The loop-tree duality at NLO and beyond

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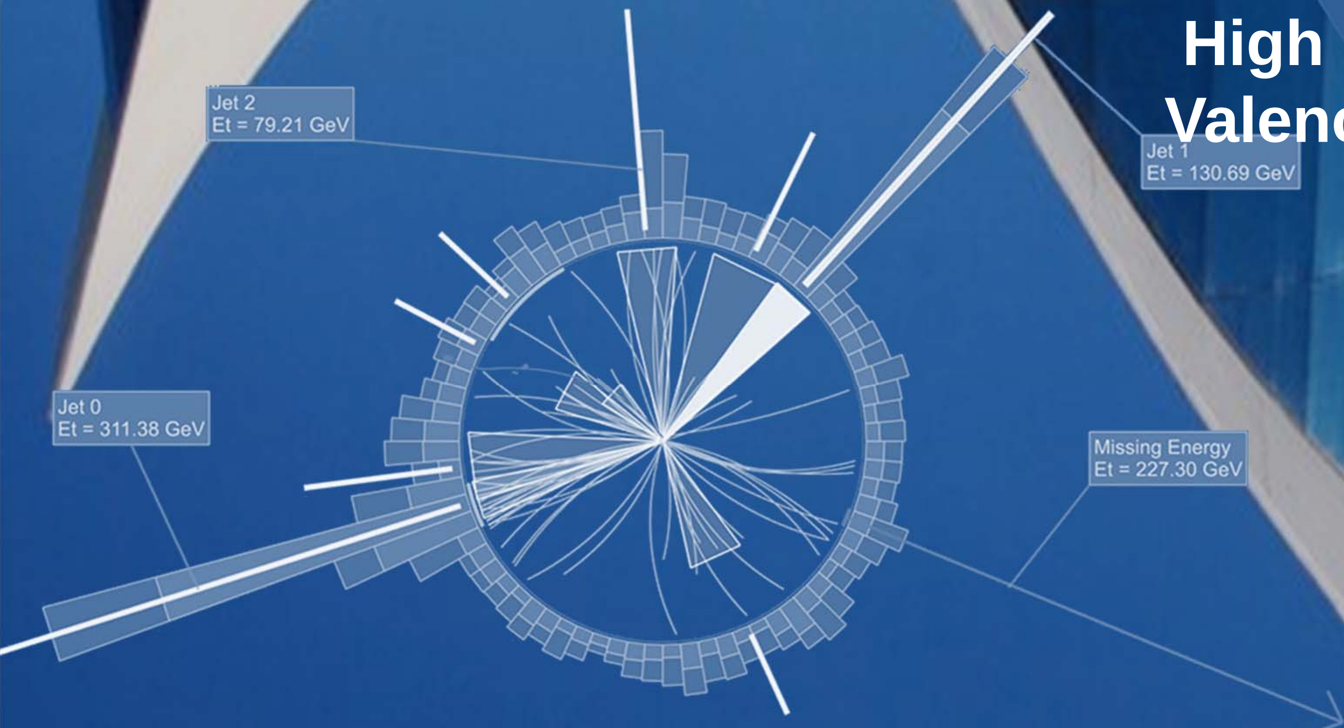


Germán
Rodrigo
IFIC
INSTITUT DE FISICA
CORPUSCULAR
CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Higher orders at the LHC

- The recent discovery of the Higgs boson at the LHC represents a great success of the SM, while the **absence so far of a clear signal of physics beyond the SM** leaves a certain degree of dissatisfaction

- **increased relevance of high-precision theoretical predictions** for the analysis of known phenomena and for finding innovative strategies to achieve new discoveries

- domain of perturbative calculations in QFT has shown an extraordinary progress in the recent years

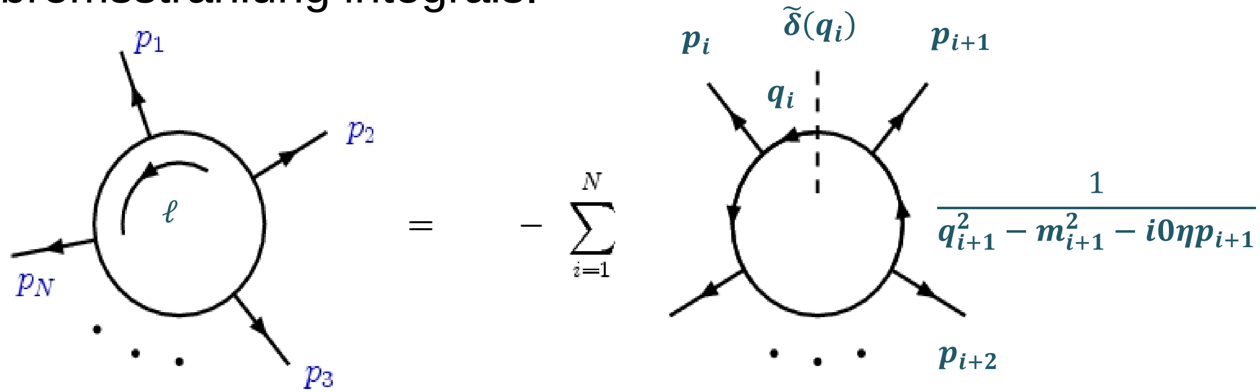
- Several tools for automated calculations at NLO, and lot of advances at NNLO (and beyond)

- Still, the **cancellation of infrared singularities** by the coherent sum over different **real and virtual** soft and collinear partonic configurations in the final state is the main source of cumbersomeness

Single cuts only: by-passing Feynman Tree Theorem

[Catani, Gleisberg, Krauss, GR, Winter, JHEP 09(2008)064]

duality relation between one-loop integrals (one-loop scattering amplitudes) with an arbitrary number of external legs (momenta) and corresponding **single-cut** bremsstrahlung integrals.



