Dilepton Invariant Mass Spectrum and the Decay rate in $B^\pm \to \pi^\pm \mu^+ \mu^-$

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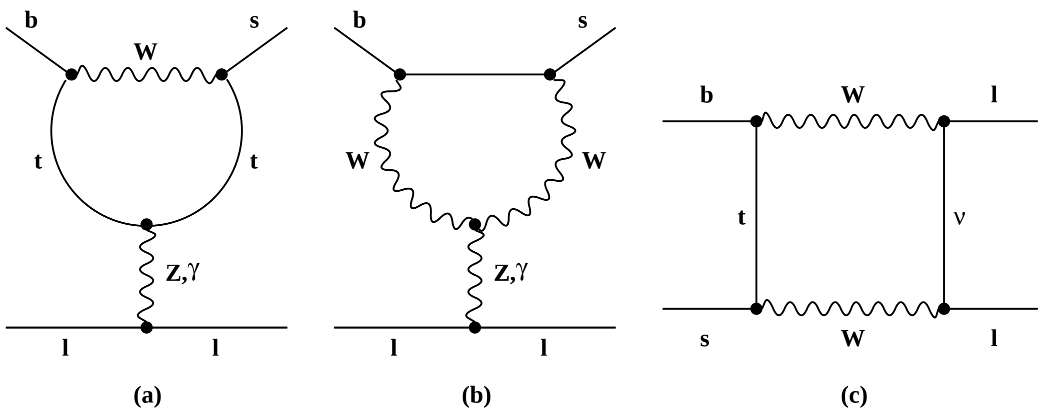
Based on the paper: AA, A. Parkhomenko, A. Rusov, Phys. Rev. D89, 094021 (2014)

Effective Weak $b \to d$ Hamiltonian

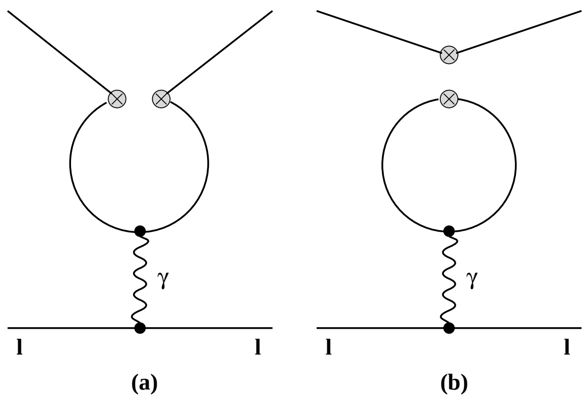
$$H_{\rm eff}^{(b\to d)} = -\frac{4G_F}{\sqrt{2}}\Bigg[V_{tb}^* V_{td} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$
$$+V_{ub}^* V_{ud} \sum_{i=1}^{2} C_i(\mu)\Big(\mathcal{O}_i(\mu) - \mathcal{O}_i^{(u)}(\mu)\Big)\Bigg] + \text{h.c.}$$

- G_F (Fermi constant), $C_i(\mu)$ (Wilson coefficients), and $\mathcal{O}_i(\mu)$ (dimension-six operators) are the same (modulo $s \to d$) as in $H_{\rm eff}^{(b\to s)}$
- However, the CKM structure of the matrix elements more interesting in $H_{\rm eff}^{(b\to d)}$, as $V_{tb}^* V_{td} \sim V_{ub}^* V_{ud} \sim \lambda^3$ are of the same order in $\lambda = \sin\theta_{12}$
- Anticipate sizable CP-violating asymmetries in $b \to d$ transitions compared to $b \to s$

Some representative diagrams in $b \to s\ell^+\ell^-$



Diagrams in the full theory



Diagrams in the effective theory

Operator Basis

- Tree operators

$$\mathcal{O}_1 = \left(\bar{d}_L \gamma_\mu T^A c_L\right)\left(\bar{c}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2 = \left(\bar{d}_L \gamma_\mu c_L\right)\left(\bar{c}_L \gamma^\mu b_L\right)$$

$$\mathcal{O}_1^{(u)} = \left(\bar{d}_L \gamma_\mu T^A u_L\right)\left(\bar{u}_L \gamma^\mu T^A b_L\right), \quad \mathcal{O}_2^{(u)} = \left(\bar{d}_L \gamma_\mu u_L\right)\left(\bar{u}_L \gamma^\mu b_L\right)$$

- Dipole operators

$$\mathcal{O}_7 = \frac{e\, m_b}{g_{\rm st}^2}\left(\bar{d}_L \sigma^{\mu\nu} b_R\right) F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{m_b}{g_{\rm st}}\left(\bar{d}_L \sigma^{\mu\nu} T^A b_R\right) G^A_{\mu\nu}$$

- Semileptonic operators

$$\mathcal{O}_9 = \frac{e^2}{g_{\rm st}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \ell\right), \quad \mathcal{O}_{10} = \frac{e^2}{g_{\rm st}^2}\left(\bar{d}_L \gamma^\mu b_L\right)\sum_\ell \left(\bar{\ell}\gamma_\mu \gamma_5 \ell\right)$$

