Higher order corrections to inclusive semileptonic B decays

Andrea Alberti

University of Torino

In collaboration with:

P. Gambino, S. Nandi and K. J. Healey

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Overview

Semileptonic B decays provide the most precise determination of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$.

With the rapid approach of the Belle II experiment, theoretical uncertainties should be reduced wherever possible.

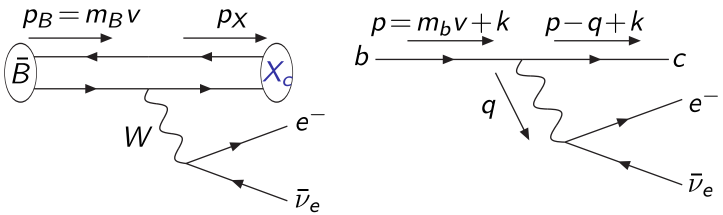
Using an Operator Product Expansion the inclusive decay rate and the moments can be expressed as a double series: perturbative (in α_S) and nonperturbative (in m_b^{-1}).

$$\Gamma_{B\to X_c e\nu} = \Gamma_0 \left[1 + c^{(1)} \frac{\alpha_S}{\pi} + c^{(2)} \left(\frac{\alpha_S}{\pi}\right)^2 + c_\pi^{(0)} \frac{\mu_\pi^2}{m_b^2} + c_G^{(0)} \frac{\mu_G^2}{m_b^2} + \ldots \right]$$

Only a subset of $O(\alpha_S/m_b^2)$ corrections had been computed so far.

- T. Becher, H. Boos and E. Lunghi, (2007)

Inclusive Semileptonic $\bar{B}$ Decay into X_c



Here is the process under exam, we are summing over X_c.
Notice that the b quark is slightly off-shell: $p_\mu = m_b v_\mu + k_\mu$.
The triple differential decay distribution will be:

$$\frac{d\Gamma}{dq^2 dE_\nu dE_e} = 2G_F^2 |V_{cb}|^2 W_{\alpha\beta} L^{\alpha\beta}$$

where $W_{\alpha\beta}$ is the hadronic tensor, thus defined:

$$W^{\alpha\beta} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) < \bar{B}| J_L^{\dagger\alpha} |X_c >< X_c| J_L^\beta |\bar{B} >$$

Time Ordered Product and OPE

Tensor $W^{\alpha\beta}$ can be related to a T-ordered product of currents T:

$$W_i = -\frac{\operatorname{Im} T_i}{\pi}; \quad T^{\alpha\beta} = -i \int \frac{d^4x}{2m_B} e^{-iq\cdot x} < \bar{B}| T\left[J_L^{\dagger\alpha}(x) J_L^{\beta}(0) \right] |\bar{B}>$$

By doing so, we can employ the Operator Product Expansion to express such product as a series of local operators:

$$T\left[J_L^{\dagger\alpha}(x) J_L^{\beta}(0) \right] = \sum_i C_i(x) O_i(0)$$

The idea is to take the expression corresponding to the current product, Taylor expand it in k_μ and obtain a series in m_b:

$$T^i = \sum_{n\geq 3} \sum_j \left(\frac{1}{m_b} \right)^{n-3} c^{ij}_{(n)} < \bar{B}| O_j^{i(n)} |\bar{B}>$$

HQET - what Operators to look for

At $k=0$ there's just one QCD operator, $O_b = \bar{b}\gamma_\mu b \rightarrow <O_b>= v_\mu$

At higher dimension, it's better to take advantage of HQET:

- only one operator at dimension-four

$$<\bar{b}_v iD_\mu b_v>= O\left(\frac{1}{m_b}\right) \qquad \bar{b}_v(iv\cdot D)b_v = \frac{1}{2m_b}\bar{b}_v \not{D}\not{D}\, b_v$$

- the kinetic operator at dimension-five

$$<\bar{b}_v iD_{(\mu}iD_{\nu)}b_v>= \frac{\lambda_1}{3}(g_{\mu\nu}-v_\mu v_\nu) + O\left(\frac{1}{m_b}\right)$$

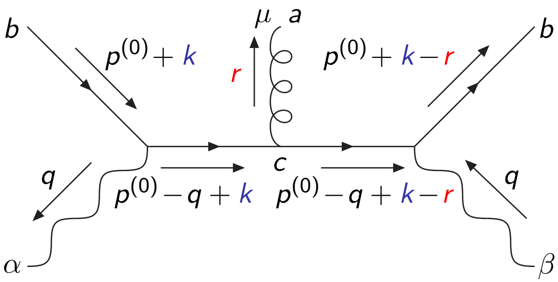
- the choromagnetic operator still at dimension-five

$$<\bar{b}_v iD_{[\mu}iD_{\alpha]}\sigma^{\mu\beta}b_v>= -2\lambda_2(g_\alpha^\beta - v_\alpha v^\beta) + O\left(\frac{1}{m_b}\right)$$

Soft External Gluon

The Taylor expansion is symmetric in $k_\mu k_\nu$ by construction, which means we're able to find $iD_{(\mu}iD_{\nu)}$, not $iD_{[\mu}iD_{\nu]}$ (that's λ_2)

We can add a soft gluon to our process, with small virtuality r_μ



and then expand the new expression unitedly in k_μ and r_μ.

