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α_s from τ decays: higher orders and perturbative behaviour

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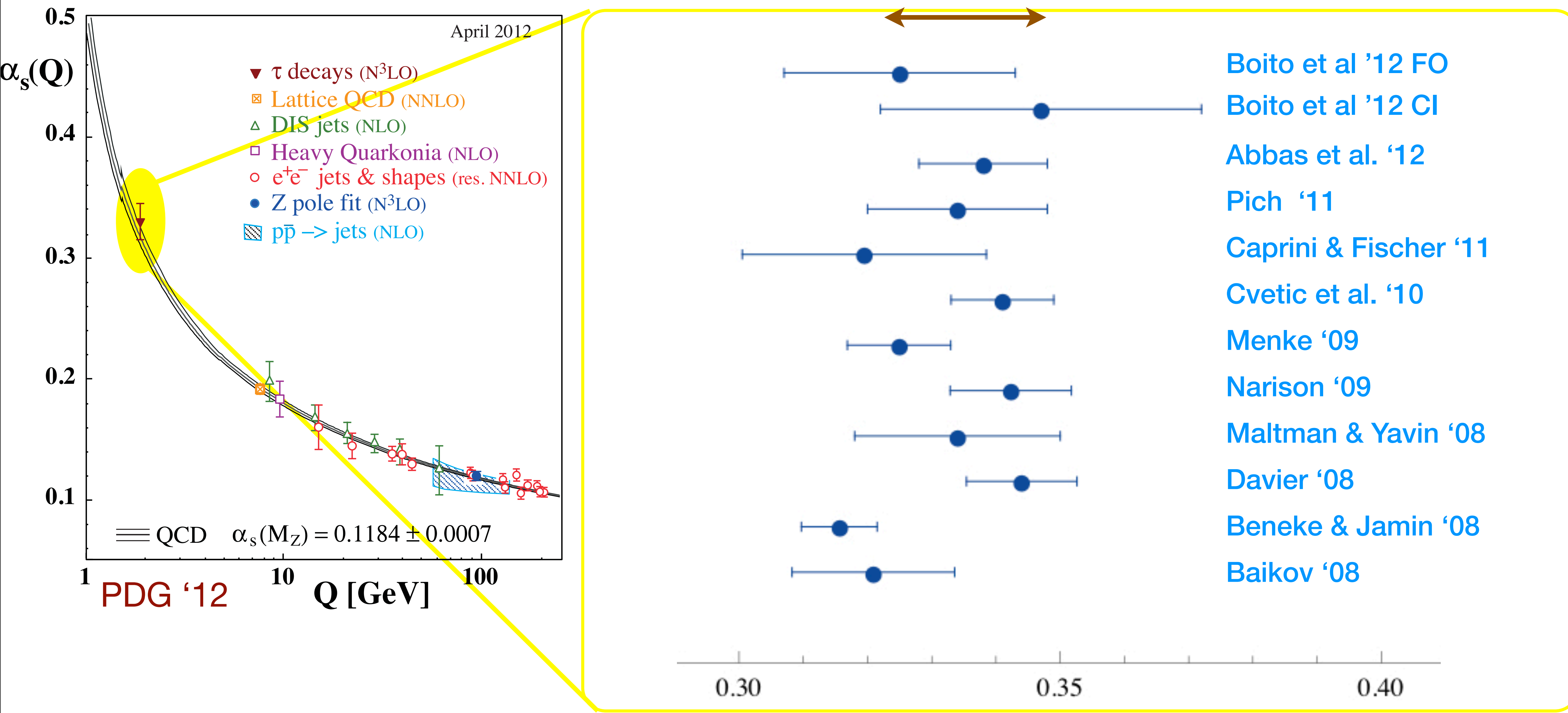
-- M Beneke, DB, M Jamin, *JHEP* **1301** (2013)
-- DB, M Golterman, M Jamin, A Mahdavi, K Maltman, J Osborne, S Peris, *Phys Rev D* **85** (2012)
-- DB, O Catà, M Golterman, M Jamin, K Maltman, J Osborne, S Peris, *Phys Rev D* **84** (2011)
-- DB, PoS Confinement X (2012)

ICHEP14, Valencia, 2 - 9 of July 2014

- **Introduction**
- **Recent developments in alpha_s from tau decays**
  - Moments, higher orders and FOPT vs CIPT
  - Duality Violations
  - Updated ALEPH data (correct correlation matrices)
- **Prospects**
  - New analysis of updated ALEPH data (in progress, no new alpha_s value)

Introduction

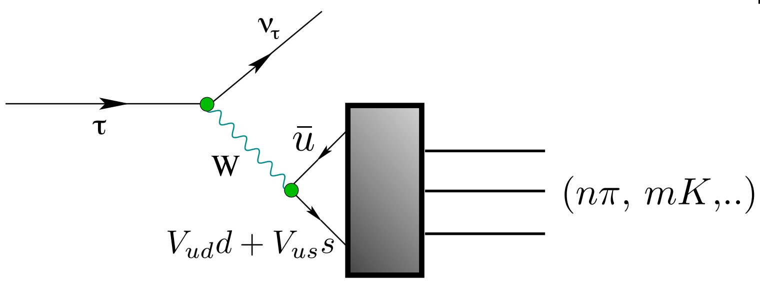
Prescription for the RG improvement (~7% error)

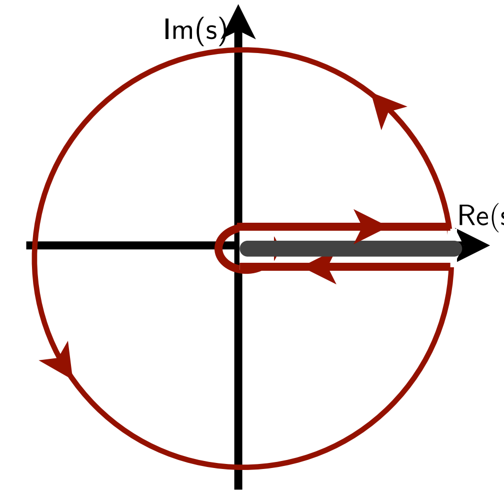


- Spread in the results reflect (mainly) details of the theoretical input.
- There are still open questions (Renormalization Group Improvement, duality violations, ...)

