Study about Neutrino Oscillation and Dissipative Effect in

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Introduction

- In this work, we show an easy way to treat neutrinos as an open system when the standard matter interaction needs to be taken into account [1]. We use a different point view than what is found in literature to study neutrino oscillation [2]. We also show some results on dissipative effects using the proposed LBNE configuration.

Overview of Formalism

- The quantum dynamical equation in this formalism is written as

$$\frac{d\rho_\nu(t)}{dt} = -i[H_{osc}, \rho_\nu(t)] + D[\rho_\nu(t)]\,, \qquad (1)$$

where H_{osc} is a piece of the total Hamiltonian that can be given by $H_{tot} = H_{osc} + H_R + H_{int}$, such that,

$$H_{osc} = H_{mass} + H_{matter}\,, \qquad (2)$$

and it can be expressed as $H_{osc} = diag.\{\tilde{E}_1, \tilde{E}_2, \tilde{E}_3\}$ if we consider the effective mass basis. This becomes similar the formalism to the neutrino oscillation with dissipative effects considering its propagation being in constant matter or in vacuum [3].

- The dissipative part can be written as

$$D[\rho_\nu] = \frac{1}{2}\sum_{k=1}^{N^2-1}\left(\left[V_k, \rho_\nu V_k^\dagger\right] + \left[V_k\rho_\nu, V_k^\dagger\right]\right), \qquad (3)$$

where the operators V_k act on neutrinos state only and must be such that $\sum_k V_k^\dagger V_k = \mathbb{1}$ for the trace of ρ_ν to be preserved.

- The evolution with the equation (1), where the term $D[\rho]$ is given by (3), ever converts a density matrix into density matrix and, in this case, the time evolution is complete positive. It means it can transform pure states into mixed states due to dissipation effects [4]. Thus, an important requirement is that the Von Neumann entropy, $S = -Tr[\rho_\nu ln\rho_\nu]$, increases with time for the density matrix of interest subsystem. This is possible if we have $V_k^\dagger = V_k$ [5].

- Equation (1) can be expanded in matrix basis of the $SU(3)$ with $V_k = a_\eta^k \lambda_\eta$ and it is rewritten as

$$\dot{\rho}_\vartheta \lambda_\vartheta = f_{ijk} H_i \rho_j \lambda_\vartheta \delta_{\vartheta k} + \rho_\nu D_{\vartheta\nu}\lambda_\vartheta\,, \qquad (4)$$

and the additional matrix, $D_{\mu\nu}$, must be real, symmetric and positive. For complete positivity, we have 36 phenomenological parameters bounded among each other by inequalities which guarantee positivity. We can use some suitable physical constraint in order to restrain the number of the new phenomenological parameters.

- One constraint is obtained supposing the decoherence is a possible phenomenon. For this, we must have $Tr[H\rho] = 0$, i. e., the energy is conserved inside of interest subsystem. With this restriction, we obtain a quantum dissipator that can be written as

$$D_{\mu\nu} = -diag\{0, 2(\vec{a}_3)^2, 2(\vec{a}_3)^2, 0, \frac{1}{2}(\vec{a}_3+\vec{a}_8)^2, \frac{1}{2}(\vec{a}_3+\vec{a}_8)^2, \frac{1}{2}(\vec{a}_3-\vec{a}_8)^2, \frac{1}{2}(\vec{a}_3-\vec{a}_8)^2, 0\}\,, \qquad (5)$$

and we can break this symmetry putting non zero values in the entries D_{33} and D_{88}.

- The other constraint includes relaxation effects and it is obtained when the elements D_{33} and D_{88} non null. Then, the most effective quantum dissipator is given by

$$D_{\mu\mu} = -diag\{0, \Gamma_{12}, \Gamma_{12}, \Gamma_{33}, \Gamma_{13}, \Gamma_{13}, \Gamma_{23}, \Gamma_{23}, \Gamma_{88}\}\,, \qquad (6)$$

with the following constraint given by the complete positivity

$$\begin{aligned}
&\Gamma_{12} = 2a_1^2 + 2a_3^2 + a_5^2 + a_7^2 \geq 0;\\
&\Gamma_{33} = 4a_1^2 + a_5^2 + a_7^2 \geq 0;\\
&\Gamma_{13} = a_1^2 + \tfrac{1}{2}\left(2\left(2a_5^2 + a_7^2\right) + (a[3] + a[8])^2\right) \geq 0;\\
&\Gamma_{23} = \tfrac{1}{2}\left(2a_1^2 + 2\left(a_5^2 + 2a_7^2\right) + (a[3] - a[8])^2\right) \geq 0;\\
&\Gamma_{88} = 3\left(a_5^2 + a_7^2\right) \geq 0,
\end{aligned} \qquad (7)$$

where we rewrite the $D_{\mu\mu}$ elements in terms of the Γ_{ij} that include decoherence effect and Γ_{ii} that include the relaxation effect.

