

**Vacuum stability**  
**Higgs and Top masses, and New Physics**

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V. Branchina, E. Messina, Phys.Rev.Lett.111, 241801 (2013)

V. Branchina, arXiv:1405.7864

work in progress with : E. Messina, A. Platania, M. Sher

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## Discovery of the Higgs boson : $M_H = 125 - 126 \text{ GeV}$

Experimental data consistent with Standard Model predictions

No sign of new physics

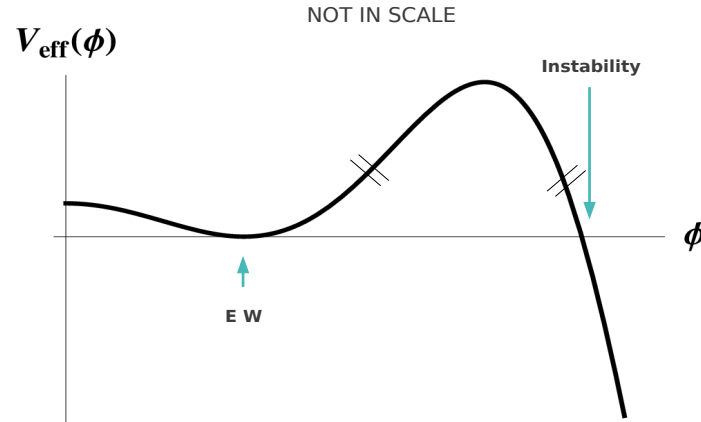
Boost new interest and work on earlier speculations

Possibility for new physics to show up only at very high energies

Largely explored scenario: new physics only appears at  $M_P$

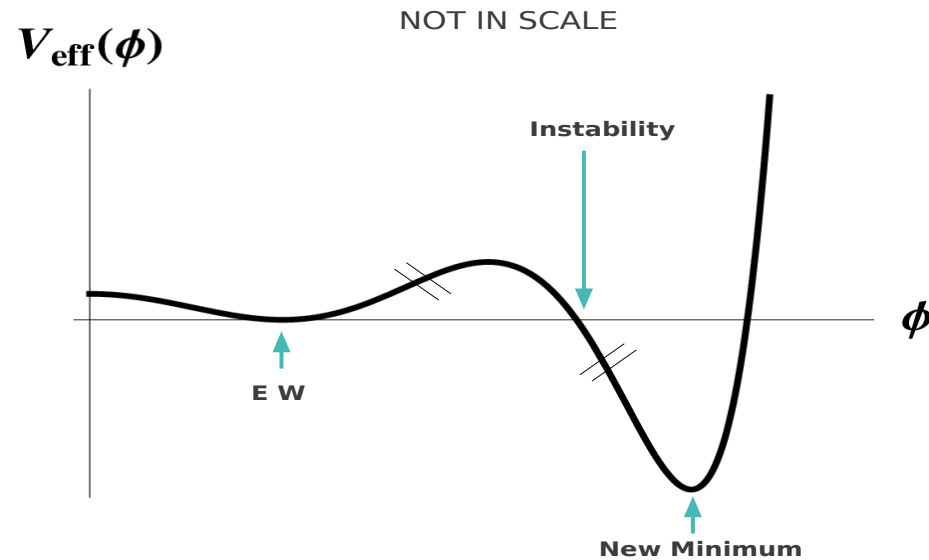
Where do these ideas come from?

## Higgs One-Loop Effective Potential $V^{1l}(\phi)$



$$\begin{aligned}
 V^{1l}(\phi) = & \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{1}{64\pi^2} \left[ \left(m^2 + \frac{\lambda}{2}\phi^2\right)^2 \left(\ln\left(\frac{m^2 + \frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right) \right. \\
 & + 3 \left(m^2 + \frac{\lambda}{6}\phi^2\right)^2 \left(\ln\left(\frac{m^2 + \frac{\lambda}{6}\phi^2}{\mu^2}\right) - \frac{3}{2}\right) + 6 \frac{g_1^4}{16}\phi^4 \left(\ln\left(\frac{\frac{1}{4}g_1^2\phi^2}{\mu^2}\right) - \frac{5}{6}\right) \\
 & \left. + 3 \frac{(g_1^2 + g_2^2)^2}{16}\phi^4 \left(\ln\left(\frac{\frac{1}{4}(g_1^2 + g_2^2)\phi^2}{\mu^2}\right) - \frac{5}{6}\right) - 12 h_t^4\phi^4 \left(\ln\left(\frac{h_t^2\phi^2}{\mu^2}\right) - \frac{3}{2}\right) \right]
 \end{aligned}$$

## RG Improved Effective Potential $V_{eff}(\phi)$

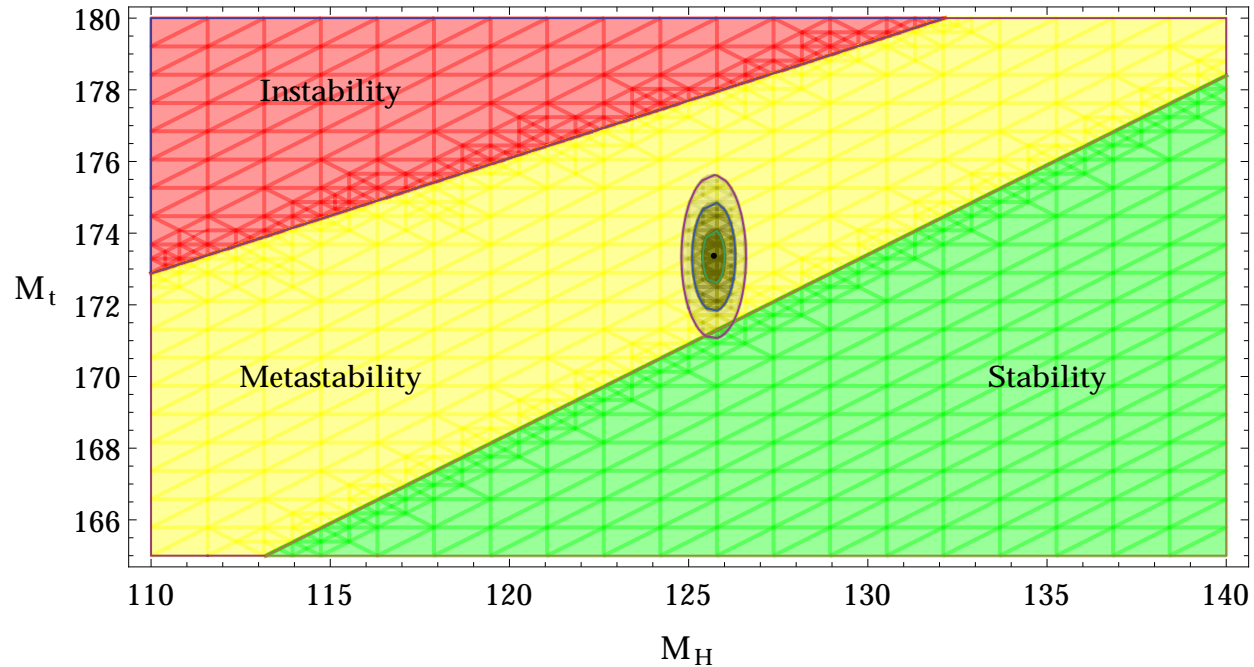


Depending on  $M_H$  and  $M_t$ , the second minimum at  $\phi_{min}^{(2)}$  can be :

(1) **lower** than the EW minimum (as in the figure) ; (2) at the **same level** of the EW minimum ; (3) **higher** than the EW minimum.

