Volume (in-)dependence for $SU(N)$ gauge theories with twisted boundary conditions

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One slide on Eguchi-Kawai reduction

- Eguchi, Kawai, 82: Consider two pure gauge lattice theories:
  - $S_{gauge} = \beta \sum_{x,\mu<\nu}(1 - \frac{1}{N}\mathrm{ReTr}\, U^{\square}_{x,\mu\nu})$
  - $S_{EK} \quad = \beta \sum_{\mu<\nu}(1 - \frac{1}{N}\mathrm{ReTr}\, U^{\square}_{\mu\nu})$

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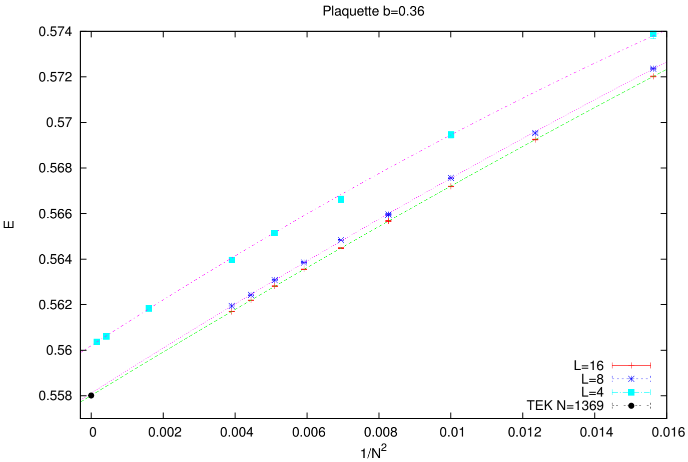
- They satisfy the same loop equations in $N \to \infty \Leftrightarrow$ no spontaneous breaking of $(\mathbb{Z}_N)^4 \to U(1)^4$ center symmetry
- But, in fact the symmetry *is* broken (Bhanot, Heller, Neuberger, 82).
- Fix 1 (Narayanan, Neuberger, 2003): Partial reduction (in L^4 box with big enough L; note that $L \to \infty$ in the continuum limit)
- Fix 2 (Gonzalez-Arroyo, Okawa, 83, 10): add twisted boundary conditions to the model.

(Another) one slide on Eguchi-Kawai reduction



Comparison of plaquette in TEK and p.b.c. lattice calculations (from Gonzalez-Arroyo's Lattice 2014 talk).

Statement of the problem & notation

- Problem: calculate glueball masses in pure gauge $SU(N)$ theory on spatial two-torus of fixed lattice size L with twisted boundary conditions.
- For numerical investigation: lattice model with Wilson action, periodic b.c. in time:

$$S = Nb \sum_{n \in \mathbb{Z}^3_{(L,L,T)}} \sum_{\mu \neq \nu} (N - z^*_{\mu\nu}(n) P_{\mu\nu}(n)),$$

  where $z_{\mu\nu}(n) = \exp(i\epsilon_{ij}\frac{2\pi k}{N})$ at corner plaquettes in each (1,2)-plane, and 1 everywhere else.
- Inverse 't Hooft coupling: $b = 1/g^2 N$
- Integer $\bar{k}$ defined as: $k\bar{k} = 1 \pmod{N}$

Motivation

Garcia-Perez, Gonzalez-Arroyo, Okawa, 13, 14:

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- Electric flux energies (calculated from Polyakov loop correlators) in the theory only depend on NL and twist parameters.
- More precisely: on $x = \frac{NL}{4\pi b}$ and $\tilde{\theta} = \frac{2\pi \bar{k}}{N}$
- Shown in perturbation theory to all orders, and also by lattice simulations in wide range of b.
- Can avoid tachyonic instabilities by keeping $k, \bar{k} \propto N$, just as in Twisted Eguchi-Kawai model (Gonzalez-Arroyo, Okawa, 10)
- Can be thought of as a strong form of TEK-like volume independence, also valid for finite N.

