



Millicharged neutrino with anomalous magnetic moment in rotating magnetized matter

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Weak and electromagnetic interactions of neutrino with matter and magnetic field

Weak and electromagnetic interactions of a neutrino with nontrivial electromagnetic properties with a dense matter and strong electromagnetic fields are of particular interest for astrophysics since they can originate new astrophysical effects and phenomena and open a window to physics beyond the Standard Model.

Electromagnetic properties of massive neutrinos are the puzzle of the modern elementary particle physics [1]. In particular, we consider the Dirac neutrino with nonzero millicharge q_ν and anomalous magnetic moment μ .

Weak interactions of a neutrino with matter composed of neutrons, electrons and protons are described by the effective Lagrangian $\Delta L_{eff} = -f^\mu (\bar{\nu} \gamma_\mu \frac{1+\gamma_5}{2} \nu)$, where matter potential f^μ depends on background particles densities, velocities and polarizations [2].

The effective approach of the investigation of a neutrino behavior in extreme background conditions is the method of exact solutions of a modified Dirac equation [3]. The most general form of the modified Dirac equation that describes the millicharged neutrino with anomalous magnetic moment in magnetized matter is given by [4,5]

$$\left(\gamma_\mu P^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - \frac{i}{2} \mu \sigma_{\mu\nu} F^{\mu\nu} - m \right) \Psi(x) = 0$$

where $P^\mu = p^\mu + q_0 A^\mu$ (it is supposed that the neutrino has a negative electric charge $q_\nu = -q_0$).

Quantum states of millicharged neutrino with anomalous magnetic moment in magnetized matter

Consider the millicharged neutrino with anomalous magnetic moment in the dense unpolarized static neutron matter and constant magnetic field. In this case the vector magnetic potential and matter potential are given by $A^\mu = (0, -\frac{yB}{2}, \frac{xB}{2}, 0)$ and $f^\mu = -Gn_n(1, 0, 0, 0)$ correspondingly, where $G = \frac{G_F}{\sqrt{2}}$ (G_F is the Fermi constant).

The Dirac equation can be rewritten in the form $i\frac{\partial}{\partial t}\Psi(x) = H\Psi(x)$ with the Hamiltonian

$$H = \gamma_0 \boldsymbol{\gamma} \mathbf{p} + \gamma_0 m + \gamma_0 \sigma_3 \mu B - (1 + \gamma_5) \frac{Gn_n}{2}$$

The exact solution for the neutrino wave function has the form

$$\Psi(x) = \sqrt{\frac{q_0 B}{2\pi L}} e^{-i(p_0 t - p_3 z)} \begin{pmatrix} C_1 \mathcal{L}_s^{l-1}(\frac{q_0 B}{2} r^2) e^{i(l-1)\varphi} \\ i C_2 \mathcal{L}_s^l(\frac{q_0 B}{2} r^2) e^{i l \varphi} \\ C_3 \mathcal{L}_s^{l-1}(\frac{q_0 B}{2} r^2) e^{i(l-1)\varphi} \\ i C_4 \mathcal{L}_s^l(\frac{q_0 B}{2} r^2) e^{i l \varphi} \end{pmatrix}$$

where $\mathcal{L}_s^l(\frac{q_0 B}{2} r^2)$ are the Laguerre functions. To determine spin coefficients C_i we introduce a new type of spin operator which is a weight superposition of the operators of transverse and longitudinal polarizations

$$S = S_{Tr} \cos \alpha + S_{long} \sin \alpha, \quad \sin \alpha = \frac{Gn_n}{\sqrt{(Gn_n)^2 + (2\mu B)^2}}$$

The spin operator commutes with the Hamiltonian and yields the spin integral of motion

$$S = \frac{\zeta}{m} \sqrt{(m \cos \alpha + p_3 \sin \alpha)^2 + 2Nq_0 B}$$

where $\zeta = \pm 1$ determines spin polarization states.