$V_{eff}(\phi)$  is obtained by considering SM interactions only

## Phase Diagram in the $M_H - M_t$ plane

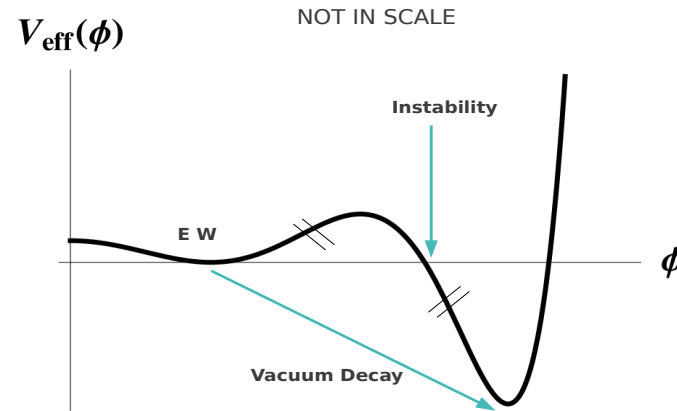


**Stability** region :  $V_{eff}(v) < V_{eff}(\phi_{min}^{(2)})$ . **Meta-stability** region :  $\tau > T_U$ .

**Instability** region :  $\tau < T_U$ . Dashed line :  $V_{eff}(v) = V_{eff}(\phi_{min}^{(2)})$ . Dashed - dotted line :  $M_H$  and  $M_t$  such that  $\tau = T_U$ .

**Remember** :  $V_{eff}(\phi)$  obtained by considering SM interactions only

## Tunnelling : EW vacuum lifetime



Tunnelling between the **Metastable EW Vacuum** and the **True Vacuum**.

As long as **EW vacuum lifetime** larger than the age of the Universe ...

.... we may well live in the **Meta-Stable (EW) Vacuum** ....

(**metastability scenario**)

EW vacuum lifetime ( = **Tunneling Time**  $\tau$  )

$$\Gamma = \frac{1}{\tau} = T_U^3 \frac{S[\phi_b]^2}{4\pi^2} \left| \frac{\det' [-\partial^2 + V''(\phi_b)]}{\det [-\partial^2 + V''(v)]} \right|^{-1/2} e^{-S[\phi_b]}$$

$\phi_b(r)$  : **Bounce Solution**

**Solution to the Euclidean Equation of Motion with appropriate boundary conditions**

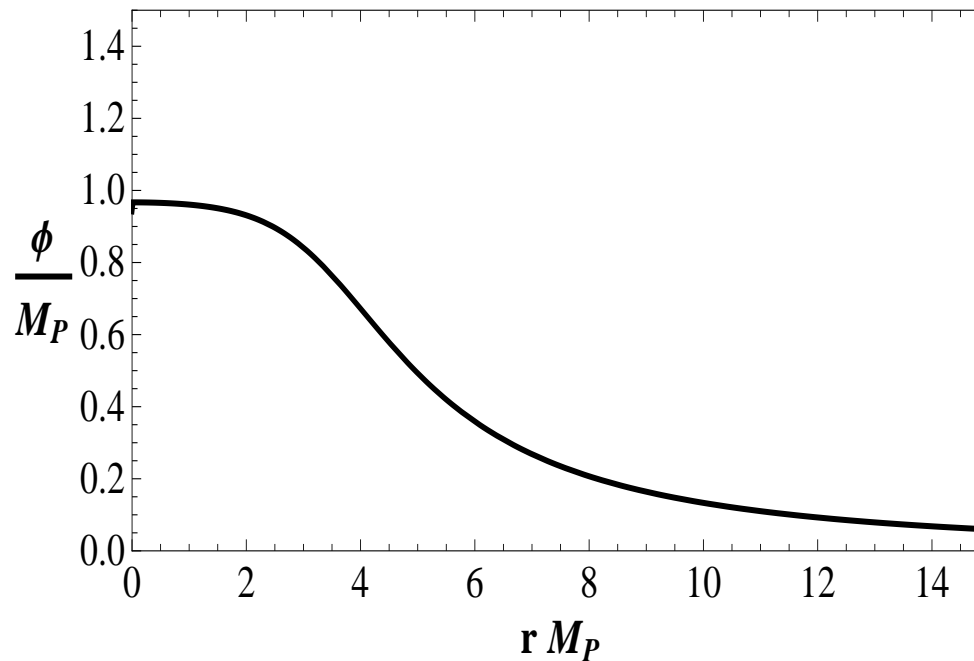
S. Coleman, Phys. Rev. D 15 (1977) 2929

C.G.Callan, S.Coleman, Phys. Rev. D 16 (1977) 1762

Bounces :

$$\phi_b(r) = \sqrt{\frac{2}{|\lambda|}} \frac{2R}{r^2 + R^2}$$

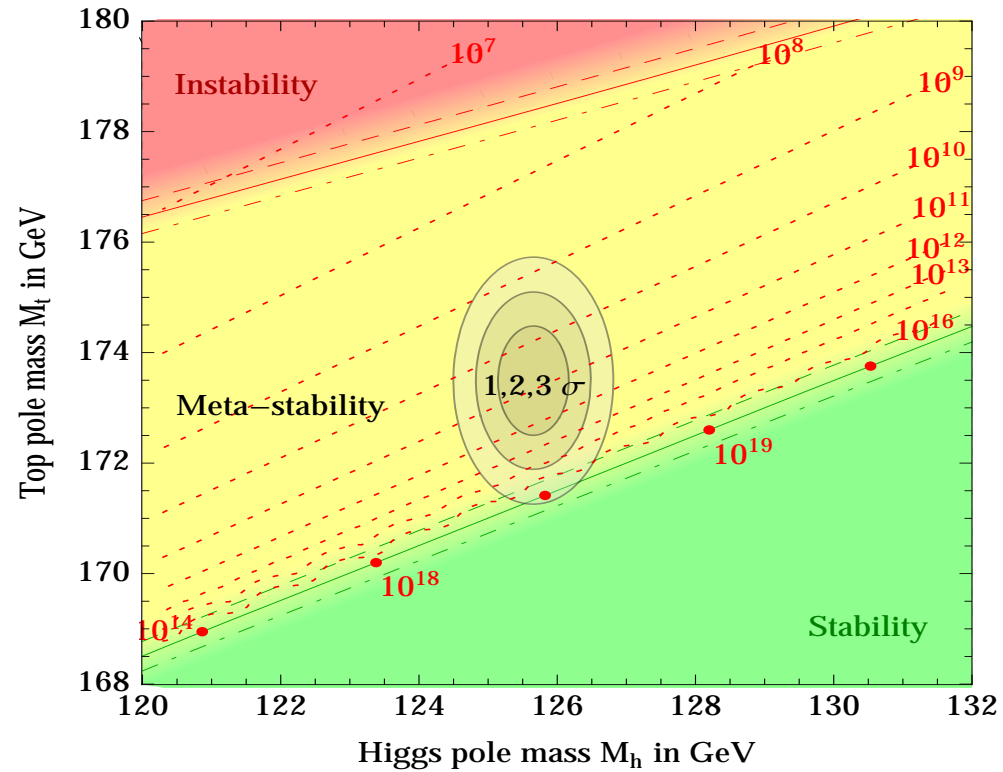
$R =$  bounce size – Classical degeneracy :  $S[\phi_b] = \frac{8\pi^2}{3|\lambda|}$