What we calculate

- Goal (long term): verify if this result holds also in the zero electric flux (glueball) sector.
- Goal (for this talk): calculate the mass of the lightest 0^{++} glueball as a function of b, for 2 different values of N.
- We take two theories chosen so that LN and $\tilde{\theta}$ are close:

1. $N = 5$, $L = 14$, $\bar{k} = 2$ ($NL = 70$, $\tilde{\theta} \approx 2.513$)
2. $N = 17$, $L = 4$, $\bar{k} = 7$ ($NL = 68$, $\tilde{\theta} \approx 2.587$)

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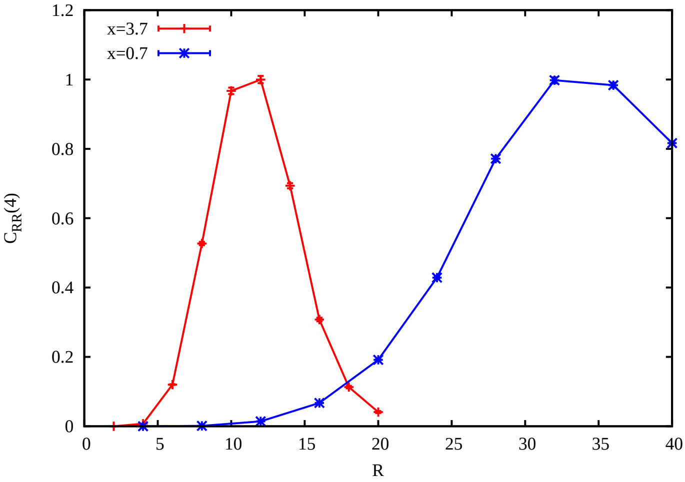
- Knowledge from earlier works:
  - $x \lesssim 0.5$ ($b \gtrsim 10$) - perturbative, small volume region
  - $0.5 \lesssim x \lesssim 4$ ($1.5 \lesssim b \lesssim 10$) - intermediate region
  - $x \gtrsim 4$ ($b \lesssim 1.5$) - large volume region

Intent: probe glueball mass in all 3 regions.

How we calculate

- Use correlations of rectangular Wilson loops and moduli of multi-winding Polyakov loops $|\mathrm{Tr}\, P^n|^2$
- Use 3 different levels of smearing and large loops, trying to follow the physical size of the glueball (including loops larger than L for small and moderate x)
- Construct: $C_{ij}(t) = \sum_{t'} \langle O_i(t'+t) O_j(t') \rangle - \langle O_i(t'+t) \rangle \langle O_j(t') \rangle$

How we calculate



$C_{RR}(4)$ is the (normalized) correlator of $W(R,R)$ at distance 4 lattice sites, $N=5$, $L=14$, $\bar{k}=2$.

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- Construct: $C_{ij}(t) = \sum_{t'} \langle O_i(t'+t) O_j(t') \rangle - \langle O_i(t'+t) \rangle \langle O_j(t') \rangle$
- Do GEVP:

$$C(t_1)v = C(t_0)\lambda v$$

  to find v, use them to change the basis $C(t) \to \tilde{C}(t)\ \forall t$ and fit to diagonal elements of $\tilde{C}(t)$ (after finding the plateau)
- Technicalities: use ≈ 12 operators for $C_{ij}(t)$, estimate if basis allows reliable GEVP by first solving it on *non-symmetrized* $C(t)$, use quad precision for GEVP and basis change

Results: work in progress

- Beware: results preliminary, all errors only statistical.

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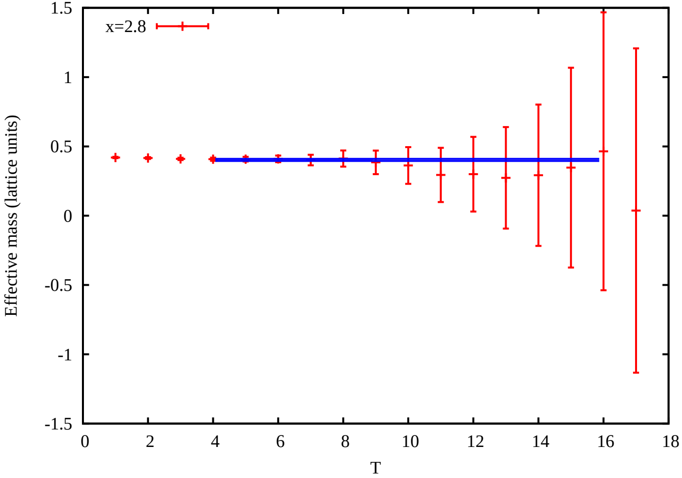
- Since last week: the small-x region under better control by simulations using replicas and T extended from 36 to 72.