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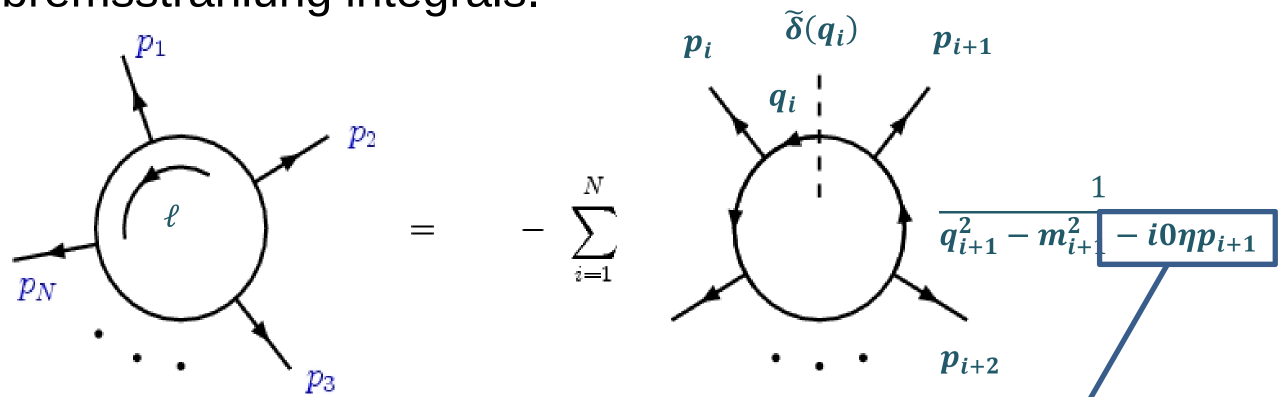
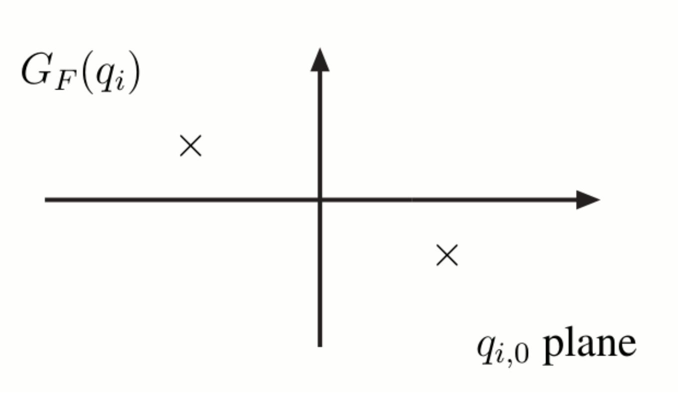
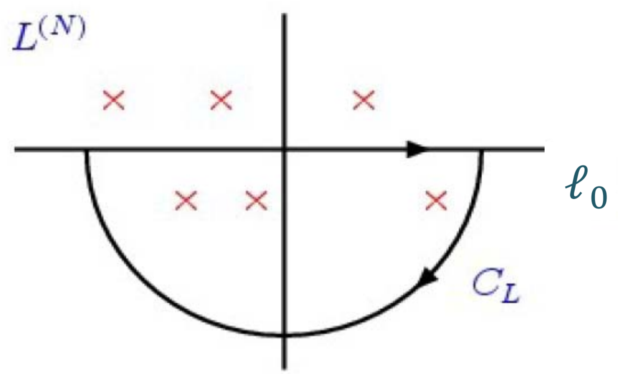


- the duality relation is realised by modifying the customary **+i0 prescription** of the Feynman propagators;
- the new +i0 prescription thus compensates for the absence of multiple-cut contributions that appear in the Feynman Tree Theorem;
- for scattering amplitudes in any relativistic, local and unitary quantum field theory;
- **recast virtual corrections** in a form that closely parallels the contribution of real corrections.

The loop-tree duality theorem



Cauchy residue theorem
in the loop energy complex plane



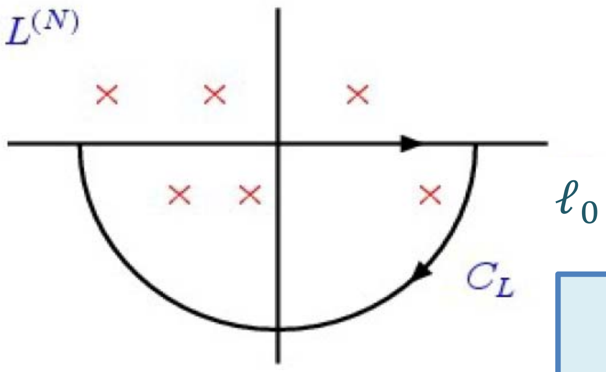
Feynman Propagator +i0: positive frequencies are propagated forward in time, and negative backward

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

selects residues with definite **positive** energy and negative imaginary part (indeed in any coordinate system)

- Extension beyond one-loop [Bierenbaum et al. 10]
- Assuming only **single poles,** multiple poles can be avoided by a convenient choice of gauge (gauge propagators) [Catani et al. 08] or by applying IBP to reduce multiplicity (beyond one-loop) [Bierenbaum et al. 13]

The loop-tree duality theorem



- the one-loop integral represented as a linear combination of *N* single-cut **phase-space** integrals

$$\int_\ell \prod G_F(q_i) = -\int_\ell \sum \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

- where $\int_\ell = -i \int \frac{d^d\ell}{(2\pi)^d}$
- $\tilde{\delta}(q_i) = 2\pi i\, \delta_+(\boldsymbol{q_i^2 - m_i^2})$ sets internal line on-shell;
- $G_D(q_i;q_j) = \frac{1}{\boldsymbol{q_j^2 - m_j^2 - i0\eta k_{ji}}}$ dual propagator, $\boldsymbol{k_{ji} = q_j - q_i}$;
- Lorentz-covariant dual prescription with η a **future-like** vector: $\eta^2 \geq 0$ with $\eta_0 > 0$;
- different choices of *η* are equivalent to different choices of the coordinate system;
- the dependence on *η* cancels in the sum of dual integrals.

