Probing CP violation in $B_s^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays

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Dalitz plot sensitivity for CKM angle $\gamma$

The search for new sources of $CP$ violation and New Physics are among the main goals of current particle physics research.

- Dalitz plot analyses provide additional tools, e.g. for the extraction of the CKM angle $\gamma$.

\[ B^0_{(s)} \rightarrow K^{+} \pi^{0} \pi^{0} \]

\[ B^0, B^0_s \]

\[ \bar{u} \rightarrow W \rightarrow \bar{s}, d \]

\[ \pi^{0} \]

\[ u \rightarrow d, s \]

\[ B^0_{(s)} \rightarrow K^{0} \pi^{+} \pi^{-} \]

\[ B^0, B^0_s \]

\[ \bar{b} \rightarrow W \rightarrow \bar{u}, \bar{s}, d \]

\[ \pi^{+} \]

\[ K^{*+}, \pi^{+} \]

\[ d, s \]

\[ \delta_j \text{ and } \alpha_j \text{ of } K^{*+}\pi^- \text{ and } K^{*0}\pi^0 (K^+\pi^-\pi^0) \]

\[ B^0_{(s)} \rightarrow K^{0} S \pi^{+} \pi^{-} \]

\[ \bar{b} \rightarrow W \rightarrow \bar{u}, \bar{s}, d \]

\[ \pi^- \]

\[ \pi^{0} \]

\[ d, s \]

\[ K^{*-} \]

\[ \delta_j \text{ between } K^{*-}\pi^+ \text{ and } K^{*-}\pi^- \text{ and } (K^0 S \pi^+\pi^-) \]

\[ A(K^{*+}\pi^-) + \sqrt{2}A(K^{*0}\pi^0) = -V_{ub}^{*}V_{us}(E_1 + E_2) \]

\[ A(K^{*-}\pi^+) + \sqrt{2}A(K^{*-0}\pi^0) = -V_{ub}V_{us}^{*}(E_1 + E_2) \]

\[ R^* = \frac{V_{ub}V_{us}^{*}}{V_{ub}^{*}V_{us}} = e^{-2i\gamma} \]

* Up to corrections due to $P_{EW}$
Dalitz plot sensitivity for CKM angle $\gamma$

The search for new sources of CP violation and New Physics are among the main goals of current particle physics research.

- Dalitz plot analyses provide additional tools, e.g. for the extraction of the CKM angle $\gamma$.

**$B^0$ case:**

- $B^0 \to K^* \pi$ is penguin dominated
- $\to$ large uncertainties in the subtraction

**$B^0_s$ case:**

- Amplitudes are tree-dominated
  $\to$ uncertainties on $\gamma$ due to $P_{EW}$ are smaller.
- Experimentally challenging for LHCb ($K^+\pi^-\pi^0$)

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CP violation in $B^0_s \rightarrow K^0_S \pi^+ \pi^-$ decays

With the first observation of the decay modes $B^0_s \rightarrow K^0_S \pi^+ \pi^-$ and $B^0_s \rightarrow K^0_S K^\pm \pi^\mp$ by LHCb, an interesting field for CP violation effect studies as well as CKM phase measurement is available.

Premise: investigate the CP violation sensitivity for different model dependent approaches and the relative phase in the $K^{\ast \pm}$ resonance for the $B^0_s \rightarrow K^0_S \pi^+ \pi^-$. 

Method I: Untagged and time integrated
Method II: Untagged and time-dependent
Method III: Tagged time-dependent

In the limit of tree-dominance, the relative phase for $B^0_s \rightarrow K^0_S \rho^0 \ (2\gamma + \phi_s)$ can be measured. Sensitivity may also be investigated (even though penguin effects are not expected to be negligible).