Degeneracy removed at the Quantum Level

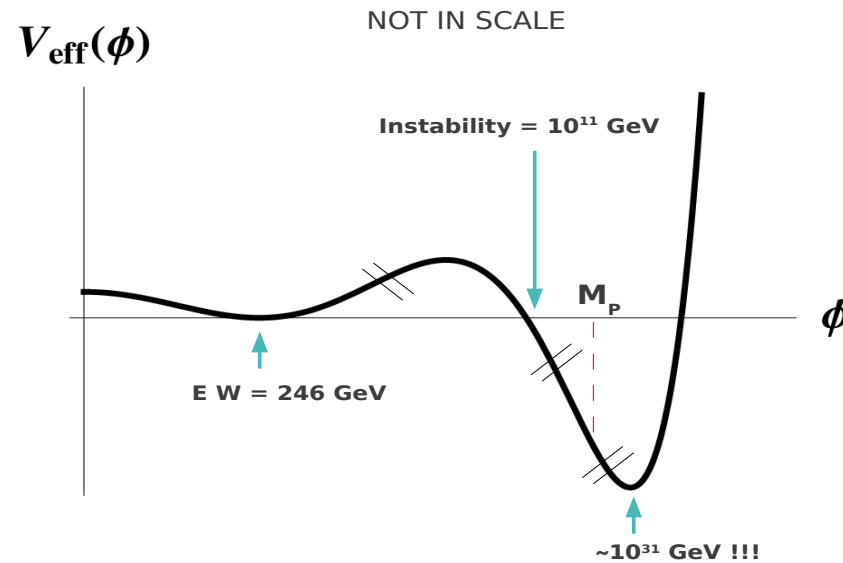


## With this Heavy Artillery $\Rightarrow$ Phase Diagram



Buttazzo, Degrandi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP 1312 (2013) 089.

Probably worth to know that for  $M_H \sim 126 \text{ GeV}$  and  $M_t \sim 173 \text{ GeV}$



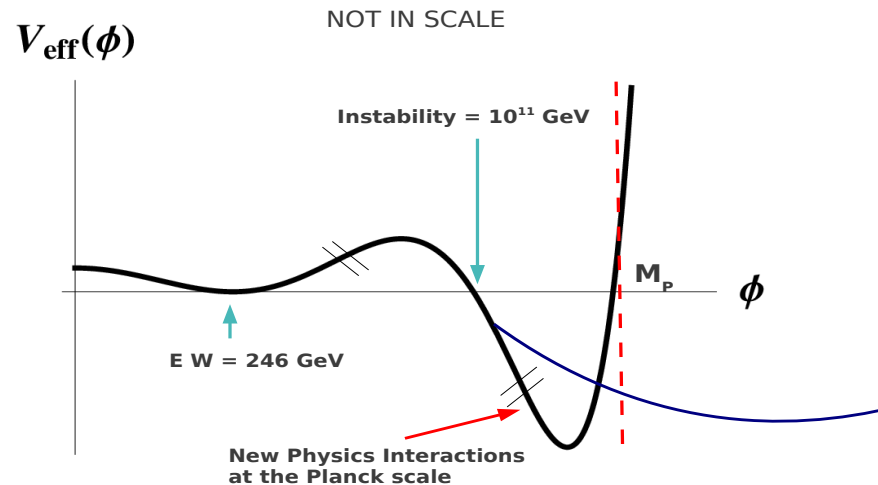
New minimum at  $\phi_{\text{min}}^{(2)} \sim 10^{30} \text{ GeV} !!!!$

SM Effective Potential extrapolated well above  $M_P$  !!!

Does it make any sense ??? Is this a problem or not ???

To make sense out of this potential, people have some arguments ...

1. New Physics Interactions that appear at the Planck scale  $M_P$  eventually stabilize the potential around  $M_P$



2. These New Physics Interactions present at the Planck scale do not affect the EW vacuum lifetime  $\tau$  (can be neglected when computing  $\tau$ )

(a) - Instability scale much lower than Planck scale  $\Rightarrow$  suppression  $\left(\frac{\Delta_{inst}}{M_P}\right)^n$

(b) - For tunnelling, only height of the barrier and turning points matter

**Let us consider New Physics at  $M_P$**

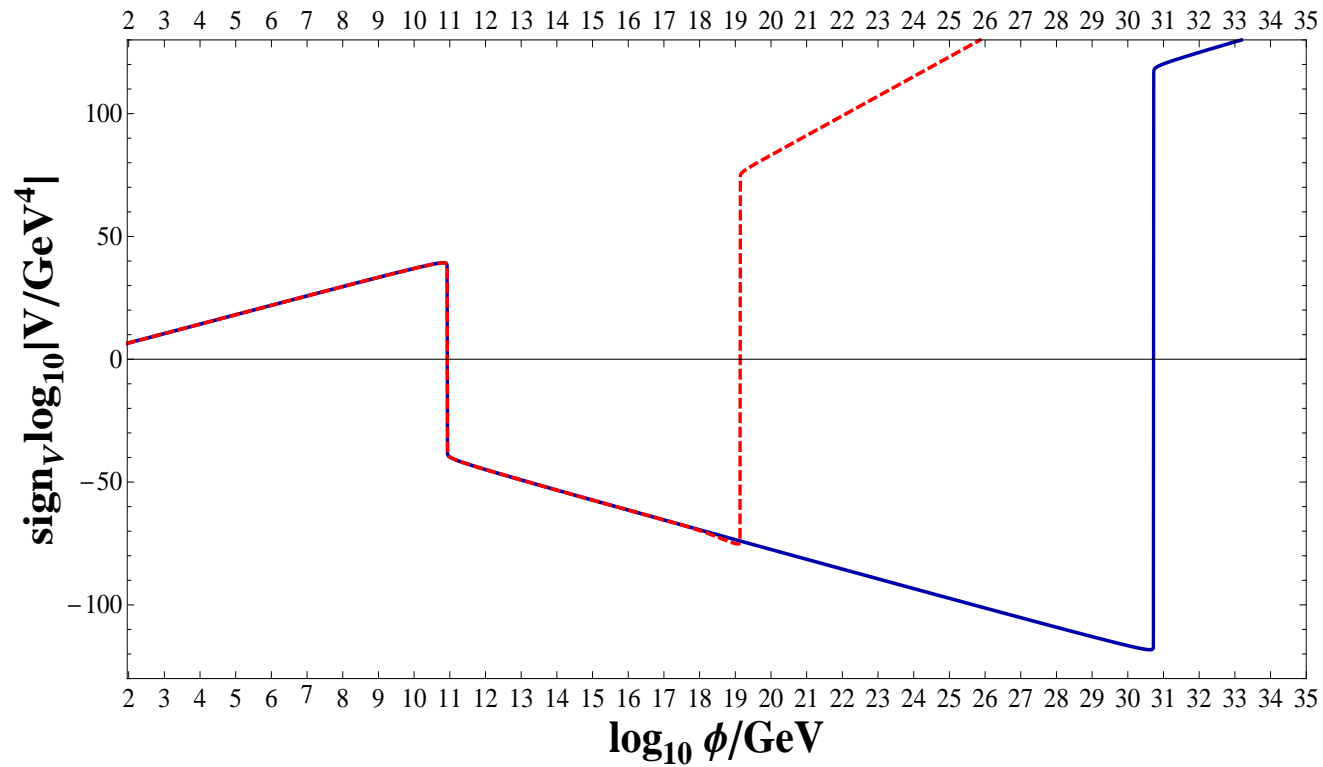
**Add, for instance,  $\phi^6$  and  $\phi^8$  to the SM Higgs potential:**

