Hadronic total cross sections, Wilson loop correlators and the QCD spectrum

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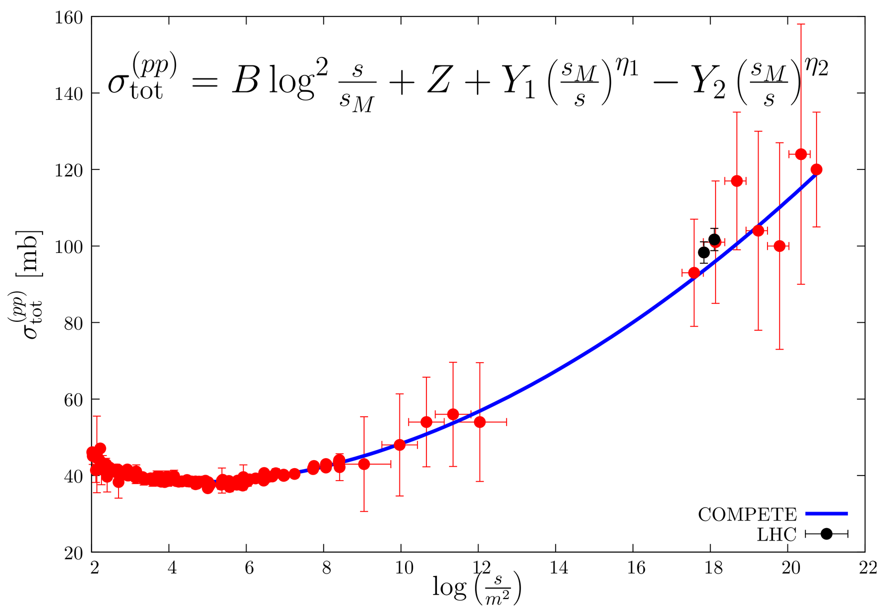
Total Cross Sections



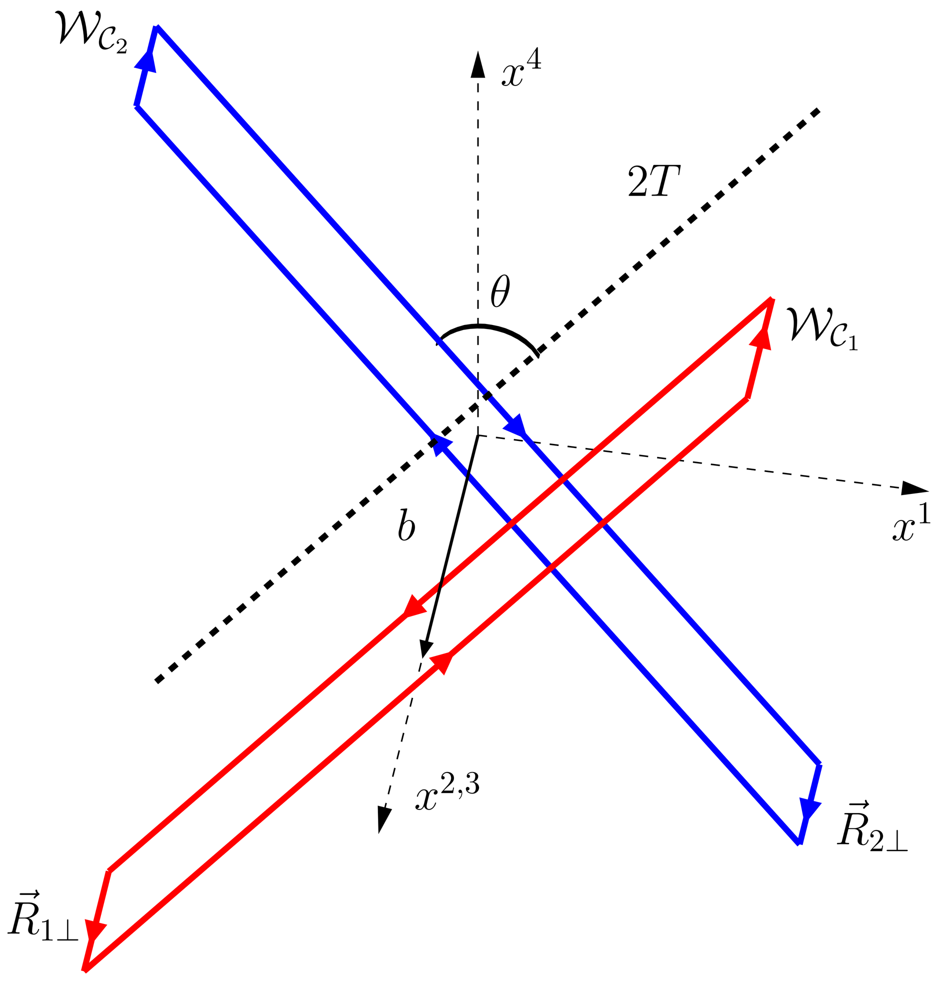
Hadronic total cross sections rise at high energy like $\sigma_{tot} \sim B\log^2 s$ with universal prefactor $B \simeq 0.3\,\text{mb}$. This behaviour respects unitarity as encoded in the Froissart-Łukaszuk-Martin bound $\sigma_{tot} \leq \frac{\pi}{m_\pi^2}\log^2\frac{s}{s_0}$, as $B \ll \pi/m_\pi^2$.
Deriving σ_{tot} from first principles of QCD and explaining the universality of B would be highly desirable. Understanding the rise of σ_{tot} is part of the problem of soft high-energy scattering, characterised by $|t| \leq 1\,\text{GeV}^2 \ll s$. In this regime perturbation theory is not fully reliable, and a nonperturbative approach is needed.

