Unitarity and causality constraints in composite Higgs models

- Outline

- $W_L W_L$ scattering in extensions of the Standard Model:
  - Anomalous couplings $\leftrightarrow$ Unitarity Violations
  - Unitarization of $W_L W_L$ scattering: Inverse Amplitude Method (IAM)

- What can the unitarized $W W$ scattering tell us about H/W anomalous couplings?
  - Dynamical Resonances and indirect constraint of anomalous couplings

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Introduction: \[ W_L^+ W_L^- \rightarrow ZZ \] in extended EW sector

**SM**

\[ W^+ W^- Z Z \]

**BSM**

\[ W^+ W^- Z Z \]

\[ \propto a^2 - 1 \]

\[ \propto a_4, a_5 \]

O(p^4) operators

From current LHC bounds, still room for anomalous Higgs/W couplings and/or corrections to W/Z self couplings.

\[ a = [0.67, 1.33] \]
\[ a_4 = [-0.09, 0.10], a_5 = [-0.23, 0.26] \]

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{2} \text{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} \text{Tr} B_{\mu\nu} B^{\mu\nu} + a_4 \left( g^2 \text{Tr} W_{\mu} W_{\nu} \right)^2 + a_5 \left( g^2 \text{Tr} W_{\mu} W^\nu \right)^2 + \ldots \]

\[ + \frac{v^2 g^2}{4} \left( 1 + 2a \left( \frac{h}{v} \right) + b \left( \frac{h}{v} \right)^2 \right) \text{Tr} W_{\mu} W^{\mu} - V(h) \]

**SM**

\[ a = b = 1 \]
\[ a_4 = a_5 = 0 \]

Unitary Gauge

Custodial Sym.
BUT just small deviations of these couplings from SM values imply a big problem: $W_L W_L$ scattering breaks unitarity.

Unitarity violation $\leftrightarrow$ possible resonances

Then, unitarization techniques to $W_L W_L$ scattering useful to diagnose possible resonances and to probe them at LHC $\rightarrow$ indirect constraint of anomalous couplings!
IAM approximately **resum** amplitudes to require unitarity

- First, we project on defined *isospin* and spin amplitudes

\[
t_{IJ}(s) = \frac{1}{64\pi} \int_{-1}^{1} d\cos\theta P_{J}(\cos\theta) A_{I}(W_{L}W_{L} \rightarrow ZZ)
\]

- We study amplitudes up to loop order \(O(p^{4})\)

\[
t_{IJ}(s) \approx t_{IJ}^{(2)}(s) + t_{IJ}^{(4)}(s) + \text{tree terms} + a_{4,5} \text{ terms}
\]

+ more diagrams

+ more diagrams
Inverse Amplitude Method (IAM)

- IAM approximately resum amplitudes to require unitarity
  - First, we project on defined Isospin and spin amplitudes
    \[ t_{IJ}(s) = \frac{1}{64\pi} \int_{-1}^{1} d\cos\theta P_j(\cos\theta) A_I(W_LW_L \rightarrow ZZ) \]
  - We study amplitudes up to loop order, \( O(p^4) \)
    \[ t_{IJ}(s) \approx t_{IJ}^{(2)}(s) + t_{IJ}^{(4)}(s) \]
    unitarity violation
    \[ \propto s \quad \propto s^2 \log s \]
  - We resum \( t_{IJ}(s) \) to define a unitary amplitude
    \[ t_{IJ}(s) \approx \frac{t_{IJ}^{(2)}}{1 - t_{IJ}^{(4)} / t_{IJ}^{(2)}} \quad |t_{IJ}(s)| < 1 \]
IAM approximately **resum** amplitudes to **require unitarity**

- First, we project on defined *Isospin* and *spin* amplitudes

\[
t_{IJ}(s) = \frac{1}{64\pi} \int_{-1}^{1} d \cos \theta P_J (\cos \theta) A_I (W_L W_L \rightarrow ZZ)
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- We resum \(t_{IJ}(s)\) to define a unitary amplitude

IAM method

\[
t_{IJ}(s) \approx \frac{t_{IJ}^{(2)}}{1 - t_{IJ}^{(4)} / t_{IJ}^{(2)}} \quad |t_{IJ}(s)| < 1
\]

- **Criticism**: Many unitarization methods exist: IAM, K-matrix approach, N/D expansion, Roy equations

- **While results can slightly differ, the overall picture is consistent among methods**

- **Successful applications in pion and nuclear physics.**
Looking for dynamical resonances

Unitarity IAM amplitude

\[ t_{IJ}(s) \approx \frac{t_{IJ}^{(2)}}{1 - t_{IJ}^{(4)}/t_{IJ}^{(2)}} \]

function of anomalous couplings

\[ \text{Unitarization:} \]

Now, we look for poles in \( t_{IJ}(s) \) in the second Riemann sheet,

\[ \frac{1}{t_{IJ}(s_R)} = 0 \quad \text{at} \quad \sqrt{s_R} = M - i \Gamma/2 \quad \text{and} \quad \Gamma > 0 \]

We study the following three channels

- \( t_{00} \rightarrow \) Scalar isoscalar
- \( t_{11} \rightarrow \) Vector isovector
- \( t_{20} \rightarrow \) Scalar isotensor

MIND: Resonances only those poles with \( \Gamma > 0 \).

