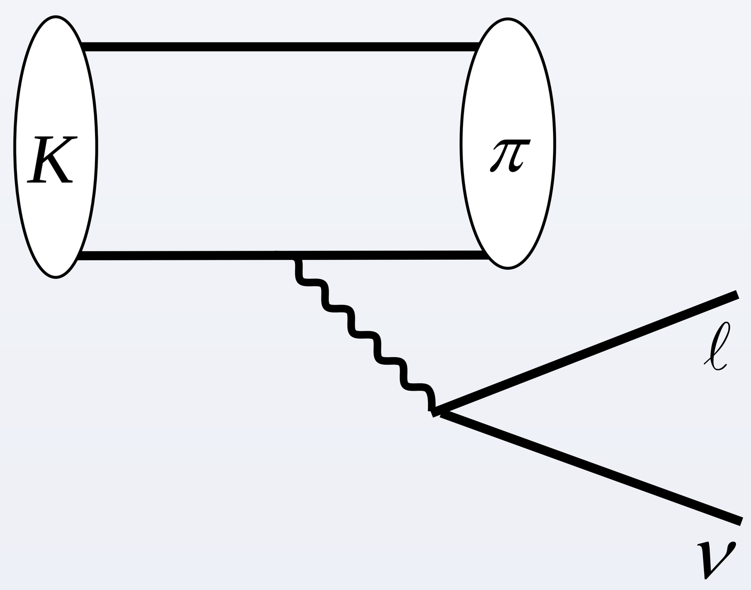
Kaon semileptonic vector form factor with $N_f = 2+1+1$ Twisted Mass fermions

N.Carrasco[a], P. Lami[a,b], V. Lubicz[a,b], E. Picca[a,b], L. Riggio[a], F. Sanfilippo[c], S. Simula[a], C. Tarantino[a,b]

INFN Roma Tre[a], Dipartimento Matematica e Fisica Università degli Studi Roma Tre[b], School of Physics and Astronomy University of Southampton[c].



Hadronic part of kaon semileptonic decay depends on the vector current matrix element

$$\langle \pi(p')|V_\mu|K(p)\rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2)$$

$$q^2 = (p_\mu - p'_\mu)^2$$

$$\Gamma(K\to\pi\ell\nu) \propto |f_+(0)V_{us}|^2$$

Determining $f_+(0)$ from Lattice QCD allows us to know $|V_{us}|$

Simulation details:

- Twisted Mass action at maximal twist at Nf=2+1+1 (strange and charm sea quarks contributions!)
- Osterwalder-Seiler valence quark action
- Iwasaki gluonic action
- Three different values of the lattice spacing: $0.06fm \div 0.09fm$
- Pion masses in range $210 \div 410\ MeV$
- Strange quark mass close to the physical value
- Pions and kaons simulated at several values of momentum

PRACE project PRA027

Analysis:

Determination of vector current matrix elements from 3-points correlation functions:

$$C_\mu^{K\pi}(t_x, t_y, \vec{p}, \vec{p}') \xrightarrow[(t_x - t_y)\to\infty]{t_x\to\infty} \frac{\sqrt{Z_K Z_\pi}}{4E_K E_\pi} \langle \pi(p')|V_\mu|K(p)\rangle e^{-E_K t_x - E_\pi(t_x - t_y)}$$