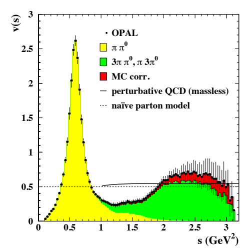
- Sum rules for the spectral functions

(in tau decays) Braaten, Narison, and Pich, 1992

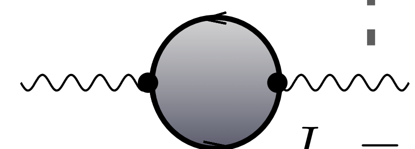




$$\int_0^{s_0} ds\, w(s)\, \frac{1}{\pi}\mathrm{Im}\tilde{\Pi}(s) = \frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \tilde{\Pi}(z)$$



experiment
(OPAL and ALEPH)

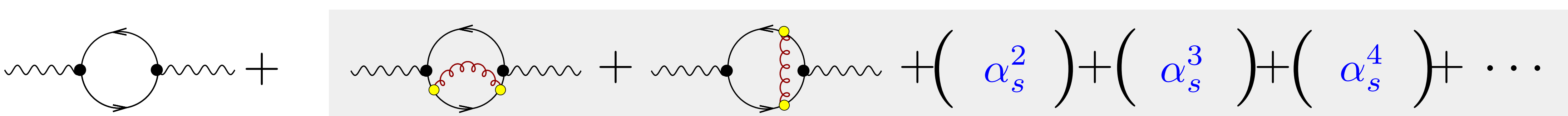


theory
$J_\mu = \bar{u}\gamma_\mu(\gamma_5)q(x)$

- Contributions to the sum rule (theory side)

$$R^{w_i}_{V/A}(s_0) = \frac{N_c}{2}\, S_{EW}|V_{ud}|^2\left[\delta^{\rm tree}_{w_i} + \delta^{(0)}_{w_i}(s_0) + \sum_{D\geq 2}\delta^{(D)}_{w_i,V/A}(s_0) + \delta^{\rm DV}_{w_i,V/A}(s_0)\right]$$

OPE DVs



α_s^4 : Baikov, Chetyrkin, Kühn 2008

Recent developments in alpha_s from tau decays

Systematic study of the weight function dependence

Beneke, DB, Jamin, '13

$$\int_0^{s_0} ds\, \boxed{w(s)}\, \frac{1}{\pi} \mathrm{Im}\tilde{\Pi}(s) = \frac{-1}{2\pi i} \oint_{|z|=s_0} dz\, \boxed{w(z)}\, \tilde{\Pi}(z)$$

- Convergence of pt. series is weight function dependent. What are *ideal* weight functions?
- Several moments used in literature. Renormalization group: what's best CIPT or FOPT?
  (**C**ontour **I**mproved **P**erturbation **T**heory or **F**ixed **O**rder **P**erturbation **T**heory)

Duality violations

$$R^{w_i}_{V/A}(s_0) = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left[\delta^{\mathrm{tree}}_{w_i} + \delta^{(0)}_{w_i}(s_0) + \sum_{D\geq 2} \delta^{(D)}_{w_i,V/A}(s_0) + \boxed{\delta^{\mathrm{DV}}_{w_i,V/A}(s_0)} \right]$$

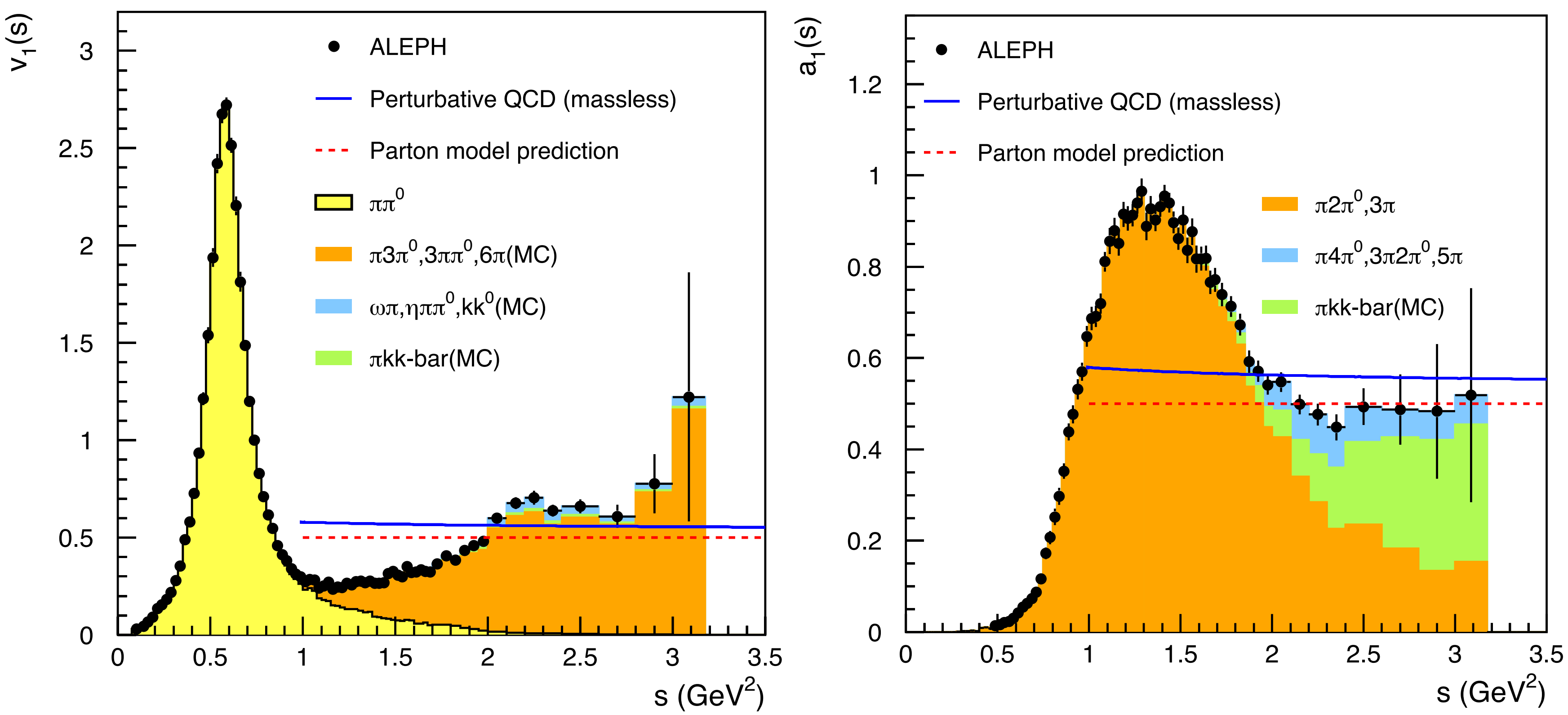
Catà, Golterman, & Peris, '05, '09; Boito et al '11, '12.

- Resonance effects close to the real axis.
- Must take them into account (at the very least for error calculation).
- One must resort to reasonable models based on QCD.
- Source of uncertainty not considered in the "standard" analysis.

