τ hadronic spectral function moments in a nonpower QCD perturbation theory

Gauhar Abbas[1], B. Ananthanarayan[2], I. Caprini[3], J. Fischer[4]

[1] IFIC, Universitat de València – CSIC, Apt. Correus 22085, E-46071 València, Spain
[2] Centre for High Energy Physics, Indian Institute of Science, Bangalore, India
[3] Horia Hulubei National Institute for Physics and Nuclear Engineering, Bucharest, Romania
[4] Institute of Physics, Academy of Sciences of the Czech Republic, CZ-182 21 Prague 8, Czech Republic

Abstract

The moments of the hadronic spectral functions are of interest for the extraction of the strong coupling and other QCD parameters from the hadronic decays of the τ lepton. We consider the perturbative behavior of these moments in the framework of a QCD nonpower perturbation theory, defined by the technique of series acceleration by conformal mappings, which simultaneously implements renormalization-group summation and has a tame large-order behavior. Two recently proposed models of the Adler function are employed to generate the higher order coefficients of the perturbation series and to predict the exact values of the moments, required for testing the properties of the perturbative expansions. We show that the contour-improved nonpower perturbation theories and the renormalization-group-summed nonpower perturbation theories have very good convergence properties for a large class of moments of the so-called "reference model", including moments that are poorly described by the standard expansions.

- The perturbative contribution

$$\delta^{(0)}_{w_i}(s_0) = \frac{1}{2\pi i}\oint\limits_{|s|=s_0} \frac{ds}{s} W_i(s/s_0)\widehat{D}_{\rm pert}(s),$$

where some weights $W_i(x)$ are given on the right column and $\widehat{D}_{\rm pert}(s)$ is the perturbative part of the reduced Adler function

$$\widehat{D}(s) \equiv -s\, {\rm d}\Pi^{(1+0)}(s)/{\rm d}s - 1.$$

- A natural expansion for the Adler function is called 'fixed-order perturbation theory' (FOPT)

$$\widehat{D}_{\rm FOPT}(s) = \sum_{n\geq 1}(a_s(\mu^2))^n[c_{n,1} + \sum_{k=2}^{n} k c_{n,k}\left(\ln\frac{-s}{\mu^2}\right)^{k-1}],$$

where $a_s(\mu^2) = \alpha_s(\mu^2)/\pi$.

- By setting $\mu^2 = -s$ in the expansion, one obtains the 'contour-improved perturbation theory' (CIPT) expansion

$$\widehat{D}_{\rm CIPT}(s) = \sum_{n\geq 1} c_{n,1}\,(a_s(-s))^n,$$

where the running coupling $a_s(-s)$ is determined by solving the RG equation

$$s\frac{da_s(-s)}{ds} = \beta(a_s), \quad \text{and}$$

$$c_{1,1}=1,\ c_{2,1}=1.640,\ c_{3,1}=6.371,\ c_{4,1}=49.076,\ c_{5,1}=283\ (\textit{estimated}),$$
$$\beta_0 = 9/4,\ \beta_1 = 4,\ \beta_2 = 10.0599,\ \beta_3 = 47.228.$$

- The renormalization-group summed (RGSPT) expansion generalizes the summation of leading logarithms to non-leading logs, by summing all the terms available from RG invariance and can be written as

$$\widehat{D}_{\rm RGSPT}(s) = \sum_{n\geq 1}(\widetilde{a}_s(-s))^n[c_{n,1} + \sum_{j=1}^{n-1} c_{j,1}d_{n,j}(y)],$$

where

$$\widetilde{a}_s(-s) = \frac{a_s(\mu^2)}{1+\beta_0 a_s(\mu^2)\ln(-s/\mu^2)}$$

is the solution of the RG equation to one loop, $d_{n,j}(y)$ are calculable functions and $y \equiv 1+\beta_0 a_s(\mu^2)\ln(-s/\mu^2)$.

- We consider two models where the Adler function is

$$\widehat{D}(s) = \frac{1}{\beta_0}{\rm PV}\int_0^\infty \exp\left(\frac{-u}{\beta_0\tilde{a}_s(-s)}\right)B(u)\,{\rm d}u,$$
$$\frac{B(u)}{\pi} = B_1^{\rm UV}(u) + B_2^{\rm IR}(u) + B_3^{\rm IR}(u) + d_0^{\rm PO} + d_1^{\rm PO}u.$$

- The free parameters are fixed by imposing the known information about the series. Then all higher order coefficients can be predicted.
- The reference model (RM) is parameterized by the UV and first IR renormalons (Beneke and Jamin, 2008).
- In the alternative model (AM), the first IR renormalon is removed by hand (Boito, Beneke and Jamin, 2013).
- The weights shown in the figures are
$W_1(x) = 2(1-x)$, $W_6(x) = 2(1-x)$=$(1-x)^2$,
$W_{16}(x) = 2(1-x)\ \frac{1}{210}(1-x)^4(13+52x+130x^2+120x^3)$.
- We employ series acceleration by conformal mapping for moments of the Adler function. The technique was first proposed by Ciulli and Fischer (1961).
- The large-order behaviour of the expansion is improved by analytic continuation in the Borel plane. We consider the functions

$$\widetilde{w}_{lm}(u) = \frac{\sqrt{1+u/l}-\sqrt{1-u/m}}{\sqrt{1+u/l}+\sqrt{1-u/m}},\quad S_{lm}(u) = \left(1-\frac{\widetilde{w}_{lm}(u)}{\widetilde{w}_{lm}(-1)}\right)^{\gamma_1}\left(1-\frac{\widetilde{w}_{lm}(u)}{\widetilde{w}_{lm}(2)}\right)^{\gamma_2},$$

where l, m are positive integers satisfying $l\geq 1$ and $m\geq 2$.

