Determination of α_s from the low-z parton-to-hadron fragmentation functions

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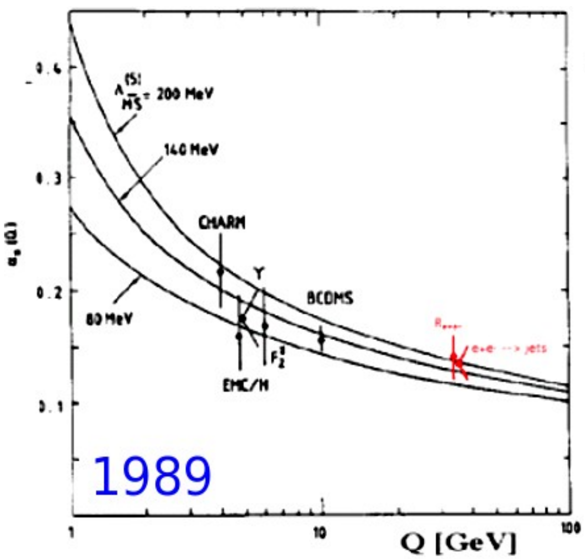
(*) Based on work with R.Perez-Ramos: arXiv:1310.8534 (JHEP to appear)

Determination of the QCD coupling α_s

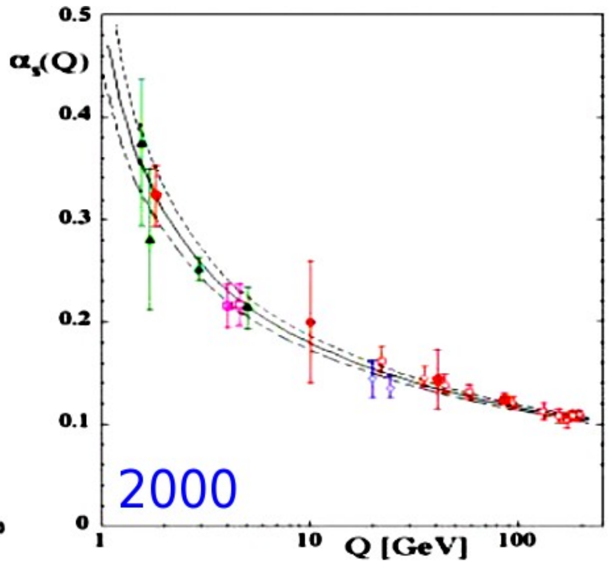
α_s = Single free parameter in QCD (in the $m_q \to 0$ limit). Determined at a given ref. scale (e.g. m_Z). Decreases as $\sim\ln(Q^2/\Lambda^2)$, with $\Lambda\sim0.25$ GeV

- Least precisely known of all couplings.
- Impacts all LHC x-sections.
- Key for SM precision fits (e.g. uncertainties H-b,c Yukawa).
- Key for BSM physics (e.g. couplings at GUT scale).



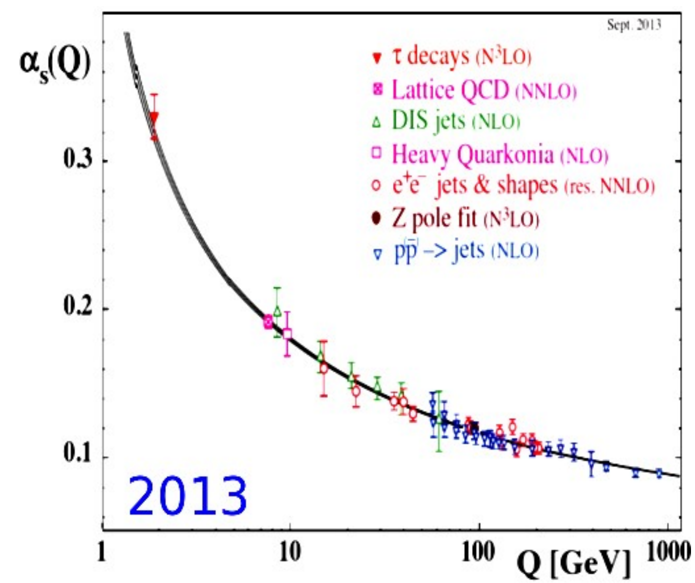
$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$\alpha_s(M_Z) = 0.1184 \pm 0.0031$ (NNLO)

S. B. , J. Phys. G 26, 2000

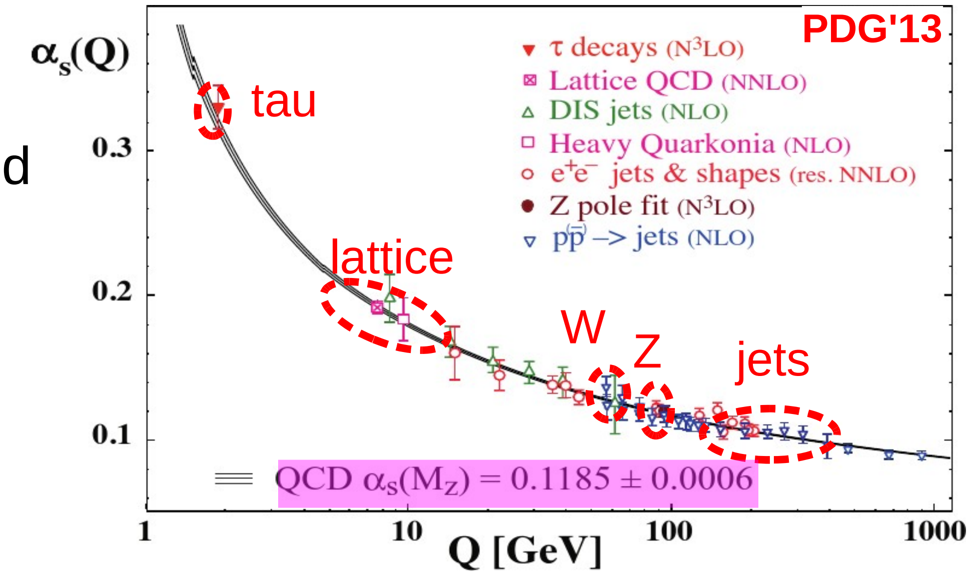


$\alpha_s(M_Z) = 0.1185 \pm 0.0006$ (NNLO)

Current uncertainty: ±0.6%
("disputed" by some: ±1%)

Determination of the QCD coupling α_s

α_s = Single free parameter in QCD (in the $m_q \to 0$ limit). Determined at a given ref. scale (e.g. m_Z). Decreases as $\sim\ln(Q^2/\Lambda^2)$, with Λ~0.25 GeV.



1. Hadronic τ decays: $R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm np})$ **(N³LO)**

2. Lattice QCD: Various short-distance quantities: $K^{\rm NP} = K^{\rm PT} = \sum_{i=0}^{n} c_i \alpha_s^i$ **(NNLO)**

3. Hadronic Z,W decays: $R_Z \equiv \frac{\Gamma(Z \to {\rm h})}{\Gamma(Z \to l)} = R_Z^{\rm EW} N_C (1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm m} + \delta_{\rm np})$ **(N³LO)**

4. DIS had. observ.: PDFs, sigma(jet): $\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z}, Q^2\right) D_j^h(z,Q^2)$ **(NNLO,NLO)**

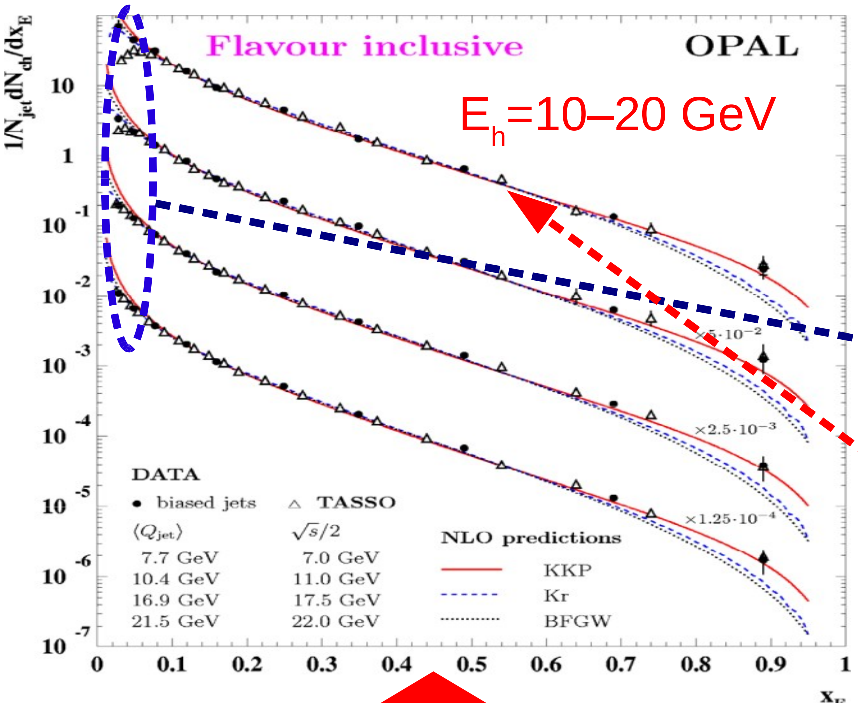
5. e⁺e⁻ had. observ.: Event-shapes, jet rates: $\frac{1}{\sigma}\frac{d\sigma}{dY} = \frac{dA}{dY}\hat{\alpha}_S + \frac{dB}{dY}\hat{\alpha}_S^2 + \frac{dC}{dY}\hat{\alpha}_S^3$ **(NNLO)**

