NATURAL QUARK MASS STRUCTURE IN A U(1)' GAUGE EXTENSION

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THE PROBLEM

The Standard Model (SM) is based in the gauge symmetry : $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

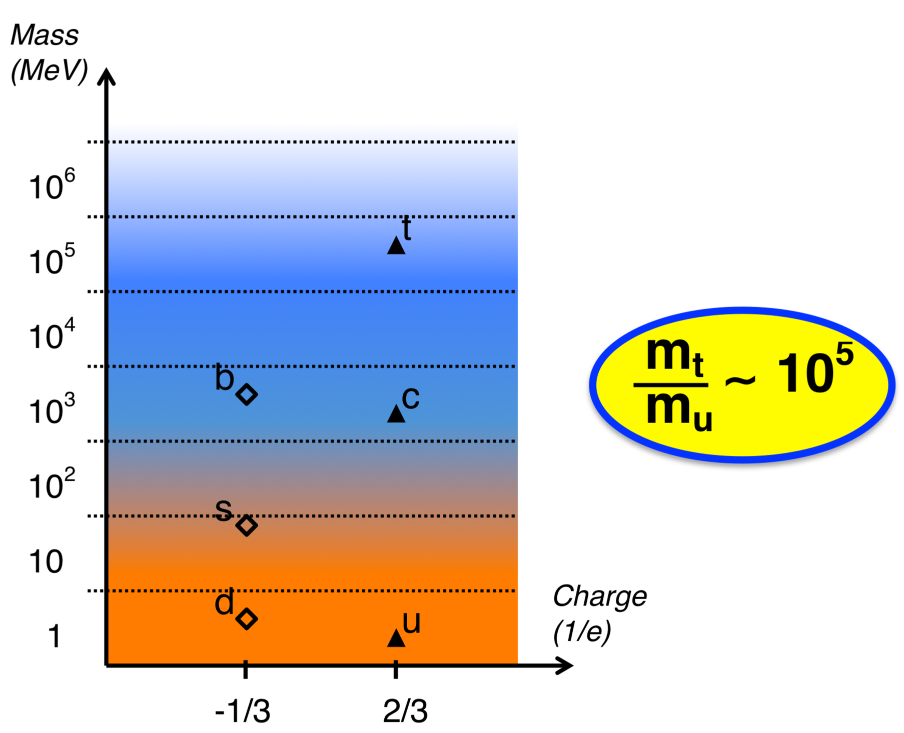
In the SM, the fermion masses are provided by Yukawa interactions consistent with the above symmetry and with the inclusion of only **one scalar doublet ϕ with Vacuum Expectation Value (VEV) $v = 246$ GeV.**

Calling q_L^i the left-handed quark doublet, U_R^i and D_R^i the right-handed up- and down-type singlets, and $i = 1, 2, 3$ the three phenomenological families, the Yukawa Lagrangian reads

$$-\mathcal{L}_Q = \overline{q_L^i}(\phi h^D)_{ij} D_R^j + \overline{q_L^i}(\widetilde{\phi} h^U)_{ij} U_R^j + h.c.,$$

After the symmetry breaking, the above Lagrangian leads to the mass matrices:

$$M_{U,D} = \frac{v}{\sqrt{2}} h^{U,D} = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$$



After diagonalization of the above mixing matrices, the mass eigenvalues are proportional to:

$$m_Q \sim v$$

However, the experiments show that the mass of the heaviest quark (top) is about 5 order of magnitude larger than the lightest quark (up). This require "unnatural" fine tuning of the Yukawa parameters. Why?

THE MODEL

Let us assume that the gauge symmetry contains an additional abelian symmetry, such that : $G' = G_{SM} \otimes U(1)_X$

The values of the extra charge X, shown in part A of Tab. 1, are **non-universal** in the left-handed quark sector: for $i = 1$ has $X_1 = 1/3$ while $X_{2,3} = 0$ for $i = 2, 3$.

