0016]

look also [arXiv:1303.2262]

\[ \alpha_s(M_{\tau}) \]

\[ \alpha_s(M_Z) \]

\[ \alpha_s(M_{\tau}) \]
MC event generator GENEVA finds better agreement with peak data using small $\alpha_s$

[Alioli, Bauer, … JHEP 1309 (2013) 120]

Using Pythia 8 shower, tune 1 for hadronization and $\alpha_s(m_Z) = 0.1135$
DIS analyses of ABM get similarly low and precise determinations.

We need to analyze more event-shapes to validate our results.

- Tail of thrust
- Moments of thrust distribution
- Tail of C-parameter (to appear soon)
- Tail of Heavy Jet Mass (w.i.p)
- Moments of C-parameter (w.i.p) and HJM
C-parameter definition

linearized momentum tensor

\[ \Theta^{\alpha \beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i p_i^\alpha p_i^\beta / |\vec{p}_i| \]  

with eigenvalues

\[ \lambda_1, \lambda_2, \lambda_3 \]

\[ \lambda_1 + \lambda_2 + \lambda_3 = 1 \]

\[ C = 3 \left( \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 \right) = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2} \]

- Double sum
- Does not require minimization

Continuous transition from 2-jet to 3-jet, ... multi-jet events

dijet \hspace{1cm} C \sim 0

three jets \hspace{1cm} C \sim 0.75

spherical \hspace{1cm} C \sim 1

\[ Q = 91.2 \text{ GeV} \]
Factorization & log resummation
Resummation of large logarithms

Event shapes are not inclusive quantities

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dC} = -\frac{2\alpha_s}{3\pi} \frac{1}{C} \left(3 + 4 \log \frac{C}{6} + \ldots\right)
\]

Large logs at small \(C\)

Invalidates perturbative expression for small \(C\)

\[
\alpha_s \log \frac{C}{6} \sim \mathcal{O}(1)
\]

One has to reorganize the expansion by considering

Counting more clear in the exponent of cumulant

\[
\log \Sigma(C_c) = \alpha_s (\log^2 C_c + \log C_c + 1)
\]

\[
\alpha_s^2 (\log^3 C_c + \log^2 C_c + \log C_c + 1)
\]

\[
\alpha_s^3 (\log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1)
\]

\[
\alpha_s^4 (\log^5 C_c + \log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1)
\]

\[
\vdots
\]

\[
\Sigma(C_c) \equiv \int_0^{C_c} dC \frac{1}{\sigma_0} \frac{d\sigma}{dC}
\]

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]

State of the art

LL NLL N^{2}LL N^{3}LL not known!
Resummation of large logarithms

Event shapes are not inclusive quantities

\[ \frac{1}{\sigma_0} \frac{d\sigma}{dC} = - \frac{2\alpha_s}{3\pi} \frac{1}{C} \left( 3 + 4 \log \frac{C}{6} + \ldots \right) \]

One has to reorganize the expansion by considering

Counting more clear in the exponent of cumulant

\[ \log \Sigma(C_c) = \alpha_s \left( \log^2 C_c + \log C_c + 1 \right) \]
\[ \alpha_s^2 \left( \log^3 C_c + \log^2 C_c + \log C_c + 1 \right) \]
\[ \alpha_s^3 \left( \log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1 \right) \]
\[ \alpha_s^4 \left( \log^5 C_c + \log^4 C_c + \log^3 C_c + \log^2 C_c + \log C_c + 1 \right) \]
\[ \ldots \]

[Hoang, VM, Schwartz, Stewart]
[Becher, Schwartz]
[Chien, Schwartz]
[Abbate, Fickinger, Hoang, VM, Stewart]
[Hoang, Kolodrubetz, VM, Stewart]
**Factorization theorem for event shapes**

\[ \frac{1}{\sigma_0} \frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O} \left( e^0, \frac{\Lambda_{\text{QCD}}}{Q} \right) \]

Universal Wilson Coefficient

Jet function

Soft function

Nonsingular terms, power corrections

Calculable in perturbation theory

Perturbative and nonperturbative components

Leading power correction comes from soft function

\[ S_e = \hat{S}_e \otimes F_e \]

[Bauer, Lee, Fleming, Sterman]

[Hoang & Stewart]

more on this later

\[ \frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} \otimes F_e \]
Renormalization group evolution

The hierarchy among the scales depends on the position on the spectrum

hard scale

\[ \mu_H \sim Q \]

\[ \log^n \left( \frac{Q}{\mu} \right) \]

jet scale

\[ \mu_J \sim Q \sqrt{\tau} \]

\[ \log^n \left( \frac{Q^2 \tau}{\mu^2} \right) \]

soft scale

\[ \mu_S \sim Q \tau \]

\[ \log^n \left( \frac{Q \tau}{\mu} \right) \]

\[ \log \left( \frac{\Lambda_{QCD}}{Q \tau} \right) \]

Use profile function to describe the whole distribution
The hierarchy among the scales depends on the position on the spectrum.
Singular & nonsingular terms
Theoretical knowledge