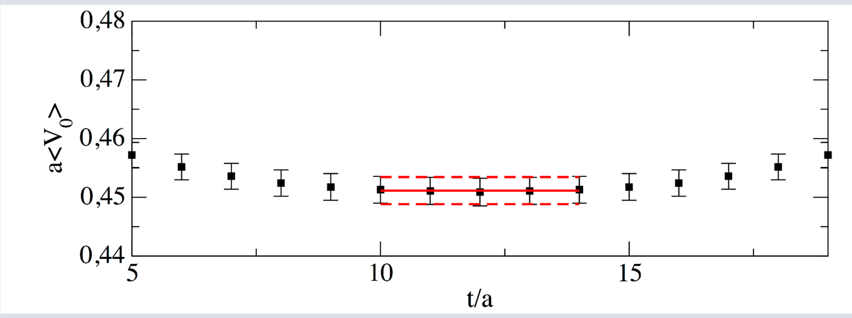
We define the following double ratio

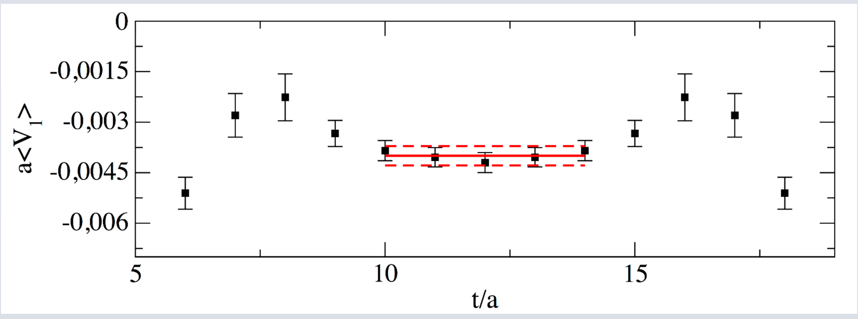
$$R_\mu = \frac{C_\mu^{K\pi}(t,\vec{p},\vec{p}')}{C_\mu^{\pi\pi}(t,\vec{p}',\vec{p}')} \frac{C_\mu^{\pi K}(t,\vec{p}',\vec{p})}{C_\mu^{KK}(t,\vec{p},\vec{p})} \xrightarrow{t\to\infty} \frac{\langle \pi(p')|V_\mu|K(p)\rangle\langle K(p)|V_\mu|\pi(p')\rangle}{\langle \pi(p')|V_\mu|\pi(p')\rangle\langle K(p)|V_\mu|K(p)\rangle}$$

$\beta = 1.90$
$L/a = 24$
$a\mu_l = 0.0080$
$a\mu_s = 0.0225$
$p_\pi = -87\text{MeV}$
$p_K = 87\text{MeV}$

$$\langle \pi(p')|V_0|K(p)\rangle = \langle V_0\rangle = 2\sqrt{R_0}\sqrt{EE'}$$

$$\langle \pi(p')|V_i|K(p)\rangle = \langle V_i\rangle = 2\sqrt{R_i}\sqrt{p_i p'_i}$$



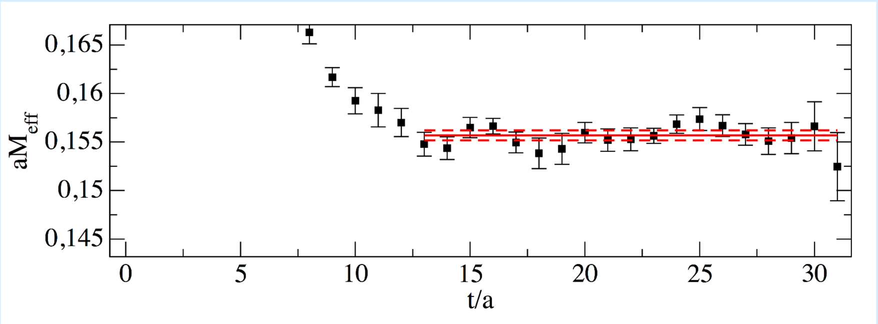


Meson masses determined from 2-points correlation functions

$$C^{K(\pi)}(t,\vec{p}(\vec{p}')) \xrightarrow{t\to\infty} \frac{Z_{K(\pi)}}{2E_{K(\pi)}} e^{-E_{K(\pi)}t}$$

Pion with zero momentum

$\beta = 1.95$
$L/a = 64$
$a\mu_l = 0.0055$
$p_\pi = 0$



Thus we can determine the form factors

$$f_+(q^2) = \frac{(E-E')\langle V_i\rangle - (p_i - p'_i)\langle V_0\rangle}{2Ep'_i - 2E'p_i} \qquad f_-(q^2) = \frac{(p_i + p'_i)\langle V_0\rangle - (E+E')\langle V_i\rangle}{2Ep'_i - 2E'p_i}$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m^2_K - m^2_\pi} f_-(q^2)$$