Singularities of the loop integrand

[Buchta, Chachamis, Draggiotis, Malamos, GR, arXiv:1405.7850 hep-ph]

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- Although the existence of singular points in the loop momentum-space is not enough to ensure the presence of singularities:

  - threshold singularities are integrable: contour deformation for numerically stable integration [Soper, Nagy, Krämer, Kilian, Kleinschmidt, Weinzierl´s talk 4-dimensional]

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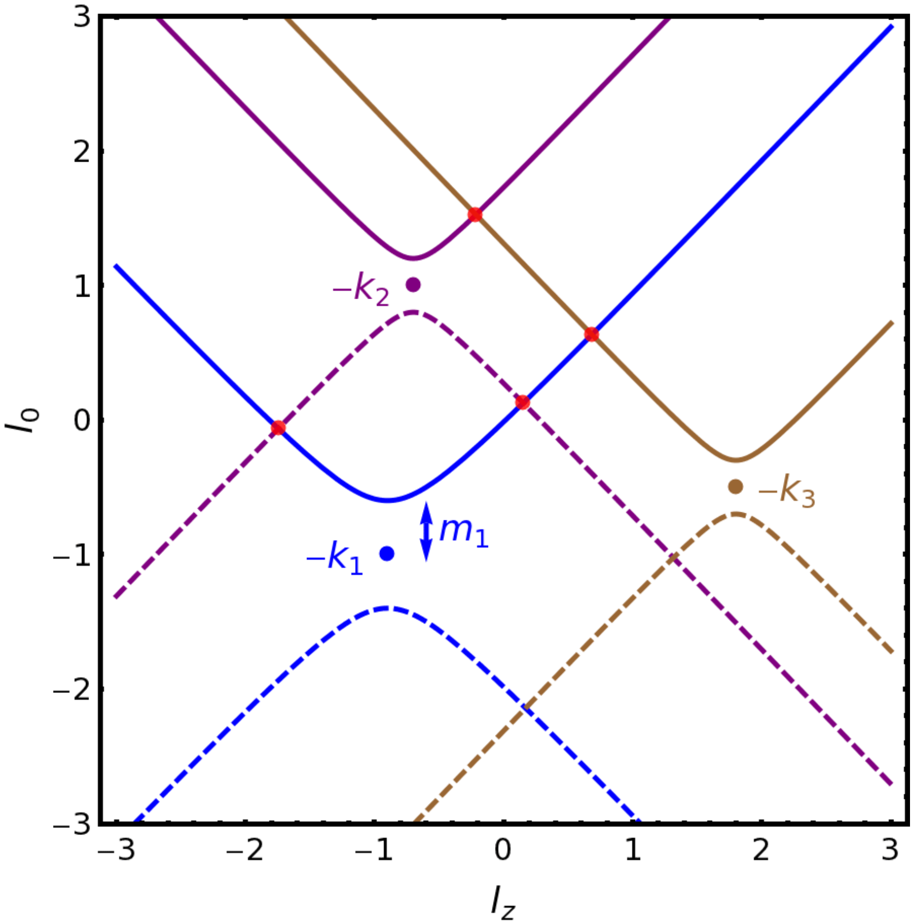
- Loop singularities arise when subsets of internal lines go on-shell. (assuming UV singularities have been subtracted)

Singularities of the loop integrand



The loop integrand becomes singular

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$$

at **hyperboloids** with origin in $-k_{i,\mu}$ $(q_i = \ell + k_i)$

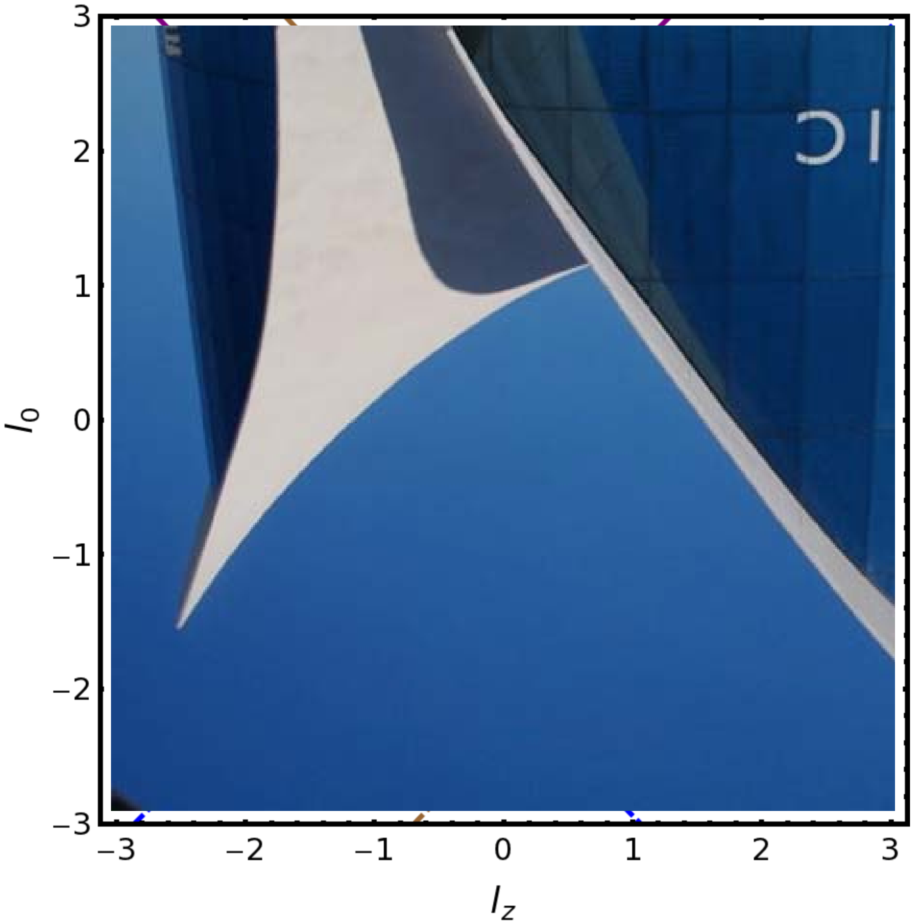
Forward light-cone (solid)