Energy states of the neutrino in the matter and magnetic field are given by

$$p_0 = -\frac{Gn_n}{2} + \varepsilon \sqrt{p_3^2 + 2Nq_0 B + m^2 + \left(\frac{Gn_n}{2}\right)^2 + (\mu B)^2 + 2mS \sqrt{\left(\frac{Gn_n}{2}\right)^2 + (\mu B)^2}}$$

where $\varepsilon = \pm 1$. Finally, the spin coefficients is obtained in the following from

$$C_1 = \frac{1}{2} \sqrt{1 + \frac{m \cos \alpha + p_3 \sin \alpha}{mS} \sqrt{1 - \sin(\alpha - \beta)}} \\ C_2 = \frac{1}{2} \delta_1 \zeta \sqrt{1 - \frac{m \cos \alpha + p_3 \sin \alpha}{mS} \sqrt{1 - \sin(\alpha + \beta)}} \\ C_3 = \frac{1}{2} \delta_2 \zeta \sqrt{1 + \frac{m \cos \alpha + p_3 \sin \alpha}{mS} \sqrt{1 + \sin(\alpha - \beta)}} \\ C_4 = \frac{1}{2} \delta_3 \zeta \sqrt{1 - \frac{m \cos \alpha + p_3 \sin \alpha}{mS} \sqrt{1 + \sin(\alpha + \beta)}}$$

where we use notations $\delta_1 = \text{sgn}[\sin \alpha - \cos \beta]$, $\delta_2 = \text{sgn}[\cos(\alpha - \beta)]$, $\delta_3 = \text{sgn}[\cos \alpha + \sin \beta]$ and introduce a new angle

$$\cos \beta = \frac{p_3 \cos \alpha - m \sin \alpha}{p_0 + \frac{Gn_n}{2}}$$

The obtained exact solution of the modified Dirac equation describes the millicharged neutrino with anomalous magnetic moment in the dense unpolarized static neutron matter and constant magnetic field [4,5].

Quantum states of millicharged neutrino in rotating magnetized matter

Consider the millicharged neutrino in dense unpolarized rotating neutron matter and constant magnetic field. In this case the vector magnetic potential and matter potential are given by $A^\mu = (0, -\frac{yB}{2}, \frac{xB}{2}, 0)$ and $f^\mu = -Gn_n(1, -\gamma\omega, x\omega, 0)$ and correspondingly, where ω is an angular velocity of matter rotation. We also consider the particular case of coincided directions of the magnetic field \mathbf{B} and matter rotation $\boldsymbol{\omega}$.

The Hamiltonian in the limit of zero neutrino mass and magnetic moment (which are, in fact, negligible quantities) has the form

$$H = \gamma_0 \boldsymbol{\gamma} \mathbf{p} - (1 + \gamma_5) (1 + \gamma_0 \gamma_1 \omega y - \gamma_0 \gamma_2 \omega x) \frac{Gn_n}{2}$$

To describe neutrino spin properties of the neutrino in a moving matter we introduce a new type of the spin operator

$$S = \frac{\boldsymbol{\Sigma} \mathbf{P}}{m} - (\gamma_0 \boldsymbol{\gamma} \mathbf{v} - \boldsymbol{\Sigma} \mathbf{v}) \frac{Gn_n}{2m}$$

which consists of the operator of the longitudinal polarization and of the additional term that accounts for the matter motion. Using the spin operator one can obtain the neutrino energy spectrum

$$p_0 = -\frac{Gn_n}{2} + \varepsilon \zeta \eta \left(mS + \frac{Gn_n}{2} \right), \quad mS = \zeta \sqrt{p_3^2 + 2Nq_0 B}$$

where $\zeta = \pm 1$ determines spin polarization states, $\varepsilon = \pm 1$ and $\eta = \text{sign}(1 + \frac{Gn_n}{2mS})$. We also introduce the effective neutrino electric charge q and effective magnetic field \mathbf{B} which are given by the expression $qB = q_0 B + (1 - \varepsilon \zeta \eta) Gn_n \omega$. For the neutrino wave function we get

$$\Psi(x) = \frac{e^{-i(p_0 t - p_3 z)}}{2} \sqrt{\frac{qB}{\pi L}} \begin{pmatrix} \frac{1 - \varepsilon \zeta \eta}{2} \psi \\ \frac{1 + \varepsilon \zeta \eta}{2} \psi \end{pmatrix}, \quad \psi = \begin{pmatrix} \sqrt{1 + \frac{p_3}{mS}} \mathcal{L}_s^{l-1}(\frac{qB}{2} r^2) e^{i(l-1)\varphi} \\ i \zeta \sqrt{1 - \frac{p_3}{mS}} \mathcal{L}_s^l(\frac{qB}{2} r^2) e^{i l \varphi} \end{pmatrix}$$

