\( \alpha_s \) from \( \tau \) decays: higher orders and perturbative behaviour

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-- DB, PoS Confinement X (2012)
Outline

- Introduction

- Recent developments in alpha_s from tau decays
  - Moments, higher orders and FOPT vs CIPT
  - Duality Violations
  - Updated ALEPH data (correct correlation matrices)

- Prospects
  - New analysis of updated ALEPH data (in progress, no new alpha_s value)
Spread in the results reflect (mainly) details of the theoretical input.

There are still open questions (Renormalization Group Improvement, duality violations, ...)

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Introduction

- Sum rules for the spectral functions
  (in tau decays) Braaten, Narison, and Pich, 1992

\[
\int_0^{s_0} ds \, w(s) \frac{1}{\pi} \text{Im} \tilde{\Pi}(s) = \frac{-1}{2\pi i} \int \frac{dz}{|z|=s_0} w(z) \tilde{\Pi}(z)
\]

- Contributions to the sum rule (theory side)

\[
R_{V/A}^{w_i}(s_0) = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left[ \delta_{w_i}^{\text{tree}} + \delta_{w_i}^{(0)}(s_0) + \sum_{D \geq 2} \delta_{w_i,V/A}^{(D)}(s_0) + \delta_{w_i,V/A}^{DV}(s_0) \right]
\]

\[
\alpha_s^2 + \alpha_s^3 + \alpha_s^4 + \cdots
\]

\[\alpha_s^4 : \text{Baikov, Chetyrkin, Kühn 2008}\]
Recent developments in $\alpha_s$ from tau decays
Recent developments in alpha_s from tau decays

- **Systematic study of the weight function dependence**

\[
\int_0^{s_0} ds w(s) \frac{1}{\pi} \text{Im}\tilde{\Pi}(s) = \frac{-1}{2\pi i} \oint_{|z|=s_0} dz w(z) \tilde{\Pi}(z)
\]

- Convergence of pt. series is weight function dependent. What are *ideal* weight functions?

- Several moments used in literature. Renormalization group: what’s best CIPT or FOPT? (Contour Improved Perturbation Theory or Fixed Order Perturbation Theory)

- **Duality violations**

\[
R_{V/A}^{w_i}(s_0) = \frac{N_c}{2} S_{EW} |V_{ud}|^2 \left[ \delta_{w_i}^{(\text{tree})} + \delta_{w_i}^{(0)}(s_0) + \sum_{D \geq 2} \delta_{w_i,V/A}^{(D)}(s_0) + \delta_{w_i,V/A}^{DV}(s_0) \right]
\]

- Resonance effects close to the real axis.
- Must take them into account (at the very least for error calculation).
- One must resort to reasonable models based on QCD.
- Source of uncertainty not considered in the “standard” analysis.

Beneke, DB, Jamin, ‘13
Catà, Golterman, & Peris, ’05, ’09; Boito et al ’11, ’12.
Davier et al 2013
Updated ALEPH data sets

- Updated ALEPH data for spectral functions

Davier, Höcker, Malaescu, Yuan, Zhang, 1312.1501v2

- Corrected covariances matrix (due to unfolding).

- Corrects a problem discovered the previous version (05/08) of the ALEPH data.

- Smaller uncertainties than the OPAL data.

Corrects a problem discovered the previous version (05/08) of the ALEPH data.

DB, Catà, Golterman, Jamin, Maltman, Osborne and Peris, ‘11

Smaller uncertainties than the OPAL data.
Perturbative series, weight functions, and RG improvement

Beneke, DB, Jamin, ‘13
A new determination of $\alpha_s$ from $\tau$ decays

- RG improvement: main theoretical uncertainty. Weight function dependent.

- Strategy: reasonable model for the leading singularities of the Borel transformed perturbative series.

\[ w(x) = 1 \]
\[ w(x) = (1 - x)^2(1 + 2x) \]
\[ w(x) = (1 - x)^3x^3(1 + 2x) \]

\[ \alpha_s(m_\tau) = 0.3186 \]

First 4 coefficients are fixed from loop computations.
A new determination of \( \alpha_s \) from \( \tau \) decays

**Conclusions depend on assumptions about higher orders**

**Reference model (RM)**  
Beneke, Jamin ‘08

\[
B[\hat{D}](u) = B[\hat{D}_1^{UV}](u) + B[\hat{D}_2^{IR}](u) + B[\hat{D}_3^{IR}](u) + d_0^{PO} + d_1^{PO} u
\]

- Model with the leading UV and the first two IR singularities.
- Favors FOPT (related to the presence of \( u = 2 \) sing).
- No IR singularity at \( u = 2 \).
- Favors CIPT.

