The Inclusive Determination of $|V_{cb}|$

Numerical Analysis and Fit to $\mathcal{O}(\alpha_s^2, \alpha_s \frac{\Lambda^2}{m_b^2})$

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with

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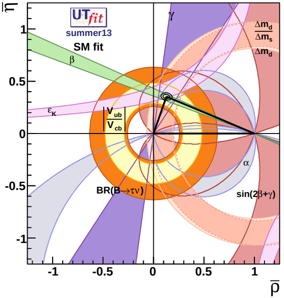
A continuation of the previous work by P. Gambino, C. Schwanda, A. Alberti

Motivation

- V_{cb} through $(B \to X_c \ell \bar{\nu})$ Important :
    - * Probe $b \to c$ transition
    - * CP violation in the SM
    - * NP search through FV Processes
- $\frac{|V_{ub}|}{|V_{cb}|}$ ratio gives one of the sides of the CKM unitarity triangle
- ϵ_k constraint on $\bar{\rho}$ and $\bar{\eta}$ sensitive to $|V_{cb}|$



- Determined from both $B \to X_c \ell \bar{\nu}$ and $B \to D^* \ell \bar{\nu}$

$|V_{cb}|_{inc} = 42.42 \pm 0.86 \times 10^{-3}$ [P.G., C.S. Phys. Rev. D 89, 014022 (2014)]

$|V_{cb}|_{ex} = 39.04 \pm 0.8 \times 10^{-3}$ [FNAL, MILC, Phys. Rev. D 89, 114504 (2014)]

- Near 3 σ discrepancy between inclusive and exclusive $B \to D^* \ell \bar{\nu}$ at zero recoil with form factors from lattice.

Inclusive $B \to X_c \ell \bar{\nu}$ Determination

Operator Product Expansion (OPE)

- Perturbative + Heavy Quark Expansion
- Heavy Quark Limit → partonic decay rate
    - * Corrections to Observables
      expressed as double expansion in $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\Lambda_{QCD}/m_b)$
- Express the observables

$$
\begin{aligned}
M_i \quad = \quad & M_i^{(0)} + M_i^{(\pi,0)} \frac{\mu_\pi^2}{m_b^2} + M_i^{(G,0)} \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} \\
& + \left(\frac{\alpha_s}{\pi}\right) \left[M_i^{(1)} + M_i^{(\pi,1)} \frac{\mu_\pi^2}{m_b^2} + M_i^{(G,1)} \frac{\mu_G^2}{m_b^2} + M_i^{(D,1)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,1)} \frac{\rho_{LS}^3}{m_b^3} \right] \\
& + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \mathcal{O}\left(m_b^{4,5}, \alpha_s^3\right)
\end{aligned}
$$

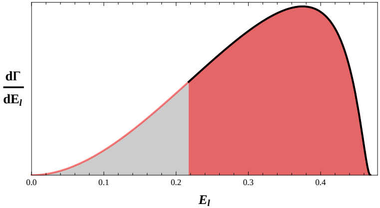
- Need information on non-perturbative parameters
- $|V_{cb}|$ extracted from width after HQE parameter fit
- $\mathcal{O}(1/mb^{4,5})$ T. Mannel, S. Turczyk, N Uraltsev [JHEP 1011 (2010) 109]
- ...too many parameters for fit, estimate small changed in $|V_{cb}|$

Relevant Observables

Leptonic Moments

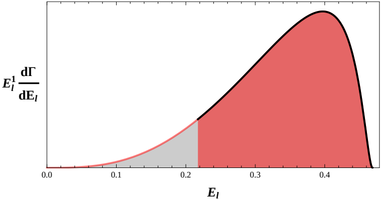
$$\langle E_\ell^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_{cut}}^{E_{max}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$

$$R^*(E_{cut}) = \frac{\int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_0^{E_{max}} \frac{d\Gamma}{dE_\ell} dE_\ell}$$