From the structure of the wave function it follows that the left-handed neutrino chiral state $\Psi_L \equiv \frac{1}{2}(1 + \gamma_5)\Psi$ corresponds to the negative neutrino spirality $\zeta = -1$ while the right-handed neutrino $\Psi_R \equiv \frac{1}{2}(1 - \gamma_5)\Psi$ corresponds to the positive spirality $\zeta = +1$. It is also obvious that the right-handed neutrino does not contain the matter terms in the obtained solutions and is attributed to a sterile neutrino.

The obtained exact solution of the modified Dirac equation describes the millicharged neutrino in rotating matter and constant magnetic field [4,5].

The energy spectrum of the active neutrinos (left-handed neutrinos and right-handed antineutrinos)

$$p_0 = \sqrt{p_3^2 + 2N(q_0 B + 2Gn_n \omega) - Gn_n}$$

is quantized due to both weak and electromagnetic interactions and represents the modified Landau levels of the millicharged neutrino in the rotating magnetized matter.

Quasi-classical treatment

On the basis of the obtained exact solutions one can consider the evolution of intensive neutrino beams in astrophysical environment in order to predict new astrophysical effects and phenomena. Herewith it is extremely important to account for both weak and electromagnetic interactions of millicharged neutrinos with the rotating magnetized matter.

Within the quasi-classical treatment we have calculated the radii of the millicharged neutrino orbits inside rotating magnetized matter

$$R = \sqrt{\frac{2N}{q_0 B + 2Gn_n \omega}}$$

that can be expressed in the completely classical form

$$R = \Omega^{-1}, \quad \Omega = \omega_m + \omega_c$$

by introducing the effective rotation frequency Ω which is determined by the cyclotron and matter induced frequencies

$$\omega_c = \frac{q_0 B}{p_0 + Gn_n}, \quad \omega_m = \frac{2Gn_n}{p_0 + Gn_n} \omega$$

It is easy to show that neutrinos with energies up to 1 eV will be trapped inside neutron stars with radius $R_S = 10 \text{ km}$ since for them $R < R_S$. Therefore we predict the effect of low energy neutrino (for instance, relic neutrinos) trapping on circular orbits inside compact astrophysical objects (for instance, neutron stars) [4].

The obtained quasi-classical neutrino circular orbits can be explained as a result of the action of the generalized *effective Lorentz force* [3,4]. The force accounts for both weak and electromagnetic interactions of the active neutrinos (left-handed neutrinos and right-handed antineutrinos) with the rotating magnetized matter. The effective Lorentz force has the following form [4]

$$\mathbf{F} = q\mathbf{E} + q[\boldsymbol{\beta} \times \mathbf{B}]$$

where

$$q\mathbf{E} = q_m \mathbf{E}_m, \quad q\mathbf{B} = -(q_m \mathbf{B}_m + q_0 B) \mathbf{e}_z$$

$\boldsymbol{\beta}$ is the neutrino speed and \mathbf{e}_z is a unit vector in the direction of the magnetic field and matter rotation. The matter induced "charge" q_m , "electric" \mathbf{B}_m and "magnetic" \mathbf{E}_m fields are given by

$$q_m = -G, \quad \mathbf{E}_m = -\nabla n_n, \quad \mathbf{B}_m = -2n_n \boldsymbol{\omega}$$

The magnetic \mathbf{B} field reproduces the ordinary electrodynamic Lorentz force and the matter induced components is due to \mathbf{E}_m and \mathbf{B}_m . Note that in case of sterile neutrinos the effective Lorentz force reduces to the ordinary electrodynamic Lorentz force.

The effective electric field \mathbf{E} is produced by the gradient of the matter number density. The matter induced charges of neutrinos and antineutrinos are of opposite signs, $q_m^\nu = -G$ and $q_m^{\bar{\nu}} = G$. Therefore, neutrinos that propagate inside the neutron matter with decreasing density are decelerated while antineutrinos are accelerated.