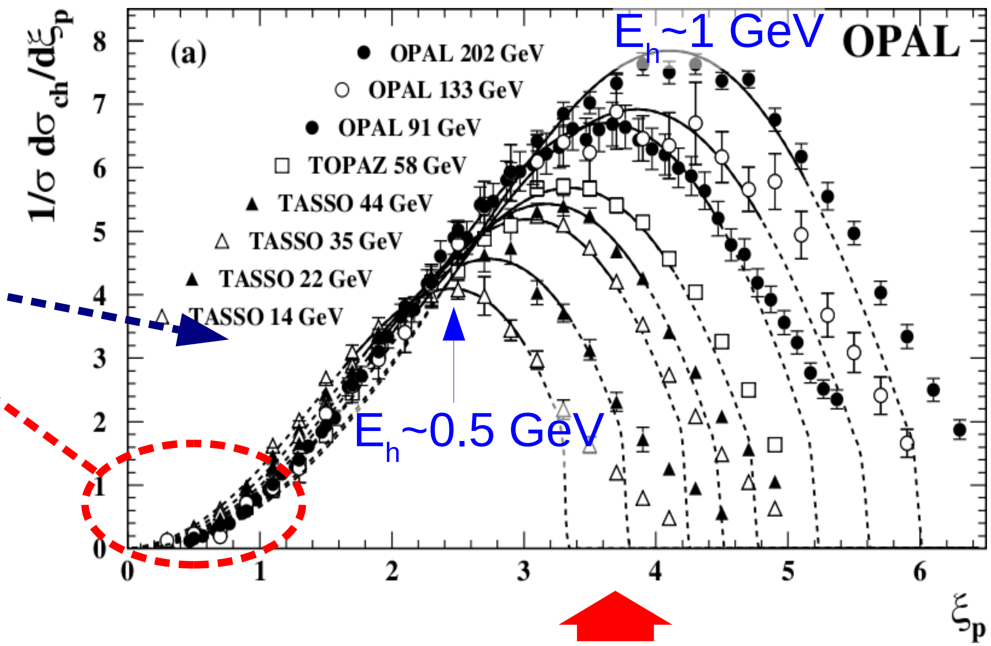
6. Other hadronic observ.: sigma(jets) in p-p, QQbar radiative decays **(NLO)**

Parton-to-hadron fragmentation functions

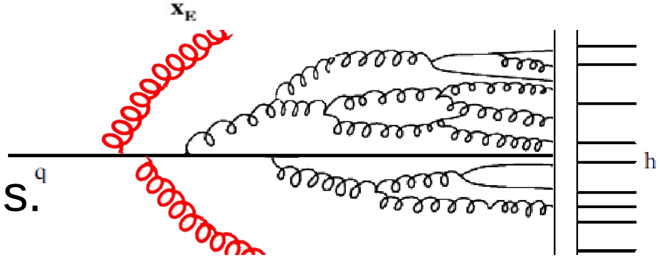
- **Hard** fragmentation function
  $z = p_{had}/p_{jet} > 0.1$
  High-p_T hadron tail in jets

- **Soft** fragmentation function
  $\xi = \log(1/z) = \log(p_{jet}/p_{had}) > 1$
  Bulk hadron production in jets





- Hard emission
- Ordered in k_T
- DGLAP evolution eqs.
  $\ln(k_T)$ evolution



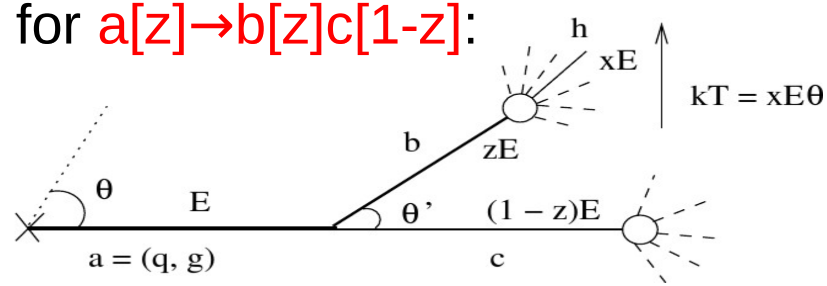
- Soft/collinear emission
- Angular ordering
- MLLA evolution eqs.
  $\ln(1/x)$ & $\ln(\theta)$ resummations

Combined QCD evolution eqs. for the FFs

- DGLAP +MLLA evolution equations for a[z]→b[z]c[1-z]:

z: energy fraction of intermediate parton

x,ω: energy fraction of final hadron

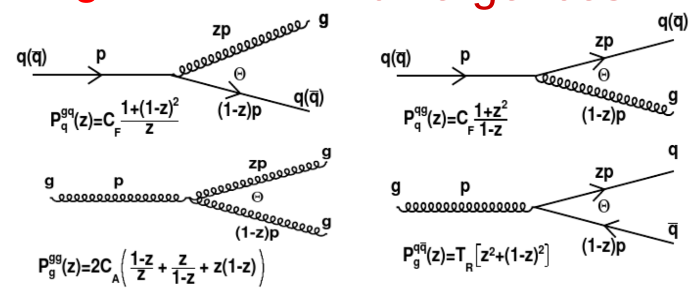


$$\frac{\partial}{\partial \ln\theta} x D_a^b(x, \ln E\theta) = \sum_c \int_0^1 dz \frac{\alpha_s(k_\perp^2)}{2\pi} P_{ac}(z) \left[\frac{x}{z} D_c^b\left(\frac{x}{z}, \ln z E\theta\right)\right]$$

QCD coupling — DGLAP splitting functions — soft&collinear divergences

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln q^2}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln\ln q^2}{\ln q^2}\right], \text{ for } \quad q^2 = \frac{k_\perp^2}{\Lambda_{QCD}^2}$$

$$\beta_0 = \frac{11}{3}N_c - \frac{4n_f T_R}{3}, \qquad \beta_1 = \frac{51}{3}N_c - \frac{38 n_f T_R}{3}$$



- Solution via Mellin moments transform:

$$\mathcal{D}(\omega, Y) = \int_0^\infty d\xi e^{-\omega\xi} D(\xi, Y), \; \hat{\xi} = \ln\frac{1}{z}, \; \hat{y} = \ln\frac{k_\perp}{Q_0}, \; \hat{\xi} + \hat{y} = \ln\frac{E\theta}{Q_0} \equiv Y$$

$$\Rightarrow \frac{\partial}{\partial Y}\mathcal{D}(\omega, Y) = \int_0^\infty d\hat{\xi} e^{-\omega\hat{\xi}} P(\hat{\xi}) \frac{\alpha_s(Y - \hat{\xi})}{2\pi} \mathcal{D}(\omega, Y - \hat{\xi}),$$

$$\text{with NLO } \alpha_s(\hat{y}) = \frac{2\pi}{\beta_0(\hat{y} + \lambda)}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln 2(\hat{y} + \lambda)}{\hat{y} + \lambda}\right], \quad \hat{y} = Y - \hat{\xi}.$$

Solution of the NMLLA+NLO* evolution eqs.

- Expressing the Mellin-transformed hadron distribution in terms of the anomalous dimension: $D \simeq C(\alpha_s(t)) \exp\left[\int^t \gamma(\alpha_s(t'))dt\right], \; t = \ln Q$
  one needs to solve the evolution equation:

$$(\omega+\gamma_\omega)\gamma_\omega - \frac{2N_c\alpha_s}{\pi} = -\beta(\alpha_s)\frac{d\gamma_\omega}{d\alpha_s} - a_1(\omega+\gamma_\omega)\frac{\alpha_s}{2\pi} - \frac{a_1}{2\pi}\beta(\alpha_s) + a_2(\omega^2+2\omega\gamma_\omega+\gamma_\omega^2)\frac{\alpha_s}{2\pi},$$

with NLO: $\beta(\alpha_s) = -\beta_0\frac{\alpha_s^2}{2\pi} - \beta_1\frac{\alpha_s^3}{4\pi^2} + \mathcal{O}(\alpha_s^4),$

Expansion in (half) orders of α_s: $\gamma \sim O_{DLA}(\sqrt{\alpha_s}) + O_{MLLA}(\alpha_s) + O_{NMLLA}(\alpha_s^{3/2}) + ...$

- Final NMLLA+NLO* solution for the anomalous dimension evolution:

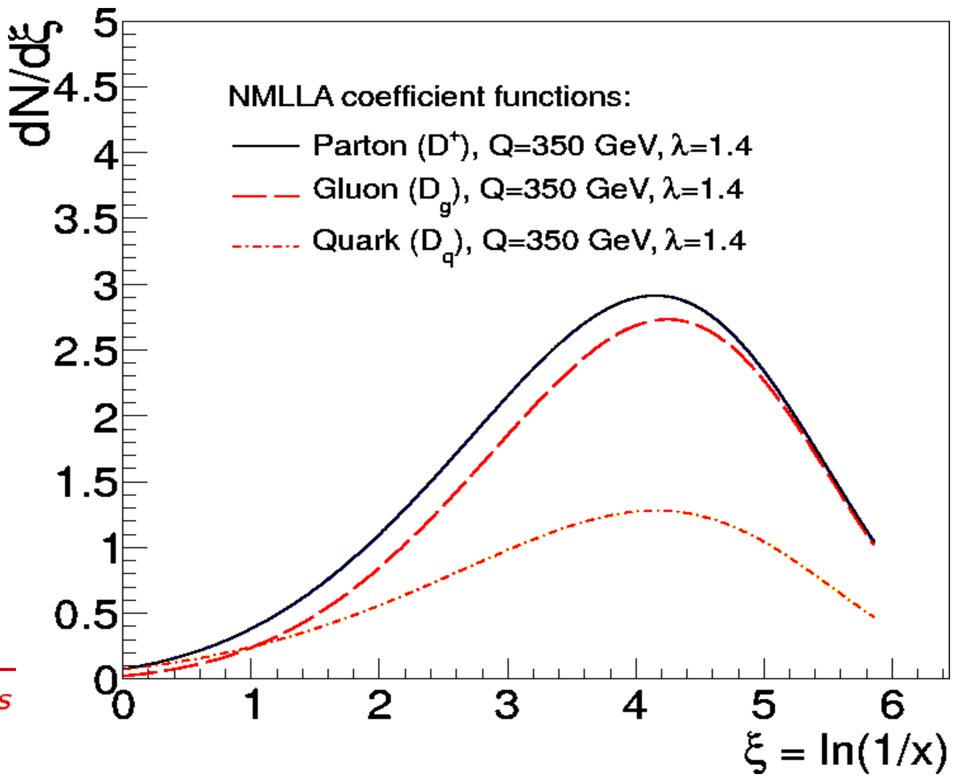
$$\gamma_\omega^{\rm NMLLA+NLO*} = \gamma_\omega^{\rm MLLA} + \frac{\gamma_0^4}{16N_c^2}\left\{a_1^2\frac{\gamma_0^2}{(\omega^2+4\gamma_0^2)^{3/2}} + \frac{a_1\beta_0}{2}\left(\frac{1}{\sqrt{\omega^2+4\gamma_0^2}} - \frac{\omega^3}{(\omega^2+4\gamma_0^2)^2}\right)\right.$$
$$+\beta_0^2\left(\frac{2\gamma_0^2}{(\omega^2+4\gamma_0^2)^{3/2}} - \frac{5\gamma_0^4}{(\omega^2+4\gamma_0^2)^{5/2}}\right) - 4N_c\frac{\beta_1}{\beta_0}\frac{\ln 2(Y+\lambda)}{\sqrt{\omega^2+4\gamma_0^2}}$$
$$\left. + \frac{1}{4}a_2\gamma_0^2\left[\frac{\omega}{(\omega^2+4\gamma_0^2)^{1/4}} + (\omega^2+4\gamma_0^2)^{1/4}\right]^2 + \mathcal{O}(\gamma_0^4).\right.$$

new higher-order terms (1st time computed)