Results: example mass plateau



Effective mass plateau for $N = 5$, $L = 14$, $\bar{k} = 2$, $b = 2$ $(x = 2.8)$, $Nmeas = 10^5$

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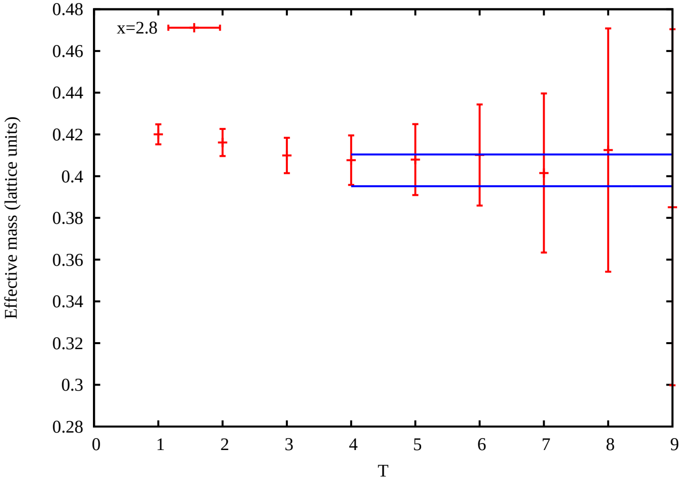

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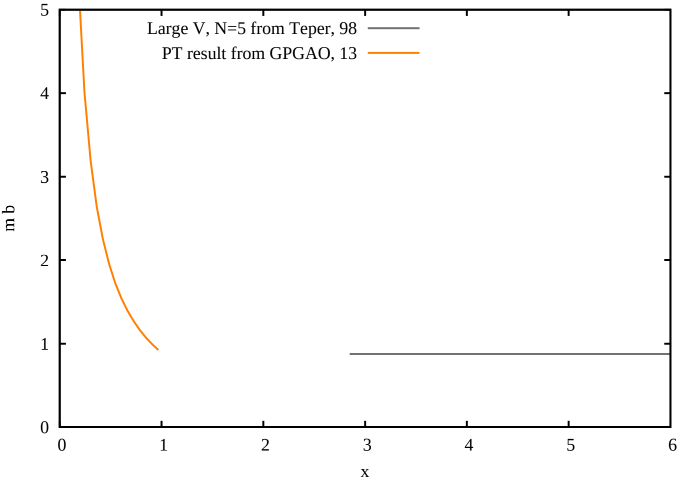

The theoretical expectations

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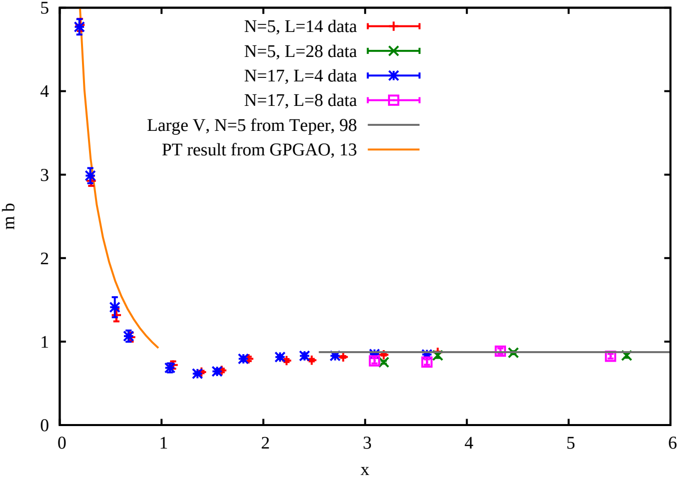

The results for both theories, note the results for doubled L in the large x region.

Conclusions

- Extracted 0^{++} glueball mass in large range of couplings with constant lattice size L for $N = 5$ and $N = 17$ with matching NL and electric flux $\tilde{\theta}$.
- Both N in good agreement in wide range of couplings, as expected by the x-scaling hypothesis!

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TODO list:

- Systematics: better extraction of plateaux, autocorrelations (slow modes in small x?)
- Add other quantum numbers (broader basis of operators?), especially 2^{++}.
- Investigate other $\bar{k}$ (and possibly N) values.

Outlook

- In principle, extension to 4 dimensions possible, done in PT (Garcia-Perez, Gonzalez-Arroyo, Okawa, Latt13)
- Need even number of twisted directions, physics governed by $N^{2/d}L$ and $\tilde{\theta}$.

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- Addition of adjoint fermions straightforward (in 4D: connection of N_f adjoints to QCD with $2N_f$ in Corrigan-Ramond limit via large-N equivalences), fundamental fermions more problematic.
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Thank you for your attention!