$\langle E_\ell^1 \rangle, \langle E_\ell^2 \rangle, \langle E_\ell^3 \rangle$ Highly Correlated

Central Leptonic Moments



$$\ell_1(E_{cut}) = \langle E_\ell \rangle_{E_\ell > E_{cut}}$$

$$\ell_{2,3}(E_{cut}) = \langle (E_\ell - \langle E_\ell \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}$$

Hadronic Moments

$$\langle (M_X^2)^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} (M_X^2)^n \frac{d\Gamma}{dM_x^2} dM_x^2}{\int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dM_x^2} dM_x^2}$$

$$h_1(E_{cut}) = \langle M_X^2 \rangle_{E_\ell > E_{cut}}$$

$$h_{2,3}(E_{cut}) = \langle (M_X^2 - \langle M_X^2 \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}$$

Experimental Observables

- Observables as $F_x(E_{cut}, m_c^2/m_b^2)$
- Express $m_b, \mu_{\pi,G}, \rho_{D,LS}$ in the "kinetic scheme" with a cutoff $\mu_{kin} = 1GeV$
- Additionally employ both kinetic and $\overline{\mathrm{MS}}$ definitions for m_c
- $\alpha_s(m_b = 4.6GeV) = 0.22$
  $\alpha_s \pm 0.005 \to \delta m_b < 1MeV$
- Additional Constraints:

Hyperfine Splitting

$$M_{B^*} - M_B = \frac{2}{3}\frac{\mu_G^2}{m_b} + O\left(\frac{\alpha_s \mu_G^2}{m_b}, \frac{1}{m_b^2}\right)$$

$$\mu_G^2 = (0.35 \pm 0.07)\,\mathrm{GeV}^2$$

Heavy Quark Sum Rules

$$\rho_{LS}^3 = (-0.15 \pm 0.10)\,\mathrm{GeV}^3$$

	Experiment	Values of $E_{cut}(\mathrm{GeV})$
R^*	BaBar	0.6, 1.2, 1.5
ℓ_1	BaBar	0.6, 0.8, 1, 1.2, 1.5
ℓ_2	BaBar	0.6, 1, 1.5
ℓ_3	BaBar	0.8, 1.2
h_1	BaBar	0.9, 1.1, 1.3, 1.5
h_2	BaBar	0.8, 1, 1.2, 1.4
h_3	BaBar	0.9, 1.3
R^*	Belle	0.6, 1.4
ℓ_1	Belle	1, 1.4
ℓ_2	Belle	0.6, 1.4
ℓ_3	Belle	0.8, 1.2
h_1	Belle	0.7, 1.1, 1.3, 1.5
h_2	Belle	0.7, 0.9, 1.3
$h_{1,2}$	CDF	0.7
$h_{1,2}$	CLEO	1, 1.5
$\ell_{1,2,3}$	DELPHI	0
$h_{1,2,3}$	DELPHI	0

Note:

- Semileptonic moments are sensitive to a linear combination of m_c and m_b: poor individual accuracy
- Generally use photon energy moments in $B \to X_s\gamma$
- Much better m_c determination from e^+e^- sum rules, lattice QCD

New Contributions:

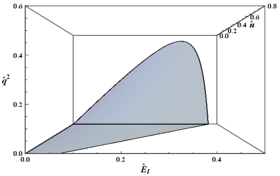
$$M_i \quad = (\dots) + \left(\frac{\alpha_s}{\pi}\right)\left[\, M_i^{(\pi,1)}\frac{\mu_\pi^2}{m_b^2} + M_i^{(G,1)}\frac{\mu_G^2}{m_b^2}\right]$$

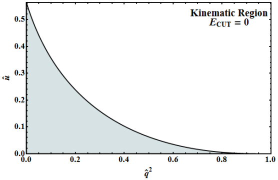
A. Alberti, P. Gambino and S. Nandi	[JHEP, 1 (2014)]
A. Alberti, T. Ewerth, P. Gambino and S. Nandi	[Nucl. Phys. B 870, 16 (2013)]
T. Becher, H. Boos, E. Lunhgi	[JHEP 0712 (2007) 062]