Similar methodology may be used for $B^0_s \rightarrow K^0_S K^\pm \pi^\mp$ decays, which is discussed later in the talk.
The decay-time distribution for $B^0_s$ and $\bar{B}^0_s$ meson decays to a final state $f$ (e.g. $B^0_s \rightarrow K^0_S \pi^+ \pi^-$) can be written as:

$$
\frac{d}{dt} \Gamma_{B^0_s \rightarrow f}(t) = \frac{N_f e^{-t/\tau(B^0_s)}}{2\tau(B^0_s)} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) + S_f \sin(\Delta m_s t) - C_f \cos(\Delta m_s t) + A^\Delta \Gamma_s \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \right]
$$

$$
\frac{d}{dt} \Gamma_{B^0_s \rightarrow f}(t) = \frac{N_f e^{-t/\tau(B^0_s)}}{2\tau(B^0_s)} \left[ \cosh \left( \frac{\Delta \Gamma_s t}{2} \right) - S_f \sin(\Delta m_s t) + C_f \cos(\Delta m_s t) + A^\Delta \Gamma_s \sinh \left( \frac{\Delta \Gamma_s t}{2} \right) \right]
$$

$$
S_f \equiv \frac{2 \Im(\lambda_f)}{1 + |\lambda_f|^2} \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad A^\Delta \Gamma_s \equiv -\frac{2 \Re(\lambda_f)}{1 + |\lambda_f|^2} \quad \lambda_f = \left| \frac{\tilde{A}_f}{A_f} \right| e^{i(\phi_f^s + \delta_f)}
$$

In the case of $B^0_s$ decays, the integration between zero and infinity leads to:

$$
N_f = \left( |A_f|^2 + |\tilde{A}_f|^2 \right) \times \frac{1 - y^2}{1 + y A^\Delta \Gamma_s}
$$

**Premise:** additional information on the phases between amplitudes from the $A^\Delta \Gamma$ term.

$$
y = \tau(B^0_s) \Delta \Gamma_s / 2
$$
A series of Toy Monte Carlo samples have been produced (*Laura++* package) using the Isobar Model with the given initial configuration:

Total amplitude: \[ A_f = \sum_{j=1}^{N} c_j F_j(f) \]

<table>
<thead>
<tr>
<th>Resonance</th>
<th>( x_j )</th>
<th>( \Delta x_j )</th>
<th>( y_j )</th>
<th>( \Delta y_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho^0(770) )</td>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( f_0^0(980) )</td>
<td>0.4 cos((5\pi/4))</td>
<td>0.0</td>
<td>0.4 sin((5\pi/3))</td>
<td>0.0</td>
</tr>
<tr>
<td>( K^*_\pm(892) )</td>
<td>1.2 cos((\pi/3))</td>
<td>0.0</td>
<td>1.2 sin((\pi/3))</td>
<td>0.0</td>
</tr>
<tr>
<td>( K^*_0(1430) )</td>
<td>1.7 cos((\pi/3))</td>
<td>0.0</td>
<td>1.7 sin((\pi/3))</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**CP Cartesian parametrisation:**
\[ c_j = (x_j \pm \Delta x_j) + i(y_j \pm \Delta y_j) \]

**Fitting scenario (\( K^0_s \rho^0 \) fixed):**

i. \( \phi_s \) floating and \( \Delta y \) fixed
ii. \( \phi_s \) fixed and \( \Delta y \) floating

nExperiments = 500
No simulation of experimental effects
(perfect acceptance, resolution, tagging)

**Three scenario of statistics:**

- Run I LHC (1+2 fb^{-1}): 500 events
- Run I+II LHC (\( \sim 10 \) fb^{-1}): 2K events
- LHC Upgrade (\( \sim 40 \) fb^{-1}): 10K events
Sensitivity CP violation studies

The sensitivity for different approaches of CP violation measurement are investigated in a number of physics scenario:

→ Three values for $\phi_s$: 0, $2\beta_s$ and $10 \times 2\beta_s$.
  - Null test: no direct CP violation
  - CP violation in the magnitude
    → $A_{CP}$ of 5%, 10%, 20% and 50%
  - CP violation due to the difference in the phase
    → Relative phase in steps of $\pi/4$ [$\pi/4$, $\pi$].
  - CP violation in both magnitude and phase:
    → Same range for $A_{CP}$ and relative phase
No direct CP violation hypothesis

\[ \phi_s = 0 \]