**Alternative Model (AM)**  
Beneke, Boito, Jamin ‘13

\[
B[\hat{D}](u) = B[\hat{D}_1^{UV}](u) + B[\hat{D}_3^{IR}](u) + B[\hat{D}_4^{IR}](u) + d_0^{PO} + d_1^{PO} u
\]

**References**

- Beneke, Boito, Jamin ‘13
- Beneke, Jamin ‘08

**Perturbative order**

- FOPT
- CIPT

**Borel sum**

**PT series, higher orders, RG improvement**
Systematic study of different weight functions used in literature

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<td>$(1 - x)^3x^3(1 + 2x)$</td>
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</table>

Beneke, Boito, Jamin '13

Good pt. behaviour.

Bad pt. behaviour.

PT series, higher orders, RG improvement

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A new determination of $\alpha_s$ from $\tau$ decays

Summary of conclusions

- A decision in favor of FOPT or CIPT depends on the higher order coefficients.

- Within reasonable assumptions **FOPT is the preferred** framework for RGI.

- Some moments are more suitable for the extraction of $\alpha_s$.

- Some of the recent extractions of $\alpha_s$ employed moments that are not optimal.

- Interesting alternatives to FO-/CIPT using knowledge about Borel plane sing.:
  - Borel sum  *Beneke and Jamin ‘08*
  - Conformal mapping  *Abbas, Ananthanarayan, Caprini, and Fischer, ‘13*
Updated ALEPH data: prospects
(preliminary)
Smaller uncertainties and correct covariances allow for a better determination of $\alpha_s$ and the non-perturbative contributions.

Example: simultaneous determination of DVs and $\alpha_s$ from the $V$ channel.

Model for DVs:
$$\frac{1}{\pi} \text{Im} \Delta(s) = e^{-\delta - \gamma s} \sin(\alpha + \beta s)$$

OPAL data

DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, S Peris, '12

Updated ALEPH data

DB, Golterman, Maltman, Osborne, S Peris, (in preparation)

$\sigma_{\alpha_s}(m_{\tau}^2) \sim 5.5\%$

$\sigma_{\alpha_s}(m_{\tau}^2) \sim 3.4\%$
Updated ALEPH data: prospects

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- OPAL data
- Updated ALEPH data

DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, S Peris, ‘12

DB, Golterman, Maltman, Osborne, S Peris, (in preparation)

Note the different scale

$\sigma_{\alpha_s}(m_{\tau}^2) \sim 5.5\%$

$\sigma_{\alpha_s}(m_{\tau}^2) \sim 3.4\%$
Final remarks

Good prospects for an independent $\alpha_s$ analysis from the updated ALEPH data

- New analysis incorporating recent knowledge.
  - Duality Violations
  - Perturbative behaviour of different moments
- Better control of the non-perturbative contribution.
- New results from the $\alpha_s$ analysis this autumn (TAU14/Aachen, September).
A new determination of $\alpha_s$ from $\tau$ decays

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Back-up slides
**Description in terms of the Adler function (derivative of $\gamma$)**

\[
D^{(1+0)}(\tau) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} a_\mu n \sum_{k=1}^{n+1} c_{n,k} \left( \log \frac{s}{\mu^2} \right)^{k-1}
\]

\[
a_\mu = \alpha(\mu)/\pi
\]

- only $c_{n,1}$ are independent (known up to $c_{4,1}$). $c_{n,k}$ depend on $c_{n,1}$ and $\beta_m$.