Large cancellation in ℓ_1, ℓ_2, ℓ_3

Requires high precision : Avoid divergences

Kinematic Cut Issues





$$E_\ell^- \leq E_\ell \leq E_\ell^+$$

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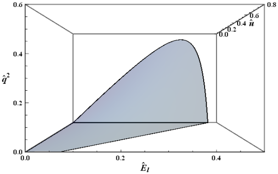

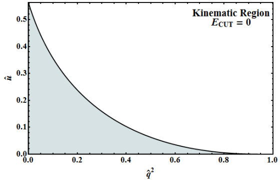

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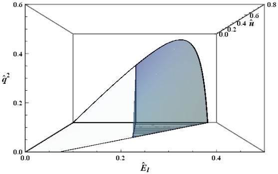
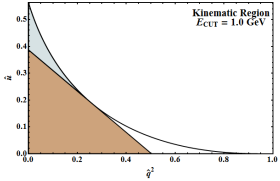


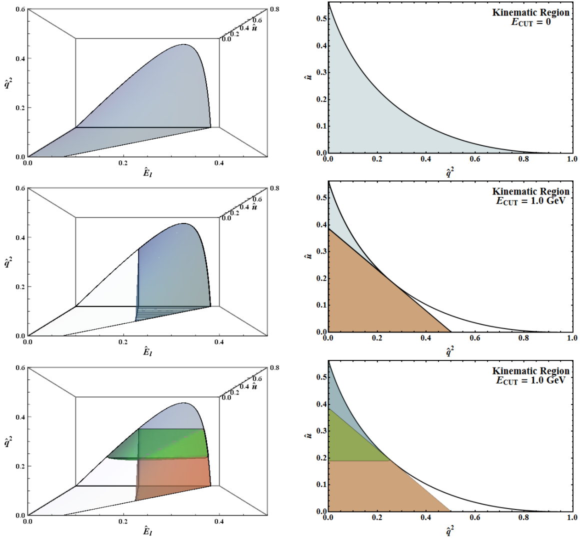


$$E_{CUT} \leq E_\ell \leq E_\ell^+$$

$$d\Gamma \propto \Theta[\hat{u}^+(\hat{q}^2) - \hat{u}] \left[\frac{1}{\hat{u}^{n>1}}\right]^+$$