$$q_{i,0}^{(+)} = \sqrt{\vec{q}_i^{\,2} + m_i^2 - i0}$$

Backward light-cone (dashed)

$$q_{i,0}^{(-)} = -q_{i,0}^{(+)}$$

Singularities of the loop integrand



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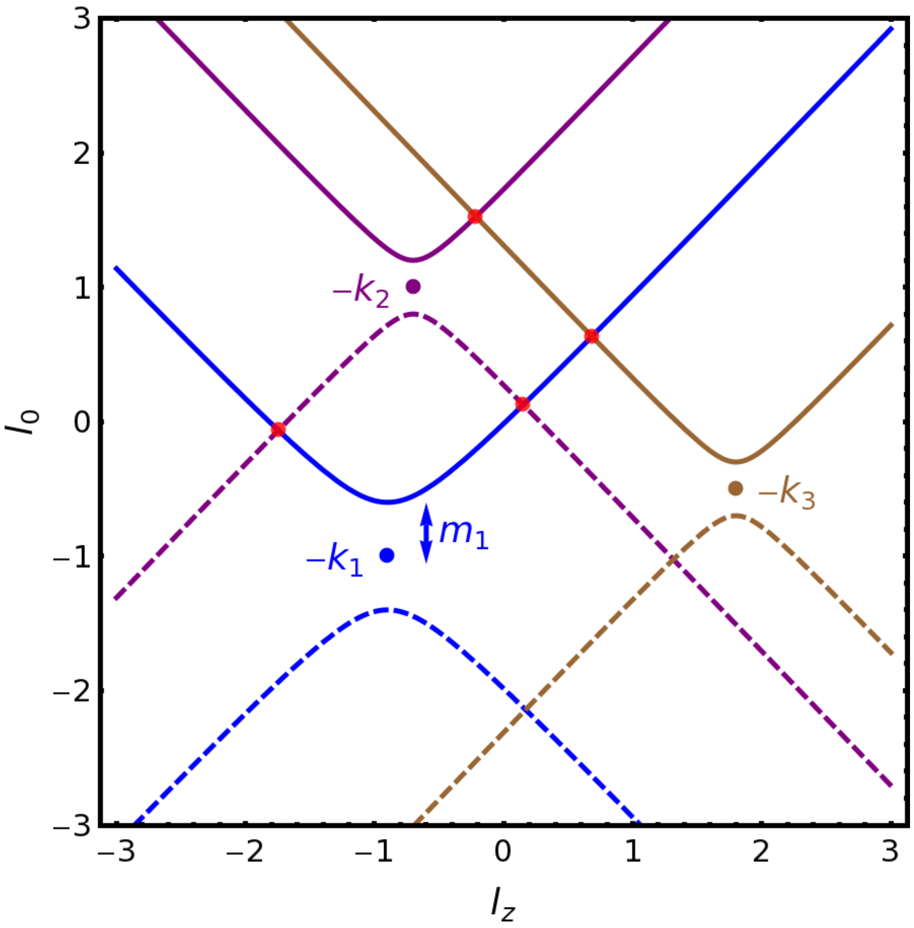
⬇

Duality: equivalent to integrate along the **forward** light-cones

Threshold singularities



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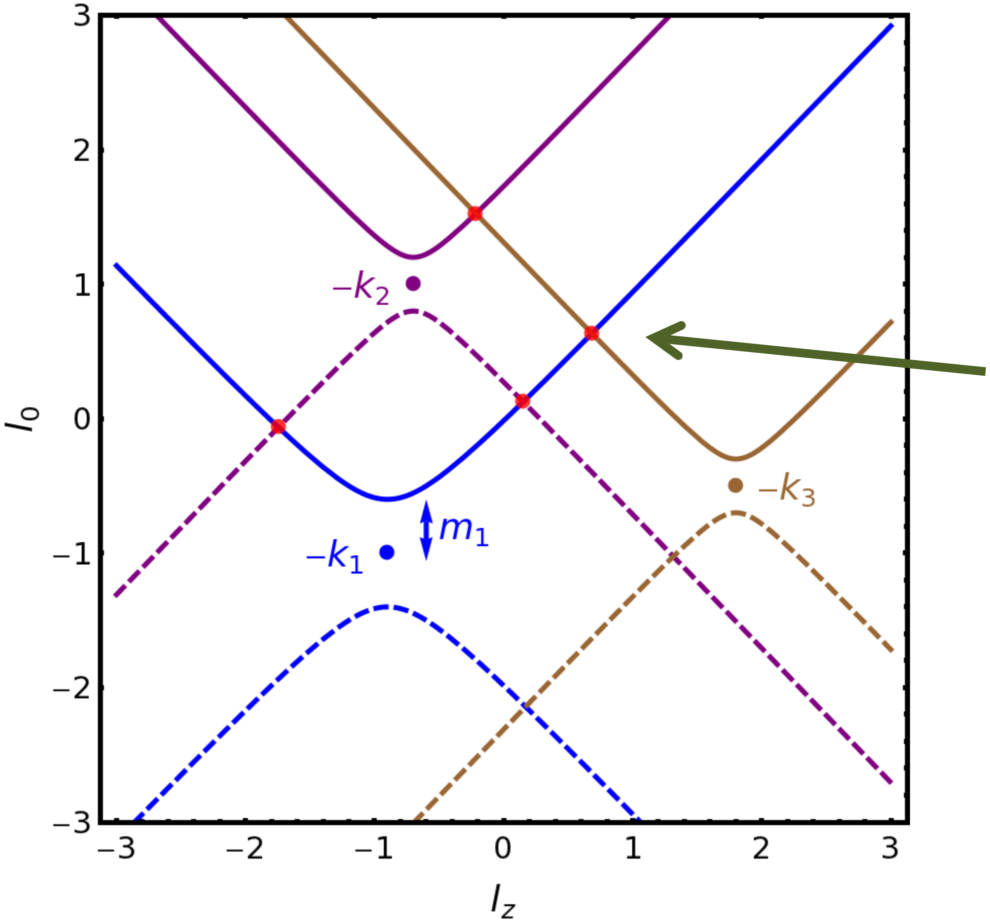