The Yukawa Lagrangian consistent with the above symmetry, with $a = 2, 3$ reads:

$$-\mathcal{L}_Q = \overline{q_L^1}(\phi_1 h^D)_{1j} D_R^j + \overline{q_L^a}(\widetilde{\phi}_1 h^U)_{aj} U_R^j + h.c.,$$

The mass matrices, which exhibits three null eigenvalues, are now of the form:

$$M_U = \frac{v_1}{\sqrt{2}} h^U = \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ * & * & * \end{pmatrix}, \quad M_D = \frac{v_1}{\sqrt{2}} h^D = \begin{pmatrix} * & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

But:

- ✘ Experimentally we do not see any massless quark.
- ✘ The local $U(1)_X$ symmetry predicts a massless gauge boson Z'. However, no such second weak neutral current has been detected.
- ✘ The Z' boson induces extra triangle chiral anomalies which are not cancelled by the X-charges from Tab. 1.

Thus, we add the extra particles shown in part B of Tab. 1, where:

- ✔ The singlets T and J^n are up- and down-like quarks, respectively, with $n = 1, 2$. They are chiral under $U(1)_X$ and vector-like under G_{sm}.
- ✔ An scalar singlet χ_0 with VEV v_χ is required to produce the symmetry breaking of the $U(1)_X$ symmetry at a large scale.
- ✔ An scalar singlet σ_0 is introduced. Since it is not essential for the symmetry breaking mechanisms, we may choose a small VEV $\langle \sigma_0 \rangle = v_\sigma \sim v$.

A. Ordinary quark and scalar content

Spectrum	G_{SM}	$U(1)_X$
$q_L^i = \begin{pmatrix} U^i \\ D^i \end{pmatrix}_L$	(3, 2, 1/3)	1/3 for $i = 1$ 0 for i = 2,3
U_R^i	(3^*, 1, 4/3)	2/3
D_R^i	(3^*, 1, -2/3)	-1/3
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \xi_1 + i\phi_1^0) \end{pmatrix}$	(1, 2, 1)	2/3

B. New extra particle content

Spectrum	G_{SM}	$U(1)_X$
T_L	(3, 1, 4/3)	1/3
T_R	(3^*, 1, 4/3)	2/3
J_L^n	(3, 1, -2/3)	0
J_R^n	(3^*, 1, -2/3)	-1/3
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \xi_2 + i\phi_2^0) \end{pmatrix}$	(1, 2, 1)	1/3
$\chi_0 = \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi + i\zeta_\chi)$	(1, 1, 0)	-1/3
$\sigma_0 = \frac{1}{\sqrt{2}}(v_\sigma + \xi_\sigma + i\zeta_\sigma)$	(1, 1, 0)	-1/3

Table 1

and the extra discrete symmetries:

- ✔ $\mathbf{Z}_2$: $\phi_2 \to -\phi_2, \quad \sigma_0 \to -\sigma_0, \quad D_R^i \to -D_R^i, \quad T_{L,R} \to -T_{L,R}$.
- ✔ $\mathbf{U}(1)_{T_3}$: $D_L^2 \to -D_L^2, \quad D_L^3 \to D_L^3$.

By requiring the above symmetries and the mixing with the extra Particles, we find extended mass matrices with the form:

$$M'_U = \begin{pmatrix} 0 & 0 & 0 & | & * \\ * & * & * & | & 0 \\ * & * & * & | & 0 \\ - & - & - & - & - \\ \times & \times & \times & | & \bullet \end{pmatrix} \quad (u \; c \; t \; T)$$

$$M'_D = \begin{pmatrix} 0 & 0 & 0 & | & * & * \\ 0 & 0 & 0 & | & * & * \\ * & * & * & | & 0 & 0 \\ - & - & - & - & - & - \\ \times & \times & \times & | & \bullet & \bullet \\ \times & \times & \times & | & \bullet & \bullet \end{pmatrix} \quad (d \; s \; b \; J^1 \; J^2)$$

Where:

- $* \sim v_{1,2}$
- $\times \sim v_\sigma$
- $\bullet \sim v_\chi$

NUMERICAL RESULTS

Let us choose:

$$M'_U = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & | & v_2 y_1 \\ v_1 Y_U & v_1 Y_U & v_1 Y_U & | & 0 \\ v_1 Y_U & v_1 Y_U & v_1 Y_t & | & 0 \\ - & - & - & - & - \\ v_\sigma c_1 & 0 & 0 & | & v_\chi h_\chi^T \end{pmatrix}, M'_D = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & | & v_1 h_D & 0 \\ 0 & 0 & 0 & | & 0 & v_1 h_D \\ v_2 Y_D & v_2 Y_D & v_2 Y_D & | & 0 & 0 \\ - & - & - & - & - & - \\ v_\sigma \Gamma_D & 0 & 0 & | & v_\chi k_{11} & 0 \\ 0 & v_\sigma \Gamma_D & 0 & | & 0 & v_\chi k_{22} \end{pmatrix}$$