Davier et al 2013

Updated ALEPH data for spectral functions

Davier, Höcker, Malaescu, Yuan, Zhang, 1312.1501v2



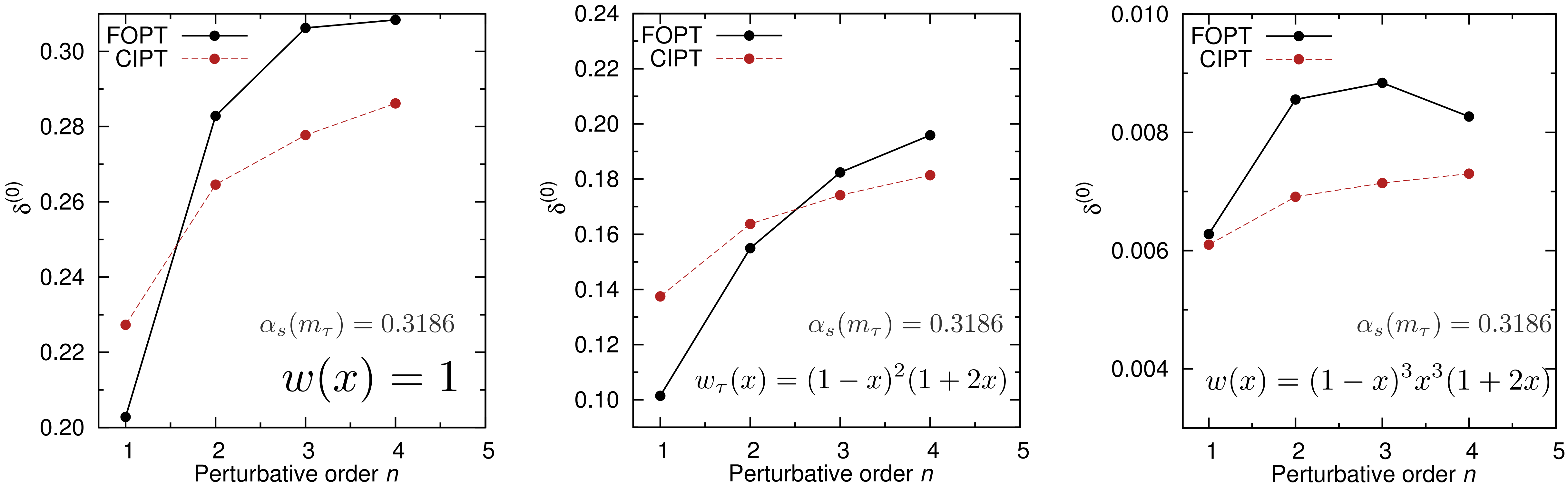
- Corrected covariances matrix (due to unfolding).
- Corrects a problem discovered the previous version (05/08) of the ALEPH data.
  DB, Catà, Golterman, Jamin, Maltman, Osborne and Peris, ‘11
- Smaller uncertainties than the OPAL data.

Perturbative series, weight functions, and RG improvement

Beneke, DB, Jamin, ‘13

FOPT vs CIPT

- RG improvement: main theoretical uncertainty. Weight function dependent.



- Strategy: reasonable model for the leading singularities of the Borel transformed perturbative series.