The effective magnetic field \mathbf{B} contains the magnetic field \mathbf{B} itself and the additional term $\mathbf{B}_m = -2n_n \boldsymbol{\omega}$ originating by the matter rotation. Note that the effective magnetic force is orthogonal to the neutrino speed $\boldsymbol{\beta}$ and, thus, *bounds both neutrinos and antineutrinos*.

Deflection of neutrinos in rotating magnetized matter

The discussed above Lorentz force can have interesting applications in astrophysics. In particular, the force reasonably disturbs the trajectories of neutrinos propagating inside the rotating magnetized matter that leads to the *spatial separation of different types of neutrinos* (neutrinos and antineutrinos and/or different flavor neutrinos and/or neutrinos with different energies) [4].

The predicted effect of the neutrino deflection in the rotating magnetized matter can explain the recent experimental data of searches of neutrino signals in the correlation with observed light signals from astrophysical transient sources (including gamma-ray bursts, core collapse supernovae and active galactic nuclei) which up to now give no results [6].

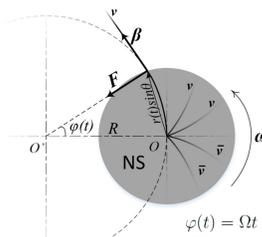
Naturally, for relativistic neutrinos the deviation angle is very small,

$$\Delta\phi \simeq R_S \Omega \sin\theta$$

where θ is the azimuthal angle of the neutrino motion and R_S characterizes the size of the source. However, due to immense distances to the astrophysical sources L the deflection of the neutrino beam is huge,

$$\Delta L \simeq \Delta\phi L$$

If we account only for the weak interactions and consider the source distance $L \sim 1 \text{ kpc}$ the deflection is about the distance between the Earth and the Sun (for $\sin \theta = 1$). Therefore, a neutrino beam can be reasonably deflected by the effective Lorentz force and cannot accompany the light signal in terrestrial observations even in the case of their initially coincided directions. Thus, the effect discussed above can explain the ANTARES experimental results [6].



Light of (milli)Charged Neutrino (LCν)

The effective Lorentz force should lead to the neutrino acceleration and the corresponding emission of the electromagnetic radiation due the neutrino electric millicharge. In the most general case of the neutrino motion in nonuniform magnetized rotating matter the effective Lorentz force originates the neutrino acceleration

$$\mathbf{a} = \frac{1}{m} (G \nabla n_n + (q_0 B + 2Gn_n \omega) [\boldsymbol{\beta} \times \mathbf{e}_z])$$

The electromagnetic radiation power of the neutrino has the classical form

$$I_{LC\nu} = \frac{2q_0^2}{3} \left(\frac{\dot{\boldsymbol{\beta}}^2}{(1 - \beta^2)^2} + \frac{(\boldsymbol{\beta} \dot{\boldsymbol{\beta}})^2}{(1 - \beta^2)^3} \right)$$

We term the considered new mechanism of the neutrino electromagnetic radiation due to the neutrino millicharge, that can be emitted in the presence of the nonuniform rotating matter and electromagnetic fields, the "Light of (milli)Charged Neutrino" (LCν). Note, that the phenomenon exists even in the absence of the electromagnetic fields, when the acceleration is produced only due to the weak interactions of neutrinos with background particles. So that the proposed new mechanism is of different nature than the one of the cyclotron radiation of charged particles in magnetic fields.

The LCν radiation power due to effective electric and magnetic fields are given by [4]

$$I_{LC\nu}^{\mathcal{E}} = \frac{2q_0^2}{3m^2} (G \nabla n_n)^2, \quad I_{LC\nu}^{\mathcal{B}} = \frac{2q_0^2}{3m^2} (q_0 B + 2Gn_n \omega)^2 \gamma^2$$

correspondingly.

The presented new mechanism of the neutrino electromagnetic radiation is of interest for astrophysics and, in particular, for a supernova explosion when a great amount of neutrinos propagate in dense magnetized rotating matter.

Neutrino Star Turning (vST)

Consider the propagation of the neutrinos inside a star on the basis of the obtained solutions for neutrinos in the rotating magnetized matter. During the propagation the effective Lorentz force disturbs the neutrino trajectories. Obviously, there is also the feedback of the neutrinos on the star. Therefore, our prediction is as follows: neutrinos that propagate inside a star should effect the star rotation. This new effect we have termed the "Neutrino Star Turning" (vST) mechanism.