After diagonalization, we find the following ratios:

$$y = \frac{Y_U}{Y_U + Y_t}, \epsilon = \frac{Y_U - Y_t}{Y_U + Y_t}$$

ϵ "measures" the level of asymmetry between the t-quark and the lighter (c and u) quarks. If $\epsilon = 0$ (i.e. $Y_U = Y_t$), the c-quark becomes massless

$$\frac{m_c}{m_t} \approx \frac{-y\epsilon}{1 + y\epsilon}$$

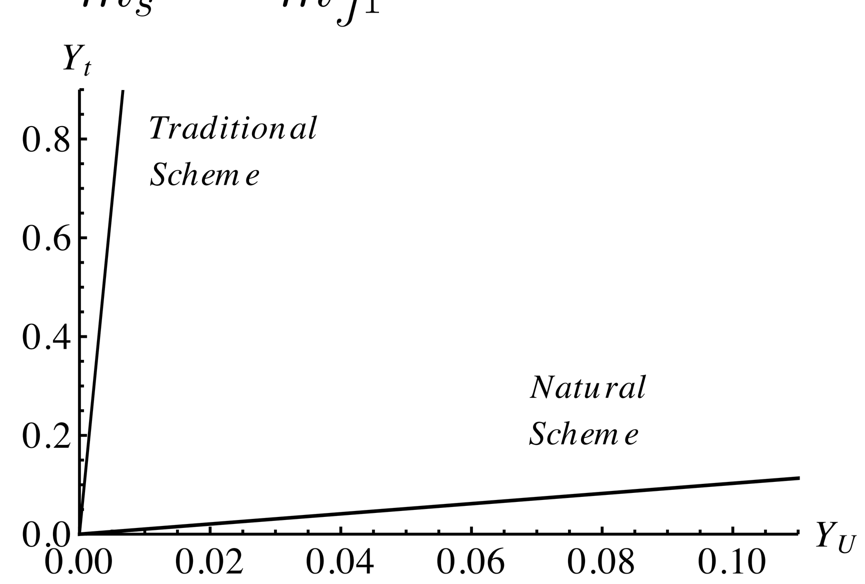
The ratio between the mass of the c- and t-quark is sensible to the asymmetry parameter. Thus, there is a scenery where the closer are the values of Y_U and Y_t, the larger is the difference between m_c and m_t. We find $y\epsilon \approx -7.3 \times 10^{-3}$

$$\frac{m_u}{m_b} = \left(\frac{y_1 c_1}{\sqrt{2} Y_D}\right) \frac{v_\sigma}{m_T}$$

The ratio between the mass of the u- and b-quark is inverse in the heavy T-quark mass $m_T \sim v_\chi$. Thus, a natural hierarchy between u and b arises.

$$\frac{m_d}{m_s} = \frac{m_{J^2}}{m_{J^1}}$$

The ratio between the d- and s-quark masses is determined only by the mass splitting of the heavy quarks J^1 and J^2. We find $m_{J^1} \approx 20 m_{J^2}$



This figure shows the top quark coupling Y_t as function of the light-quark coupling Y_U, which exhibits two possible solutions. First, the *traditional scheme*, where $Y_t/Y_U \approx 135.05$ and $\epsilon \approx -0.985$ is required to fit the experimental masses. Second, we obtain a "*natural scheme*" where $Y_t/Y_U \approx 1.03$, which is consistent with a symmetry where degenerated up-type Yukawa couplings is favored, with $\epsilon \approx -0.015$.

CONCLUSION: Predictable mass ratios were obtained with few free parameters, and hierarchical structures arose "naturally" without large tuning of the Yukawa couplings.

References

[1] For details and further references, see R. Martinez, J Nisperuza, F. Ochoa, J.P. Rubio, Phys. Rev. D 89, 056008 (2014) [arXiv:1303.2734 [hep-ph]]