sing. of $B[R](t)$

$$\frac{c}{(1+at)^b} + \cdots$$

→

$$B[R](t)$$

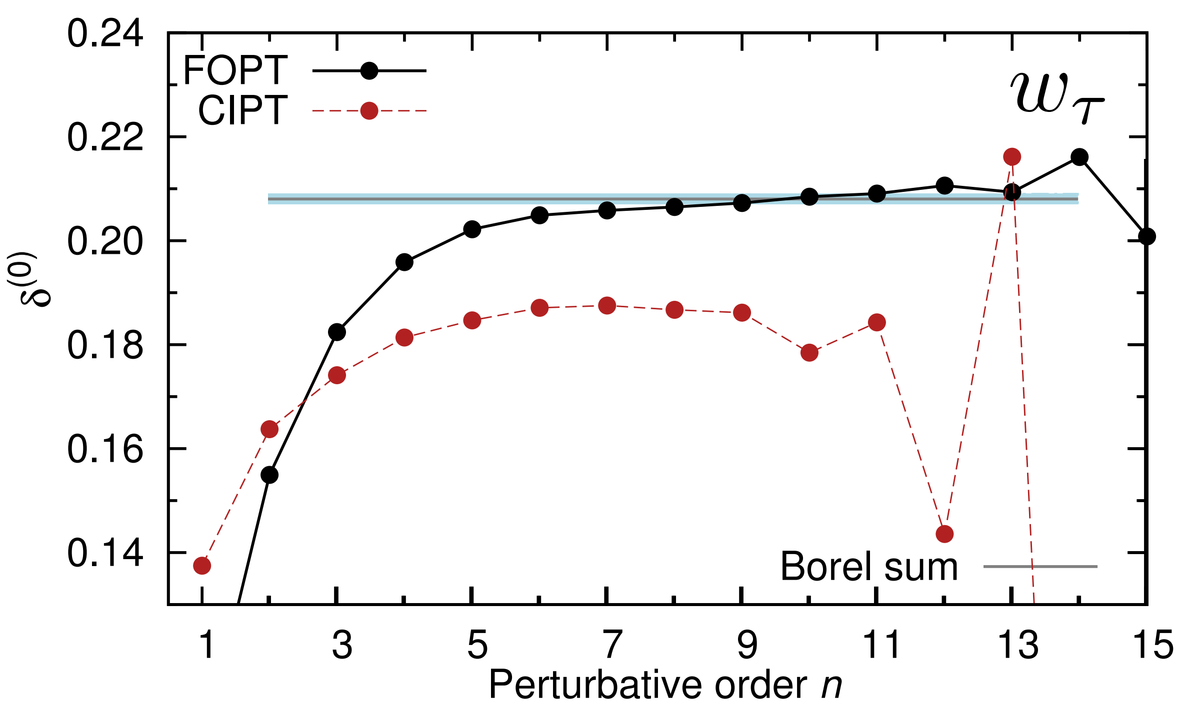
→

$$\sum_n^\infty r_n \alpha_{(s)}^{n+1}$$

First 4 coefficients are fixed from loop computations

Reference model (RM) Beneke, Jamin '08

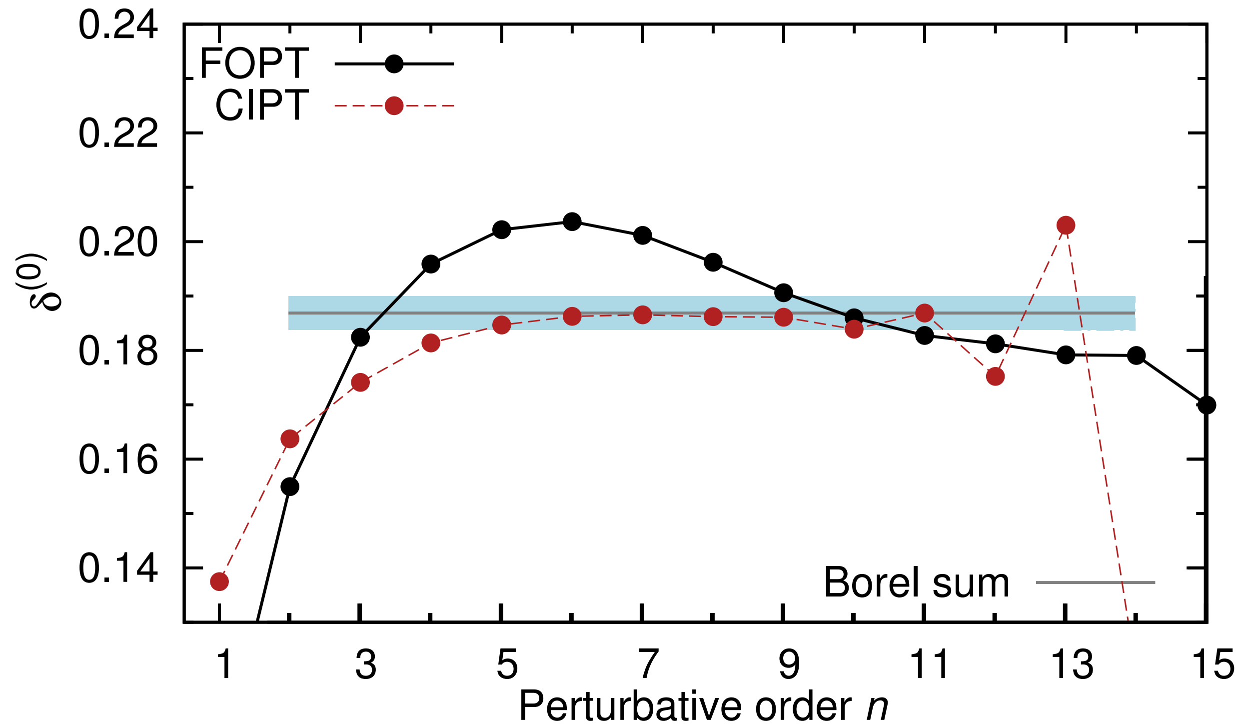
$$B[\widehat{D}](u) = B[\widehat{D}_1^{\rm UV}](u) + B[\widehat{D}_2^{\rm IR}](u) + B[\widehat{D}_3^{\rm IR}](u) + d_0^{\rm PO} + d_1^{\rm PO}\, u$$



- Model with the leading UV and the first two IR singularities.
- Favors FOPT (related to the presence of u = 2 sing).

Alternative Model (AM) Beneke, Boito, Jamin '13

$$B[\widehat{D}](u) = B[\widehat{D}_1^{\rm UV}](u) + B[\widehat{D}_3^{\rm IR}](u) + B[\widehat{D}_4^{\rm IR}](u) + d_0^{\rm PO} + d_1^{\rm PO}\, u$$



- No IR singularity at u = 2.
- Favors CIPT.

Conclusions depend on assumptions about higher orders

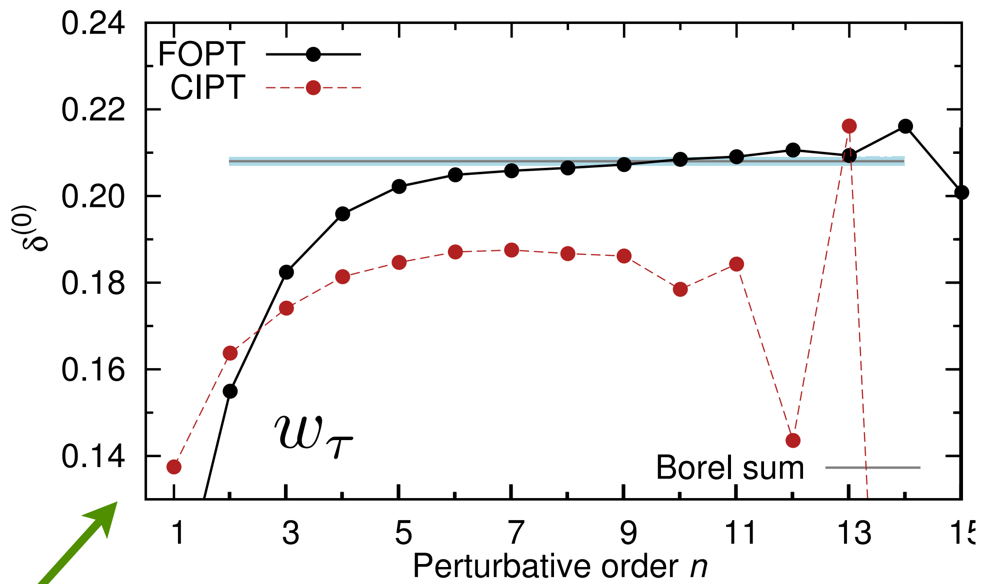
- Systematic study of different weight functions used in literature

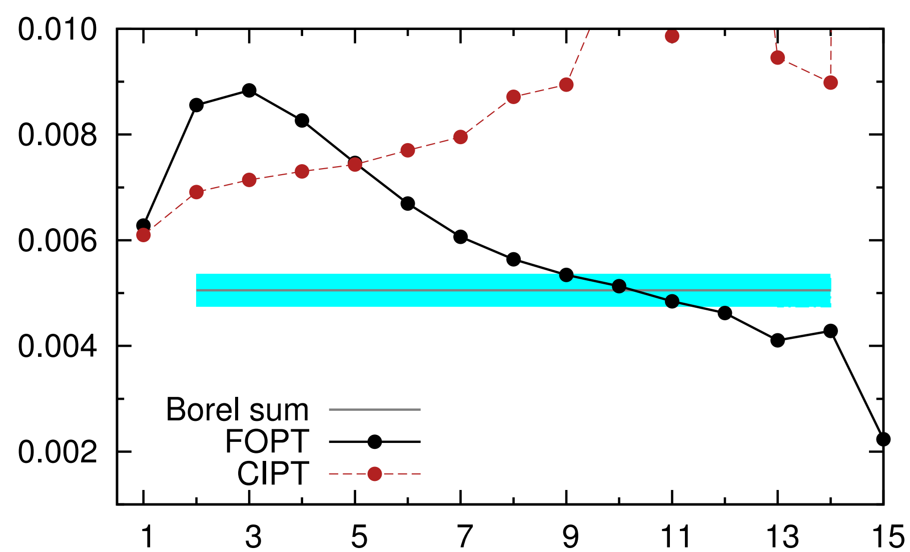
k	$w_k(x)$
1	1
2	x
3	x^2
4	x^3
5	x^4
6	$1-x$
7	$1-x^2$
8	$1-x^3$
9	$1-\frac{3x}{2}+\frac{x^3}{2}$
10	$(1-x)^2$
11	$(1-x)^3$
w_τ	$(1-x)^2(1+2x)$
13	$(1-x)^3(1+2x)$
14	$(1-x)^2 x$
15	$(1-x)^3 x(1+2x)$
16	$(1-x)^3 x^2(1+2x)$
17	$(1-x)^3 x^3(1+2x)$

Good pt. behaviour.

