Inclusion of isospin breaking effects in lattice simulations

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(University of Southampton)
Lattice 2014 last week in Columbia University (NYC, USA)
Slides: https://indico.bnl.gov/conferenceDisplay.py?confId=736
What’s new?
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- [MILC, 2014] — C. Bernard talk at Lattice 2014
  - update of quark masses and Dashen’s theorem corrections using electro-quenched simulations
  - new insights on finite-volume effects
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  - study of the $\Sigma^0 - \Lambda^0$ system
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   • finite-volume corrections to hadron masses in NREFTs
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- [QCDSF, 2014] — G. Schierholz talk at Lattice 2014
  - new full $N_f = 1+1+1$ QCD+QED simulations
  - preliminary results for the baryon octet splittings
What’s new?

  • new full $N_f = 1+1+1$ QCD+QED simulations
  • preliminary results for the baryon octet splittings
  • new set of $N_f = 1+1+1+1$ full QCD+QED simulations
  • extensive analytical/numerical study of finite-volume effects
  • high precision computation of the hadron spectrum splittings (continuum, infinite volume and physical point extrapolation)
• Motivations
• Lattice QCD+QED
• Update on electro-quenched results
• Isospin splittings in the hadron spectrum
• Summary & outlook
Motivations
Isospin symmetry breaking

- Isospin symmetric world: up and down quarks are particles with identical physical properties.
Isospin symmetry breaking

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- Isospin symmetry is explicitly broken by:
  - the up and down quark mass difference
    \[ \frac{|m_u - m_d|}{\Lambda_{QCD}} \simeq 0.01 \]
  - the up and down electric charge difference
    \[ \alpha \simeq 0.0073 \]

<table>
<thead>
<tr>
<th></th>
<th>up</th>
<th>down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (MeV)</td>
<td>2.3(+0.7</td>
<td>-0.5)</td>
</tr>
<tr>
<td>Charge (e)</td>
<td>2/3</td>
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source: [PDG, 2013]
Nucleon mass splitting

- Well known experimentally:

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- initial condition for Big-Bang nucleosynthesis
Dashen’s theorem

In the SU(3) chiral limit [Dashen, 1969]:

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- \( \varepsilon \) is important to determine light quark mass ratios
Lattice QCD+QED
Lattice QCD

- Lattice QCD simulation: Monte-Carlo estimation of discretised QCD functional integrals
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- Extremely expensive, but ab-initio
Non-compact lattice QED

- Naively discretised Maxwell action:

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- No mass gap: large finite volume effects expected
Zero-mode subtraction

Finite volume: **momentum quantisation**

\[
\alpha \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \cdots \quad \leftrightarrow \quad \frac{\alpha}{V} \sum_k \frac{1}{k^2} \cdots
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Contains a straight 1/0!
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- **Many possible schemes:**
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- **Different schemes:** **different finite volume behaviours**
- Some more interesting than others
QED_{TL} zero-mode subtraction

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  Mostly used in all simulations so far
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- **With QED\textsubscript{TL},** the $T \to \infty$, $L = \text{cst.}$ limit can diverge:

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\frac{\alpha}{V} \sum_{k \neq 0} \frac{1}{k^2} \cdots \quad \leftrightarrow \quad \frac{\alpha}{L^3} \int \frac{dk_0}{2\pi} \sum_{k} \frac{1}{k^2} \cdots
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- **QED\textsubscript{TL}** does not have reflection positivity
**Example — 1-loop QED\textsubscript{TL} [BMW\textsc{c}, 2014]:**

\[ m(T, L) \sim m \left\{ 1 - q^2 \alpha \left[ \frac{\kappa}{2mL} \left( 1 + \frac{2}{mL} \left[ 1 - \frac{\pi}{2\kappa} \frac{T}{L} \right] \right) \right] - \frac{3\pi}{(mL)^3} \left[ 1 - \frac{\coth(mT)}{2} \right] - \frac{3\pi}{2(mL)^4} \frac{L}{T} \right\} \]

up to exponential corrections, with \( \kappa = 2.83729 \ldots \)
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- Divergent finite volume effects with $T \to \infty, L = \text{cst}$.
- Same behaviour independently discovered by MILC
QED\textsubscript{L} zero-mode subtraction

- QED\textsubscript{L}: \( A_\mu(k_0, 0) = 0 \)

inspired from [Hayakawa & Uno, 2008]
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- \textbf{QED}_L finite volume effects:

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inverse powers of \( L \), independent of \( T \)
Finite-volume effects

Pure QED simulations (quenched) from [BMWc, 2014]
Finite-volume effects

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- [BMWc, 2014]: Ward identities: NLO is universal
Electro-quenched approximation

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- **Greatly reduce** the computational cost
Electro-quenched approximation:

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- Non-unitary theory (partially quenched)
- Greatly reduce the computational cost
- Missing contributions are large-$N_c$ and SU(3) flavour suppressed: $O(10\%)$ of EM effects
Update on electro-quenched results
EQ results for the baryon spectrum