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H(Q, \mu)$</td>
<td>Hard function known at 3 loops</td>
</tr>
<tr>
<td>$J_n(s, \mu)$</td>
<td>Jet function known at two loops</td>
</tr>
<tr>
<td></td>
<td>Running known at three loops</td>
</tr>
<tr>
<td>$S_C(\ell, \mu)$</td>
<td>Soft function known analytically at one loop,</td>
</tr>
<tr>
<td></td>
<td>numerically at two loops</td>
</tr>
<tr>
<td></td>
<td>Running known at three loops</td>
</tr>
</tbody>
</table>

- Fixed-order predictions known at three loops
- Mass corrections known at $N^2$LL and two loops
- FS QED corrections known at $N^3$LL
C-parameter soft function computation

[ Kolodrubetz, Hoang, VM, Stewart]

Analytic computation of soft function at 1-loop

\[ S_{e}^{1\text{-loop}}(\ell) = \frac{2\alpha_s C_F e^{\epsilon E}}{\mu \pi \Gamma(1-\epsilon)} \left( \frac{\ell}{\mu} \right)^{-1-2\epsilon} I_e(\epsilon) \]

universal formula for all event shapes

\[ I_{\tau}(\epsilon) = \frac{1}{\epsilon} \quad I_{C}(\epsilon) = \frac{1}{2} \frac{\Gamma(\epsilon)^2}{\Gamma(2\epsilon)} \]

Numerical determination at 2-loops using Event2
Non-singular terms

\[
\frac{d\hat{\sigma}_{ns}}{dC} = \frac{d\hat{\sigma}^{\text{FO}}_{\text{full}}}{dC} - \frac{d\hat{\sigma}^{\text{FO}}_{s}}{dC}
\]

full FO

SCET with fixed scales

\[
\frac{d\sigma_{ns}}{dC} = \frac{d\hat{\sigma}_{ns}}{dC} \otimes F_C
\]
same shape function as singular terms

LO (analytically)

\[
1 \frac{d\hat{\sigma}_{ns}}{\sigma_0} = \frac{\alpha_s(Q)}{2\pi} f_1(C) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 f_2(C) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 f_3(C) + \ldots
\]

NLO (from Event2)

NNLO (from EERAD3)
Power Corrections
&
full results
OPE for non-perturbative corrections

\[ S_e(e) = \langle 0 | \overline{Y}_{\nu}^{\dagger} Y_{\nu}^{\dagger} \delta(\ell - Q\hat{e}) Y_{\nu} \overline{Y}_{\nu} | 0 \rangle \]

[Lee & Sterman]

For \( e \gg \frac{\Lambda_{\text{QCD}}}{Q} \)

Shape function can be expanded in the tail

\[ F_e(\ell) \simeq \delta(\ell) - \Omega_1 \delta'(\ell) \]

\[ \Omega_1 = \langle 0 | \overline{Y}_{\nu}^{\dagger} Y_{\nu}^{\dagger} Q\hat{e} Y_{\nu} \overline{Y}_{\nu} | 0 \rangle \]

\[
\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} - \frac{\Omega_1}{Q} \frac{d}{de} \frac{d\hat{\sigma}}{de} \simeq \frac{d\hat{\sigma}}{de} \left( e - \frac{\Omega_1}{Q} \right) + \mathcal{O} \left[ \left( \frac{\Lambda_{\text{QCD}}}{Q e} \right)^2 \right]
\]

Universality:

\[ \Omega_1^e = c_e \Omega_1^p \]

Leading power corrections proportional to each other, calculable coefficient

We define the gap scheme for \( \Omega_1 \) in which it is renormalon-free
• Hadron mass effects are $O(1)$, cannot be treated as a correction

• Universality broken by hadron masses.

\[
e(N) = \frac{1}{Q} \sum_{i \in N} m_i \ f_e(r_i, y_i)
\]

\[
\Omega_1^e = c_e \int dr \ g_e(r) \Omega_1(r)
\]

\[
ge_e(r) = \frac{1}{c_e} \int dy \ f_e(r, y)
\]

\[
c_e = \int_{-\infty}^{\infty} dy \ f_e(1, y)
\]

• Power corrections carry anomalous dimension

\[
\gamma^{\Omega_1} = -\frac{\alpha_s C_A}{\pi} \log(1 - r^2)
\]
Cross section convergence

**Fixed Order**

- $O(\alpha_s^3)$
- $O(\alpha_s^2)$
- $O(\alpha_s)$

$Q = 91.2$ GeV

**Renormalon Subtraction**

- $N^3LL$
- $N^2LL$
- $NLL$

$Q = 91.2$ GeV

**Power Corrections**

- $N^3LL$
- $N^2LL$
- $NLL$

$Q = 91.2$ GeV

**Log Resummation**

- $N^3LL$
- $N^2LL$
- $NLL$

$Q = 91.2$ GeV

*Note: The graphs show the convergence of cross sections with respect to $C$ for different orders of perturbation theory and resummation schemes.*
Fits for $\alpha s$
We perform global fits for energies between 35 and 206 GeV. We restrict ourselves to the tail of the distribution.
**α_s** determination: C-parameter tail fits