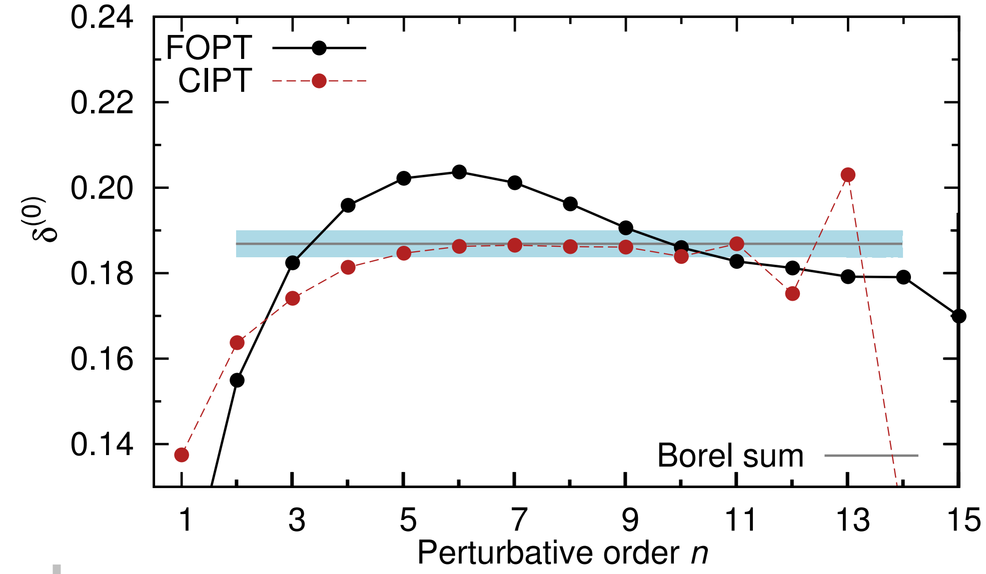
Bad pt. behaviour.

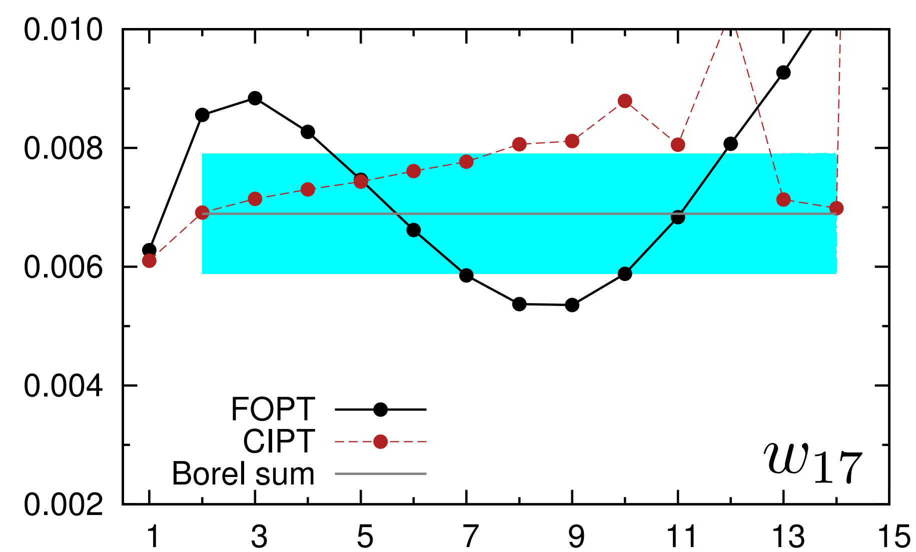
RM





AM





Beneke, Boito, Jamin '13

Summary of conclusions

- A decision in favor of FOPT or CIPT depends on the higher order coefficients.

- Within reasonable assumptions **FOPT is** the **preferred** framework for RGI.

- Some moments are more suitable for the extraction of α_s.

- Some of the recent extractions of α_s employed moments that are not optimal.

- Interesting alternatives to FO-/CIPT using knowledge about Borel plane sing.:
  - Borel sum Beneke and Jamin ‘08
  - Conformal mapping Abbas, Ananthanarayan, Caprini, and Fischer, ‘13

Updated ALEPH data: prospects
(preliminary)

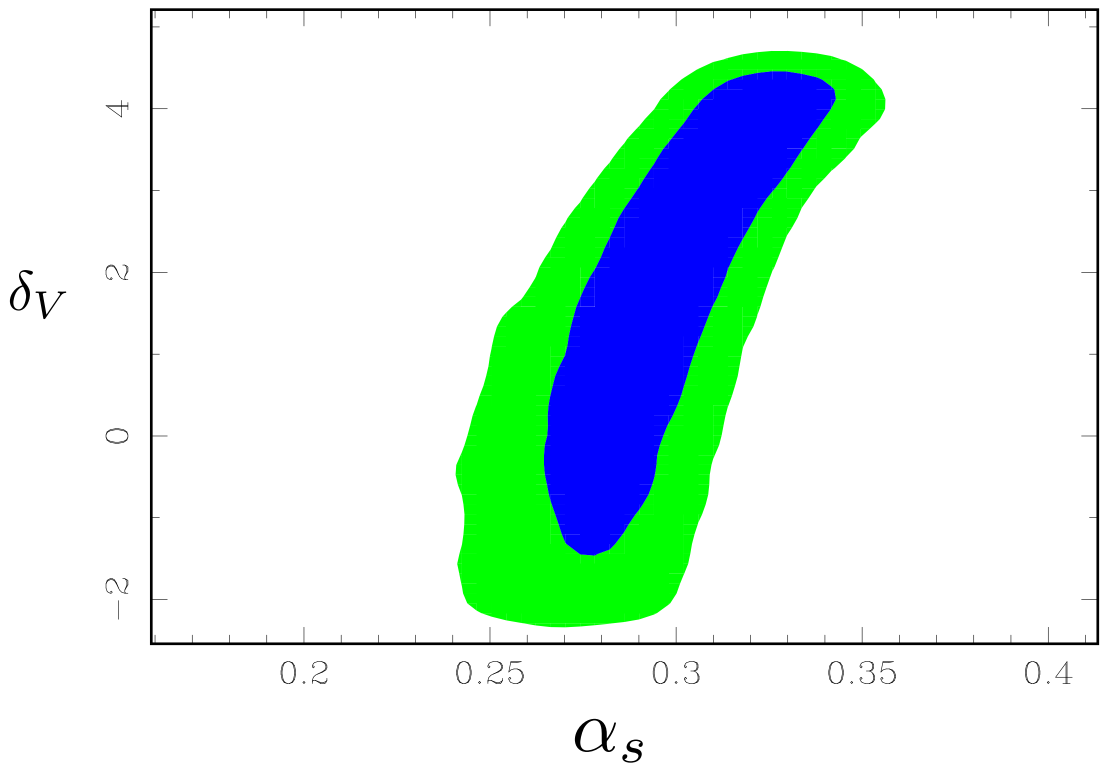
- Smaller uncertainties and correct covariances allow for a better determination of alpha_s and the non-perturbative contributions

- Example: simultaneous determination of DVs and alpha_s from the *V* channel.