- As in LBNE the matter potential has an important role, we will use the same analytical approximation scheme that was used in the Ref. [6]. In this scheme the matter parameters are given in terms of the usual effective parameters and this way, it is easy to apply this formalism to arrive the same expressions for survival and appearance probabilities as was given in Ref. [7] to the vacuum.

- In this situation, we can write the survival probability as

$$P_{\nu_\mu\to\nu_\mu} = \frac{1}{3} + e^{-\Gamma_{ii}x} f_{ii} + e^{-\Gamma_{ij}x} f_{ij} \cos\left[\frac{\Delta\tilde{m}_{ij}^2}{2E}x\right], \qquad (8)$$

and appearance probability as

$$\begin{aligned}
P_{\nu_\mu\to\nu_e} = {} & \frac{1}{3} + e^{-\Gamma_{33}x}A_{11} - e^{-\Gamma_{88}x}A_{33} + e^{-\Gamma_{12}x}\\
& \times\left(A_{12}\sin\left[\frac{\Delta\tilde{m}_{21}^2}{2E}x\right] + B_{12}\cos\left[\frac{\Delta\tilde{m}_{21}^2}{2E}x\right]\right)\\
& + \left(A_{13}\cos\left[\delta + \frac{\Delta\tilde{m}_{31}^2}{2E}x\right] + B_{13}\cos\left[\frac{\Delta\tilde{m}_{31}^2}{2E}x\right]\right)\\
& \times e^{-\Gamma_{13}x} + e^{-\Gamma_{23}x} \times\\
& \left(A_{23}\cos\left[\delta + \frac{\Delta\tilde{m}_{32}^2}{2E}x\right] + B_{23}\cos\left[\frac{\Delta\tilde{m}_{32}^2}{2E}x\right]\right),
\end{aligned} \qquad (9)$$

where $f_{ij} = f_{ij}(\vec{\theta}_{eff})$, $A_{ij} = A_{ij}(\vec{\theta}_{eff})$ and $B_{ij} = B_{ij}(\vec{\theta}_{eff})$.

Dissipative Effects and Neutrinos in LBNE

- In this section, we are going to apply the formalism exposed before, using the LBNE configuration to see the neutrinos behavior. In order to know the effects that each one of the 5 phenomenological parameters brings to the neutrino oscillation, we take one of them with typical value that changes the neutrino behavior and, in each possible set, some parameters are taken null as is possible, and other with values that do not change appreciably the neutrino behavior, but which are necessary to keep the complete positivity constraint.

- So, considering the LBNE, the distance between the source and the detector is 1300 km and to oscillation parameters we have used the following values: $m_{12} = 8 \times 10^{-23}$ GeV, $m_{31} = 2.5 \times 10^{-23}$ GeV, $\theta_{23} = 0.74$, $\theta_{12} = 0.58$, $\theta_{13} = 0.14$, $\delta = 0$ and the usual Earth density. For a specific Γ_{ij} that we are interested, we vary its value about three different values as we can see in the figures bellow. All of plots were made considering normal hierarchy.

- In particular, the Fig. (1) and (2) show the relaxation effects. They allow a bigger appearance neutrinos than the standard pattern to all of energy bins. This happens because the relation effects change the constant terms in the probabilities and they are responsible for becoming the probabilities a perfect mixing of the state, i. e., in the rate $1/3 : 1/3 : 1/3$. This happens when the propagation achieves its asymptotic limit $x \to \infty$. However, because they eliminate the constant terms in the probabilities, the maximum and minimum values of the probabilities change too and this new exotic appearance is duo to this as well as the behavior for survival probabilities.

- The decoherence effects, Γ_{12}, Γ_{13}, Γ_{23}, are represented in the Fig. (3), (4) and (5). On the survival plots, we can see that in the case of the LBNE, the decoherence effect from Γ_{23} will be most effective than Γ_{12}, Γ_{13}, and this is expected because the main oscillation term comes from the usual parameters that describe transitions between the families ν_μ and ν_τ. However, from energy above of 10 Gev the behavior due to Γ_{12}, Γ_{13} parameters can change the probabilities in an exotic way because on the resonance region the behavior depend on the ν_e parameters.