Wilson Coefficients in the SM

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of Other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

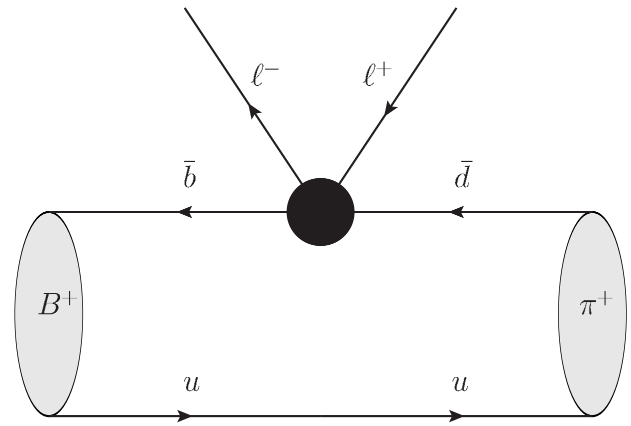
$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

- Three-loop running is used for α_s coupling with $\Lambda_{\overline{\text{MS}}}^{(5)} = 220$ MeV

$B \to \pi$ *transition matrix elements*

Momentum transfer:

$q = p_B - p_\pi = p_{\ell^+} + p_{\ell^-}$



$$\langle \pi(p_\pi)|\bar{b}\gamma^\mu d|B(p_B)\rangle = f_+(q^2)\,(p_B^\mu + p_\pi^\mu) + \left[f_0(q^2) - f_+(q^2)\right]\frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

$$\langle \pi(p_\pi)|\bar{b}\sigma^{\mu\nu}q_\nu d|B(p_B)\rangle = \frac{i f_T(q^2)}{m_B + m_\pi}\left[(p_B^\mu + p_\pi^\mu)\,q^2 - q^\mu\left(m_B^2 - m_\pi^2\right)\right]$$

- Dominant theoretical uncertainty is in the form factors $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$; require non-perturbative techniques, such as Lattice QCD
- Their current determination is the main focus of this talk

$B^+ \to \pi^+\ell^+\ell^-$ *differential branching fraction*

Differential branching fraction

$$\frac{d\mathrm{Br}\,(B^+ \to \pi^+\ell^+\ell^-)}{dq^2} = \frac{G_F^2 \alpha_{\rm em}^2 \tau_B}{1024\pi^5 m_B^3} |V_{tb}V_{td}^*|^2 \sqrt{\lambda(q^2)} \sqrt{1 - \frac{4m_\ell^2}{q^2}} F(q^2)$$

Dynamical function

$$F(q^2) = \frac{2}{3}\lambda(q^2)\left(1 + \frac{2m_\ell^2}{q^2}\right) \left| C_9^{\rm eff} f_+(q^2) + \frac{2m_b}{m_B + m_\pi} C_7^{\rm eff} f_T(q^2) \right|^2$$

$$+ \frac{2}{3}\lambda(q^2)\left(1 - \frac{4m_\ell^2}{q^2}\right) |C_{10}^{\rm eff}|^2 f_+^2(q^2) + \frac{4m_\ell^2}{q^2} \left(m_B^2 - m_\pi^2\right)^2 |C_{10}^{\rm eff}|^2 f_0^2(q^2)$$

$C_i^{\rm eff}(q^2)$ are effective Wilson coefficients. They contain, apart from the Wilson coefficients $C_i(\mu)$, also explicit perturbative improvements

$B \to \pi \ell^+ \nu_\ell$ decay

$$\langle \pi | \bar{u} \gamma^\mu b | B \rangle = f_+(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right) + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

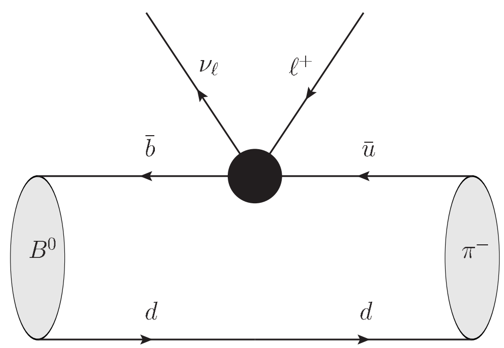
- $f_0(q^2)$ contribution is suppressed by m_ℓ^2/m_B^2 for $\ell = e, \mu$
- Differential decay width

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$

with $\lambda(q^2) = \left(m_B^2 + m_\pi^2 - q^2\right)^2 - 4 m_B^2 m_\pi^2$

- Assuming Isospin symmetry: $f_+(q^2)$ and $f_0(q^2)$ in charged current $B \to \pi \ell \nu_\ell$ and neutral current $B \to \pi \ell^+ \ell^-$ decays are equal
- Global fit of the CKM matrix elements [PDG, 2012]

$$|V_{ub}| = (3.51^{+0.15}_{-0.14}) \times 10^{-3}$$



Parametrizations of the $f_+(q^2)$ form factor

- The Becirevic-Kaidalov (BK) parametrization

$$f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m^2_{B^*})(1 - \alpha_{BK}\, q^2/m^2_{B^*})}$$