NMLLA parton/gluon/quark single inclusive FFs

- Quark FF: $\mathcal{D}_q(\omega, Y, \lambda) \approx C_q^g(\Omega)\mathcal{D}^+(\omega, Y, \lambda), \quad C_q^g(\Omega) = \frac{P_{qg}(\Omega)}{P_{++}(\Omega) - P_{--}(\Omega)}$
- Gluon FF: $\mathcal{D}_g(\omega, Y, \lambda) \approx C_g^g(\Omega)\mathcal{D}^+(\omega, Y, \lambda), \quad C_g^g(\Omega) = \frac{P_{++}(\Omega) - P_{qq}(\Omega)}{P_{++}(\Omega) - P_{--}(\Omega)}$

- Jet of virtuality Q = 350 GeV evolved down to scale $\lambda = \ln(Q_0/\Lambda_{QCD}) = 1.4$ i.e. to $Q_0 \sim 0.8$ GeV



- NMLLA+NLO ratio of g/q multiplicities:

$$\frac{\mathcal{N}_g}{\mathcal{N}_q} = \frac{N_c}{C_F}\left(1 - r_1\gamma_0 - r_2\gamma_0^2\right), \gamma_0 \sim \sqrt{\alpha_s}$$

[Consistent with I.Dremin et al. Phys.Rep. 349(2001)]

Distorted Gaussian FF parametrization

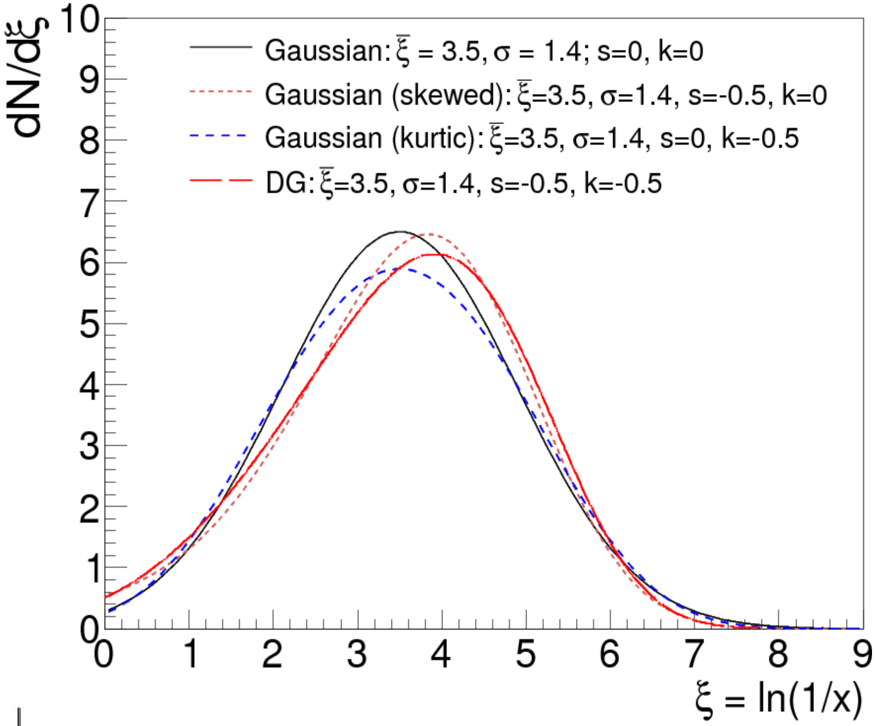
- The hadron distribution in jets can be generically expressed as a Distorted Gaussian:

$$D^+(\xi, Y, \lambda) = \frac{\mathcal{N}}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right], \ \delta = \frac{(\xi-\bar{\xi})}{\sigma}$$

- DG components:
  - ➔ Mean multiplicity: $\mathcal{N} = \mathcal{D}^+(\omega = 0, Y, \lambda)$
  - ➔ Peak position: $\bar{\xi}$ (mean)

$$\xi_{\max} - \bar{\xi} = -\frac{1}{2}\sigma s\left(1 - \frac{1}{4}\frac{k_5}{s} + \frac{5}{6}k\right)$$

  - ➔ Dispersion (width): σ
  - ➔ Skewness: s
  - ➔ Kurtosis: k



- DG moments from anomalous dim.:

$$K_{n\geq 0} = \int_0^Y dy \left(-\frac{\partial}{\partial\omega}\right)^n \gamma_\omega(\alpha_s(y+\lambda))\Big|_{\omega=0}$$

$$\mathcal{N} = K_0, \quad \bar{\xi} = K_1, \quad \sigma = \sqrt{K_2}, \quad s = \frac{K_3}{\sigma^3}, \quad k = \frac{K_4}{\sigma^4}$$

NMLLA+NLO* evolution of the (DG) FF moments

~jet energy ~energy cut-off of shower

Final expressions as a function of $Y = \ln(E\theta/Q_0)$ and $\lambda = \ln(Q_0/\Lambda_{QCD})$:

- Multiplicity: $$\mathcal{N}(Y) \propto \exp\left[2.50217\left(\sqrt{Y+\lambda}-\sqrt{\lambda}\right) - 0.491546\ln\frac{Y+\lambda}{\lambda} - (0.06889 - 0.41151\ln(Y+\lambda))\frac{1}{\sqrt{Y+\lambda}} + (0.06889 - 0.41151\ln\lambda)\frac{1}{\sqrt{\lambda}}\right]$$
- Peak position: $$\xi_{\max}(Y) = 0.5Y + 0.592722\left(\sqrt{Y+\lambda}-\sqrt{\lambda}\right) - \frac{1}{2}\sigma\, s + 0.002\ln\frac{Y+\lambda}{\lambda}$$
- Width: $$\sigma(Y) = 0.36499\sqrt{(Y+\lambda)^{3/2}-\lambda^{3/2}}\left\{1 - 0.299739 f_1(Y,\lambda)\frac{1}{\sqrt{Y+\lambda}} - [1.12479 f_2(Y,\lambda) + 0.0449219 f_1^2(Y,\lambda) + (0.32239 - 0.246692\ln(Y+\lambda))\, f_3(Y,\lambda)]\frac{1}{Y+\lambda}\right\}.$$
- Skewness: $$s(Y) = -\frac{1.94704}{\sqrt{(Y+\lambda)^{3/2}-\lambda^{3/2}}}\left[1 - 0.299739 f_1(Y,\lambda)\frac{1}{\sqrt{Y+\lambda}}\right]$$
- Kurtosis: $$k(Y) = -\frac{2.15812}{\sqrt{Y+\lambda}}\frac{1-\left(\frac{\lambda}{Y+\lambda}\right)^{5/2}}{\left[1-\left(\frac{\lambda}{Y+\lambda}\right)^{3/2}\right]^2}\left\{1 + [1.19896 f_1(Y,\lambda) - 1.99826 f_4(Y,\lambda)]\frac{1}{\sqrt{Y+\lambda}} + [1.07813 f_1^2(Y,\lambda) + 4.49915 f_2(Y,\lambda) + 1.28956 f_3(Y,\lambda) - 2.39583 f_1(Y,\lambda) f_4(Y,\lambda) - 3.76231 f_5(Y,\lambda) + 0.0217751 f_6(Y,\lambda) - (0.986767 f_3(Y,\lambda) - 0.822306 f_6(Y,\lambda))\ln(Y+\lambda)]\frac{1}{Y+\lambda}\right\}.$$

Evolution of the (DG) FF: limiting spectrum

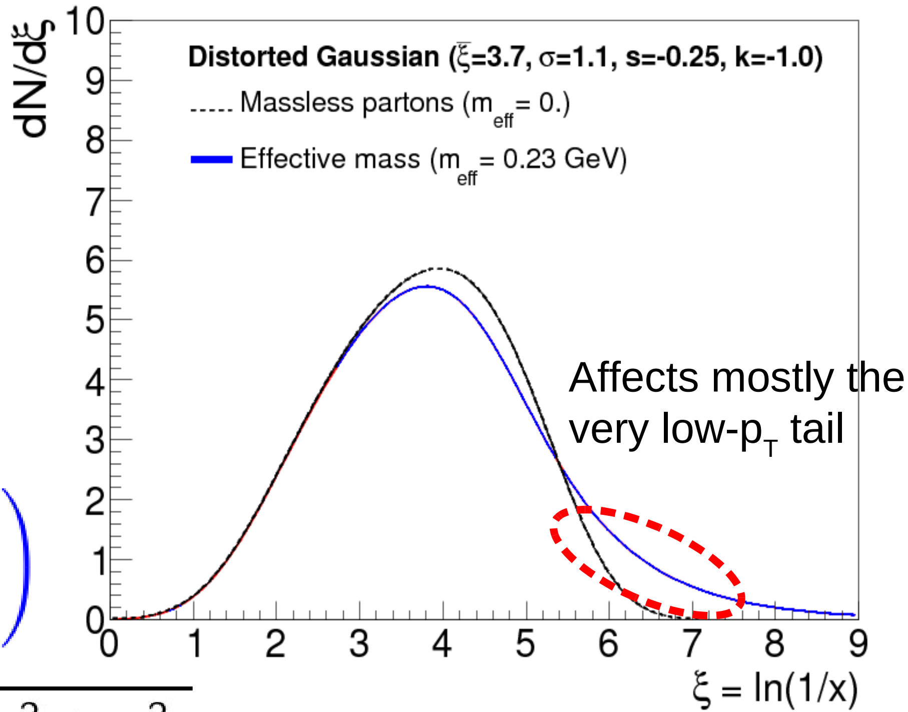
- Final expressions evolved down to Λ_{QCD}: $Y = \ln(2\sqrt{s}/\Lambda_{QCD})$, $Q_0 = \Lambda_{QCD}$

$$\begin{aligned}
\mathcal{N}(Y) &= \mathcal{K}^{\rm ch} \exp\left[2.50217\sqrt{Y} - 0.491546\ln Y - (0.06889 - 0.41151\ln Y)\frac{1}{\sqrt{Y}}\right.\\
&+ \left.(0.00068 - 0.161658\ln Y)\frac{1}{Y}\right],\\
\bar{\xi}(Y) &= 0.5Y + 0.592722\sqrt{Y} + 0.002\ln Y,\\
\xi_{\max}(Y) &= 0.5Y + 0.592722\sqrt{Y} - 0.351319 + 0.002\ln Y,\\
\sigma(Y) &= 0.36499Y^{3/4}\left[1 - 0.299739\frac{1}{\sqrt{Y}} - (1.4921 - 0.246692\ln Y)\frac{1}{Y} + \frac{1.98667}{Y^{3/2}}\right]\\
s(Y) &= -\frac{1.94704}{Y^{3/4}}\left[1 - 0.299739\frac{1}{\sqrt{Y}} - \frac{1.64393}{Y}\right],\\
k(Y) &= -\frac{2.15812}{\sqrt{Y}}\left[1 - 0.799305\frac{1}{\sqrt{Y}} + (0.730466 - 0.164461\ln Y)\frac{1}{Y} - \frac{8.05771}{Y^{3/2}}\right]
\end{aligned}$$

- Evolution of all moments depend on 1 single free parameter: Λ_{QCD}, which can then be extracted from fits of exp. e^+e^-,e-p $\rightarrow$ jets(hadrons) data

Data vs. theory: Mass & $N_{flavours}$ corrections

- FF distribution measurement for massive hadrons (ξ_p), but theory derived for massless partons/hadrons ($\xi = \xi_E$).