- The appearance probabilities, we have the most interesting behavior due to the dissipative effects. Essentially, for understanding the strange behavior that was brought by Γ_{12} andΓ_{23} in the appearance probability, we need to keep in mind that decoherence effect acts in the decoupling of the oscillation between the families that are related by these parameters. So, in the Fig. (3) we can see that before 2 GeV Γ_{12} increases the appearance probability, while after 2 Gev Γ_{12} decreases it. In the Fig. (5), the Γ_{23} always decreases the maximum probability, but it increases all the minimum.

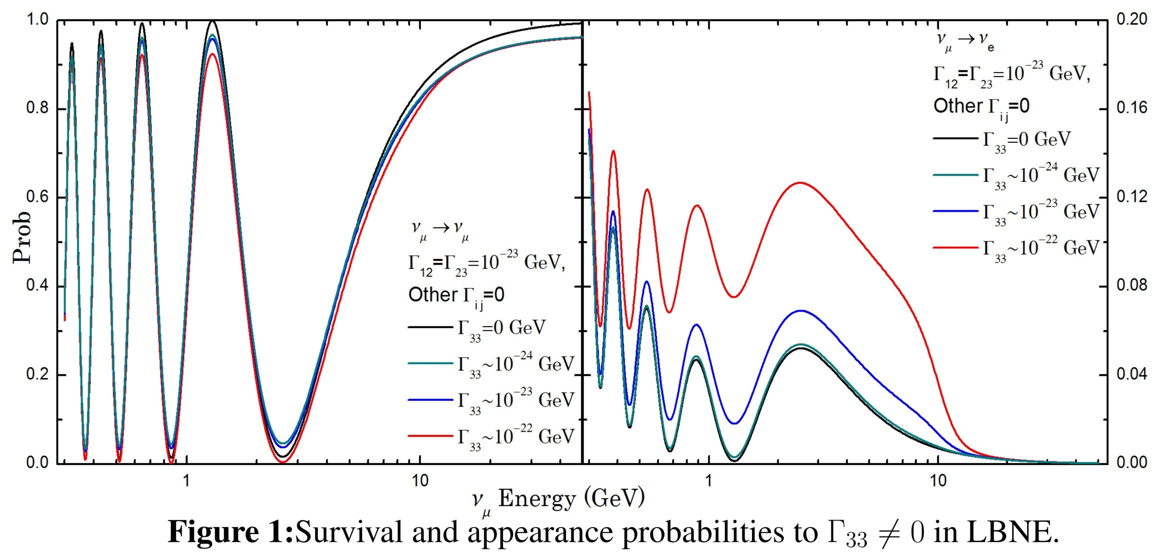

Figure 1: Survival and appearance probabilities to $\Gamma_{33} \neq 0$ in LBNE.

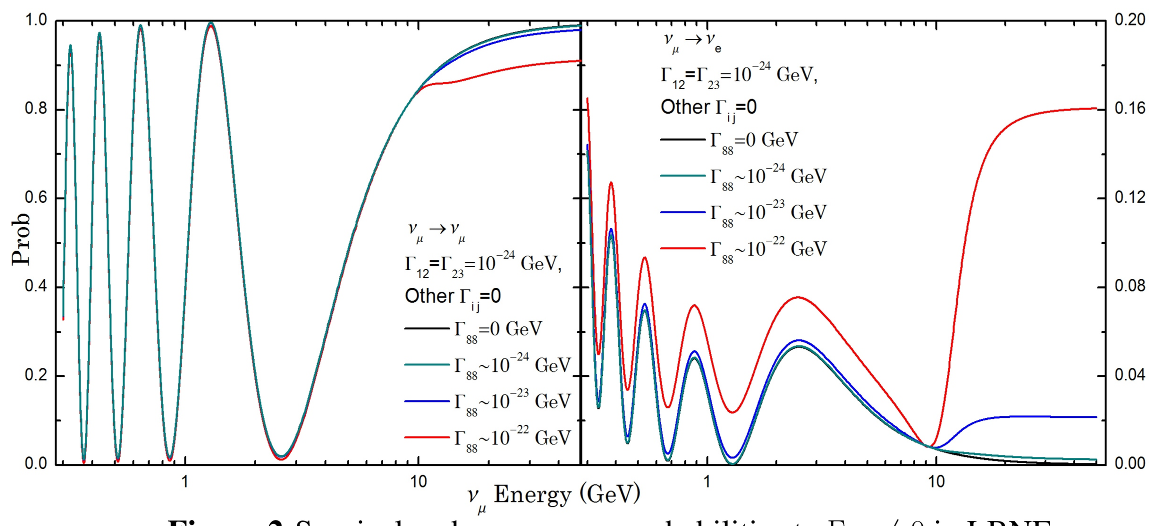

Figure 2: Survival and appearance probabilities to $\Gamma_{88} \neq 0$ in LBNE.