Study of f_+ and f_0 as functions of the transferred momentum q^2

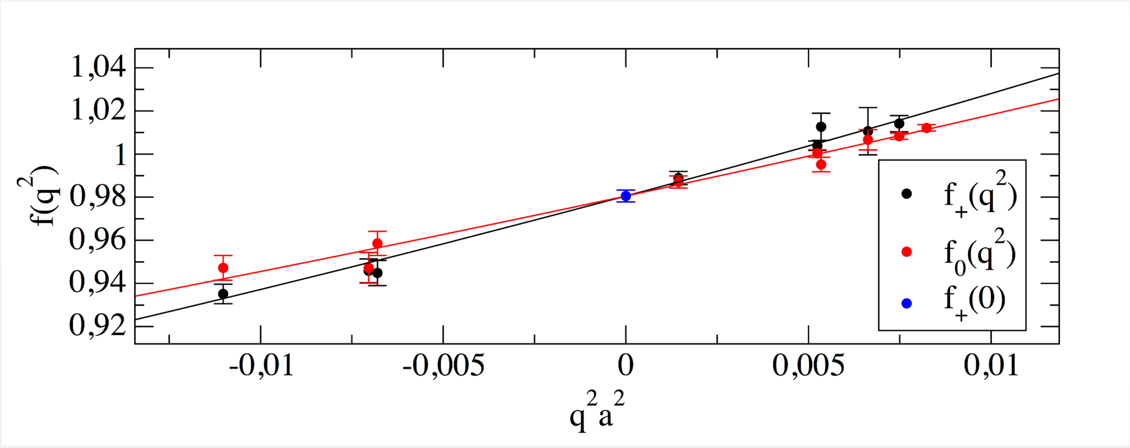
$$f_+(q^2) = \frac{a_0 + a_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_V{}^2}} \qquad f_0(q^2) = \frac{b_0 + b_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_S{}^2}}$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \qquad t_+ = (m_k + m_\pi)^2 \qquad t_0 = (m_k + m_\pi)(\sqrt{m_k} - \sqrt{m_\pi})^2$$

We also tried other ansatz, obtaining very similar results

Interpolation to $q^2 = 0$, where $f_+(0) = f_0(0)$

$\beta = 1.90$
$L/a = 24$
$a\mu_l = 0.0060$
$a\mu_s = 0.0225$



$f_+(0)$ data was then interpolated to the physical value of the strange quark mass[1]

Chiral and continuum extrapolation

Two different ansatz

SU(2) ChPT

$$f_+(0) = F^+{}_0\left(1 - \frac{3}{4}\xi\log\xi + P_2\xi + P_3 a^2\right) \qquad \xi = \frac{M_\pi^2}{(4\pi f_\pi)^2}$$