Method I (time–integrated): \( B_s^0 (\bar{B}_s^0) \)

Method II (untagged): \( B_s^0 (\bar{B}_s^0) \)

Method III (tagged): \( B_s^0 (\bar{B}_s^0) \)
CP violation in the K*(892) resonance

\[ \phi_s = 0 \]

\[ \phi_s = 2\beta_s \]

\[ \phi_s = 2\beta_s \]

\[ \phi_s = 0 \]

\[ \phi_s = 0 \]

\[ \phi_s = 2\beta_s \]

Method I (time-integrated): \( B_s^0 (\bar{B}_s^0) \)
Method II (untagged): \( B_s^0 (\bar{B}_s^0) \)
Method III (tagged): \( B_s^0 (\bar{B}_s^0) \)
Fit to $\phi_s$ with CP violation in K*(892)

Perfect tagging gives an order of magnitude improvement in sensitivity
Alternative fit : CP violation in $\rho^0(770)$

In the same way that the mixing phase can be extracted by fixing $\Delta y$, one can fix $\phi_s = 2\beta_s$, and try to measure the components for the $\rho^0(770)$:

As on the previous slide, the untagged method has limited sensitivity to the relative phase.

Method II (untagged): $B_s^0(\bar{B}_s^0)$  Method III (tagged) : $B_s^0(\bar{B}_s^0)$
**ToyMC results in a nutshell**

The summary of the Toy Monte Carlo results are collected in the table below (for LHC run I+II):

<table>
<thead>
<tr>
<th>$K^{*\pm}(892)$</th>
<th>$CP$</th>
<th>$\sigma_{err}$ (tag) (%)</th>
<th>$\sigma_{err}$ (untag)/(tag)</th>
<th>$\sigma_{err}$ (t-indep)/(tag)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_j$</td>
<td>$\bar{a}_j$</td>
<td>$\delta_j$</td>
</tr>
<tr>
<td>$A_{CP} = 20%$ ((\phi_s = 0))</td>
<td>4.6</td>
<td>3.7</td>
<td>12.3</td>
<td>11.6</td>
</tr>
<tr>
<td>$A_{CP} = 50%$ ((\phi_s = 2\beta_s))</td>
<td>5.1</td>
<td>3.3</td>
<td>15.0</td>
<td>12.2</td>
</tr>
<tr>
<td>$\Delta\delta = \pi/4$ ((\phi_s = 0))</td>
<td>4.2</td>
<td>4.3</td>
<td>12.2</td>
<td>7.7</td>
</tr>
<tr>
<td>$\Delta\delta = 3\pi/4$ ((\phi_s = 2\beta_s))</td>
<td>4.2</td>
<td>4.0</td>
<td>12.4</td>
<td>4.7</td>
</tr>
<tr>
<td>$A_{CP} = 5% + \Delta\delta = \pi/4$ ((\phi_s = 0))</td>
<td>4.5</td>
<td>3.9</td>
<td>11.4</td>
<td>8.3</td>
</tr>
<tr>
<td>$A_{CP} = 50% + \Delta\delta = \pi$ ((\phi_s = 2\beta_s))</td>
<td>5.2</td>
<td>3.6</td>
<td>14.5</td>
<td>7.1</td>
</tr>
</tbody>
</table>

- These studies indicate that good precision can be achieved for the relative magnitude and phase of the $K^{*\pm}(892)\pi^\mp$ amplitude in all scenarios investigated (even in the time-integrated).
- Untagged approach has limited sensitivity to determine $\phi_s \rightarrow$ tagged analysis will be required.
- Considering the LHCb tagging efficiency power of $\sim 5\%$, the tagged approach still provides $\sim 2.5$ better sensitivity to $\phi_s$ than the untagged case.
Related method for $B^0_s \rightarrow K^0_S K\pi$ decays

In addition to $B^0_s \rightarrow K^0_S \pi^+\pi^-$ decays, a more attractive (experimentally) approach is to perform a simultaneous fit of the two different $B^0 \rightarrow K^0_S K^+\pi^-$ and $B^0 \rightarrow K^0_S K^-\pi^+$ final states.