$$V(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

**Higgs Effective Potential modified :**

$$V_{eff}^{new}(\phi) = V_{eff}(\phi) + \frac{\lambda_6(\phi)}{6M_P^2} \xi(\phi)^6 \phi^6 + \frac{\lambda_8(\phi)}{8M_P^4} \xi(\phi)^8 \phi^8$$

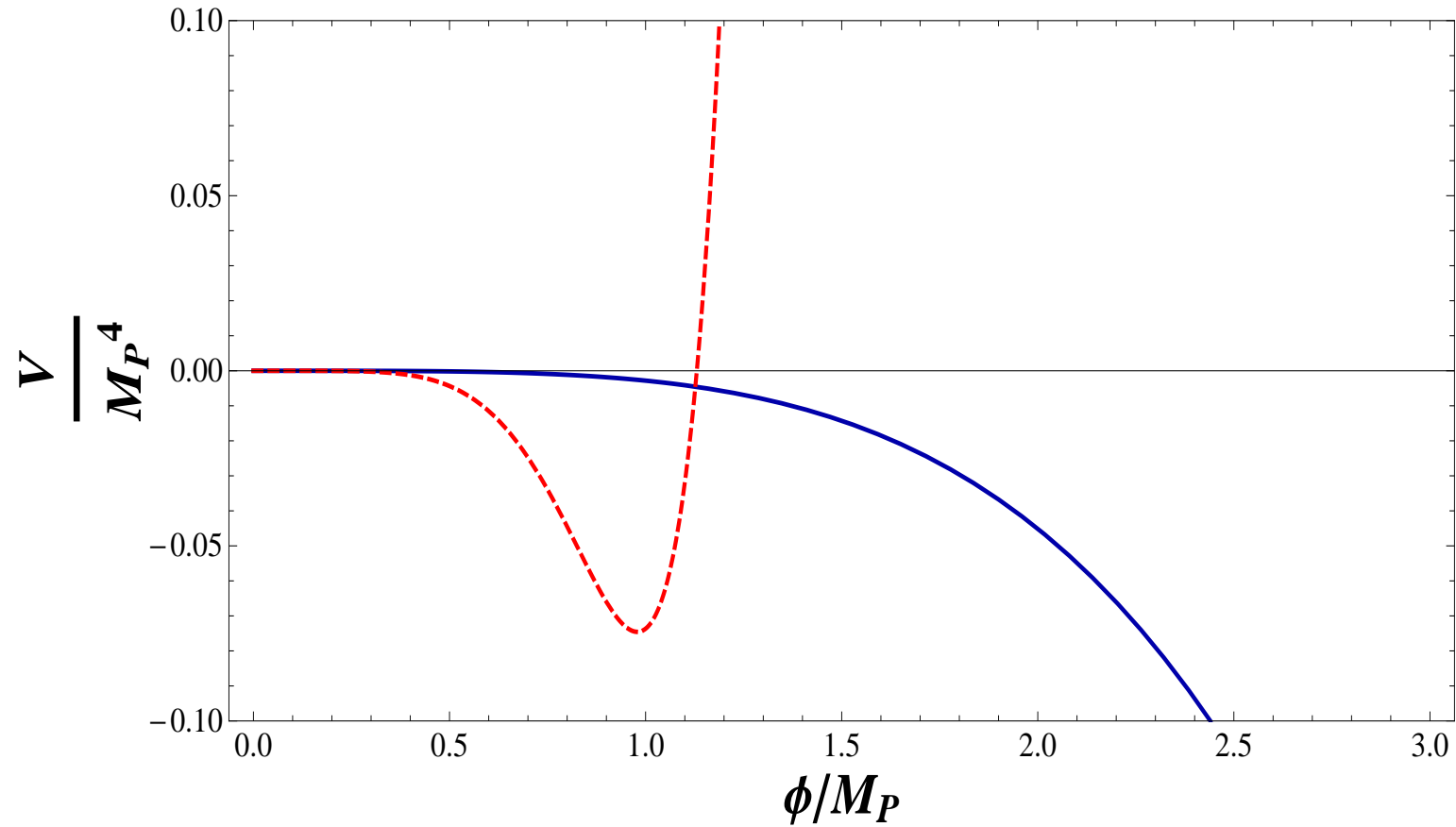
**Effective Potential**     $M_H \sim 126$      $M_t \sim 173$     **Log-Log Plot**



**Blue line :**     $V_{eff}(\phi)$     no higher order terms

**Red line :**     $V_{eff}^{new}(\phi)$     with  $\lambda_6(M_P) = -2$      $\lambda_8(M_P) = 2.1$

## Zoom around the Planck scale



**Blue line :**  $V_{eff}(\phi)$  no higher order terms

**Red line :**  $V_{eff}^{new}(\phi)$  with  $\lambda_6(M_P) = -2$   $\lambda_8(M_P) = 2.1$

We have a New Potential  $\Rightarrow$  we have to consider new bounce configurations for the computation of the tunnelling time

$$V(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

In the computation of the EW vacuum lifetime :

Competition between

Old Bounce  $\phi_b^{(Old)}(r)$  and the New Bounce  $\phi_b^{(New)}(r)$

**New Physics not included : Only  $\phi_b^{(old)}$  (Literature case)**

$$\Gamma = \frac{1}{\tau} = \frac{1}{T_U} \left[ \frac{S[\phi_b^{(old)}]^2}{4\pi^2} \frac{T_U^4}{R_M^4} e^{-S[\phi_b^{(old)}]} \right] \times [e^{-\Delta S_1}]$$