- The Ball-Zwicky (BZ) parametrization

$$f_+(q^2) = f_+(0)\left[\frac{1}{1 - q^2/m^2_{B^*}} + \frac{r_{BZ}\, q^2/m^2_{B^*}}{(1 - q^2/m^2_{B^*})(1 - \alpha_{BZ}\, q^2/m^2_{B^*})}\right]$$

- The Boyd-Grinstein-Lebed (BGL) parametrization

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2, q_0^2)} \sum_{k=0}^{k_{\max}} a_k \left[z(q^2, q_0^2)\right]^k$$

Blaschke factor $P(q^2) = z(q^2, m^2_{B^*}) \implies$ pole at $q^2 = m^2_{B^*}$

$$z(q^2, q_0^2) = \frac{\sqrt{m_+^2 - q^2} - \sqrt{m_+^2 - q_0^2}}{\sqrt{m_+^2 - q^2} + \sqrt{m_+^2 - q_0^2}}, \quad m_+ = m_B + m_\pi$$

q_0^2 and $\phi(q^2, q_0^2)$ are taken to ensure a fast convergence

- The Bourrely-Caprini-Lellouch (BCL) parametrization

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{k_{\max}} b_k$$
$$\times \left(\left[z(q^2, q_0^2)\right]^k - (-1)^{k-k_{\max}-1} \frac{k}{k_{\max}+1} \left[z(q^2, q_0^2)\right]^{k_{\max}+1} \right)$$

$\chi^2_{\min}$/ndf and p-value for the $f_+(q^2)$ fits

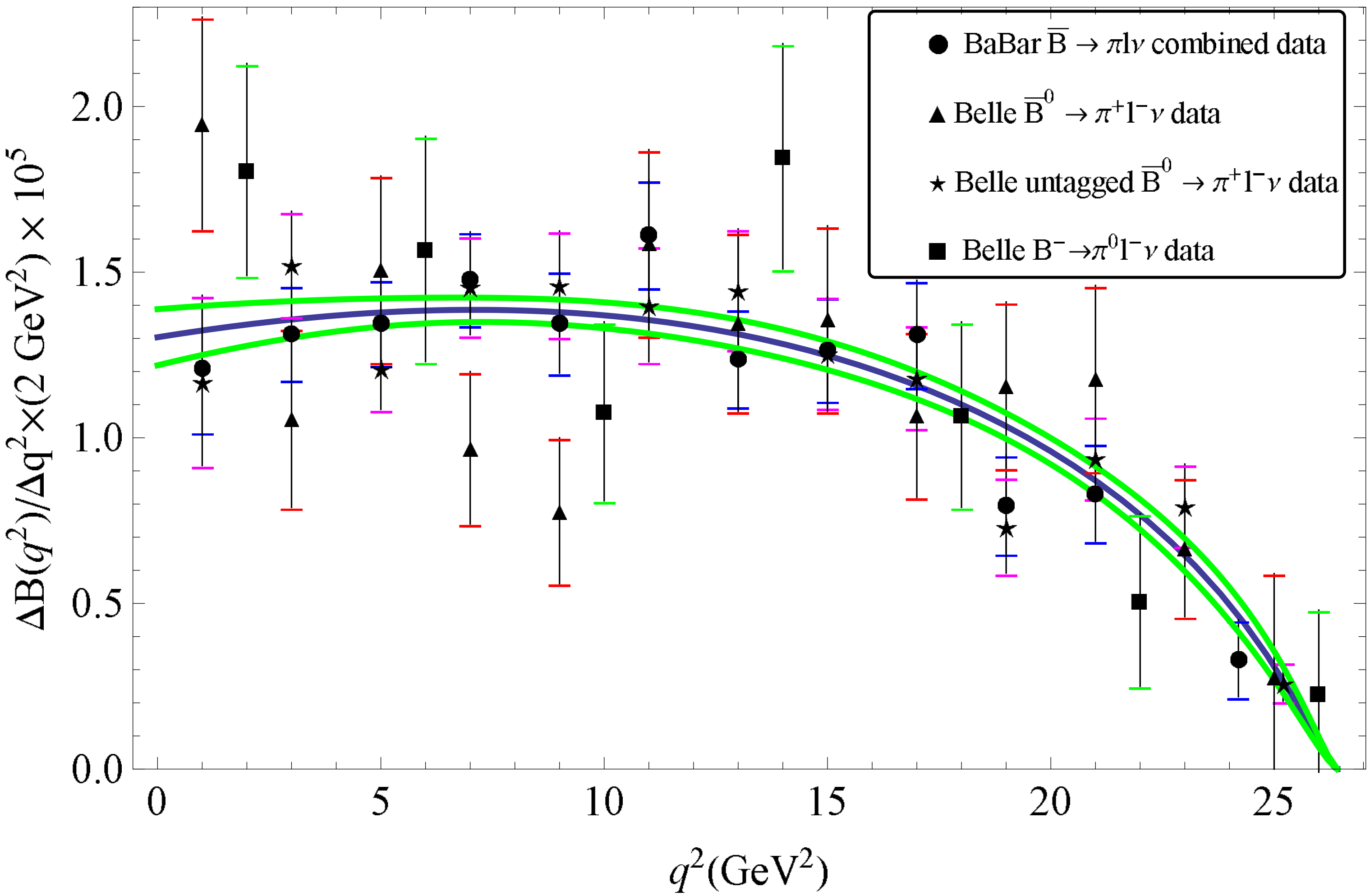
- χ^2-distribution function for N experimental points [PDG, 2012]