Model for DVs: $$\frac{1}{\pi}\mathrm{Im}\Delta(s) = e^{-\delta-\gamma\, s}\sin\left(\alpha+\beta\, s\right)$$

- OPAL data

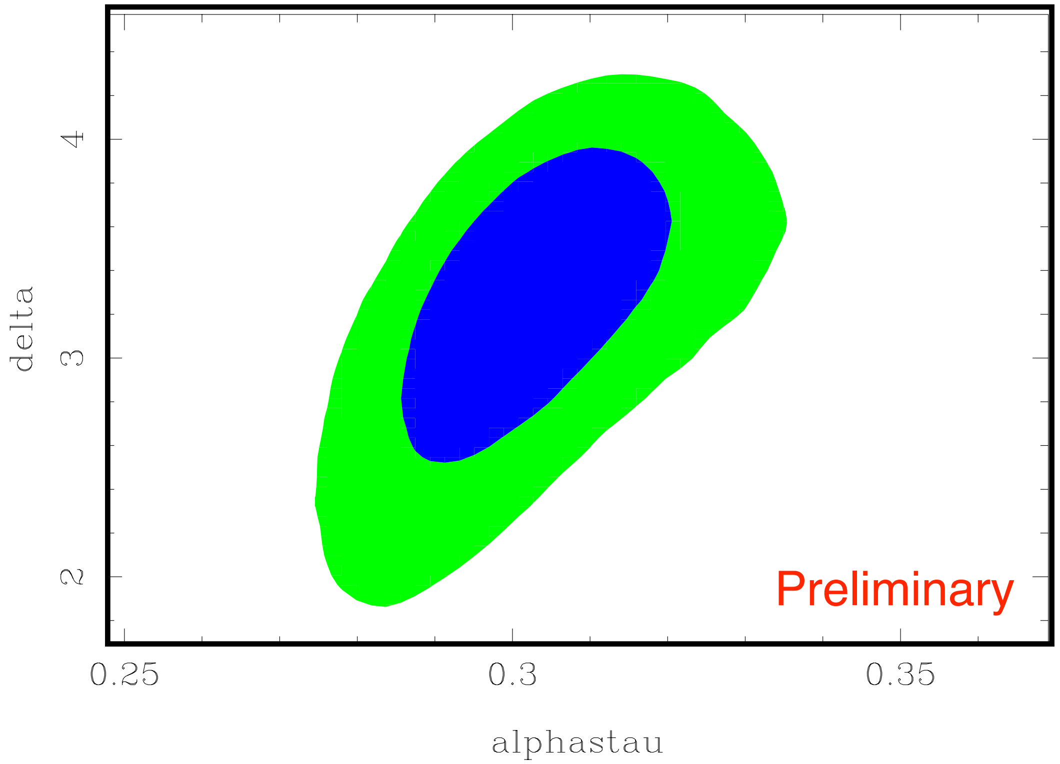
DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, S Peris, '12



$$\sigma_{\alpha_s(m_\tau^2)} \sim 5.5\%$$

- Updated ALEPH data

DB, Golterman, Maltman, Osborne, S Peris, (in preparation)



$$\sigma_{\alpha_s(m_\tau^2)} \sim 3.4\%$$

Updated ALEPH data: prospects

- Smaller uncertainties and correct covariances allow for a better determination of alpha_s and the non-perturbative contributions

- Example: simultaneous determination of DVs and alpha_s from the *V* channel.

Model for DVs: $$\frac{1}{\pi}\mathrm{Im}\Delta(s) = e^{-\delta-\gamma\, s}\sin\left(\alpha+\beta\, s\right)$$

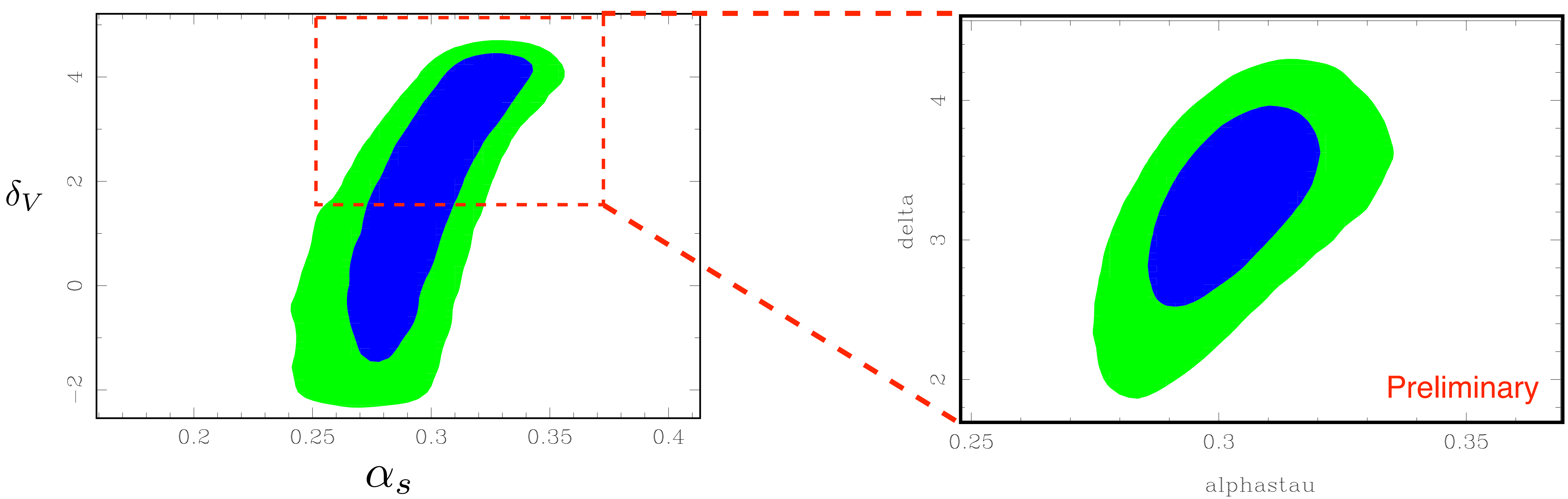
- OPAL data

DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, S Peris, '12

- Updated ALEPH data

DB, Golterman, Maltman, Osborne, S Peris, (in preparation)

Note the different scale



$\sigma_{\alpha_s(m_\tau^2)} \sim 5.5\%$

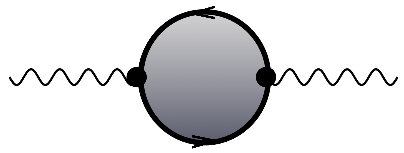
$\sigma_{\alpha_s(m_\tau^2)} \sim 3.4\%$

Good prospects for an independent alpha_s analysis from the updated ALEPH data

- New analysis incorporating recent knowledge.
    - Duality Violations
    - Perturbative behaviour of different moments
- Better control of the non-perturbative contribution.
- New results from the alpha_s analysis this autumn (TAU14/Aachen, September).