- ▷ Cuts introduce a false divergence
  - ▷ Easy algebraically, bad numerically

Kinematic Cut Issues



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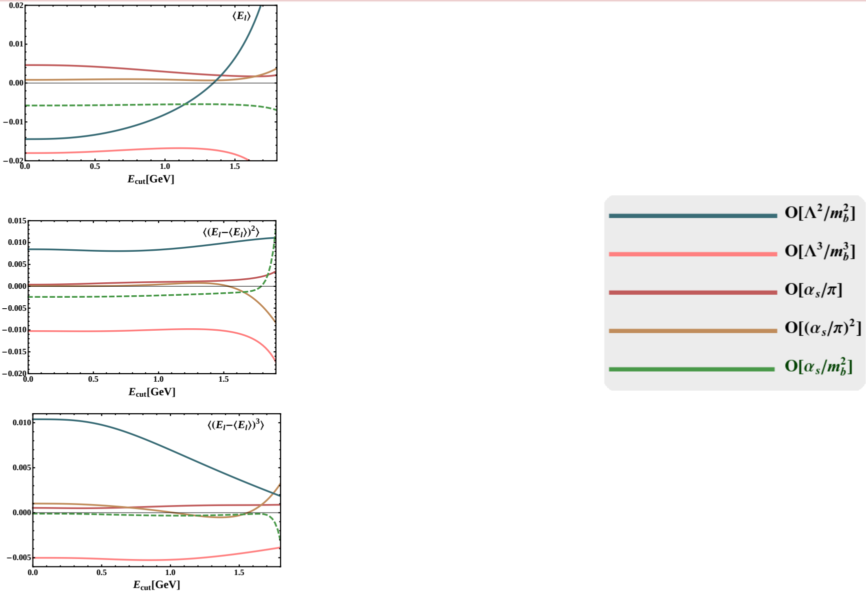
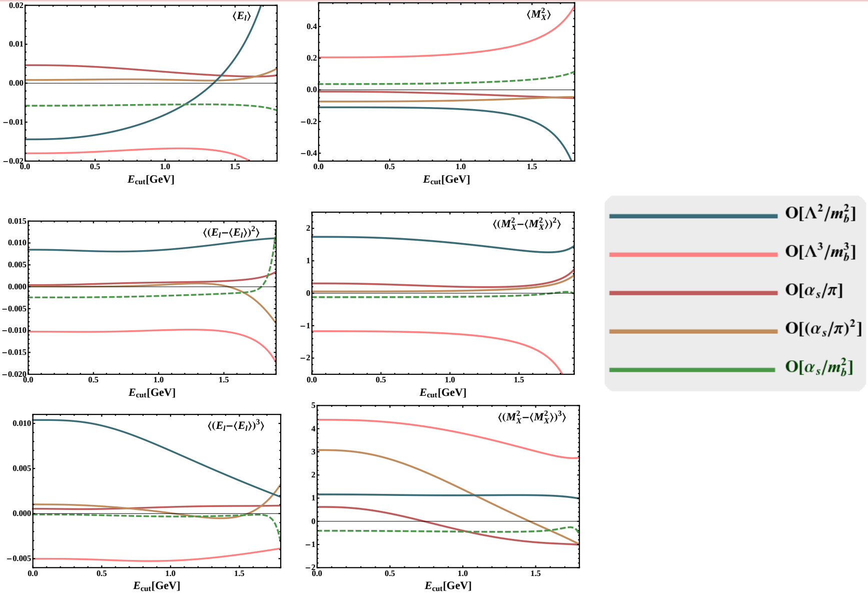
$$d\Gamma \propto \Theta[\hat{u}^+(\hat{q}^2) - \hat{u}] \left[\frac{1}{\hat{u}^{n>1}}\right]^+$$

- ▷ Split and recombine regions
  - ▷ Divergences in $\hat{q}^2/\hat{u}$ unit-box slow with poor numerical precision
  - ▷ Needs algebraic integration of divergences

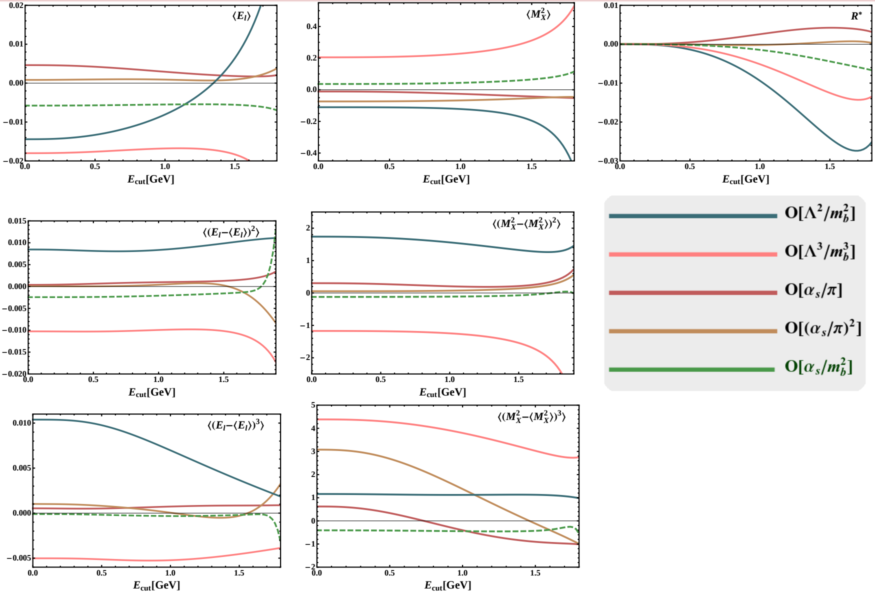
New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: I1, I2 ,I3