The idea of star angular momentum losses due to the neutrino emission was proposed for the first time in [7]. In these and other subsequent papers only the slow-down of the star rotation is considered. We have developed a new approach to the description of neutrino propagation inside a rotating star on the basis of the introduced vST mechanism that can lead to both acceleration and deceleration of the star rotation.

To estimate the efficiency of the predicted vST mechanism we consider the impact of the escaping neutrinos on the star rotation. In case of relativistic neutrinos we obtain the shift of the star angular velocity in the following form [4]

$$|\Delta\omega| = \frac{5N_\nu}{6M_S} (q_0 B + 2Gn_n \omega)$$

where N_ν is a number of emitted neutrinos.

The impacts of the weak and electromagnetic interactions on the vST mechanism are defined by the matter induced and cyclotron frequencies accordingly. Recall that the matter induced charges of neutrinos and antineutrinos are of opposite signs, $q_m^\nu = -G$ and $q_m^{\bar{\nu}} = G$. Therefore, the weak interactions of neutrinos with the star matter spin down the star rotation ($\omega < 0$) while for antineutrinos the rotation is spined up ($\omega > 0$). The electromagnetic interaction of the negative millicharged neutrinos and positive millicharged antineutrinos with the star magnetic field reinforces these effects.

Only due to the weak interactions the vST mechanism shifts the star rotation according to the following relation

$$\frac{|\Delta\omega|}{\omega_0} = 4 \times 10^{-66} N_\nu \left(\frac{1.4 M_\odot}{M_S} \right) \left(\frac{\rho_n}{10^{14} \text{ g/cm}^3} \right)$$

while the electromagnetic interactions yields

$$\frac{|\Delta\omega|}{\omega_0} = 7.6 \varepsilon \times 10^{18} \left(\frac{P_0}{10 \text{ s}} \right) \left(\frac{N_\nu}{10^{58}} \right) \left(\frac{1.4 M_\odot}{M_S} \right) \left(\frac{B}{10^{14} \text{ G}} \right)$$

where ε is given by the expression $q_0 = \varepsilon e_0$ (e_0 is the absolute value of the electron charge).

From the obtained estimations it follows that the impact of the weak interactions on the star angular velocity is of interest only in case of sufficiently large neutrino emission. However, the consideration of the electromagnetic interactions can significantly reinforce the discussed effect.

New astrophysical limit on neutrino millicharge

One of the most strongest astrophysical neutrino sources is a supernova explosion with total $N_\nu \sim 10^{58}$ of emitted neutrinos. Therefore, the impact of the vST mechanism on the pulsar rotation rate during the formation of the pulsar in the supernova explosion is of particular interest.

The possible existence of a nonzero neutrino millicharge should not significantly change the rotation rate of a born pulsar. From the straightforward demand $|\omega| < \omega_0$ and we have obtained a new limit on the neutrino millicharge [4]

$$q_0 < 1.3 \times 10^{-19} e_0$$

that is, in fact, one of the *most severe astrophysical limits on the neutrino millicharge* [1].

Neutrino Star Turning and pulsar glitches

The shift of a star rotation due to neutrino emission recalls a very intriguing phenomenon that occurs during a life of pulsars. Pulsar timing observations exhibit the effect of sporadic sudden increases of the pulsar rotation frequency (*pulsar glitches*). Possible explanations of the phenomenon are given in many papers.

However, very recently the observation of the "anti-glitch" phenomenon, that is a sudden decrease of the rotation frequency, has been reported [8]. The observation of the "anti-glitch" requires reexamination of the nature of the pulsars frequency shifts. The vST mechanism proposed above can be used to explain both the glitch and "anti-glitch" phenomena. In particular, neutrinos (or antineutrinos) with a negative millicharged propagating inside a pulsar spin down the pulsar rotation rate. From our estimations it follows that the proposed vST mechanism with about 10^{51} total emitted neutrinos with the millicharge $q_0 = 10^{-18} e_0$ can explain the recent "anti-glitch" event of the magnetar 1E 2259+586 [4].