Back-up slides

RGImprovement

- Description in terms of the Adler function (derivative of )

$$D^{(1+0)}_{\rm pert}(s) = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\, c_{n,k} \left(\log\frac{-s}{\mu^2}\right)^{k-1} \qquad a_\mu = \alpha(\mu)/\pi$$

- only $c_{n,1}$ are independent (known up to $c_{4,1}$). $c_{n,k}$ depend on $c_{n,1}$ and β_m .
- Prescriptions for the RG improvement

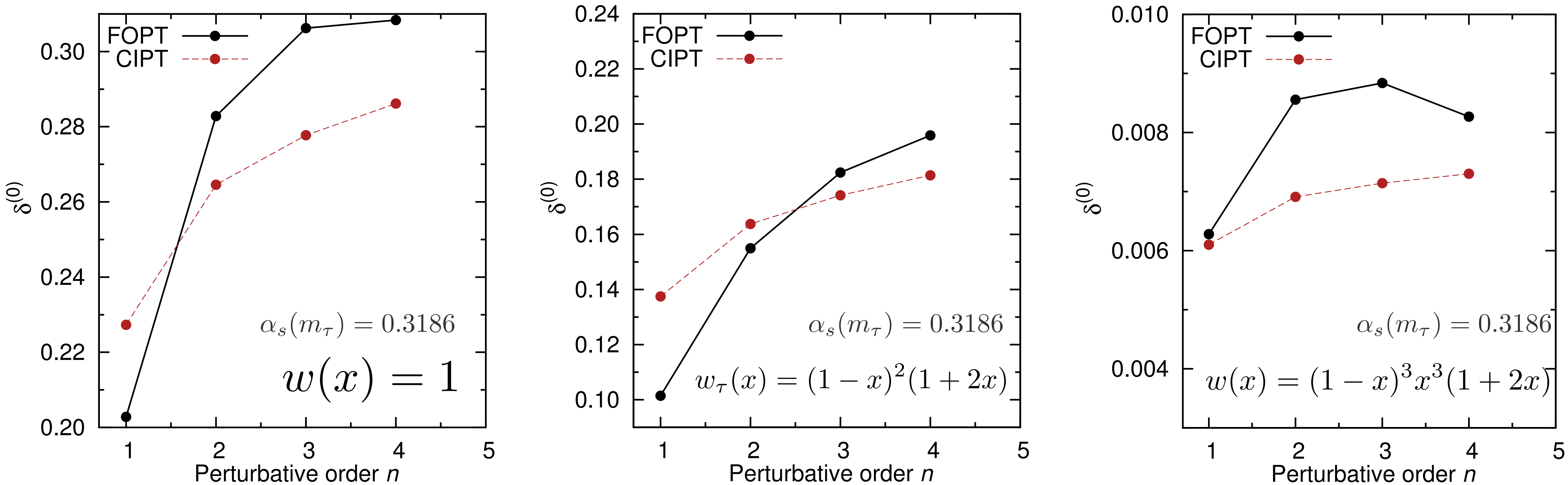
FOPT
$\mu = s_0$

$$\delta^{(0)}_{{\rm FO},w_i} = \sum_{n=1}^{\infty} a(s_0)^n \sum_{k=1}^{n} k\, c_{n,k}\, J^{{\rm FO},w_i}_{k-1} \qquad J^{{\rm FO},w_i}_n \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x} W_i(x)\log^n(-x)$$

CIPT
$\mu = -s_0 x$

$$\delta^{(0)}_{{\rm CI},w_i} = \sum_{n=1}^{\infty} c_{n,1}\, J^{{\rm CI},w_i}_n(s_0) \qquad J^{{\rm CI},w_i}_n(s_0) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x} W_i(x)\, a^n(-s_0 x)$$

Le Diberder and Pich ‘92



Borel transform

$$R \sim \sum_{n}^{\infty} r_n \alpha_{(s)}^{n+1}$$ divergent but (hopefully) **asymptotic**

Dyson 1952

$R \downarrow$? $\qquad$ $r_n \downarrow$ *in QFT we only know the expansion*

- Define the Borel transformed series

$$B[R](t) \equiv \sum_{n=0}^{\infty} r_n \frac{t^n}{n!} \quad \text{which can be "summed"} \Longrightarrow \quad \tilde{R} \equiv \int_0^{\infty} dt\, \mathrm{e}^{-t/\alpha} B[R](t)$$

- Divergent behaviour encoded in the singularities of $B[R](t)$

(review) Beneke 1999

Strategy:

sing. of $B[R](t)$: $\dfrac{c}{(1+at)^b} + \cdots$ *(renormalons)* → $B[R](t)$ → $\sum_{n}^{\infty} r_n \alpha_{(s)}^{n+1}$

$$D_{\rm pert}^{(1+0)}(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} \left(\frac{c_{n,1}}{\pi^n}\right) \alpha_Q^n$$

- General structure of large-order behavior (believed to be) known

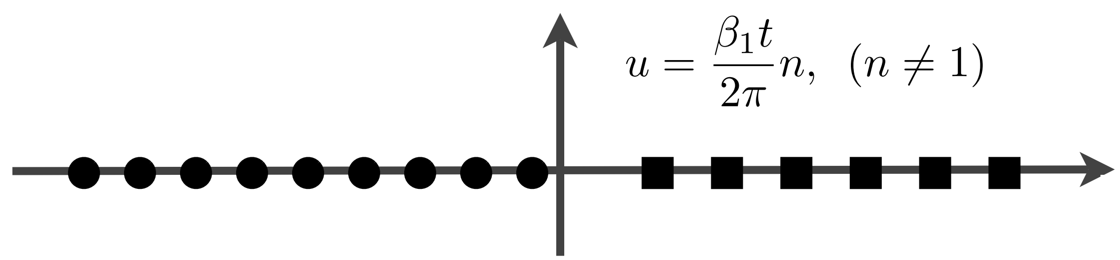
Borel transformed Adler function

(review) Beneke 1999

$$B[\hat{D}](t) \equiv \sum_{n=0}^{\infty} \frac{c_{n,1}}{\pi^n} \frac{t^n}{n!}$$