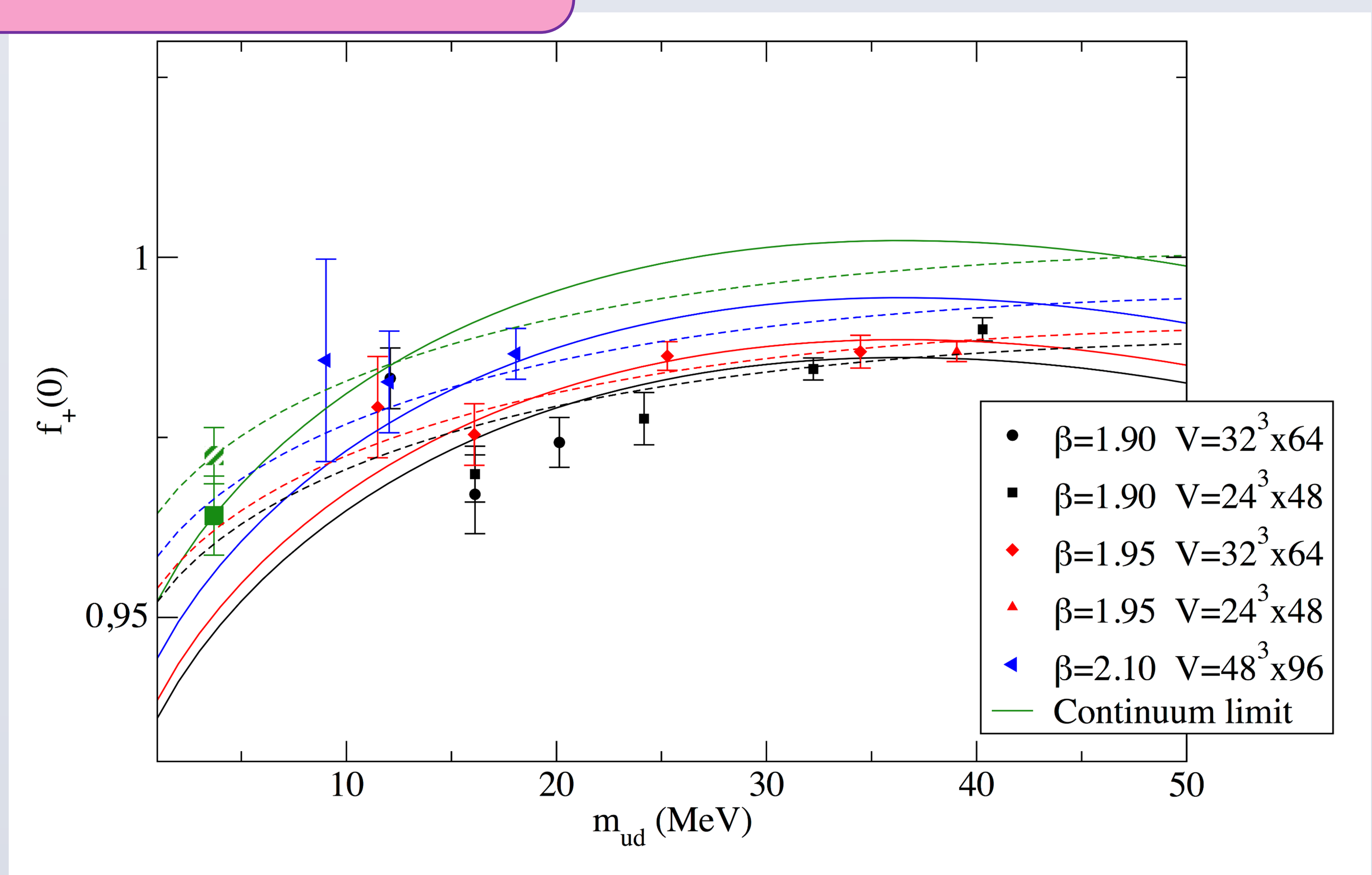
SU(3) ChPT

$$f_+(0) = 1 + f_2 + \Delta f$$

$$f_2^{\text{full QCD}} = \frac{3}{2}H_{\pi K} + \frac{3}{2}H_{\eta K} \qquad H_{PQ} = -\frac{1}{64\pi^2 f_\pi^2}\left[M_P^2 + M_Q^2 + \frac{2M_P^2 M_Q^2}{M_P^2 - M_Q^2}\log\frac{M_Q^2}{M_P^2}\right]$$

$$\Delta f = (M_K^2 - M_\pi^2)^2\left[\Delta_0 + \Delta_1(M_K^2 + M_\pi^2) + \Delta_3 a^2\right]$$

Chiral and continuum extrapolation of $f_0(q^2)$
- **SU(2): solid line**
- **SU(3): dashed line**



Our result

$$f_+(0) = 0.9683(50)_{stat+fit}(42)_{Chiral} = 0.9683(65)$$

Experimental average[2]

$$V_{us}f_+(0) = 0.2163(5)$$

Thus we obtain

$$V_{us} = 0.2234(16)$$

Testing the SM

$$|V_{ud}| = 0.97425(22)$$

from nuclear beta decay[3]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9991(8)$$

$|V_{ub}|^2 \sim 10^{-5}$

Contact information

Paolo Lami
Università degli Studi Roma Tre
INFN Sezione Roma Tre
Email: lami@fis.uniroma3.it

[1] N. Carrasco et al. ''Up, down, strange and charm quark masses with Nf=2+1+1 twisted mass Lattice QCD'' arxiv:1403.4504
[2] M. Antonelli et al. ''An evaluation of $|V_{us}|$ and precise tests of the SM from world data ...'' arxiv:1005.2323
[3] J.C. Hardy, I.S. Towner ''Superallowed 0+ to 0+ nuclear beta decays:...'' PhysRevC.79.055502

RESEARCH POSTER PRESENTATION DESIGN © 2012
www.PosterPresentations.com