$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{\rm h}}{d\xi_p} \propto \frac{p_h}{E_h} D^+(\xi, Y)$$

$$\xi = \ln(1/x) = \ln\left(\frac{\sqrt{s}/2}{\sqrt{(s/4)e^{-2\xi_p} + m_{\rm eff}^2}}\right)$$

$$p_h = (\sqrt{s}/2)\cdot \exp -\xi_p \qquad E_h = \sqrt{p_h^2 + m_{\rm eff}^2}$$

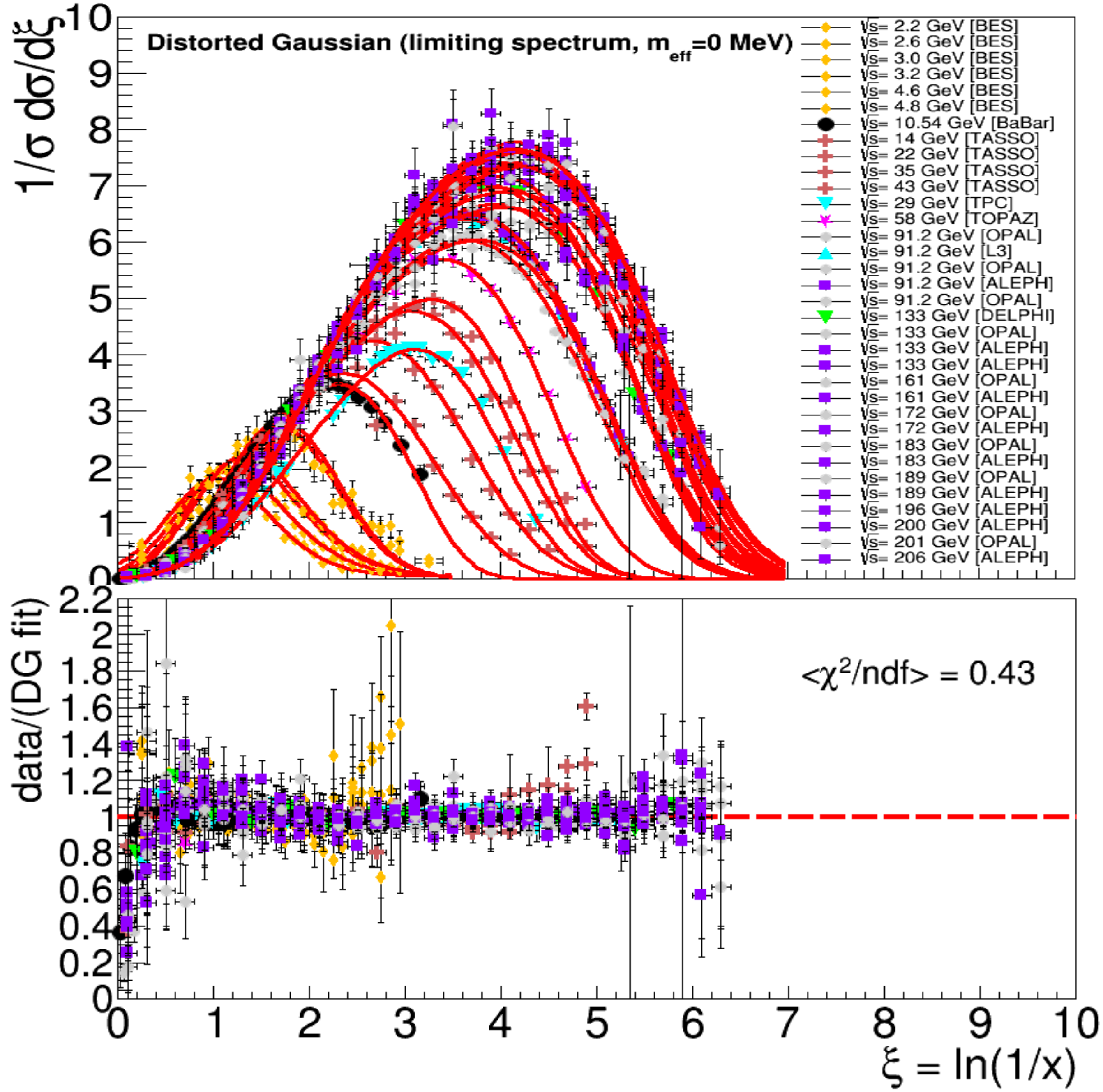
Mass effects gauged by varying data fits with m_{eff} = 0 – 0.36 GeV

- Theoretical expressions depend on number of active flavours (N_f).

  N_f= 5 assumed. Small corrections in evolution applied at thresholds:
  N_f=3 Eq. for data at $\sqrt{s} < m_c \sim 1.3$ GeV
  N_f=4 Eq. for data at $m_c < \sqrt{s} < m_b \sim 4.2$ GeV

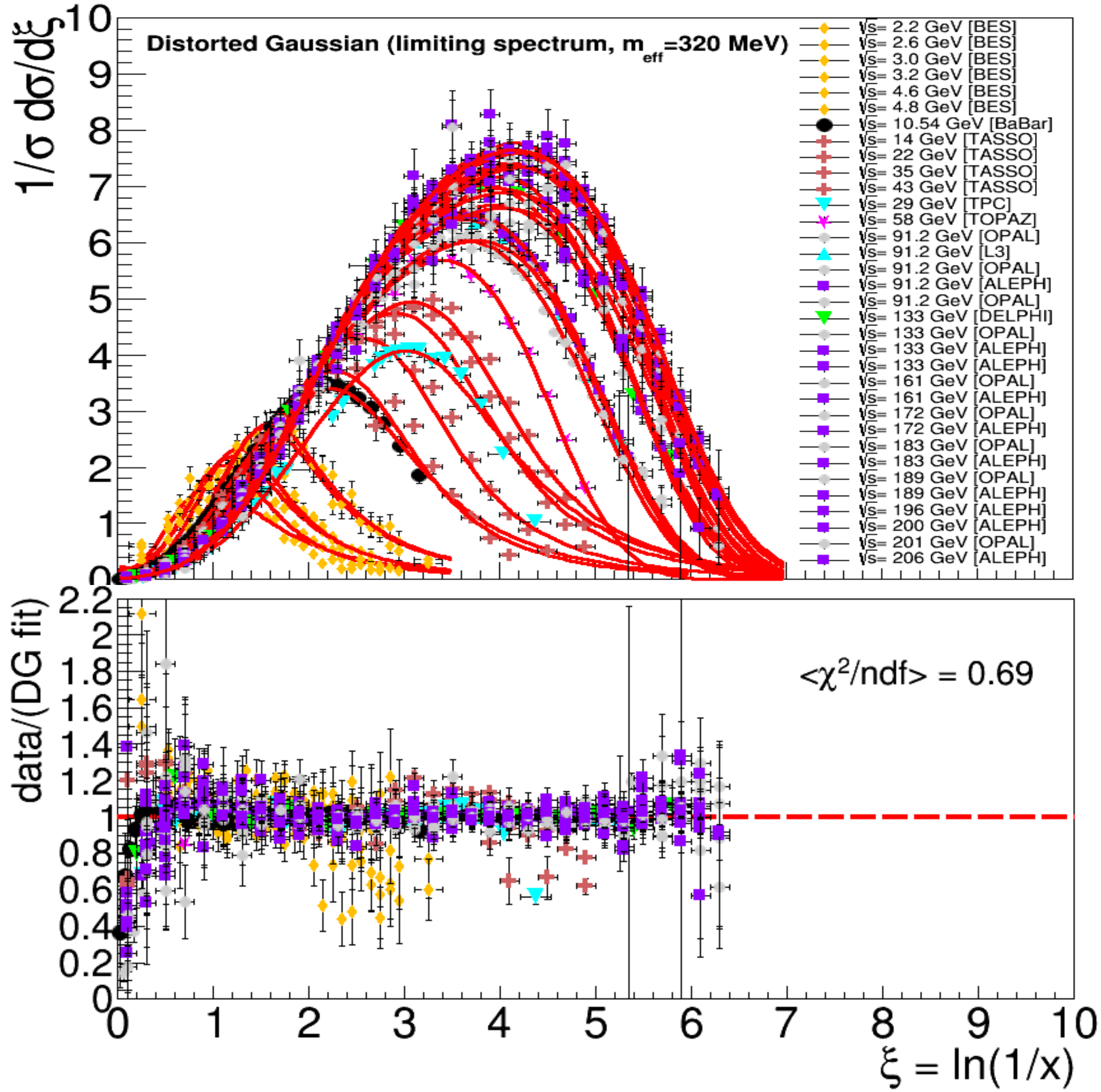
Distorted Gaussian fits to e^+e^- FFs (m_{eff} = 0 GeV)

- 34 e^+e^- data-sets at $\sqrt{s} = 2.2 - 206$ GeV
  ~1200 data points
- Peak shifts to right, width increases, moderate non-Gaussian tails
- Excellent fit to DG at all energies, with 5 free parameters: N_{ch}, ξ_{max}, σ, s, k