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - F(x_i; \alpha_1, \ldots, \alpha_k))^2}{\sigma_i^2},$$

- $F(x_i; \alpha_1, \ldots, \alpha_k)$ denotes theoretical estimates of partial BF $y = \Delta\mathcal{B}(q^2)/\Delta q^2$

	BK	BZ	BGL	BCL
BaBar 2011	9.93/10 (45%)	4.80/9 (85%)	4.12/9 (90%)	3.75/9 (93%)
BaBar 2012	8.68/10 (56%)	5.50/9 (79%)	5.65/9 (77%)	5.73/9 (77%)
Belle 2011	15.86/11 (15%)	14.55/10 (15%)	12.97/10 (23%)	14.44/10 (15%)
Belle 2013	24.41/18 (14%)	23.55/17 (13%)	24.16/17 (12%)	23.26/17 (14%)
BaBar & Belle	44.99/43 (39%)	44.91/42 (35%)	44.56/42 (36%)	44.77/42 (36%)

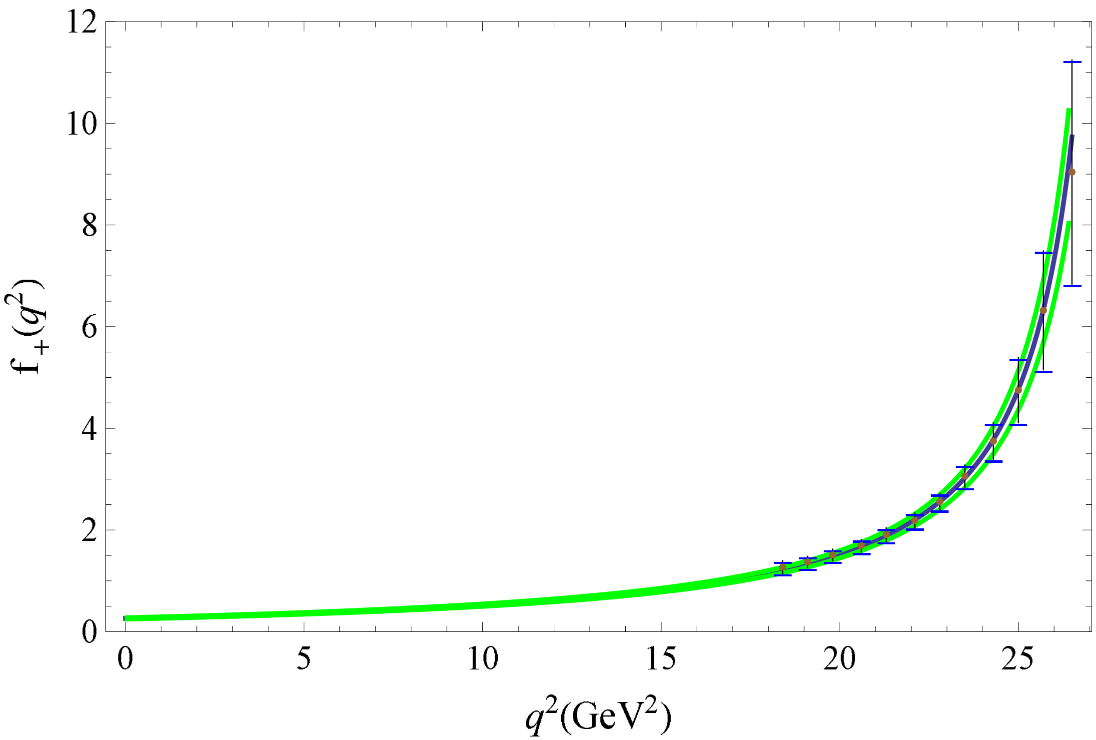
Fitting of the form-factor shape



Fitting of the form factor shape

BGL parametrization ($k_{max} = 2$):

$$a_0 = 0.0209 \pm 0.0004$$
$$a_1 = -0.0306 \pm 0.0031$$
$$a_2 = -0.0473 \pm 0.0189$$