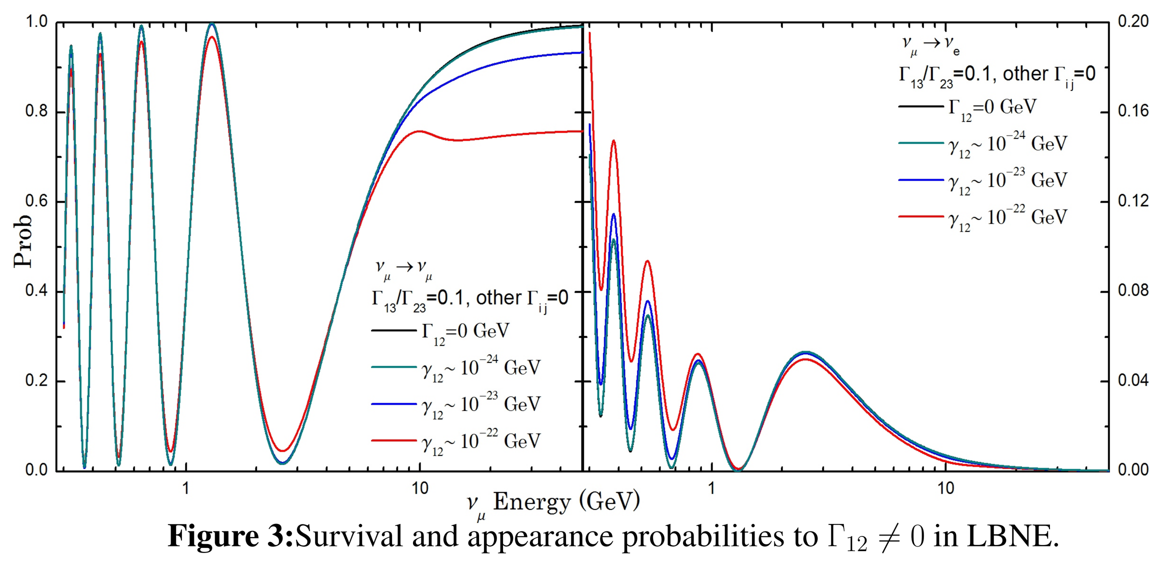

Figure 3: Survival and appearance probabilities to $\Gamma_{12} \neq 0$ in LBNE.

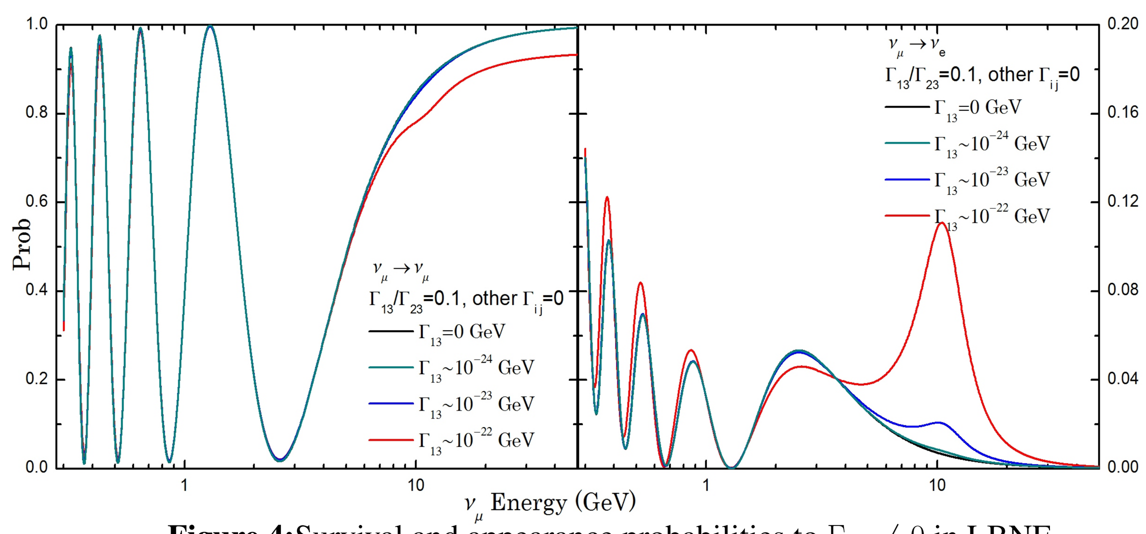

Figure 4: Survival and appearance probabilities to $\Gamma_{13} \neq 0$ in LBNE.