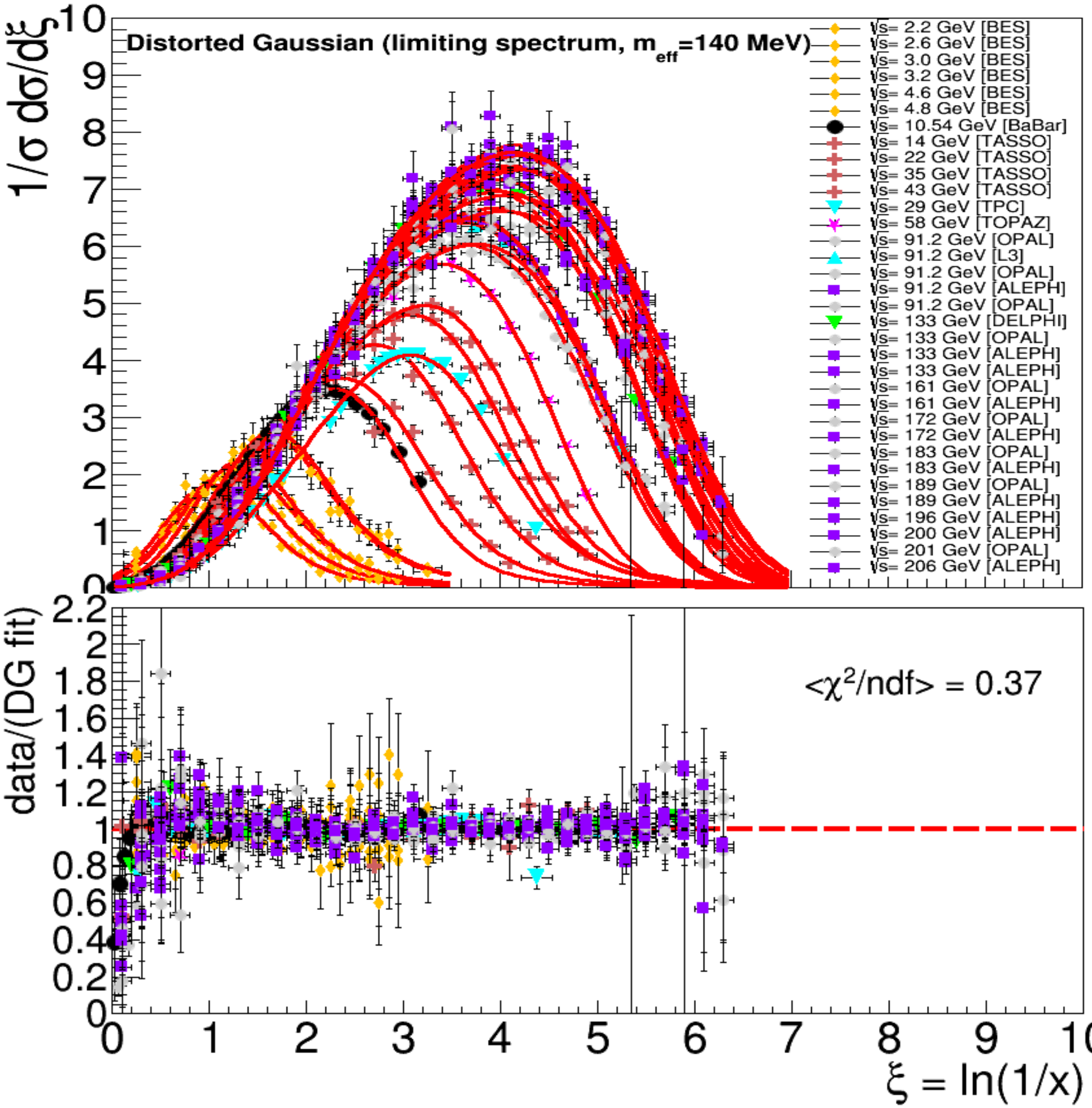
Distorted Gaussian fits to e^+e^- FFs (m_{eff}=0.36 GeV)

- 34 e^+e^- data-sets at $\sqrt{s}$ = 2.2 – 206 GeV
  ~1200 data points
- Peak shifts to right, width increases, moderate non-Gaussian tails
- Excellent fit to DG at all energies, with 5 free parameters: N_{ch}, ξ_{max}, σ, s, k



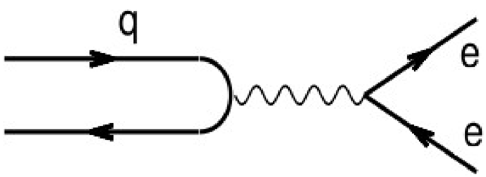
Distorted Gaussian fits to e^+e^- FFs (m_{eff}=0.14 GeV)

- 34 e^+e^- data-sets at $\sqrt{s} = 2.2 - 206$ GeV ~1200 data points
- Peak shifts to right, width increases, moderate non-Gaussian tails
- Excellent fit to DG at all energies, with 5 free parameters: N_{ch}, ξ_{max}, σ, s, k
- Best χ^2/ndf for $m_{eff} \sim m_\pi$



Distorted Gaussian fits to DIS FFs (m_{eff}=0.14 GeV)

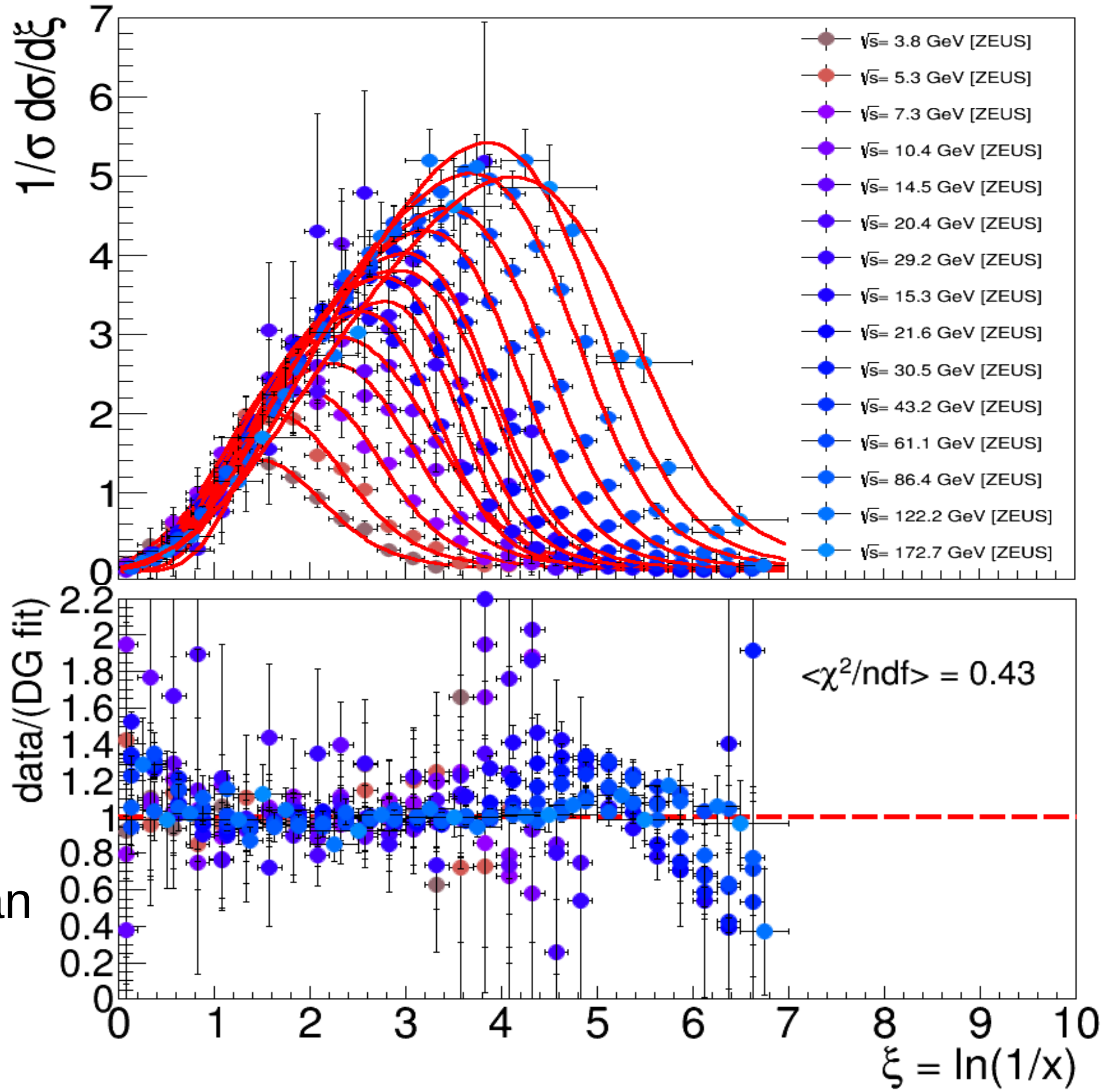
- Breit frame in DIS:



"Brick wall" frame:
Incoming quark scatters off photon & returns along same axis

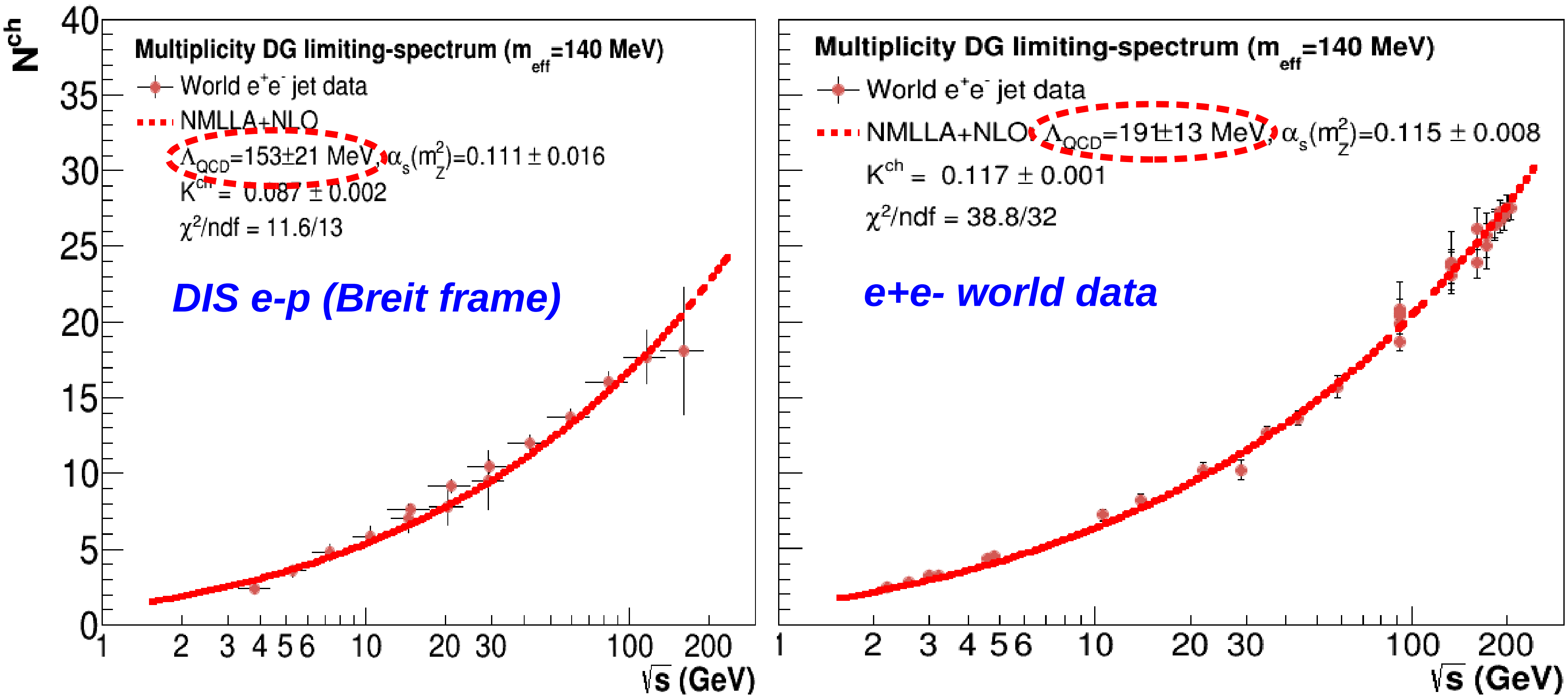
- 15 ZEUS data-sets at $\sqrt{s} = 3.8 - 173$ GeV
  ~250 data points
  (H1 data to be added)

- Good fits to DG but larger uncertainties than e^+e^- measurements



Evolution of the FF moments vs. √s: Multiplicity

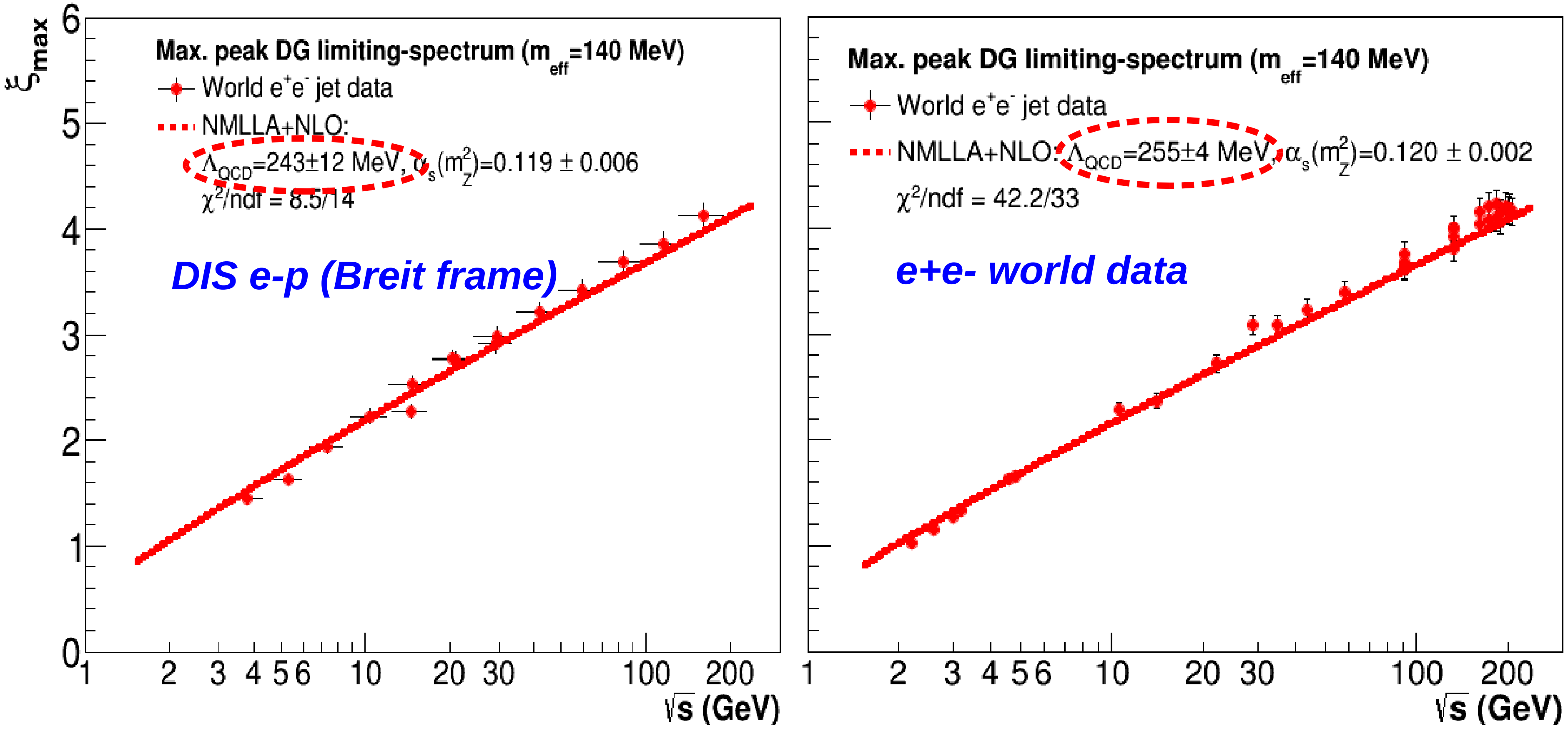
$$\mathcal{N}(Y) = \mathcal{K}^{ch} \exp\left[2.50217\sqrt{Y} - 0.491546 \ln Y - (0.06889 - 0.41151 \ln Y)\frac{1}{\sqrt{Y}} + (0.00068 - 0.161658 \ln Y)\frac{1}{Y}\right], \quad Y = \ln\left(\frac{\sqrt{s}}{2\Lambda_{QCD}}\right)$$



- Good data vs. (NMLLA+NLO) agreement for multiplicity evolution (χ^2/ndf~1.3) from N_{ch}~1–30: K_{ch}~0.12 (local hadron-parton duality norm.)
- DIS N_{ch}(e-p) ~ lower than $N_{ch}(e^+e^-)$, but with larger uncertainties

Evolution of the FF moments vs. √s: Peak

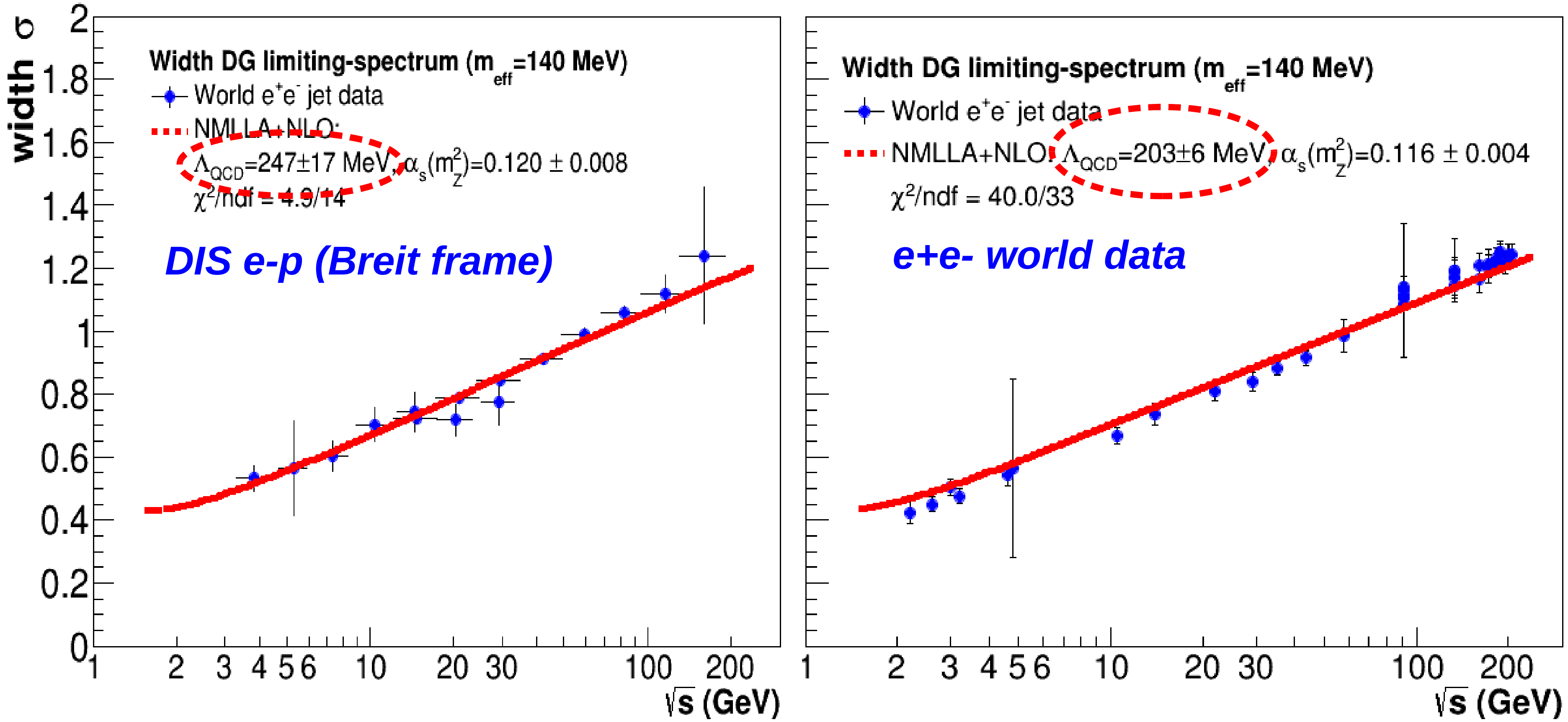
$$\xi_{\max}(Y) = 0.5Y + 0.592722\sqrt{Y} - 0.351319 + 0.002\ln Y, \quad Y = \ln\left(\frac{\sqrt{s}}{2\Lambda_{\rm QCD}}\right)$$



- Good data vs (NMLLA+NLO*) agreement for peak evolution (χ^2/ndf~1.3)
- Consistent DIS e-p and e^+e^- peak positions & extracted Λ_{QCD}.