The dual integrand becomes singular when a dual propagator becomes singular (two or more propagators on-shell)

- Integrating along the forward light-cone $\tilde{\delta}(q_1)$ one pass **from outside to inside** the light-cone hyperboloid of $-k_3$
- Integrating along the forward light-cone $\tilde{\delta}(q_3)$ one pass **from inside to outside** the light-cone hyperboloid of $-k_1$
- Opposite sign ➔ **singularity cancels** among dual integrals

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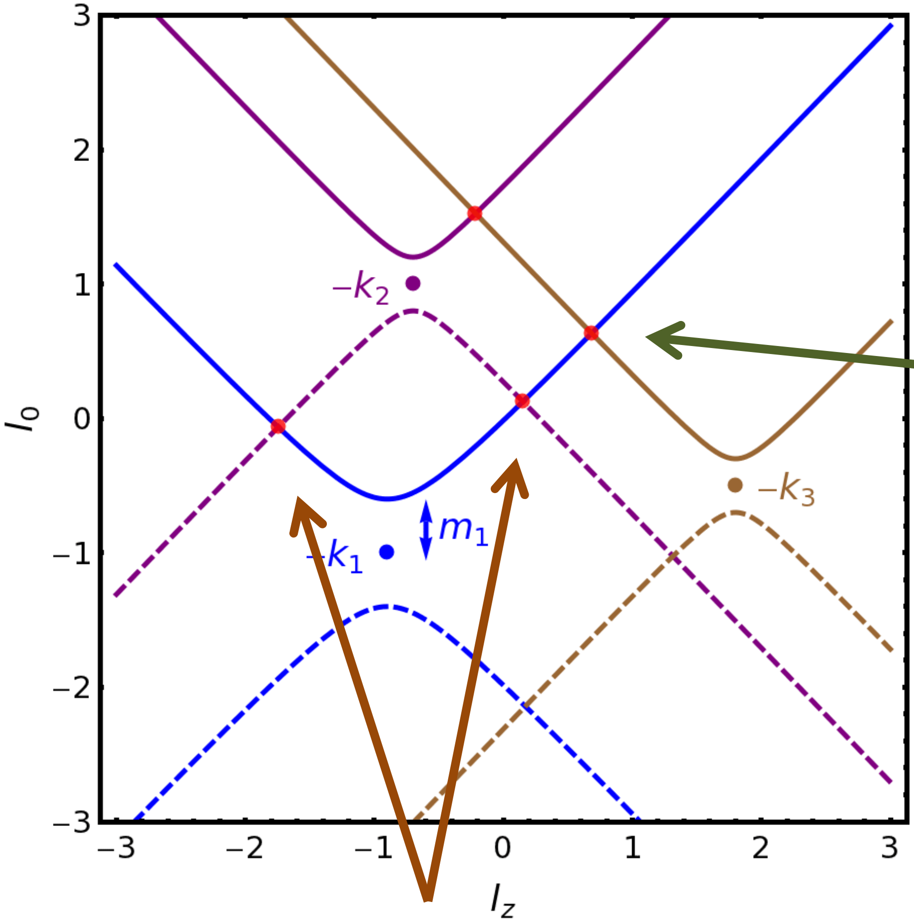

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- Meet singularities along the forward light-cone $\tilde{\delta}(q_1)$ but not along the forward light-cone $\tilde{\delta}(q_2)$
- Singularity remains: time-like separated propagators with lower energy: **causally connected**

Singularities of the dual integrand

Each dual integrand rewritten as

$$\tilde{\delta}(q_i)\, G_D(q_i; q_j) = i\, 2\pi\, \frac{\delta\left(q_{i,0} - \boldsymbol{q}_{i,0}^{(+)}\right)}{2\boldsymbol{q}_{i,0}^{(+)}}\, \frac{1}{\left(\boldsymbol{q}_{i,0}^{(+)} + k_{ji,0}\right)^2 - \left(\boldsymbol{q}_{j,0}^{(+)}\right)^2}$$

where $\boldsymbol{q}_{i,0}^{(+)} = \sqrt{\vec{\boldsymbol{q}}_i^{\,2} + \boldsymbol{m}_i^2 - \boldsymbol{i0}}$ is the (positive) loop energy measured from the vertex of the light-cone, and $\boldsymbol{k}_{ji,0} = (\boldsymbol{q}_j - \boldsymbol{q}_i)_0$