Mind: resonances coupled to \( W \) and \( Z \)!!
Results

Appearance of resonances for $a$, $b$ and $a_{4,5}$ free

Our initial setup:

Singlet + Triplet of SU(2)$_V$

$H \quad W$

- Unitarization $\rightarrow$ Dynamical resonances by H/W self interactions?
Resonances: case $a \leq 1$

- Resonance appearance crucially depends on the anomalous couplings $a$.
  - 2 cases: $a \leq 1$ and $a > 1$.
- $a = 1$ or $a < 1$ common features!

### Graphical Representation

- **$a = 1$**
  - IAM: $a = 1.0, b = 1.0$
  - Allowed
  - Isoscalar
  - Isovector
  - Excluded
- **$a = 0.9$**
  - IAM: $a = 0.9, b = 1.0$
  - Excluded

- **Forbidden region** has isotensor “resonances” in which phase $\delta_{20}$ goes through $-\pi/2$ (violates causality)
Resonances: case $a \leq 1$

If excluding

$M_R < 600$ GeV
Resonances: Masses and Widths for $a \leq 1$

For $a < 1$, resonances tend to be slightly heavier and broader than for $a = 1$. The coupling $b \neq 1$ is only marginally visible in the widths (loops).
Resonances: Signal strengths for $a \leq 1$

\[ R = \frac{\sigma_R(pp \rightarrow VV \rightarrow R \rightarrow VV)}{\sigma^{SM}(pp \rightarrow VV \rightarrow H \rightarrow VV)} \]

For $M_H = M_R$

- $\sigma_R/\sigma^{SM} < 0.5$: The signal is always much lower than the one for a Higgs of the same mass.
- For $a < 1$, the signal $\sigma_R/\sigma^{SM}$ is slightly larger than the one for $a=1$.

Typically, $\sigma_R/\sigma^{SM} \sim 0.01-0.1$
Resonances : Signal strengths for $a \leq 1$

Typically, $\frac{\sigma_R}{\sigma_{SM}} \sim 0.01 - 0.1$ ($\sigma_{SM}$ only from VBF Higgs production)

Currently at LHC probing, at best $R \sim 0.1 - 1$ including gluon $H$ production
Resonances : case $a > 1$

The situation for $a < 1$ is not radically different from $a = 1$

Resonances (particularly in the vector channel) are slightly more difficult to appear per $a<1$

They tend to be slightly heavier and broader

They give a larger experimental signal (at least in the EWA)

This situation is quite different for $a > 1$
Resonances : case $a > 1$

**new features for $a > 1$:**

1) For a given $a_4$-$a_5$ values, often 2 poles $\rightarrow$ one with $\Gamma < 0$ (*causality violation*)

$\Gamma \sim -500 \text{ GeV}$

We exclude those values of $a_4$ & $a_5$ where causality is violated.
Resonances : case $a > 1$

some new features:

2) NOW, causality Violations shows up on all isospin amplitudes, $t_{00}, t_{10}, t_{20}$

True resonances if the phase shift goes through $+\pi/2$

We exclude those values of $a_4 \& a_5$ where causality is violated
Resonance: $a_4$-$a_5$ appearance plot for $a>1$

Our initial setup:
Singlet + Triplet of SU(2)$_V$

$H$ $W$

No large room for dynamical resonances by H/W self interactions
Conclusions

- Unitarity is a powerful tool to study $WW$ scattering amplitudes.

- It can help constrain anomalous couplings by helping to predict heavier resonances (>300 GeV) whose absence exclude regions of the $a_4 - a_5$ parameter space.

- Current LHC Higgs search results likely do not yet probe these IAM resonances, but may be possible in near future.
Conclusions

- Unitarity is a powerful tool to study $WW$ scattering amplitudes.

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- Current LHC Higgs search results likely do not yet probe these IAM resonances, but may be possible in near future.

THANK YOU!
Introduction: \( W_L^+ W_L^- \rightarrow ZZ \) & anomalous coupling constraints

\[
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+ \frac{\nu^2}{4} \left( 1 + 2a \left( \frac{h}{\nu} \right) + b \left( \frac{h}{\nu} \right)^2 \right) \text{Tr} D_{\mu} U^\dagger D^{\mu} U - V(h)
\]

**SM** \( a = b = 1 \) \( a_4 = a_5 = 0 \)

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\( \otimes \) BUT just small deviations of these couplings from SM values imply a big problem: *unitarity violation of the \( W_L W_L \) scattering*

\( \otimes \) Unitarity violation \( \leftrightarrow \) possible resonances \( \otimes \)

Then, unitarization techniques to \( W_L W_L \) scattering useful to diagnose possible resonances, match to LHC \( \rightarrow \) constraint indirectly the anomalous couplings!
In the SM, the Higgs boson unitarizes $W_L W_L$ scattering

$$\mathcal{L}_{SM} = -\frac{1}{2} \text{Tr} W_{\mu \nu} W^{\mu \nu} - \frac{1}{4} \text{Tr} B_{\mu \nu} B^{\mu \nu} + \frac{v^2}{4} \left( 1 + 2 \left( \frac{h}{v} \right) + \left( \frac{h}{v} \right)^2 \right) \text{Tr} D_\mu U^\dagger D^\mu U - V(h)$$

$$D_\mu U = \partial_\mu U + \frac{i}{2} g \tau^i W_\mu^i U - \frac{i}{2} g' B_\mu U \tau^3$$
Figure paper