Evolution of the FF moments vs. √s: Width

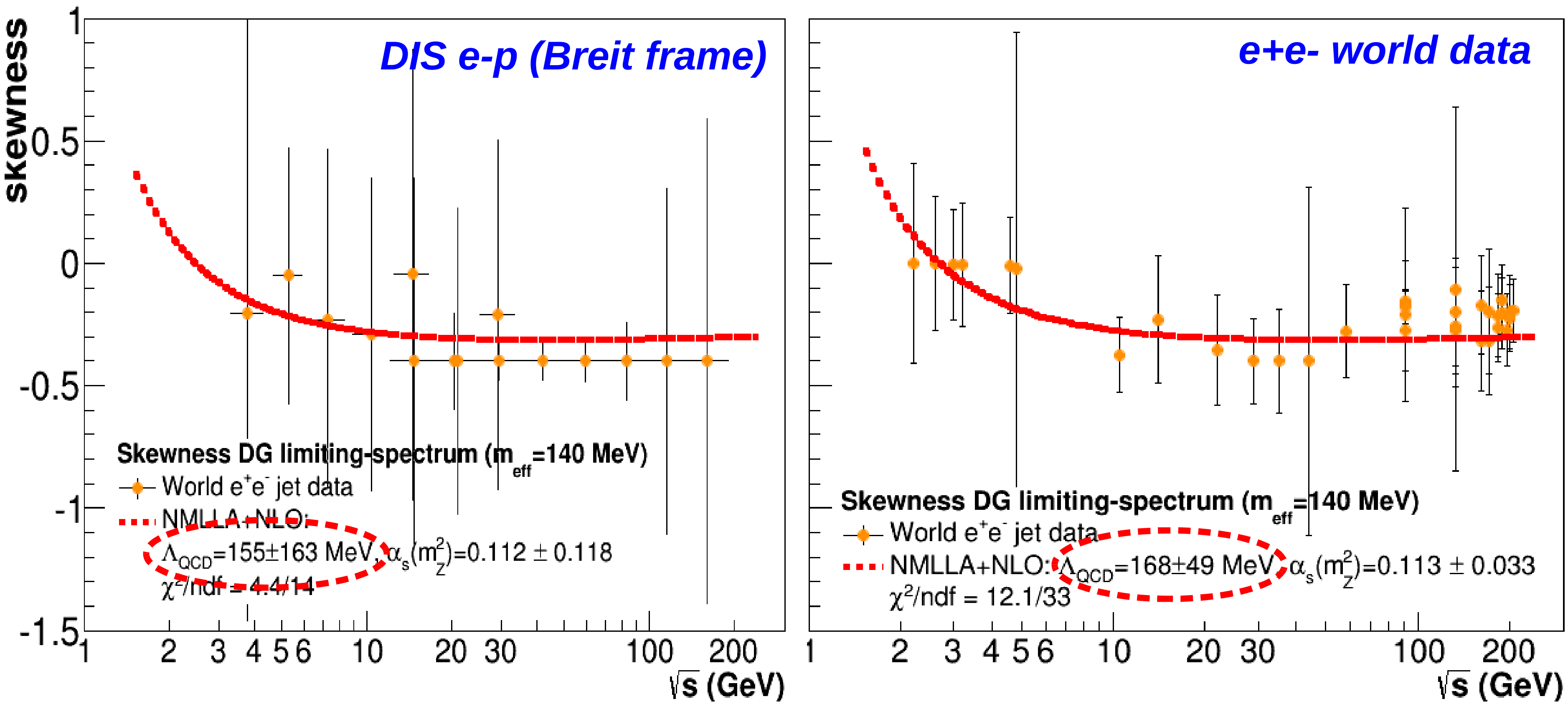
$$\sigma(Y) = 0.36499\,Y^{3/4}\left[1 - 0.299739\frac{1}{\sqrt{Y}} - (1.4921 - 0.246692\ln Y)\frac{1}{Y} + \frac{1.98667}{Y^{3/2}}\right], \quad Y = \ln\left(\frac{\sqrt{s}}{2\Lambda_{QCD}}\right)$$



- Good data vs (NMLLA+NLO*) agreement for width evolution (χ^2/ndf~1.2)
- Consistent DIS e-p and e^+e^- widths (but larger DIS uncertainties)

Evolution of the FF moments vs. √s: Skewness

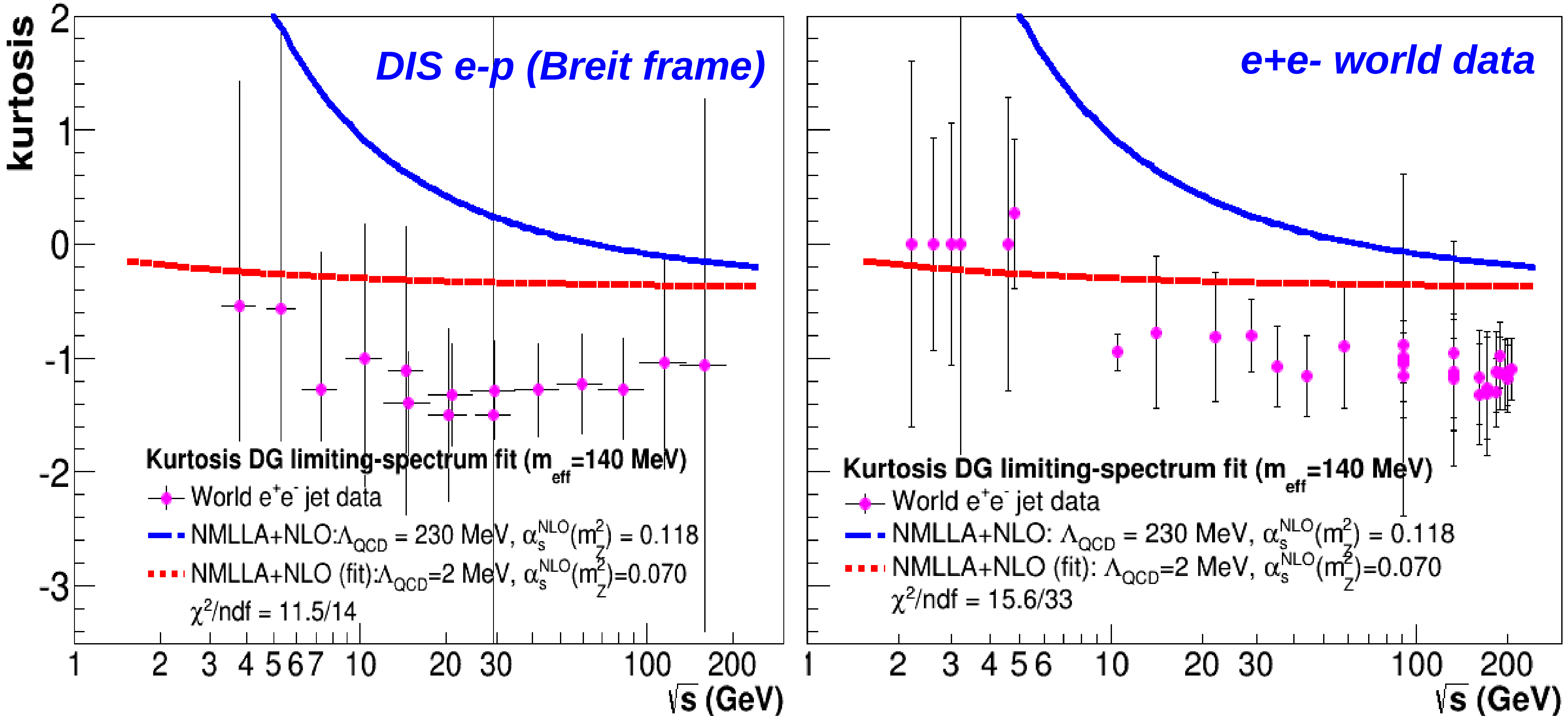
$$s(Y) = -\frac{1.94704}{Y^{3/4}}\left[1 - 0.299739\frac{1}{\sqrt{Y}} - \frac{1.64393}{Y}\right],\; Y = ln\left(\frac{\sqrt{s}}{2\Lambda_{\rm QCD}}\right)$$



- Very good data-theory agreement, though overestimated point-to-point uncertainties in skewness extraction (χ^2/ndf<<1)
- Consistent DIS e-p & e^+e^- skewness (but larger DIS uncertainties)

Evolution of the FF moments vs. √s: Kurtosis

$$k(Y) = -\frac{2.15812}{\sqrt{Y}}\left[1 - 0.799305\frac{1}{\sqrt{Y}} + (0.730466 - 0.164461\ln Y)\frac{1}{Y} - \frac{8.05771}{Y^{3/2}}\right], \quad Y = ln\left(\frac{\sqrt{s}}{2\Lambda_{\rm QCD}}\right)$$



- Data show smaller kurtosis than NMLLA+NLO* prediction. Trend maybe reproduced but clear offset. Missing higher-order corrections ?