Borel sum: $$\hat{D}(\alpha) \equiv \int_0^{\infty} dt\, \mathrm{e}^{-t/\alpha}\, B[\hat{D}](t)$$

- Singularities in the *t* plane



UV renormalons

- sign alternating
- leading sing. in the Adler function at *u* =-1
- no-sign alternation in known coeff.: small residue for the leading UV pole

IR renormalons

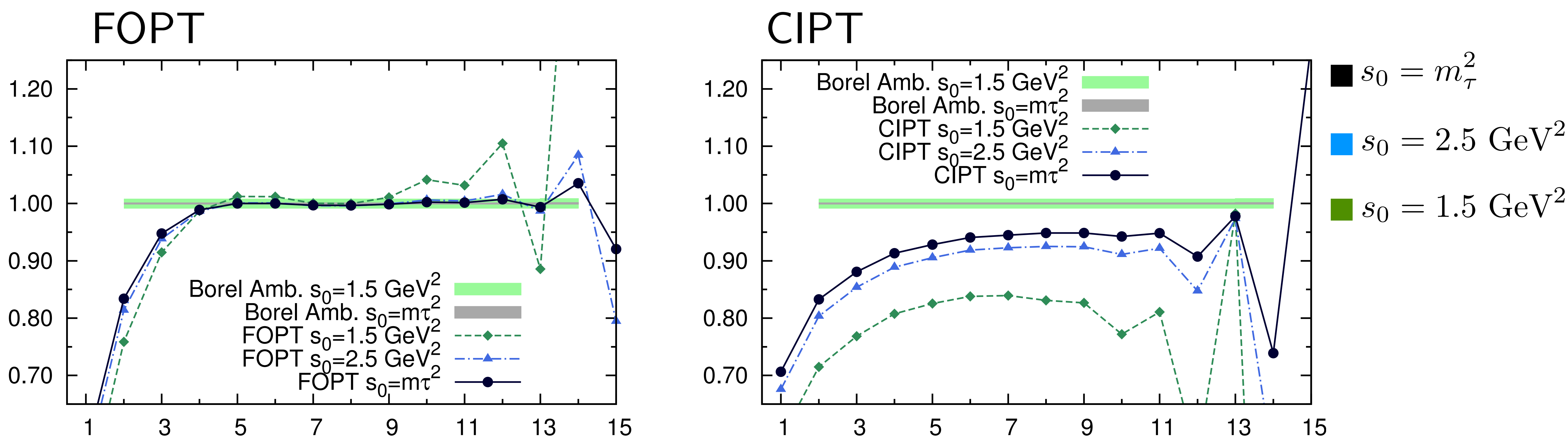
- fixed sign
- sing. at *u* = 2, 3, 4... related to dim-4, dim-6, dim-8... contributions
- *u* = 2 related to the gluon condensate

$$B[D_p] = \frac{c_p}{(p-u)^{\gamma}} \left[1 + \tilde{b}_1 (p-u) + \cdots \right]$$

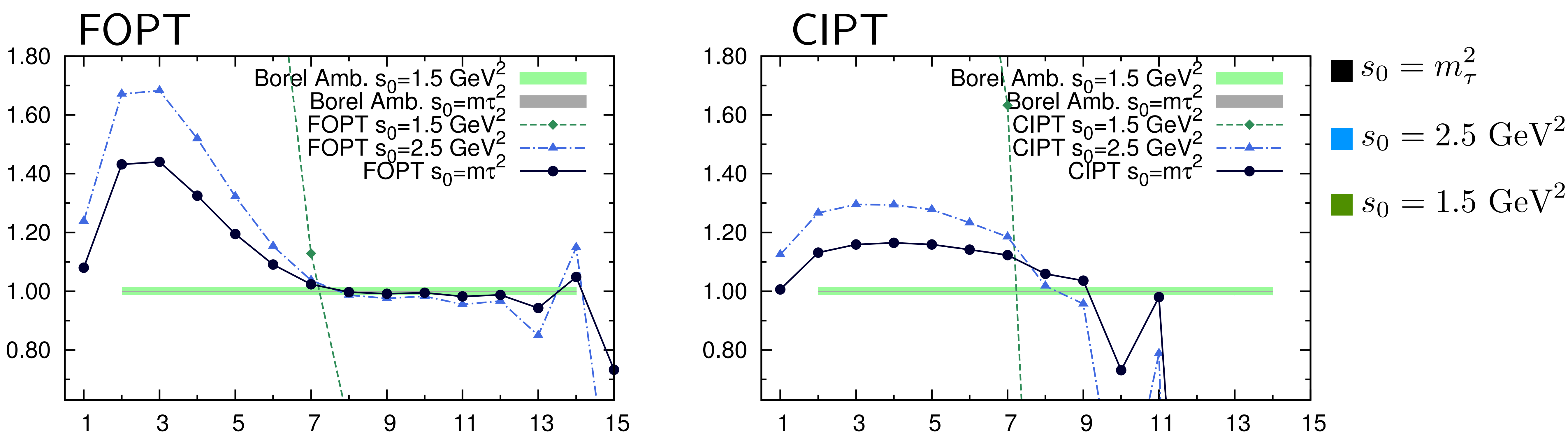
Structure of each singularity in principle calculable (up to)

Reference model

- $w(x) = 1 - x^2$



- $w(x) = (1 - x)^3 x^3 (1 + 2x)$



Reference model

Beneke & Jamin '08

- Separating the contributions in FOPT

$$\delta^{(0)}_{\mathrm{FO},w_i} = \sum_{n=1}^{\infty} \left[c_{n,1}\delta^{\mathrm{tree}}_{w_i} + g_n^{[w_i]} \right] a(s_0)^n \qquad g_n^{[w_i]} = \sum_{k=2}^{n} k\, c_{n,k} J^{\mathrm{FO},w_i}_{k-1}$$

- Result at α_s^n . FOPT sums the first *n* ***rows***. Important cancellations.

w_τ

Fixed Order

α_s^n		$c_{1,1}$	$c_{2,1}$	$c_{3,1}$	$c_{4,1}$	$c_{5,1}$	$c_{6,1}$	$c_{7,1}$	$c_{8,1}$	g_n	$\frac{c_n+g_n}{c_n}$
1		1									1
2	g_2	3.56	+ 1.64							3.56	3.17
3	g_3	8.31	+ 11.7	+ 6.37						20.0	4.14
4	g_4	−20.6	+ 30.5	+ 68.1	+ 49.1					78	2.59
⋮											⋮
6	g_6	−2924	−2858	−2280	2214	5041	3275			−807	0.754
⋮											⋮
8	g_8	14652	−29552	−145846	−502719	−393887	260511	467787	388442	−329054	0.153

- CIPT sums the first *n* ***columns to all orders***. Misses the cancellations.