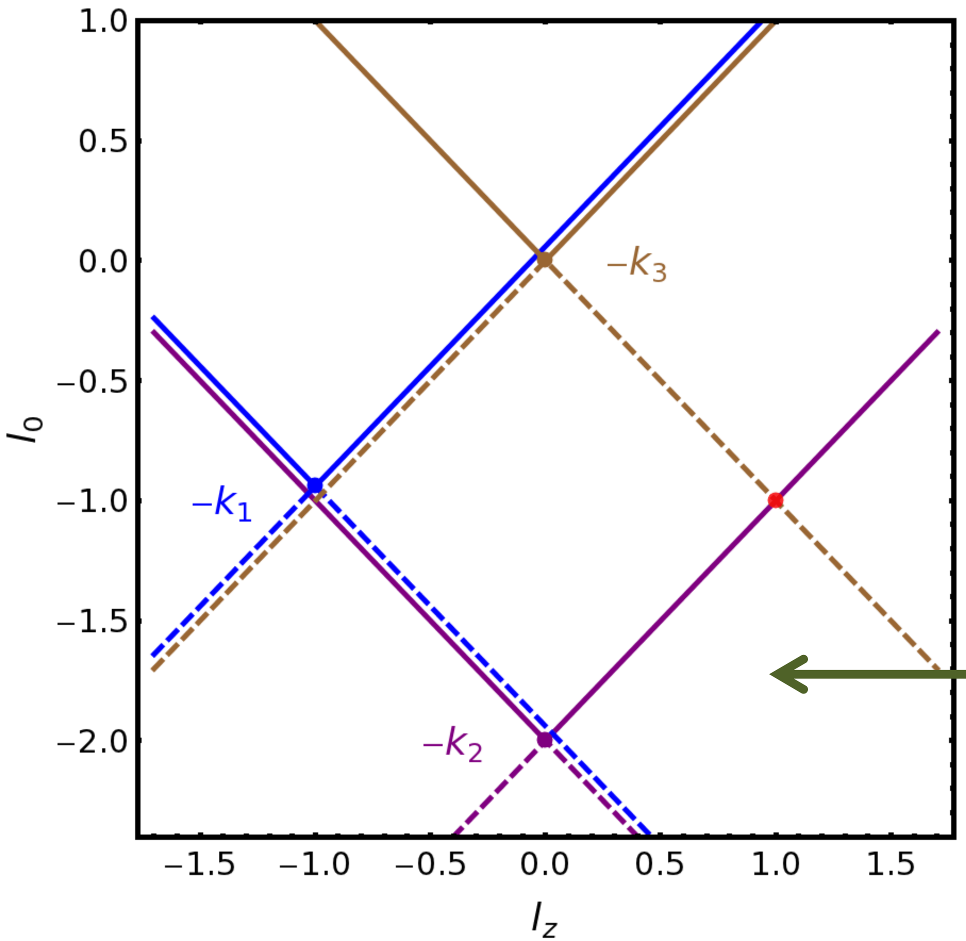
- $\boldsymbol{1/q_{i,0}^{(+)}}$ can become singular for $\boldsymbol{m_i = 0}$, but the integral $\int_l \delta\left(q_{i,0} - \boldsymbol{q}_{i,0}^{(+)}\right) / \boldsymbol{q}_{i,0}^{(+)}$ is still convergent in the IR (soft) by two powers, soft singularity with the contribution of two other propagators

- The dual propagator can become singular if

$\boldsymbol{q}_{i,0}^{(+)} + \boldsymbol{q}_{j,0}^{(+)} + \boldsymbol{k}_{ji,0} = \boldsymbol{0}$ **forward** light-cone of $-k_i$ intersects **backward** of $-k_j$

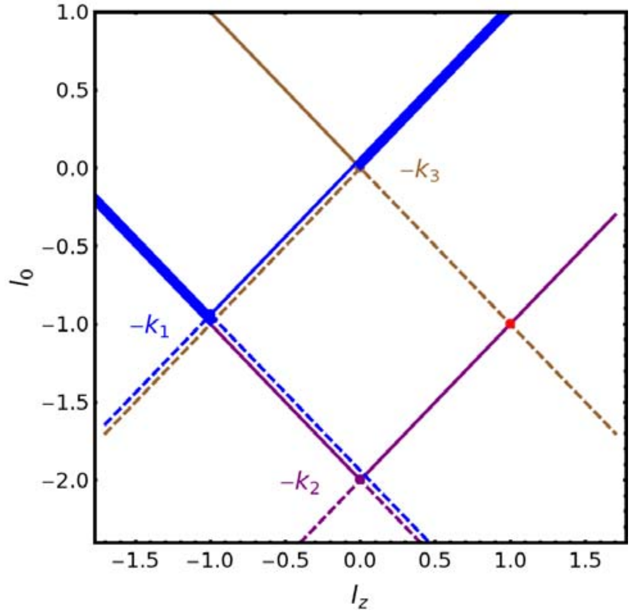
$\boldsymbol{q}_{i,0}^{(+)} - \boldsymbol{q}_{j,0}^{(+)} + \boldsymbol{k}_{ji,0} = \boldsymbol{0}$ the **two forward** light-cones intersect

IR singularities



- Massless internal line and on-shell adjacent external (massless) momenta
- The light-cone hyperboloids intersect **tangentially** over an **infinite** interval
- e.g. $e^+e^- \to q(p_1)\bar{q}(p_2)$ with $k_1 = p_1$, $k_2 = p_1 + p_2$, $k_3 = 0$