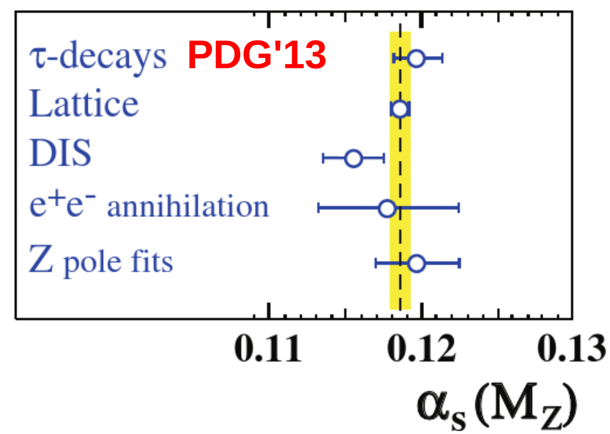
$\alpha_s(m_Z)$ from evolution of low-z FFs in e^+e^- & DIS

- Extracted values of Λ_{QCD} & associated $\alpha_s(m_Z)$ at NMLLA+NLO* accuracy ($\overline{MS}$ scheme, N_f=5) combining e^+e^- & DIS data:

DG comp:	Peak position	Multiplicity	Width	Skewness	Combined
Λ_{QCD}	255 ± 4	240 ± 75	212 ± 6	167 ± 48	241 ± 5 MeV
$\alpha_s(m_Z^2)$	0.120 ± 0.002	0.119 ± 0.04	0.117 ± 0.004	0.113 ± 0.03	0.1193 ± 0.0018

- Excellent agreemet with 2013 world-average: $\alpha_s(m_Z)$ = 0.1185±0.0006 (NNLO)

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0006$$

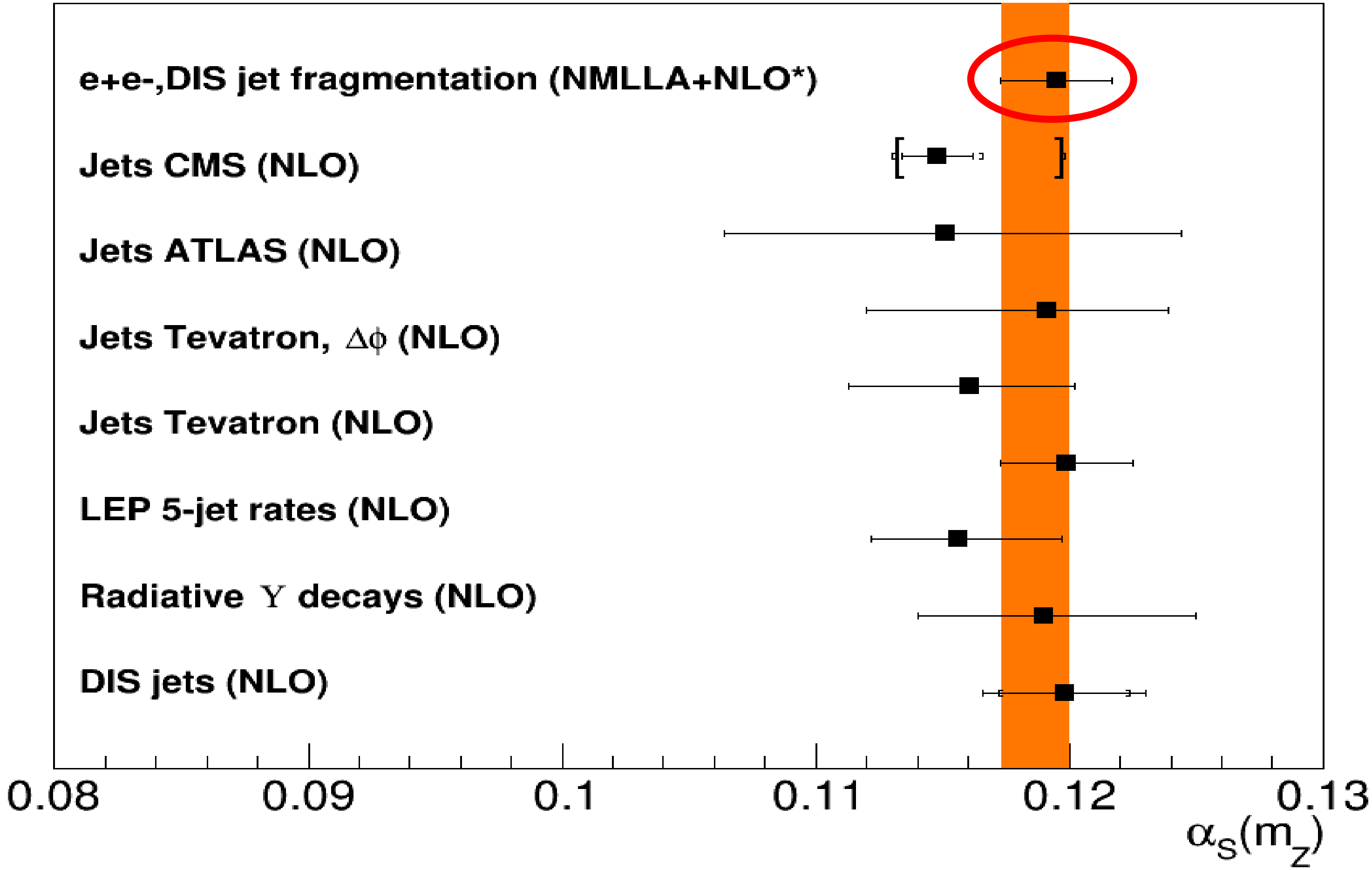


- Peak (±1.5%) & width (±3.5%) are the most precise quantities.

- Final α_s uncertainty (~1.5%) includes:
  - Mass effects variations.
  - Not (yet) scale uncertainties:
    To be determined by redoing fits varying e.g. $Q_0=\Lambda_{QCD} - 4\times\Lambda_{QCD}$
  - Non-pQCD corrections are small: method valid down to Λ_{QCD} (convergent NMLLA resummation of log(1/x) & log(θ) divergences).

Summary: $\alpha_s(m_Z)$ from evolution of low-z FFs

- This work provides the most precise measurement of α_s among those at NLO(*) accuracy (with totally different systematics):

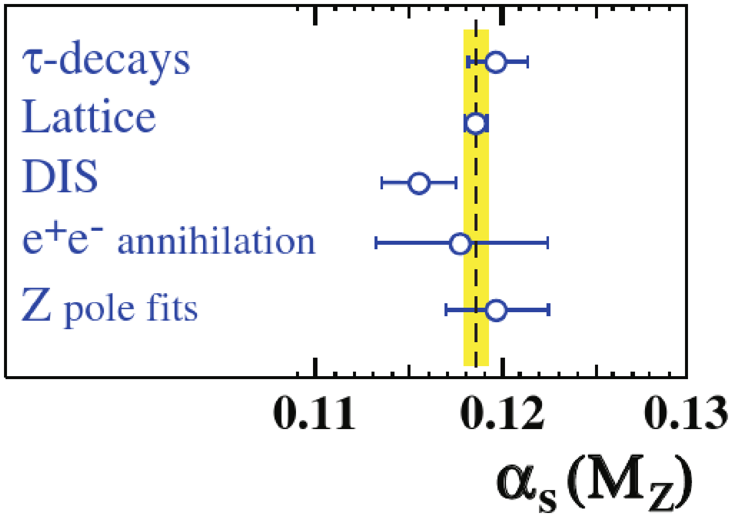


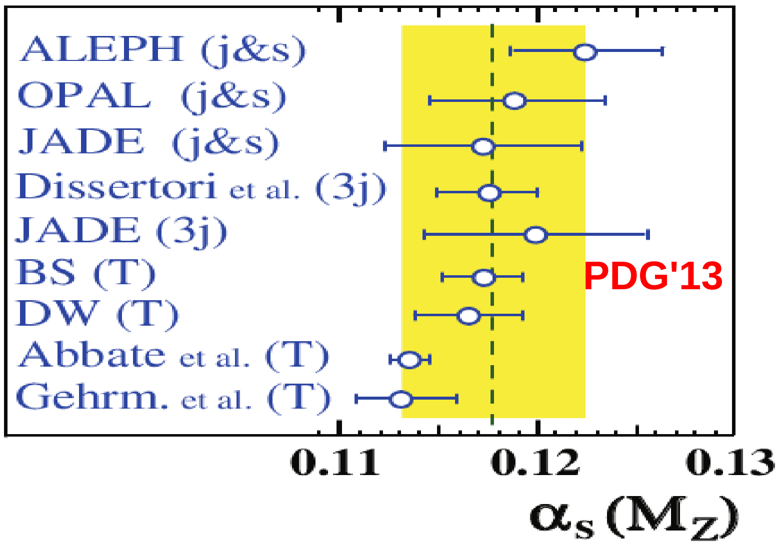
- Outlook: higher-order corrections being worked out ...

Backup slides

Multi-prong determination of α_s coupling

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0006$$





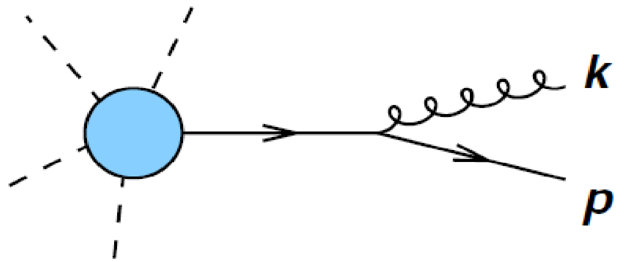
Snowmass'13, arXiv:1310.5189

Method	Current relative precision	Future relative precision
e^+e^- evt shapes	expt $\sim$ 1% (LEP) thry $\sim$ 1–3% (NNLO+up to N^{3}LL, n.p. signif.)	< 1% possible (ILC/TLEP) $\sim$ 1% (control n.p. via Q^2-dep.)
e^+e^- jet rates	expt $\sim$ 2% (LEP) thry $\sim$ 1% (NNLO, n.p. moderate)	< 1% possible (ILC/TLEP) $\sim$ 0.5% (NLL missing)
precision EW	expt $\sim$ 3% (R_Z, LEP) thry $\sim$ 0.5% (N^{3}LO, n.p. small)	0.1% (TLEP [10]), 0.5% (ILC [11]) $\sim$ 0.3% (N^{4}LO feasible, $\sim$ 10 yrs)
τ decays	expt $\sim$ 0.5% (LEP, B-factories) thry $\sim$ 2% (N^{3}LO, n.p. small)	< 0.2% possible (ILC/TLEP) $\sim$ 1% (N^{4}LO feasible, $\sim$ 10 yrs)

Parton shower evolution

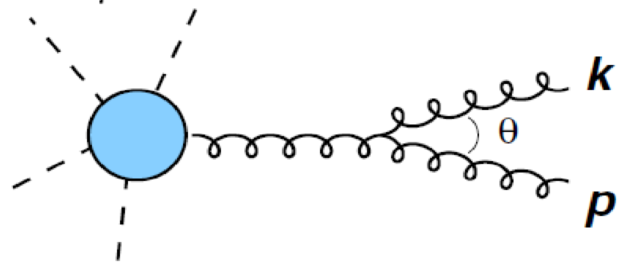
- Soft & collinear divergences are ubiquitous in all QCD processes:

Soft gluon emission from quark:



$$d\sigma \approx \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

Soft gluon emission from gluon:



$$d\sigma \approx \frac{2\alpha_s C_A}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$$

- Same divergence structures:
  For $E \rightarrow 0$: infrared (or soft) divergence
  For $\theta \rightarrow 0,\pi$: collinear divergence
  regardless of where gluon is emitted from

- Soft&collinear contributions dominate the radiation evolution of partons (jets).