Lattice data by the HPQCD Collab. are shown as vertical bars

Heavy-Quark Symmetry (HQS) relations

- Heavy quark symmetry reduces the number of independent form factors to just one: $f_+(q^2)$
- Including symmetry-breaking corrections [Beneke & Feldmann (2000)]:

$$f_0(q^2) = (1 - q^2/m_B^2) f_+(q^2) \left(1 + \frac{\alpha_s(\mu_h) C_F}{4\pi} \left(2 - 2L(q^2)\right)\right) + \frac{\alpha_s(\mu_{hc}) C_F}{4\pi} \frac{q^2}{(m_B^2 - q^2)^2} \Delta F_\pi,$$

$$f_T(q^2) = (1 + m_\pi/m_B) f_+(q^2) \left(1 + \frac{\alpha_s(\mu_h) C_F}{4\pi} \left(\ln \frac{m_b^2}{\mu^2} + 2L(q^2)\right)\right) - \frac{\alpha_s(\mu_{hc}) C_F}{4\pi} \frac{m_B(m_B + m_\pi)}{m_B^2 - q^2} \Delta F_\pi,$$

$$L(q^2) = \left(1 - \frac{m_B^2}{q^2}\right) \ln\left(1 - \frac{q^2}{m_B^2}\right), \quad \Delta F_\pi = \frac{8\pi^2 f_B f_\pi}{3 m_B} \left\langle l_+^{-1} \right\rangle_+ \left\langle \bar{u}^{-1} \right\rangle_\pi$$

- $\mu_h \sim m_b; \quad \mu_{hc} \sim \sqrt{m_b \Lambda}; \quad \Lambda \simeq 0.5$ GeV
- Fitting $f_+(q^2)$ from the charged current data on $B \to \pi \ell^+ \nu_\ell$ decay $\Longrightarrow$ Model-independent predictions of differential branching ratio (dimuon mass spectrum) in $B \to \pi \ell^+ \ell^-$ for low-q^2 values

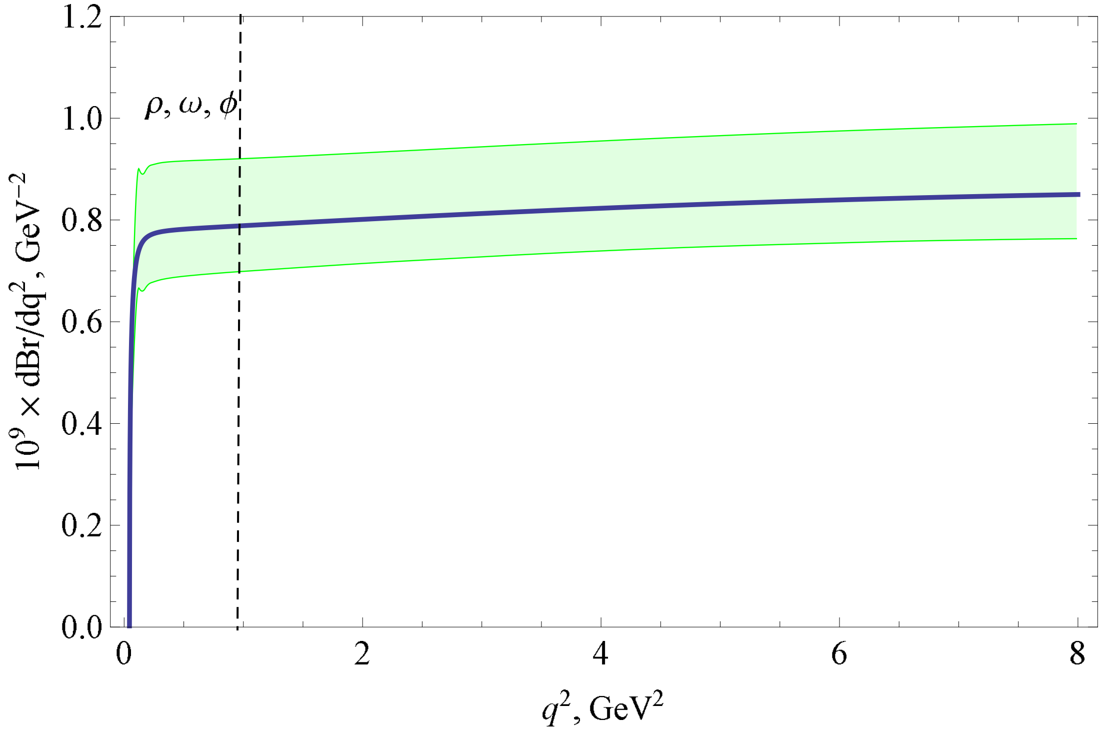
$B^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$ *at large hadronic recoil*