**New Physics included :  $\phi_b^{(new)}$  and  $\phi_b^{(old)}$  (Our case)**

$$\begin{aligned} \Gamma = \Gamma_1 + \Gamma_2 = \frac{1}{\tau_1} + \frac{1}{\tau_2} &= \frac{1}{T_U} \left[ \frac{S[\phi_b^{(old)}]^2}{4\pi^2} \frac{T_U^4}{R_M^4} e^{-S[\phi_b^{(old)}]} \right] \times [e^{-\Delta S_1}] \\ &+ \frac{1}{T_U} \left[ \frac{S[\phi_b^{(new)}]^2}{4\pi^2} \frac{T_U^4}{R^4} e^{-S[\phi_b^{(new)}]} \right] \times [e^{-\Delta S_2}] \end{aligned}$$

**Neglecting for a moment the  $\Delta S$  (quantum) contributions**

**Literature :  $S[\phi_b^{(old)}] \sim 1833 \Rightarrow \tau \sim 10^{555} T_U$**

**Our case :  $S[\phi_b^{(new)}] \sim 82 \Rightarrow \tau \sim 10^{-208} T_U$**

**Contribution from  $\phi_b^{(old)}$  exponentially suppressed !**

**New Physics Interactions at the Planck scale do matter !!!**



Quantum fluctuations do not change significantly these “classical” results

Literature : Loop contributions to  $\tau$

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|                     |                           |
|---------------------|---------------------------|
| $e^{\Delta S_H}$    | 2.87185                   |
| $e^{\Delta S_t}$    | $1.20708 \times 10^{-18}$ |
| $e^{\Delta S_{gg}}$ | $1.26746 \times 10^{50}$  |

$$\Rightarrow \quad \tau_{cl} \sim 10^{555} T_U \quad \rightarrow \quad \tau \sim 10^{588} T_U$$

Our case : Loop contributions to  $\tau$

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|                     |                          |
|---------------------|--------------------------|
| $e^{\Delta S_H}$    | $2.82295 \times 10^{10}$ |
| $e^{\Delta S_t}$    | $8.62404 \times 10^{-5}$ |
| $e^{\Delta S_{gg}}$ | $4.97869 \times 10^9$    |

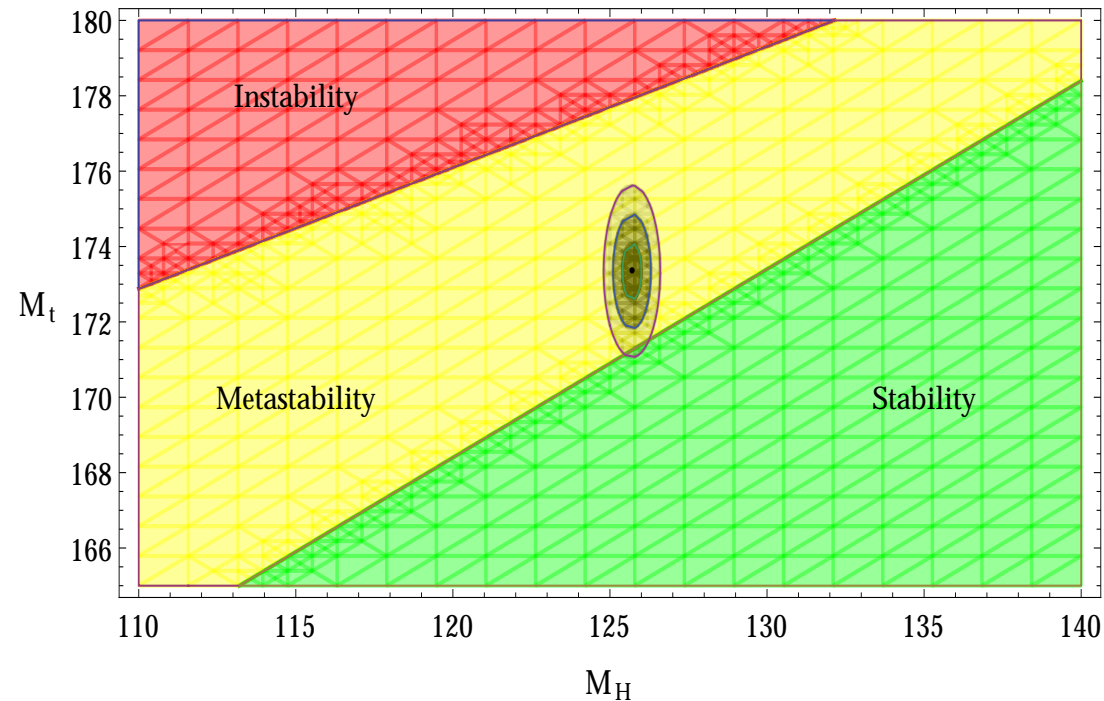
$$\Rightarrow \quad \tau_{cl} \sim 10^{-208} T_U \quad \rightarrow \quad \tau \sim 10^{-189} T_U$$

How comes that new physics can have such an impact on  $\tau$  ?

Why the arguments on the suppression of new physics do not apply ?

**Let's move to Phase Diagrams...**

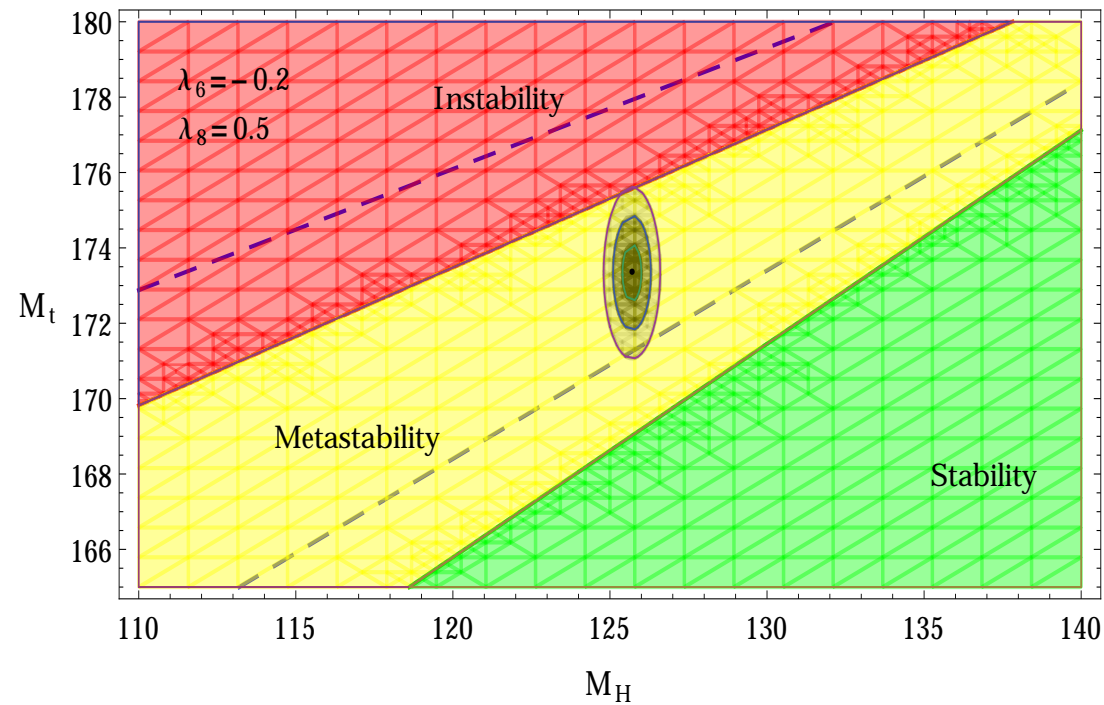
## Phase diagram with $\lambda_6 = 0$ and $\lambda_8 = 0$ - Literature case



This is the well known Phase Diagram ... (1) For  $M_H \sim 125 - 126$  GeV and  $M_t \sim 173$  we live in a metastable state ; (2)  $3\sigma$  close to the stability line (Criticality) ; (3) Precision measurements of the top mass will allow to discriminate between stable, metastable, or critical EW vacuum ...