**Prescriptions for the RG improvement**

**FOPT**

\[
\mu = s_0
\]

\[
\delta^{(0)}_{FO,w_i} = \sum_{n=1}^{\infty} a(s_0)^n \sum_{k=1}^{n} k c_{n,k} J^{FO,w_i}_{k-1}
\]

\[
J^{FO,w_i}_{n} \equiv \frac{1}{2\pi i} \int_{|x|=1} \frac{dx}{x} W_i(x) \log^n(-x)
\]

**CIPT**

\[
\mu = -s_0 x
\]

\[
\delta^{(0)}_{CI,w_i} = \sum_{n=1}^{\infty} c_{n,1} J^{CI,w_i}_{n}(s_0)
\]

\[
J^{CI,w_i}_{n}(s_0) \equiv \frac{1}{2\pi i} \int_{|x|=1} \frac{dx}{x} W_i(x) a^n(-s_0 x)
\]

*Le Diberder and Pich ‘92*

\[
\alpha_s(m_\tau) = 0.3186
\]

\[
w(x) = 1
\]

\[
w(x) = (1-x)^2(1+2x)
\]

\[
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A new determination of $\alpha_s$ from $\tau$ decays

Dyson 1952

\[ R \sim \sum_{n} \infty r_n \alpha_{(s)}^{n+1} \] divergent but (hopefully) asymptotic

in QFT we only know the expansion

- Define the Borel transformed series

\[ B[R](t) \equiv \sum_{n=0}^{\infty} r_n \frac{t^n}{n!} \] which can be “summed” \[ \tilde{R} \equiv \int_{0}^{\infty} dt \, e^{-t/\alpha} B[R](t) \]

- Divergent behaviour encoded in the singularities of $B[R](t)$

Strategy:

Dyson 1952 (review) Beneke 1999

\[ \frac{c}{(1 + at)^b} + \cdots \] (renormalons)

\[ D_{\text{pert}}^{(1+0)}(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} \left( \frac{c_{n,1}}{\pi^n} \right) \alpha_Q^n \]
General structure of large-order behavior (believed to be) known

Borel transformed Adler function

\[ B[\hat{D}](t) \equiv \sum_{n=0}^{\infty} \frac{c_{n,1}}{\pi^n} \frac{t^n}{n!} \]

Borel sum:

\[ \hat{D}(\alpha) \equiv \int_{0}^{\infty} dt \, e^{-t/\alpha} \, B[\hat{D}](t) \]

Singularities in the t plane

- UV renormalons
  - sign alternating
  - leading sing. in the Adler function at \( u = -1 \)
  - no-sign alternation in known coeff.: small residue for the leading UV pole

- IR renormalons
  - fixed sign
  - sing. at \( u = 2, 3, 4... \) related to dim-4, dim-6, dim-8... contributions
  - \( u = 2 \) related to the gluon condensate

Structure of each singularity in principle calculable (up to ...)

\[ B[D_p] = \frac{c_p}{(p-u)^\gamma} \left[ 1 + \tilde{b}_1(p-u) + \cdots \right] \]
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$w(x) = 1 - x^2$

Reference model

Energy dependence

$w(x) = (1 - x)^3 x^3 (1 + 2x)$
Why FOPT is better in the RM

Reference model

Beneke & Jamin ‘08

Separating the contributions in FOPT

\[
\delta_{FO,w_i}^{(0)} = \sum_{n=1}^{\infty} \left[ c_{n,1} \delta_{w_i}^{\text{tree}} + g_n^{[w_i]} \right] a(s_0)^n
\]

\[
g_n^{[w_i]} = \sum_{k=2}^{n} k c_{n,k} J_{k-1}^{FO,w_i}
\]

Result at \( \alpha_s^n \). FOPT sums the first \( n \) rows. Important cancellations.

\[
\begin{array}{cccccccccc}
\alpha_s^n & c_{1,1} & c_{2,1} & c_{3,1} & c_{4,1} & c_{5,1} & c_{6,1} & c_{7,1} & c_{8,1} & g_n & \frac{c_n+g_n}{c_n} \\
1 & 1 & & & & & & & & & 1 \\
2 & g_2 & 3.56 + 1.64 & & & & & & & 3.56 & 3.17 \\
3 & g_3 & 8.31 + 11.7 + 6.37 & & & & & & & 20.0 & 4.14 \\
4 & g_4 & -20.6 + 30.5 + 68.1 & + 49.1 & & & & & & 78 & 2.59 \\
\vdots & & & & & & & & & & \\
6 & g_6 & -2924 & -2858 & -2280 & 2214 & 5041 & 3275 & & -807 & 0.754 \\
\vdots & & & & & & & & & & \\
8 & g_8 & 14652 & -29552 & -145846 & -502719 & -393887 & 260511 & 467787 & 388442 & -329054 & 0.153 \\
\end{array}
\]

CIPT sums the first \( n \) columns to all orders. Misses the cancellations.