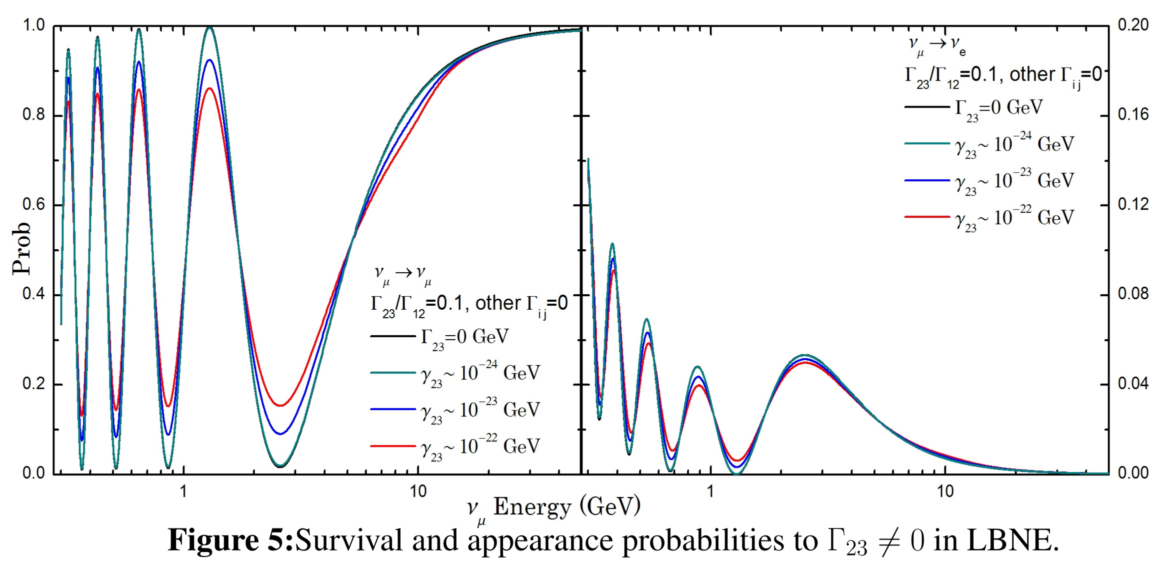

Figure 5: Survival and appearance probabilities to $\Gamma_{23} \neq 0$ in LBNE.

- Γ_{13} is the most intriguing decoherence parameter because it acts under the effective potential mass terms. So, around of 4 Gev and before, its behavior is similar of the other decorence terms, except that Γ_{13} brings the little phase that is caused if the decoherence value is high. This happens because it eliminates the oscillation where the effective potential is most effective leading the oscillation for a vacuum regime.

- The other interesting thing is a new peak that occurs around of 10 GeV that can be completely explained by means of the appearance probability to two families where it is clear that the diference between constant and the amplitude term at the ressonace region results in this peak.

Conclusion

We investigated dissipative effects in neutrinos oscillations concerning on the proposed LBNE set-up. We find that non-trivial effects can be observed at LBNE. Particularly interesting effects are expected in the "resonance region", which is just outside the energy range of LBNE neutrinos.

References

[1] G. Lindblad, Commun. Phys. **48**, 119 (1976).

[2] C. Giunti and C. W. Kim, *Fundamentals of neutrino physics and astrophysics* (Oxford University Press, New York, 2007).

[3] R. L. N. Oliveira and M. M. Guzzo, in preparation (2014).

[4] R. Alicki and K. Lendi, *Quantum dynamical semigrou-ps and applications, Lect. Notes Phys.* (Springer-Verlag, Berlim, 1987).

[5] F. Benatti and H. Narnhofer, Lett. Math. Phys. **15**, 325 (1988).

[6] M. Freund, Phys.Rev. D **64**, 053003 (2001).

[7] R. L. N. Oliveira and M. M. Guzzo, Eur. Phys. Jour. C **73**, 2434 (2013).

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