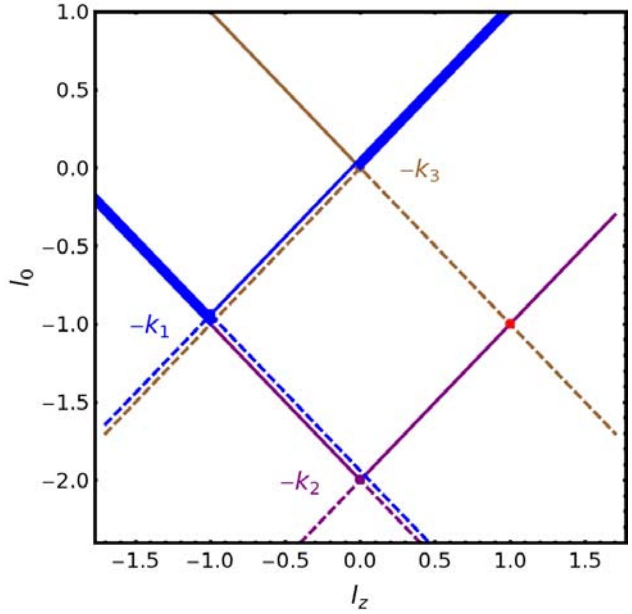
IR singularities

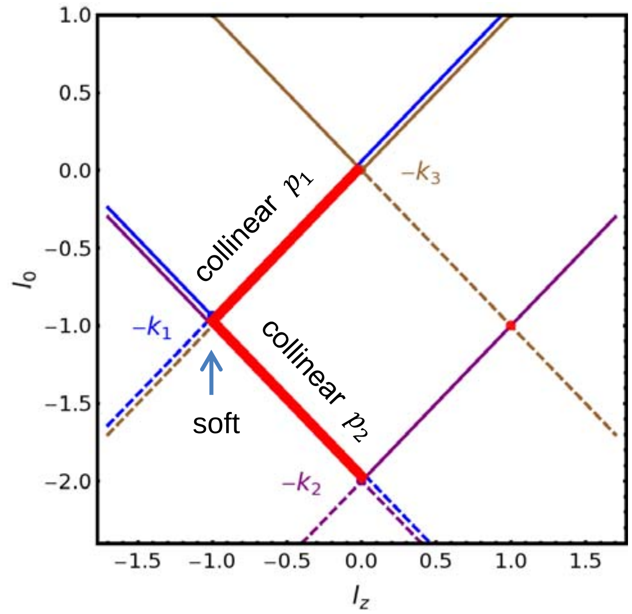


Forward with forward light-cone:

- collinear singular behaviour cancels among dual integrals

IR singularities





Forward with forward light-cone:

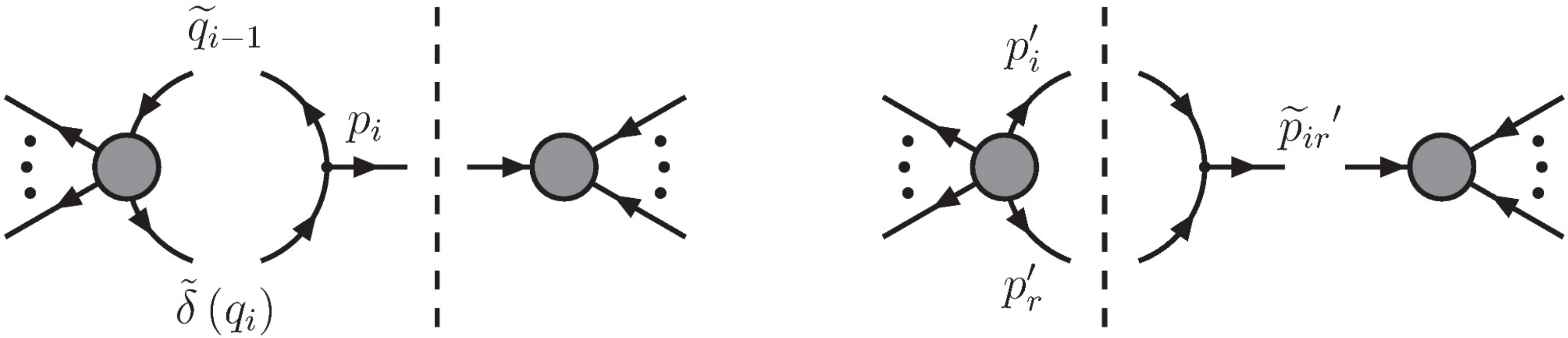
- collinear singular behaviour cancels among dual integrals

Forward with backward light-cone:

- Collinear and soft singular behaviour remains, and
- Is restricted to a **finite region** of the loop-momentum space, which is of the order of the magnitude of external momenta
- Mapping with finite size real emission phase-space

Factorization in the collinear limit

[Catani, de Florian, GR, PLB586(2004), JHEP 07(2012)026]



- dual scattering amplitude: $p_i || q_i$

$$\left| \boldsymbol{M}_N^{(1)} (p_1, \dots, p_N) \right\rangle \to \left| \boldsymbol{M}_{N+2}^{(0)} (\dots, p_i, -q_i, q_i, \dots) \right\rangle$$

$$\simeq \boldsymbol{Sp}^{(0)}(p_i, -q_i ; -\tilde{q}_{i-1}) \left| \overline{\boldsymbol{M}}_{N+1}^{(0)} (\dots, p_{i-1}, -\tilde{q}_{i-1}, q_i, \dots) \right\rangle$$