Nonperturbative approach: Wilson-loop correlators

The nonperturbative approach to *elastic meson-meson scattering* is as follows [1, 2]:

1. mesons are described as wave packets of transverse colourless dipoles;
2. in the soft high-energy regime, the dipoles travel essentially undisturbed on their classical, almost lightlike trajectories;
3. mesonic amplitudes are obtained from the dipole-dipole (dd) amplitudes after folding with the appropriate squared wave functions, $A(s,b) = \langle\!\langle A^{(dd)}(s,b;\nu)\rangle\!\rangle$.

Here b is the impact parameter, and ν denotes collectively the dipole variables (longitudinal momentum fraction, transverse size and orientation). This approach extends also to baryons if one adopts a quark-diquark picture.
At high energy the dd amplitude is given by the normalised connected correlator $\mathcal{C}_M$ of the Wilson loops (WL) running along the classical trajectories of the two dipoles, $A^{(dd)}(s,b;\nu) = -\mathcal{C}_M(\chi,b;\nu)$, with $\chi \simeq \log\frac{s}{m^2}$ the hyperbolic angle between the trajectories.



The correlator $\mathcal{C}_M$ is obtained from the correlator $\mathcal{C}_E$ of two Euclidean WL at angle θ,

$$\mathcal{C}_E(\theta,b;\nu) \equiv \lim_{T\to\infty}\frac{\langle\mathcal{W}_{\mathcal{C}_1}\mathcal{W}_{\mathcal{C}_2}\rangle}{\langle\mathcal{W}_{\mathcal{C}_1}\rangle\langle\mathcal{W}_{\mathcal{C}_2}\rangle} - 1\,,$$

through the analytic continuation (AC) [3]

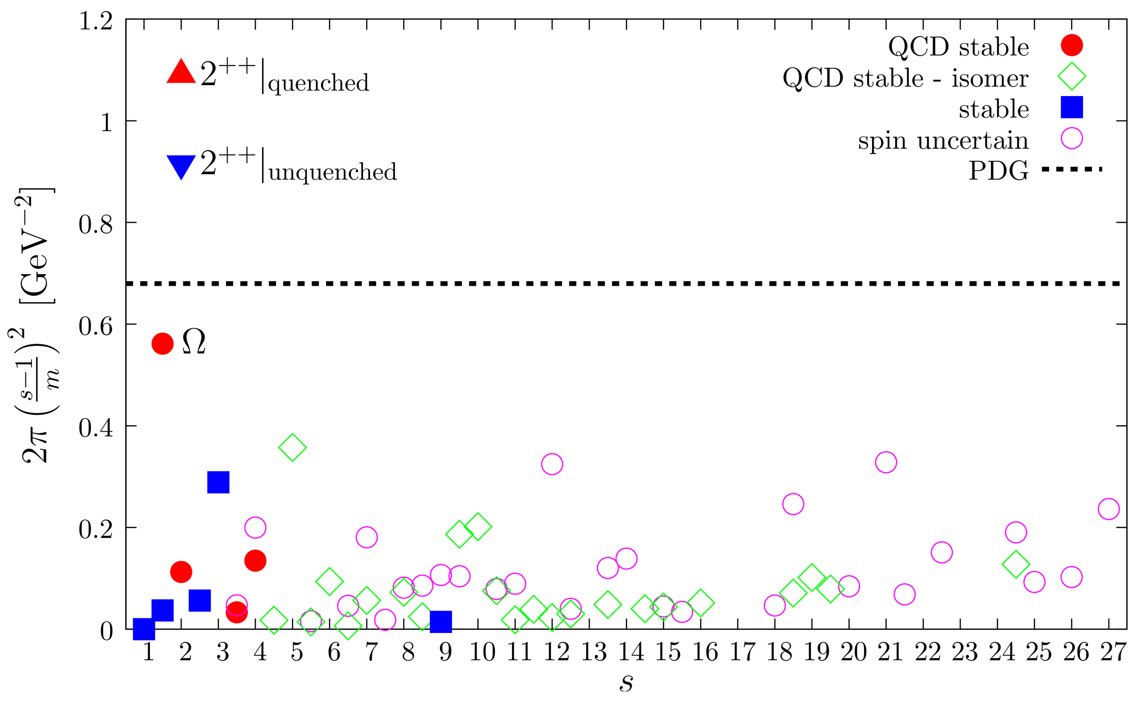
$$\mathcal{C}_M(\chi,b;\nu) = \mathcal{C}_E(\theta\to -i\chi,b;\nu)\,.$$