- Partially integrated branching fractions for $B^{\pm} \to \pi^{\pm}\ell^{+}\ell^{-}$

$$\mathrm{Br_{th}}(B^{+} \to \pi^{+}\mu^{+}\mu^{-}; 0.05\ \mathrm{GeV}^2 \leq q^2 \leq 8\ \mathrm{GeV}^2) = \left(0.65^{+0.08}_{-0.06}\right) \times 10^{-8}$$

$$\mathrm{Br_{th}}(B^{+} \to \pi^{+}\mu^{+}\mu^{-}; 1\ \mathrm{GeV}^2 \leq q^2 \leq 8\ \mathrm{GeV}^2) = \left(0.57^{+0.07}_{-0.05}\right) \times 10^{-8}$$

- Dimuon invariant mass spectrum at large hadronic recoil



Determining the form factors $f_0(q^2)$ and $f_T(q^2)$

- In the low hadronic recoil region (large-q^2) there are no relations among the form factors any more, and the three form factors $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$ are independent quantities
- Lattice QCD data on $f_T^{B\pi}(q^2)$ are either absent (Fermilab Lattice and MILC Collaborations yet to unblind their analysis [arxiv:1311.6552]), or they are preliminary (HPQCD Collaboration [arxiv:1310.3207])
- However, Lattice results on the $B \to K$ form factors $f_+^{BK}(q^2)$, $f_0^{BK}(q^2)$ and $f_T^{BK}(q^2)$ are available in large-q^2
- They can be combined with an Ansatz on $SU(3)$-symmetry breaking corrections to get the corresponding $f_T^{B\pi}(q^2)$ form factor
- In extrapolating the form factors to low-q^2, where they are known, we use the z-expansion for $f_0^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$

Estimates of $SU(3)_F$-breaking in the $f_T(q^2)$ Form Factor

- Introduce the form-factors ratio

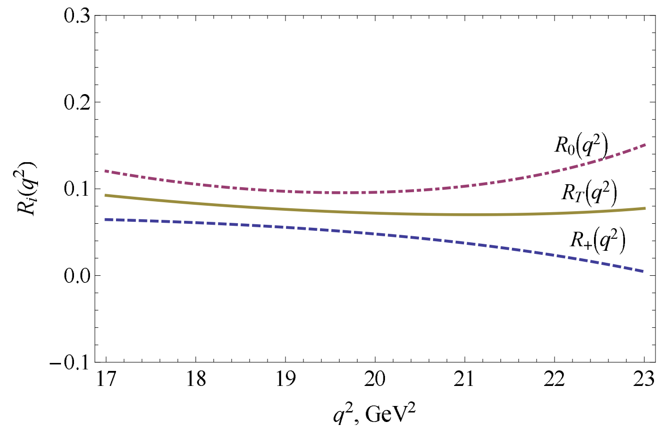
$$R_i(q^2) \equiv \frac{f_i^{BK}(q^2)}{f_i^{B\pi}(q^2)} - 1, \quad i = +, 0, T$$

- $SU(3)_F$-symmetry breaking corrections in the form factors are expected to be of $O(15\%)$
- Lattice data in large-q^2 domain available for the five form factors

$$f_+^{BK}(q^2),\ f_0^{BK}(q^2),\ f_T^{BK}(q^2),\ f_+^{B\pi}(q^2),\ f_0^{B\pi}(q^2)$$

- Our Ansatz:

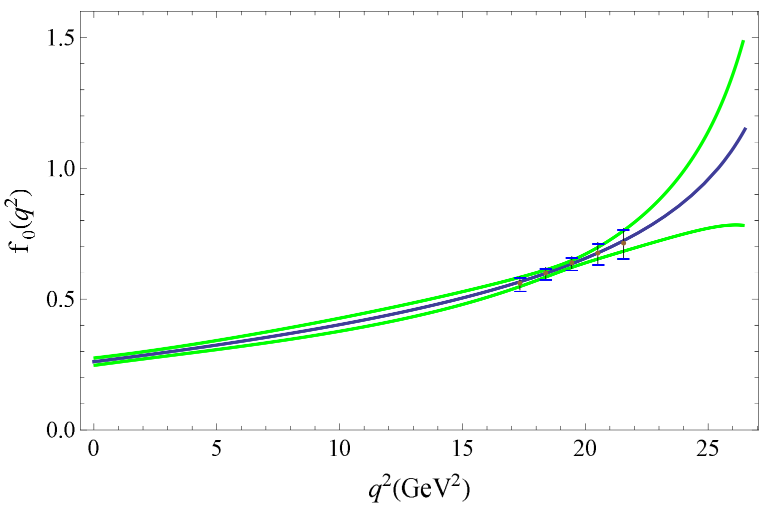
$$R_T(q^2) = \frac{1}{2}\left[R_+(q^2) + R_0(q^2)\right]$$