- From the theory side, method has similar issues as $B^0 \rightarrow K\pi\pi$.
- Less sensitivity due to the penguin dominance and the electroweak penguin effect is large.
- No need to study any mode with $\pi^0$.
- Untagged time-dependent method may be the only viable approach.
- Work in progress to produce similar studies.
General remarks

- Good sensitivity for the phase difference $B^0_s$ and $\bar{B}^0_s$ from $K^*(892)\pi$ decays can be achieved with untagged analysis approaches (e.g. for LHC Run I and Run II).

- Tagging is, however, needed to determine $\phi_s$ (i.e. relative phase between $B^0_s \rightarrow K^0_s\rho^0$ and $\bar{B}^0_s \rightarrow K^0_s\rho^0$).

- Alternative method with $B^0_s \rightarrow K^0_s K^\pm\pi^\mp$ is more attractive experimentally [work in progress].
Backup slides
The main features for the Dalitz plot generation in fit in the Laura++ package are listed below:

The generation is performed using the full time-dependent using:

\[
\begin{align*}
\text{CP polar Generation:} & \quad \bar{c}_j = a_j e^{i\delta_j} \\
\text{CP Cartesian Fit parametrisation:} & \quad \bar{c}_j = (x_j \pm \Delta x_j) + i(y_j \pm \Delta y_j)
\end{align*}
\]

Multiple solutions:

- It is possible the during the process of minimisation the fit finds multiple solutions.
- To ensure a global minimum, each fit is repeated 30 times with randomised starting parameters.
- The solution with the smallest negative log-likelihood is taken as the default result.
- This is performed for each of the 500 experiments.

\[
\begin{align*}
\text{Fit fractions:} & \quad FF_j = \frac{\int \int (|c_j F_j|^2 + |\bar{c}_j \bar{F}_j|^2) dm_{K_S^0 \pi^+}^2 + dm_{K_S^0 \pi^-}^2}{\int \int (|A|^2 + |\bar{A}|^2) dm_{K_S^0 \pi^+}^2 + dm_{K_S^0 \pi^-}^2} \\
\text{Asymmetry:} & \quad A_{CP,j} = \frac{|\bar{c}_j|^2 - |c_j|^2}{|\bar{c}_j|^2 + |c_j|^2} = \frac{-2(c_j \Delta x_j + y_j \Delta y_j)}{x_j^2 + \Delta x_j^2 + y_j^2 + \Delta y_j^2}
\end{align*}
\]
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**CP violation in f₀(980) and K*(1430)**

\[ f₀(980) \]
\[ \phi_s = 2\beta_s \]
\[ N_{\text{evts}} = 2K \]
\[ A_{CP}^{f₀(980)} = 20\% \]

\[ K*(1430) \]
\[ \phi_s = 2\beta_s \]
\[ N_{\text{evts}} = 2K \]
\[ A_{CP}^{K*(1430)} = 20\% \]

\[ \Delta \delta_{J}^{K*±(1430)} = 3\pi/4 \]
Statistical sensitivity comparison

Method I (time–integrated): $B_s^0(\bar{B}_s^0)$  
Method II (untagged): $B_s^0(\bar{B}_s^0)$  
Method III (tagged) : $B_s^0(\bar{B}_s^0)$
An alternative to the model-dependent approaches presented in this study, is to analyse methods such as Mirandizing.

\[ DP_{S_{CP}} = \frac{N_+(i) - N_-(i)}{\sqrt{N_+(i) + N_-(i)}} \]

\[ m' = \frac{1}{\pi} \arccos \left( 2 \frac{m_{\pi^+\pi^-} - m_{\pi^+\pi^-}^{\text{min}}}{m_{\pi^+\pi^-}^{\text{max}} - m_{\pi^+\pi^-}^{\text{min}}} - 1 \right) \]

\[ \theta' = \frac{1}{\pi} \theta_{\pi^+\pi^-} \]