Results: total cross sections from the hadronic spectrum

The total cross section satisfies the bound [4]

$$\sigma_{\text{tot}} \underset{s\to\infty}{\lesssim} \frac{4\pi}{\mu^2}\log^2\frac{s}{m^2} = 2B_{\text{th}}\log^2\frac{s}{m^2}\,,$$

with $\mu^{-1} = \max_a \frac{s^{(a)}-1}{m^{(a)}}$ determined from the hadronic spectrum. To determine μ we maximise over stable states of QCD *in isolation*, as electroweak effects have been neglected from the onset. States with $s^{(a)} \geq 1$ are considered, including nuclei.
Quite surprisingly, this singles out the $\Omega^\pm$ baryon, which yields $B_{\text{th}} \simeq 0.56\ \text{GeV}^{-2}$. The resulting bound on σ_{tot} is much more stringent than the original bound, and the prefactor is of the same order of magnitude of the experimental value $B_{\text{exp}} \simeq 0.680(13)\ \text{GeV}^{-2}$.



Nuclear data taken from Ref. [5]

Furthermore, our bound is not singular in the chiral limit.
If universality is achieved (see box below) then σ_{tot} is entirely determined by the hadronic spectrum, and reads [4]

$$\sigma_{\text{tot}} \underset{s\to\infty}{\simeq} \frac{2\pi}{\mu^2}\log^2\frac{s}{m^2} = B_{\text{th}}\log^2\frac{s}{m^2}\,.$$

This *prediction* for the prefactor is in fair agreement with B_{exp} (taking into account a systematic error of order 10% on B_{exp}). The same conditions leading to universal total cross sections also give universal, black-disk-like elastic scattering amplitudes.
Total cross sections are usually believed to be governed by the gluonic sector of QCD. However, in the *quenched* theory one finds from the glueball spectrum $B_Q \gtrsim 1.6B_{\text{exp}}$, which suggests large unquenching effects.

Method

The energy dependence of σ_{tot} is determined by the large-s, large-b behaviour of the amplitude through the "effective radius" of interaction b_c, beyond which the amplitude is negligible, as $\sigma_{\text{tot}} \propto b_c^2$.
On the Euclidean side, we obtain information on the b- and θ-dependencies of $\mathcal{C}_E$ by inserting a complete set of asymptotic states between the WL operators. After AC this gives us information on the s- and b-dependencies. This requires two crucial analyticity assumptions:

1. AC can be performed separately for each term in the sum;
2. WL matrix elements are analytic in θ.

A few reasonable finiteness assumptions on the WL matrix elements are also made.

Leading behaviour of $\mathcal{C}_M$

At large χ, b, the relevant correlator reads [4]

$$\mathcal{C}_M(\chi,b;\nu) \simeq \sum_{\alpha\neq 0} f_\alpha(\nu)\prod_a [w_a(\chi,b)]^{n_a(\alpha)}$$

where the sum is over states α, and $n_a(\alpha)$ is the number of particles of type a in state α. Here

$$w_a(\chi,b) = \frac{e^{[r^{(a)}\chi - b]m^{(a)}}}{\sqrt{2\pi b m^{(a)}}}\,,\qquad r^{(a)} \equiv \frac{s^{(a)}-1}{m^{(a)}}\,,$$

with $(s^{(a)}, m^{(a)})$ spin and mass of particles of type a. Particles of type a contribute only for $b \lesssim r^{(a)}\chi$, and so the effective radius of interaction is given by

$$b_c(s) = \max_a r^{(a)}\log\frac{s}{m^2} \equiv \frac{1}{\mu}\log\frac{s}{m^2}\,.$$

Large-s behaviour of σ_{tot}

We find for σ_{tot} [4]

$$\sigma_{\text{tot}} \underset{s\to\infty}{\simeq} 2\pi(1-\kappa)[b_c(s)]^2 \simeq \frac{2\pi}{\mu^2}(1-\kappa)\log^2\frac{s}{m^2}$$

with $|\kappa| \leq 1$ due to unitarity. In general κ depends on the colliding hadrons. Analyticity and crossing symmetry requirements show that universality is most naturally achieved if $\kappa = 0$, corresponding to a vanishing or oscillating correlator as $\chi \to \infty$ at fixed b.

References

[1] O. Nachtmann, Ann. Phys. **209** (1991) 436
[2] H. G. Dosch *et al.*, Phys. Rev. D **50** (1994) 1992
[3] MG and E. Meggiolaro, Phys. Lett. B **675** (2009) 123
[4] MG and E. Meggiolaro, JHEP **03** (2014) 002
[5] Audi *et al.*, Nucl. Phys. A **729** (2003) 3