## Phase diagram with $\lambda_6 = -0.2$ and $\lambda_8 = 0.5$

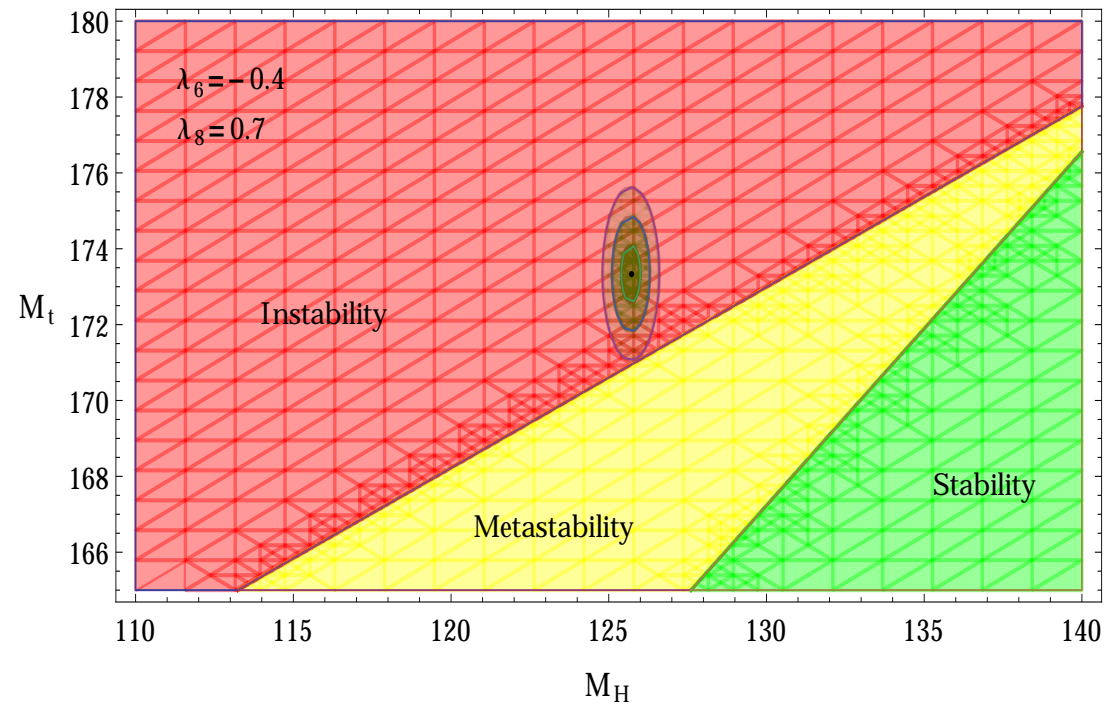
(Please note : Natural values for the coupling constants)



The strips move downwards ... The Experimental Point no longer at  $3\sigma$  from the stability line ...

**Phase diagram with  $\lambda_6 = -0.4$  and  $\lambda_8 = 0.7$**

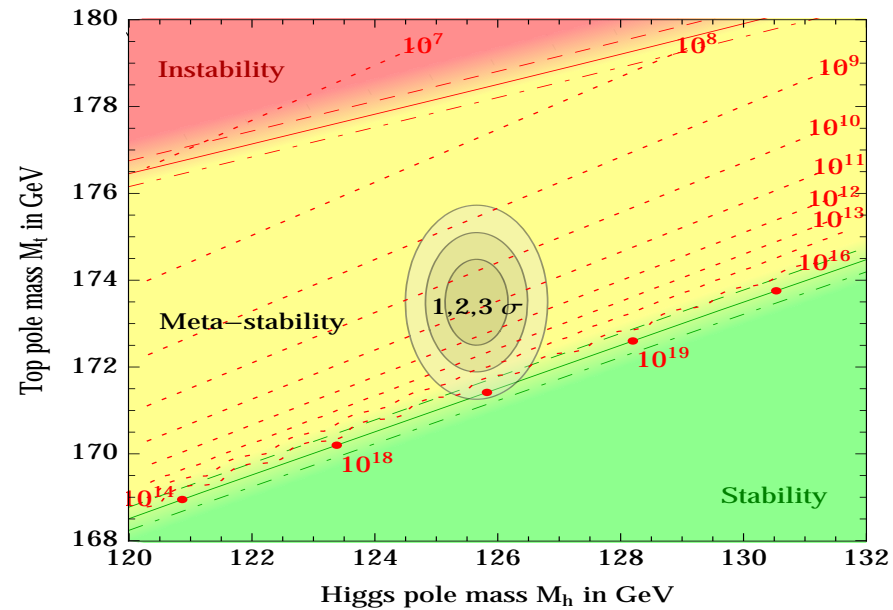
(Please note : Natural values for the coupling constants)



**Even worse !**

# Lesson

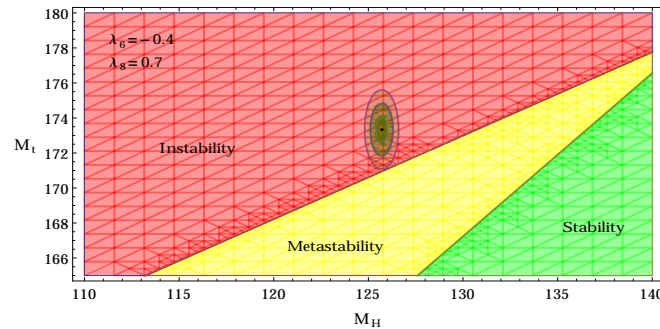
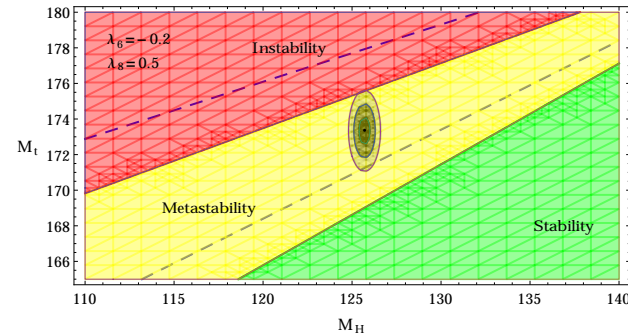
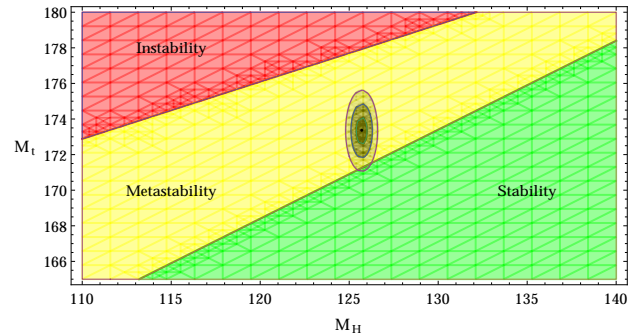
## The Phase Diagram



in not Universal !