- γ_j, $j=1,2$, are suitable exponents, defined such as to preserve the behavior of $B(u)$.
- We obtain a new class of 'RGS non-power expansions' (RGSNP)

$$\widehat{D}_{\rm RGSNP}(s) = \sum_{n\geq 0} c^{(lm)}_{n,\rm RGS}(y)\,\mathcal{W}^{(lm)}_{n,\rm RGS}(s),$$

$$\mathcal{W}^{(lm)}_{n,\rm RGS}(s) = \frac{1}{\beta_0}{\rm PV}\int_0^\infty \exp\left(\frac{-u}{\beta_0\tilde{a}_s(-s)}\right)\frac{(\widetilde{w}_{lm}(u))^n}{S_{lm}(u)}{\rm d}u.$$

- Similar expansions are obtained for CIPT and FOPT.

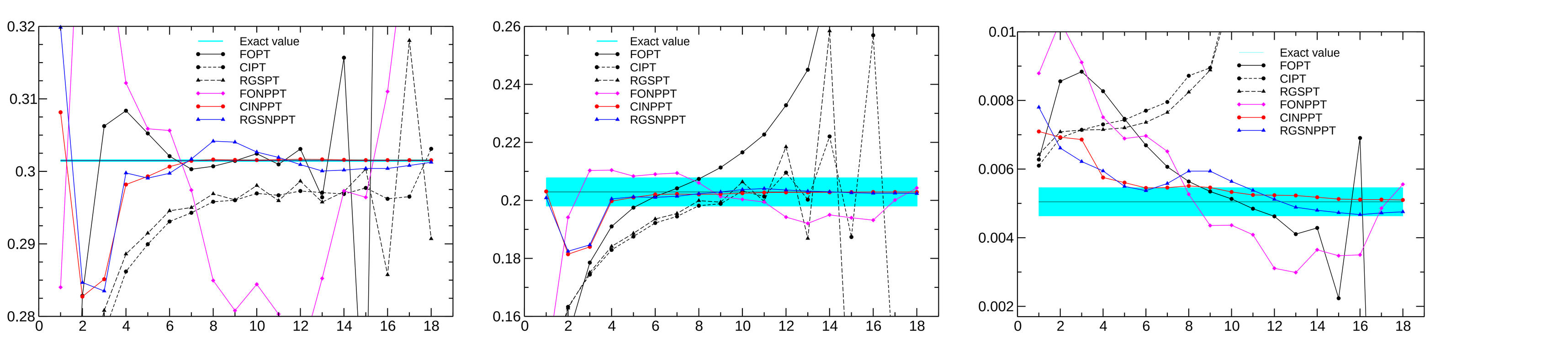

FIGURE 1: $\delta^{(0)}_{w_i}$ for the weights W_1, W_6, and W_{16} calculated for the RM, as functions of the perturbative order up to which the series was summed. The horizontal bands give the uncertainties of the exact values. We use $\alpha_s(M_\tau^2)=0.3186$.

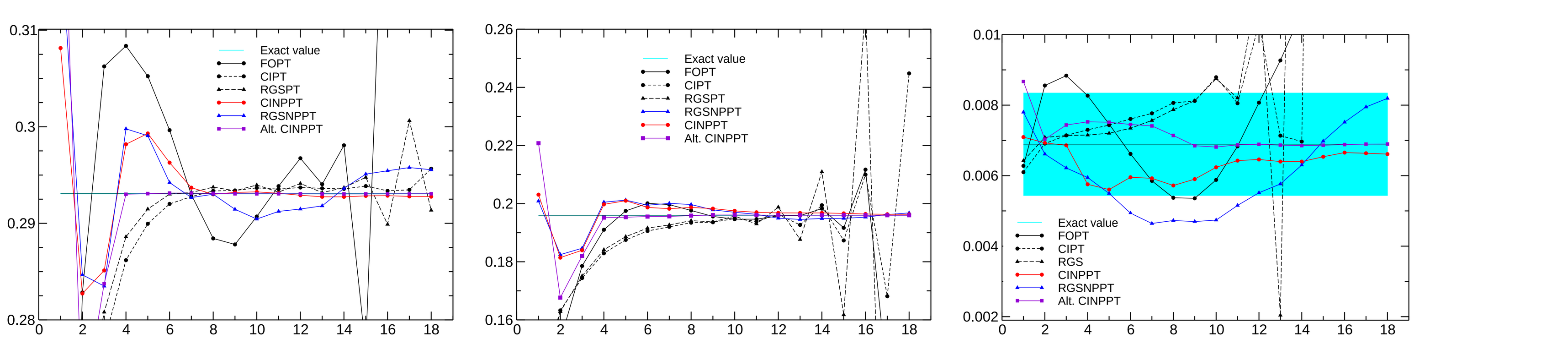

FIGURE 2: $\delta^{(0)}_{w_i}$ for the weights W_1, W_6, and W_{16} calculated for the AM, as functions of the perturbative order up to which the series was summed.

- The CINPPT and RGSNPPT expansions provide a good perturbative description of a large class of τ hadronic spectral function moments, including some for which all the standard expansions fail.
- A program that employs these expansions for the simultaneous determination of the strong coupling and other parameters of QCD from hadronic τ decays is of interest for future investigations.

References: G.A., B.A., I.C., J.F. *Phys.Rev. D88 (2013) 3, 034026*, G.A., B.A., I.C., J.F. *Phys.Rev. D87 (2013) 014008*, G.A., B.A., I.C. *Phys.Rev. D85 (2012) 094018*.