- tree-level scattering amplitude: $p_i{}' || p_r{}'$

$$\left| \boldsymbol{M}_{N+1}^{(0)} (p'_1, \dots, p'_{N+1}) \right\rangle \simeq \boldsymbol{Sp}^{(0)}(p'_i, p'_r ; \tilde{p}_{ir}{}') \left| \overline{\boldsymbol{M}}_{N}^{(0)} (\dots, p'_{i-1}, \tilde{p}_{ir}{}', p'_{i+1}, \dots) \right\rangle$$

with on-shell momenta $\tilde{\boldsymbol{q}}_{i-1}^2 = \boldsymbol{0}$, $\tilde{\boldsymbol{p}}'^2_{ir} = \boldsymbol{0}$,

can restrict the loop momenta to $\boldsymbol{q}_{i,0}^{(+)} < \boldsymbol{p}_{i,0}$

mapping between primed and un-primed external momenta

Conclusions and outlook

- the loop-tree duality method exhibits attractive theoretical issues and nice properties
- threshold and infrared singularities occurring in the intersection of forward light-cones cancel among dual integrals
- remaining loop singularities (**causally connected**) are restricted to a **finite region** of the loop momentum space, which is of the size of external momenta

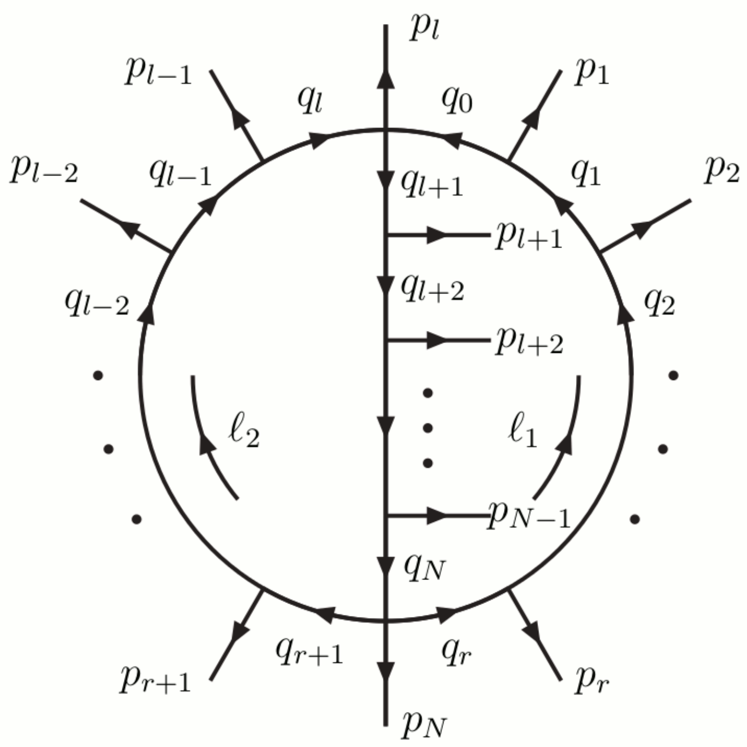
Outlook

- numerical implementation at one-loop (at work soon)
- singularities at higher orders (and numerical implementation)

Duality at two loops (and beyond)

[Bierenbaum, Catani, Draggiotis, GR, JHEP 10(2010)073]

[Bierenbaum, Buchta, Draggiotis, Malamos, GR, JHEP 03(2013)025]



- Iterative application of the duality theorem
- **Loop lines:**

$$\alpha_1 = \{0,1,\ldots,r\}$$
$$\alpha_2 = \{r+1,\ldots,l\}$$
$$\alpha_3 = \{l+1,\ldots,N\}$$

$$G_D(\alpha_k) = \sum_{i\in\alpha_k} \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

$$G_F(\alpha_k) = \prod_{i\in\alpha_k} G_F(q_i)$$

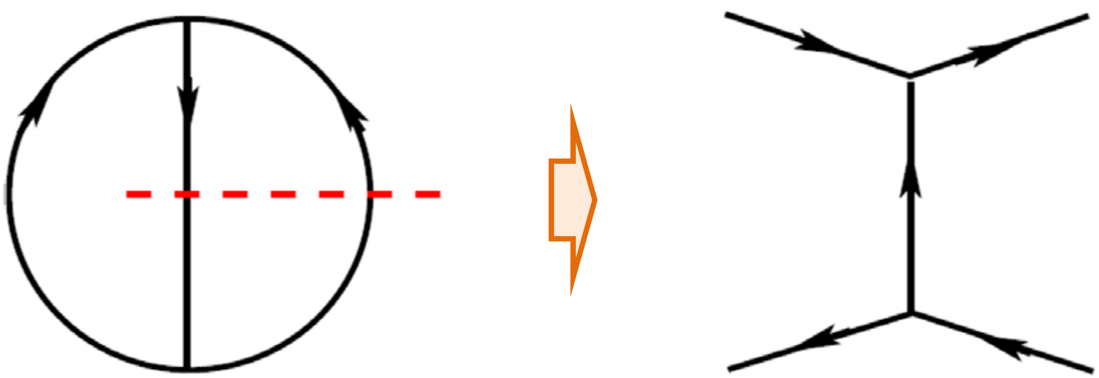
- duality applies to Feynman propagators depending on the same loop momentum

$$G_D(\alpha_1 \cup \alpha_2) = G_D(\alpha_1)G_D(\alpha_2) + G_D(\alpha_1)G_F(\alpha_2) + G_F(\alpha_1)G_D(\alpha_2)$$

Duality at two loops (and beyond)

- **Two cuts only:** open any two-loop diagram to a **tree-level diagram** (sign in $-\alpha_1$ indicates a change of momentum flow)

$$L^{(2)}(p_1, \dots, p_N) = \int_{\ell_1} \int_{\ell_2} [-G_D(\alpha_1) G_F(\alpha_2) G_D(\alpha_3) + G_D(\alpha_1)\, G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_1 \cup \alpha_2) G_D(\alpha_3)]$$



- at one-loop the complex dual prescription depends on external momenta only, however, at two loops it might depend on the integration momenta: complex dual prescription only on external momenta, requires to introduce disconnected diagrams