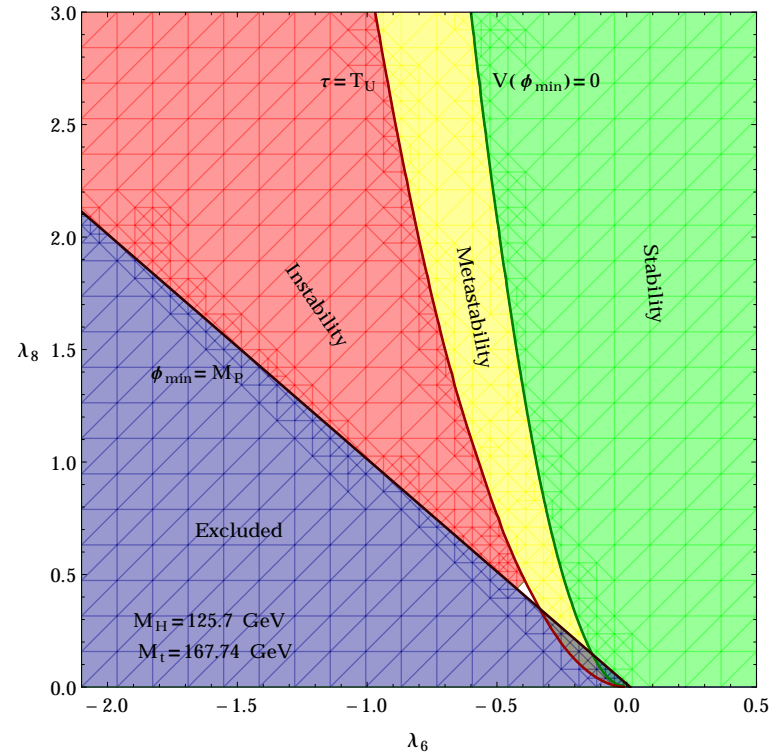
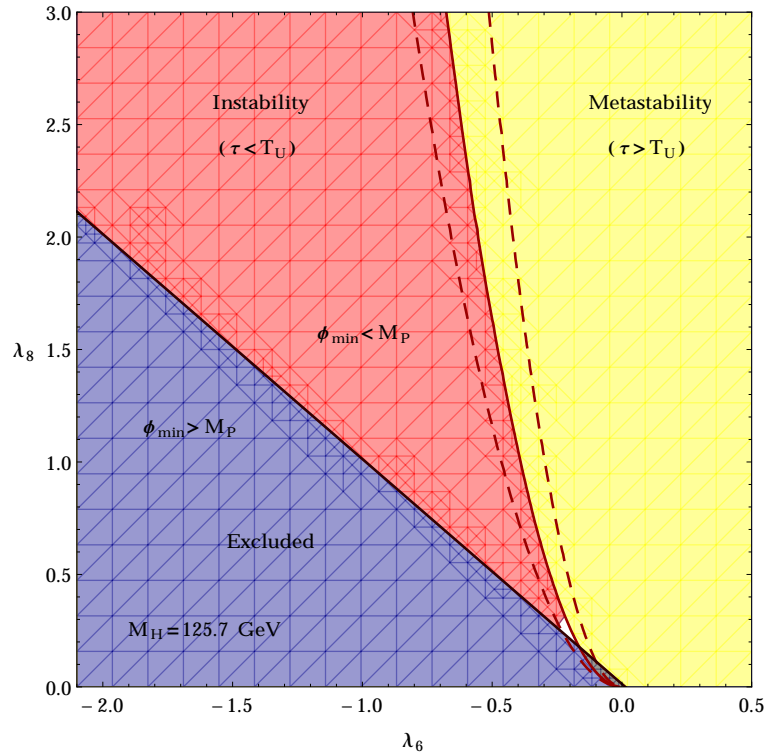
It is one out of many different possibilities ....

## “Precision Measurements of $M_t$ ”



**Precision measurements of  $M_t$  (and/or  $M_H$ ) cannot discriminate between stability, metastability or criticality !** The knowledge of  $M_t$  and  $M_H$  alone is not sufficient to decide of the **EW vacuum stability condition**. We need informations on **NEW PHYSICS** in order to asses this question !

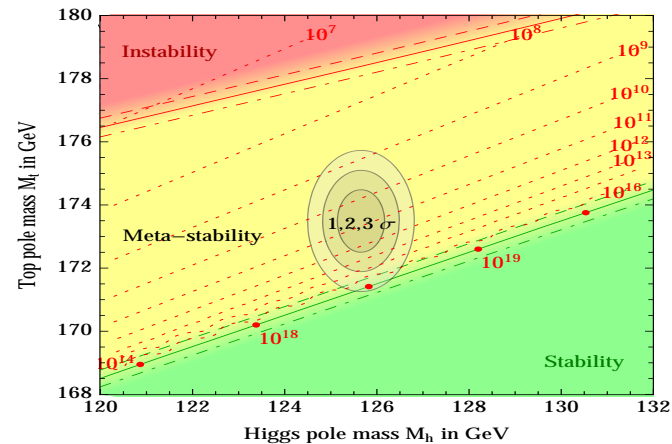
## “Precision Measurements of $M_t$ ”



Constraining allowed region in theory space - **BSM “Stability Test”**



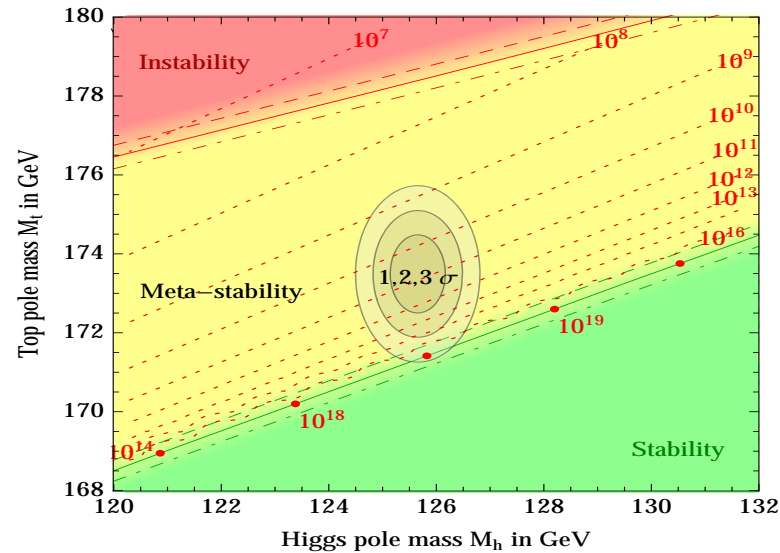
## “Near-Criticality”



Somebody considers this near-criticality of the SM vacuum as the most important message so far from experimental data on the Higgs boson

But : This “near-criticality” picture (technically  $\lambda(M_P) \sim 0$  and  $\beta(\lambda(M_P)) \sim 0$ ) can be easily screwed up by even small seeds of new physics ! Strong sensitivity to new physics, No Universality.