New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: h1, h2 ,h3



New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: R^*



Theoretical Error and Correlation Issues

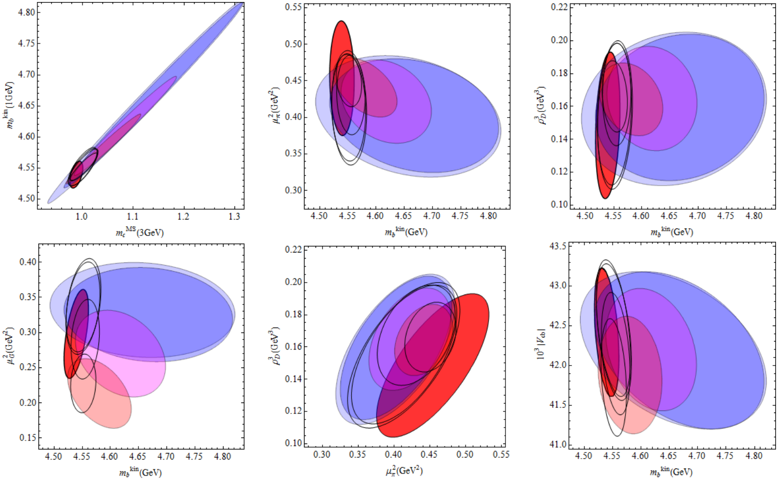
- Perform High-Precision fits for use in HFAG Fortran Routine
- Functional dependence on E_{cut} is important
- Theoretical uncertainty assigned :
  $M_i(1\,\text{GeV})$ and $M_i(1.1\,\text{GeV})$ are very close and
  highly correlated
  - * Previous fits assumed 100% correlation : too strong
  - * Dependence of observable on E_{cut} would be free from theoretical uncertainty

Standard Global Fit Theoretical Correlation Scenarios

A : 100% Correlation between M_i at different E_{cut}
B : Correlations from varying theoretical parameters
C : Constant scale factor (0.97/100 MeV Steps)
D : Functional Scale Factor ($E_0 \approx 1.75\,\text{GeV}, \Delta \approx 0.25\,\text{GeV}$) *

$$\xi(E_{cut}) = 1 - \frac{1}{2}\exp^{-\frac{E_0 - E_{cut}}{\Delta}}$$

Parameter Fits



Two-dimensional projections of the fits performed with different assumptions for the theoretical correlations. The orange, magenta, blue, light blue 1-sigma regions correspond to scenarios A,B,C,D ($\Delta = 0.25$GeV), respectively. The red corresponds to scenario D with $\mathcal{O}(\alpha_s/m_b^2)$ corrections.

Preliminary Results

	m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\,\lvert V_{cb}\rvert$
$\mathcal{O}(\alpha_s^2, m_b^{-2})$	4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
$\overline{m}_c(3\mathrm{GeV})$	0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86
	m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\,\lvert V_{cb}\rvert$
$\mathcal{O}(\alpha_s^2, \alpha_s m_b^{-2})$	4.539	0.988	**0.454**	0.148	**0.298**	-0.142	10.67	42.42
$\overline{m}_c(3\mathrm{GeV})$	0.022	0.013	0.079	0.045	0.063	0.097	0.16	0.81

New Inclusive Fit : $\lvert V_{cb}\rvert_{inc} = (42.42 \pm 0.81) \times 10^{-3}$

- Significant increase μ_π^2, decrease in μ_G^2
- Decreased theoretical error on inclusive $\lvert V_{cb}\rvert$
- Result reinforces our confidence in the inclusive method

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Progress Aim :

- Check fit in scenarios **A,B,C**
- Incorporate $\mathcal{O}(\alpha_s/m_b^3)$: calculation currently in progress

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Thank You.

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