EQ results for $\varepsilon$

[Maltman and Kotchan, 1990]
[Donoghue et al., 1993]
[Bijnens, 1993]
[Baur and Urech, 1996]
[Bijnens and Prades, 1997]
[Donoghue and Perez, 1997]
[Gao et al., 1997]
[Moussallam, 1997]
[Duncan et al., 1996] (quenched QCD)
[RBC-UKQCD, 2007]
[RBC-UKQCD, 2010]
[RM123, 2013]
[BMWc, 2014] (EQ, preliminary)
[MILC, 2014] (preliminary)
EQ results for light quark masses

- PDG 2013 band
- [Duncan et al., 1996] (quenched QCD)
- [RBC-UKQCD, 2007]
- [RBC-UKQCD, 2010]
- [RM123, 2013]
- [BMWc, 2014] (EQ, preliminary)
- [MILC, 2014] (preliminary)
- [PACS-CS, 2012]
Isospin splittings in the hadron spectrum
[QCDSF, 2014]: progress summary

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more details: G. Schierholz talk at Lattice 2014
[BMWC, 2014]: mass splitting calculation

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- **many smeared sources** per configurations (O(100))
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- Small extrapolation to the physical point (similar to [BMWc, 2013])
- Systematic error based on BMW’s histogram method. Weights are based on the goodness of the fits, flat and Akaike’s information criterion (**overfitting is penalised**)
- O(500) analyses per mass splitting
[BMWc, 2014]: finite-volume study

\[ \chi^2/\text{dof} = 0.86 \] (A)

\[ \chi^2/\text{dof} = 0.90 \] (B)

\[ (aM_{K^0})^2 - (aM_{K^+})^2 \]

LO
NLO
NNLO

1/(aL)

0.237
0.238
0.238
0.238
\[ \Delta_{CG} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi \] (Coleman-Glashow relation)
What is the mass difference between $\bar{\Xi}_{cc}^+$ and $\bar{\Xi}_{cc}^{++}$ (including sign)?)?

I do not care how you calculate it (HQET, Lattice, ...), JUST DO IT

J. Engelfried, LHC Workshop 2013, Trento

$\Delta CG = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi$ (Coleman-Glashow relation)
Results for the nucleon mass splitting

\[ (M_n - M_p)_{\text{QED}} \text{ (MeV)} \]

\[ (M_n - M_p)_{\text{QCD}} \text{ (MeV)} \]

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Summary & outlook
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Finite-size effects on masses are now well controlled.
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Summary

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- The isospin splittings in the hadron spectrum are determined with a high accuracy and full control of uncertainties
- The nucleon mass splitting is determined as a $> 5\sigma$ effect
Outlook
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- QCD+QED decay constants are gauge variant and IR divergent. How to deal with that? First lattice attempt: [C.T. Sachrajda, Lattice 2014]

- Compute corrections to matrix elements ($K_{\ell 3}, K \rightarrow \pi\pi, \ldots$)
Outlook

- Unquenched computations of the light quark masses and Dashen’s theorem corrections
- QCD+QED decay constants are gauge variant and IR divergent. How to deal with that? First lattice attempt: [C.T. Sachrajda, Lattice 2014]
- Compute corrections to matrix elements ($K_{\ell 3}$, $K \rightarrow \pi \pi, \ldots$)
- QCD+QED to compute hadronic corrections to anomalous magnetic moments.
Thank you!
Backup
# Full QCD + QED projects

<table>
<thead>
<tr>
<th></th>
<th>RBC-UKQCD</th>
<th>PACS-CS</th>
<th>QCDSF-UKQCD</th>
<th>BMWc</th>
</tr>
</thead>
<tbody>
<tr>
<td>arXiv</td>
<td>1006.1311</td>
<td>1205.2961</td>
<td>1311.4554 and Lat. 2014</td>
<td>1406.4088</td>
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<tr>
<td>fermions</td>
<td>DWF</td>
<td>clover</td>
<td>clover</td>
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<tr>
<td>$N_f$</td>
<td>2+1</td>
<td>1+1+1</td>
<td>1+1+1</td>
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<td>method</td>
<td>reweighting</td>
<td>reweighting</td>
<td>RHMC</td>
<td>RHMC</td>
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<tr>
<td>min($M_\pi$) (MeV)</td>
<td>420</td>
<td>135</td>
<td>250</td>
<td>195</td>
</tr>
<tr>
<td>$a$ (fm)</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
<td>0.06 — 0.10</td>
</tr>
<tr>
<td>$#a$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$L$ (fm)</td>
<td>1.8</td>
<td>2.9</td>
<td>1.9 — 2.6</td>
<td>2.1 — 8.3</td>
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<tr>
<td>$#L$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>11</td>
</tr>
</tbody>
</table>
[BMWc, 2014]: QED simulations

\[ L(m_{2L} - m_L) \]

\[ L_{m_L} = 2 \]

unsmeared, Wilson

smeared, clover

\( a m_L \)
[BMWc, 2014]: charge renormalisation

\[ \Delta M_{\pi}^2 [\text{MeV}^2] \]

\[ \frac{e^2}{4\pi} \]

bare
renormalized
[BMWC, 2014]: charm discretisation effects

\[ \Delta M_{\chi} [\text{MeV}] \]

\[ \Delta D \quad \chi^2 / \text{dof}=0.94 \]

\[ \Delta \Xi_{cc} \quad \chi^2 / \text{dof}=1.30 \]