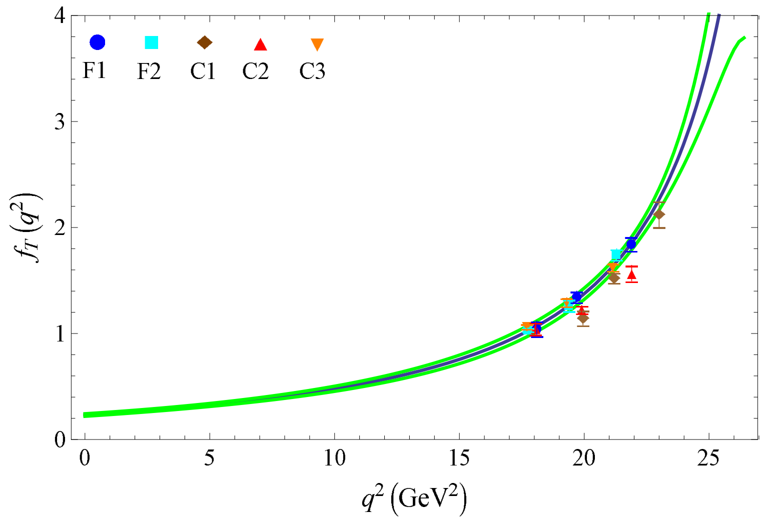


- With $f_T^{BK}(q^2)$ and $R_T(q^2)$,

$$f_T^{B\pi}(q^2) = \frac{f_T^{BK}(q^2)}{1 + R_T(q^2)}$$

Our determination of $f_0^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$ and comparison with Lattice QCD



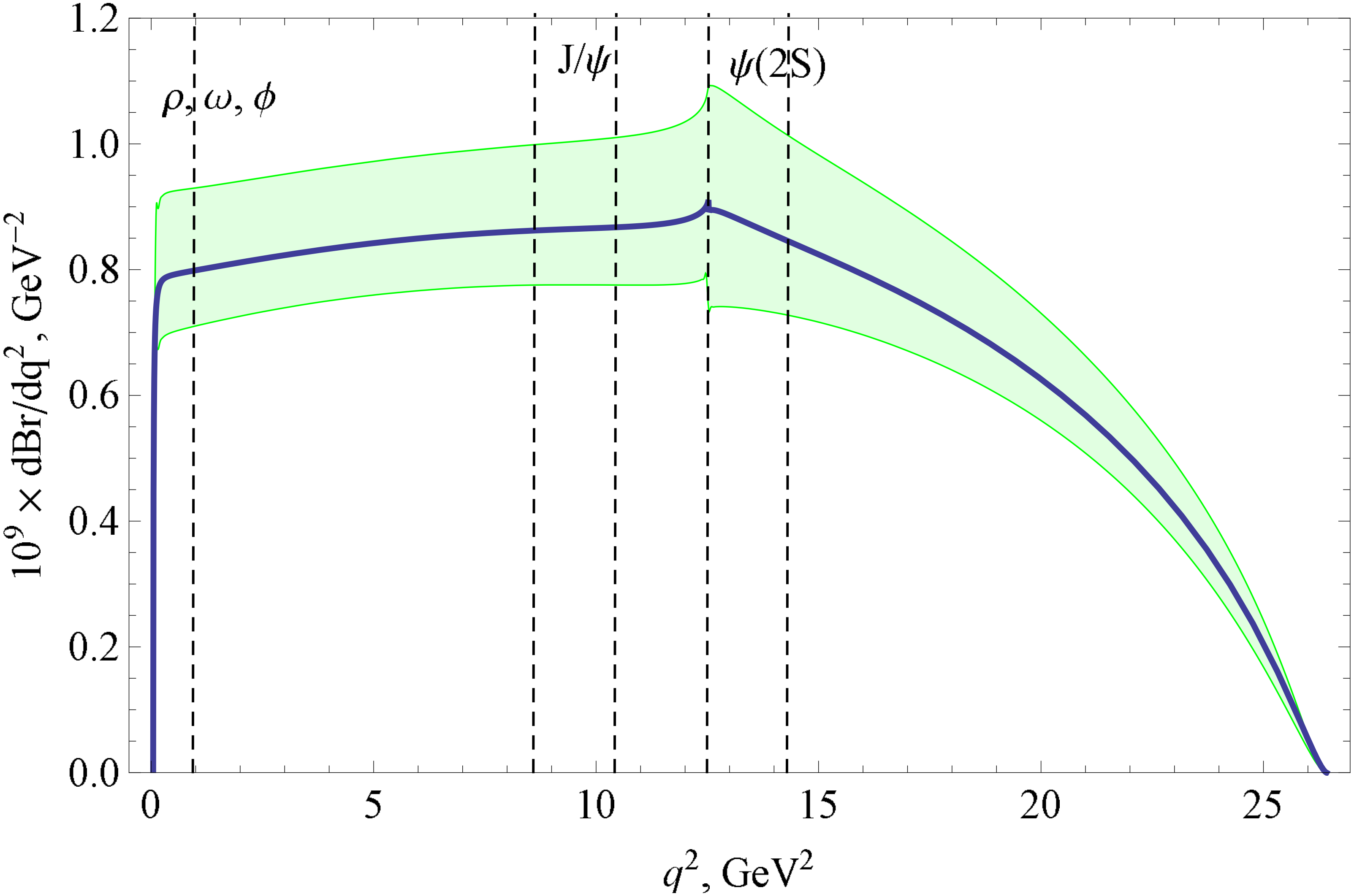


Lattice data obtained by the HPQCD Collab. are shown:

- for $f_0^{B\pi}(q^2)$ [Phys. Rev. D73 (2006) 074502; E D75 (2007) 119906]
- for $f_T^{B\pi}(q^2)$ [arXiv:1310.3207; Conf. Proceedings: C13-07-29.1 (2013)]
- In almost the entire q^2-domain, the form factors are now determined accurately. Further improvements expected from lattice $B \to \pi$ studies

$B^+ \to \pi^+ \mu^+ \mu^-$ *in the entire range of* q^2

The dimuon invariant mass spectrum at entire range of q^2



Partial branching ratios $d\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-)/dq^2$

$[q^2_{\rm min}, q^2_{\rm max}]$	$10^8 \times \mathcal{B}(q^2_{\rm min} \leq q^2 \leq q^2_{\rm max})$
$[0.05, 2.0]$	$0.15^{+0.03}_{-0.02}$
$[1, 2.0]$	$0.08^{+0.01}_{-0.01}$
$[2.0, 4.3]$	$0.19^{+0.03}_{-0.02}$
$[4.3, 8.68]$	$0.37^{+0.06}_{-0.04}$
$[10.09, 12.86]$	$0.25^{+0.04}_{-0.03}$
$[14.18, 16.0]$	$0.15^{+0.03}_{-0.02}$
$[16.0, 18.0]$	$0.15^{+0.03}_{-0.02}$
$[18.0, 22.0]$	$0.25^{+0.04}_{-0.03}$
$[22.0, 26.4]$	$0.13^{+0.02}_{-0.02}$
$[0.05, 8.0]$	$0.66^{+0.10}_{-0.07}$
$[1.0, 8.0]$	$0.58^{+0.09}_{-0.06}$
$[4m^2_\mu, (m_B - m_\pi)^2]$ (total)	$1.88^{+0.32}_{-0.21}$

SM vs. experimental data

- SM theoretical estimate of the total branching fraction:

$$\mathrm{Br}_{\mathrm{SM}}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}) = \left(1.88^{+0.32}_{-0.21}\right) \times 10^{-8}$$

- Uncertainty from the form factors reduced greatly. Residual theoretical uncertainty mainly from the scale dependence and the CKM matrix elements
- First measurements of $\mathrm{Br}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-})$ based on 1 fb^{-1} integrated luminosity is reported by the LHCb Collaboration JHEP 12 (2012) 125

$$\mathrm{Br}_{\mathrm{exp}}(B^{\pm} \to \pi^{\pm}\mu^{+}\mu^{-}) = (2.3 \pm 0.6(\mathrm{stat}) \pm 0.1(\mathrm{syst})) \times 10^{-8}$$

- Good agreement with SM-based theory within large experimental error, but very significant improvement expected from the future analysis
- Dimuon invariant mass distribution is yet to be measured; low-q^2-domain calculated precisely thanks to CC data and heavy quark symmetry
- Precise estimates of asymmetries (CPV and isospin-violating) being worked out

Backup

Results for $f_T(q^2)$ in the BGL Parametrization

- Derived from Lattice-QCD results by HPQCD Collab. on tensor $B \to K$ transition form factor

q^2, GeV^2	18.4	19.1	19.8	20.6
$f_T^{BK}(q^2)$	1.197 ± 0.047	1.307 ± 0.051	1.434 ± 0.057	1.608 ± 0.069
$R_T(q^2)$	0.080 ± 0.021	0.076 ± 0.021	0.073 ± 0.023	0.071 ± 0.023
$f_T^{B\pi}(q^2)$	1.108 ± 0.126	1.215 ± 0.115	1.337 ± 0.117	1.503 ± 0.123
q^2, GeV^2	21.3	22.1	22.8	23.5
$f_T^{BK}(q^2)$	1.793 ± 0.082	2.054 ± 0.106	2.342 ± 0.135	2.713 ± 0.176
$R_T(q^2)$	0.070 ± 0.037	0.072 ± 0.050	0.076 ± 0.067	0.083 ± 0.090
$f_T^{B\pi}(q^2)$	1.675 ± 0.144	1.916 ± 0.169	2.178 ± 0.211	2.506 ± 0.302

- These values used for derivation of $f_T(q^2)$ parameters
- $f_T(q^2)$ best-fit values and correlation matrix

$$\begin{aligned} a_0 &= 0.0458 \pm 0.0027, \\ a_1 &= -0.0234 \pm 0.0124, \\ a_2 &= -0.2103 \pm 0.1052, \end{aligned} \qquad r_{ij} = \begin{pmatrix} 1 & 0.68 & -0.90 \\ 0.68 & 1 & -0.83 \\ -0.90 & -0.83 & 1 \end{pmatrix}$$