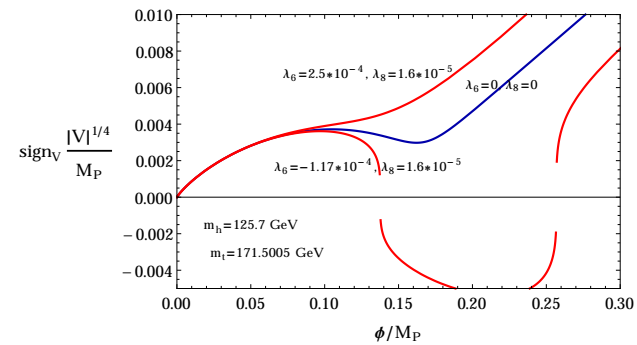
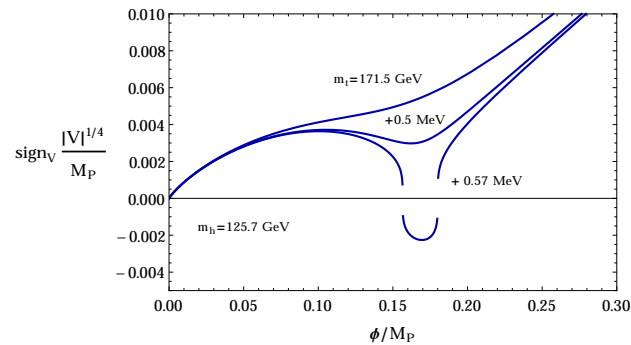
## Higgs Inflation “1”



The Higgs inflation scenario of Shaposhnikov - Bezrukov strongly relies on the realization of the criticality picture ( $\lambda(M_P) \sim 0$  and  $\beta(\lambda(M_P)) \sim 0$ ). As we have just said, even a little seed of new physics can easily screw up this picture

## Higgs Inflation “2” (Masina - Notari)

For a **narrow band of values of the top quark and Higgs boson masses**, the Standard Model Higgs potential develops a shallow local minimum higher than the EW minimum, where primordial inflation could have started



**Again : Strong sensitivity to new physics !**

## Conclusions and Outlook

- The **Stability Phase Diagram** of the EW vacuum **strongly depends** on New Physics
- **Precision Measurements** of the **Top Mass** will not allow to **discriminate** between **stability, metastability or criticality** of the EW vacuum. Phase Diagram too sensitive to New Physics
- **Higgs Inflation** in trouble. **Any small seed** of new physics screws up the picture
- Our results provide a “**BSM stability test**”. A BSM is acceptable if it provides either a **stable** EW vacuum or a **metastable** one, with lifetime larger than the age of the universe (**No  $\tau \ll T_U$  !!**). **In the past**, it was thought that the stability of the EW vacuum could be studied with **no reference to the UV completion** of the SM
- This analysis can be repeated even if the new physics scale lies **below the Planck scale**, say, for instance, **GUT scale**, or ...

**BACK UP SLIDES**

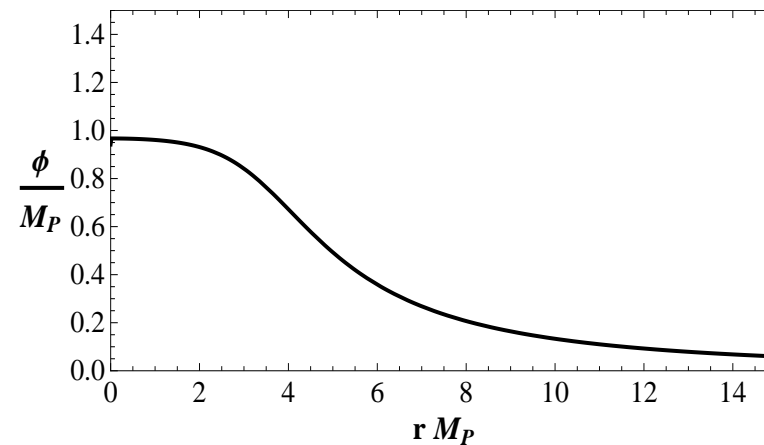
1. **New physics** appears in terms of **higher dimension operators**, and people expected their contribution to be **suppressed** as  $(\frac{\Lambda_{inst}}{M_P})^n$

But: **Tunnelling** is a **non-perturbative** phenomenon. We first select the **saddle point**, i.e. compute the **bounce** (**tree level**), and then compute the quantum fluctuations (**loop corrections**) on the top of it.

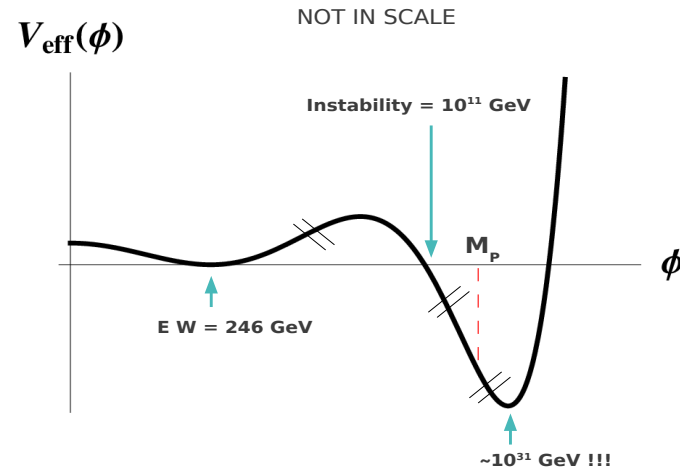
Suppression in terms of **inverse powers of  $M_P$**  (**power counting theorem**) concerns the **loop corrections**, not the saddle point (**tree level**).

Remember :  $\tau \sim e^{S[\phi_b]}$

New bounce  $\phi_b^{(2)}(r)$  , New action  $S[\phi_b^{(2)}]$  , New  $\tau$



## 2. Height of the barrier and turning points...



This is QFT with “very many” dof, not 1 dof QM  $\Rightarrow$  the potential is not  $V(\phi)$  in figure with 1 dof, but...

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} (\vec{\nabla} \phi)^2 - V(\phi) = \frac{1}{2} \dot{\phi}(\vec{x}, t)^2 - U(\phi(\vec{x}, t))$$

where  $U(\phi(\vec{x}, t))$  is :  $U(\phi(\vec{x}, t)) = V(\phi(\vec{x}, t)) - \frac{1}{2} (\vec{\nabla} \phi(\vec{x}, t))^2$

Very many dof, not 1 dof... Potential  $\sum_{\vec{x}} U(\phi(\vec{x}, t))$

The bounce is **not a constant configuration** ... **Gradients** do matter a lot!