[Graphs showing the α_s(m_Z) distribution with and without renormalon subtractions, comparing N^3LL, N^2LL, and NLL contributions.]
**$\alpha_s$ determination: C-parameter tail fits**

**No Renormalon Subtractions**

<table>
<thead>
<tr>
<th>$\alpha_s(m_Z)$</th>
<th>$\frac{d\sigma}{dC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1025 - 0.1150</td>
<td>0.3 - 0.6</td>
</tr>
</tbody>
</table>

**With Renormalon Subtractions**

<table>
<thead>
<tr>
<th>$\alpha_s(m_Z)$</th>
<th>$\frac{d\sigma}{dC}$</th>
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<td>0.1025 - 0.1150</td>
<td>0.3 - 0.6</td>
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</table>

**Preliminary**

$\alpha_s(m_Z) = 0.113 \pm 0.002$

all errors combined
Universality: thrust vs C-parameter

Renormalon subtraction improves $\alpha_s$ agreement

hadron-mass effects improve universality

we expect further improvement if NLL hadron mass effects are included
Conclusions & Outlook
Preliminary

\[ \alpha_s(m_Z) = 0.113 \pm 0.002 \]

**\(\alpha_s(m_Z)\) determination from event shape fits**

- **MC power corrections**
  - \(\tau, \rho, Y_3, B\) [Chien & Schwartz]
  - \(\rho\)

- **Analytic power corrections**
  - \(\tau\) tail fits
  - \(C\)-param

- **NLL resummation**
  - \(\tau, \rho, Y_3, B\) [Becher & Schwartz]

- **N^3LL resummation**
  - [Dissertori et al]

- **World average**
Conclusions & Outlook

- Less precision than with thrust, but good consistency check of method + universality.
- First fits ever including hadron mass effects.
- Primary massive production computation (w.i.p.).
- QED effects can be easily added (w.i.p.).
- Fits to the first moment of C-parameter (w.i.p.).
- Close the picture with fits to HJM distribution (w.i.p.).
Backup slides
### Renormalization scale setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
<th>Range of Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>1.1 GeV</td>
<td>1 to 1.3 GeV</td>
</tr>
<tr>
<td>$R_0$</td>
<td>0.7 GeV</td>
<td>0.6 to 0.9 GeV</td>
</tr>
<tr>
<td>$n_0$</td>
<td>12</td>
<td>10 to 16</td>
</tr>
<tr>
<td>$n_1$</td>
<td>25</td>
<td>22 to 28</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.67</td>
<td>0.64 to 0.7</td>
</tr>
<tr>
<td>$t_s$</td>
<td>0.83</td>
<td>0.8 to 0.86</td>
</tr>
<tr>
<td>$r$</td>
<td>0.33</td>
<td>0.26 to 0.38</td>
</tr>
<tr>
<td>$e_J$</td>
<td>0</td>
<td>$-0.5$ to $0.5$</td>
</tr>
<tr>
<td>$e_H$</td>
<td>1</td>
<td>0.5 to 2.0</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0</td>
<td>$-1, 0, 1$</td>
</tr>
</tbody>
</table>

**Scale variation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_3^{\text{cusp}}$</td>
<td>1553.06</td>
<td>$-1553.06$ to $+4659.18$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>-43.2</td>
<td>$-44.2$ to $-42.2$</td>
</tr>
<tr>
<td>$j_3$</td>
<td>0</td>
<td>$-3000$ to $+3000$</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0</td>
<td>$-500$ to $+500$</td>
</tr>
<tr>
<td>$\epsilon_{2,\text{low}}$</td>
<td>0</td>
<td>$-1, 0, 1$</td>
</tr>
<tr>
<td>$\epsilon_{2,\text{high}}$</td>
<td>0</td>
<td>$-1, 0, 1$</td>
</tr>
<tr>
<td>$\epsilon_{3,\text{low}}$</td>
<td>0</td>
<td>$-1, 0, 1$</td>
</tr>
<tr>
<td>$\epsilon_{3,\text{high}}$</td>
<td>0</td>
<td>$-1, 0, 1$</td>
</tr>
</tbody>
</table>

**Non-singular unknowns**

- $Q = 91.2$ GeV

---

**Diagrams**

- $\mu_i$ vs $C$
- $\mu_i$ vs $C$ for $Q = 91.2$ GeV
Cross section components

Singular dominates in the peak and tail

\[ \left| \frac{1}{\sigma} \frac{d\sigma_i}{dC} \right| \]

FO results reproduced in far tail

\( Q = m_Z \)

Singular

Nonsingular

Subt.

Total
\( \alpha_s \) determination: \( C \)-parameter tail fits

Strong degeneracy between \( \alpha_s \) and \( \Omega_1 \) which is broken if many values of the center of mass energy are included.

We perform global fits for energies between 35 and 206 GeV. We restrict ourselves to the tail of the distribution.