How to actually find λ_2

In the new expansion we can see terms containing a gluon field G_μ:

$$iD_{(\mu}iD_{\nu)} = i\partial_\mu i\partial_\nu - G^a_{(\mu}T^a i\partial_{\nu)} - \frac{i}{2}\partial_{(\mu}\left[G^a_{\nu)}T^a\right] + \frac{1}{2}G^a_\mu G^b_\nu\left(T^aT^b + T^bT^a\right)$$

$$iD_{[\mu}iD_{\nu]}\sigma^{\mu\alpha} = \left[i\partial_\nu\left(G^a_\mu\right) - i\partial_\mu\left(G^a_\nu\right)\right]T^a\sigma^{\mu\alpha} + G^a_\mu G^b_\nu\left(T^bT^a - T^aT^b\right)\sigma^{\mu\alpha}$$

Once we find the imprint of each operator in our new expression, we can calculate the complete coefficient for both λ_1 and λ_2 (checking in the meantime that λ_1 is the same as before)

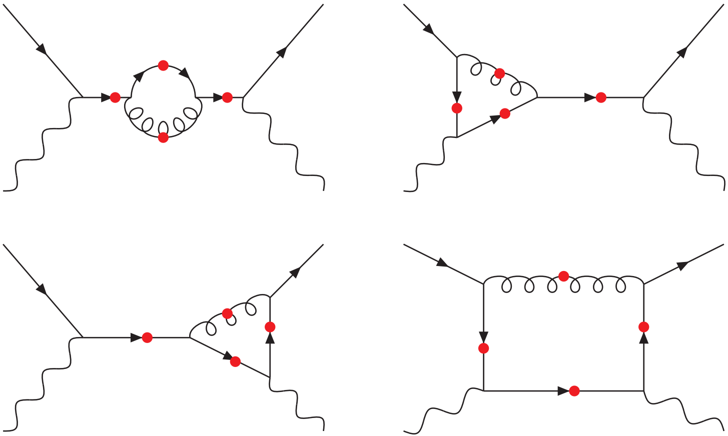
$$iD_\mu \;\rightarrow\; -t^a_\mu T^a$$

$$iD_{(\mu}iD_{\nu)} \;\rightarrow\; -\left(k_\mu t^a_\nu + k_\nu t^a_\mu\right)T^a + \frac{1}{2}\left(r_\mu t^a_\nu + r_\nu t^a_\mu\right)T^a$$

$$iD_{[\mu}iD_{\nu]}\sigma^{\mu\alpha} \;\rightarrow\; \left(r_\nu t^a_\mu - r_\mu t^a_\nu\right)T^a\sigma^{\mu\alpha}$$

One Loop Diagrams

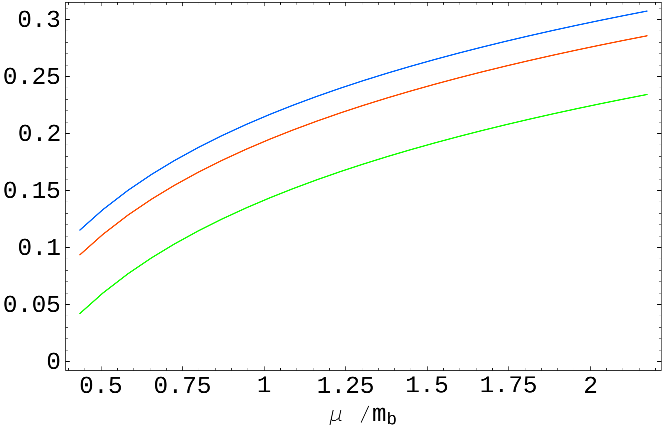
These are all the diagrams at order $O(\alpha_S)$



the red dots mark all possible insertions of an external gluon.

μ dependence

After having performed renormalization and matching, the final result still retains a dependence on the renormalization scale μ.



Relative NLO corrections to λ_2 coefficients in the width, first leptonic moment and second central leptonic moment.

Numerical Results

Once the numerical integration has been performed,
we can see how sizable these new contributions are :

- inside the total rate, the coefficient of λ_2 increases by $+7\%$

$$\Gamma_T = \Gamma_0 \left[\left(1 - 1.78\frac{\alpha_S}{\pi}\right)\left(1 + \frac{\lambda_1}{2m_b^2}\right) - \left(1.94 + 2.42\frac{\alpha_S}{\pi}\right)\frac{3\lambda_2}{m_b^2} \right]$$

- $+28\%$ in the case of the mean lepton energy

$$< E_e >= 1.41\text{GeV}\left[\left(1 - 0.02\frac{\alpha_S}{\pi}\right)\left(1 - \frac{\lambda_1}{2m_b^2}\right) - \left(1.19 + 4.20\frac{\alpha_S}{\pi}\right)\frac{3\lambda_2}{m_b^2}\right]$$

- an increase of $+23\%$ for the variance of the lepton energy

$$l_2 = 0.18\text{GeV}^2\left[1 - 0.2\frac{\alpha_S}{\pi} - \left(4.9 - 0.4\frac{\alpha_S}{\pi}\right)\frac{\lambda_1}{m_b^2} - \left(2.9 + 8.4\frac{\alpha_S}{\pi}\right)\frac{3\lambda_2}{m_b^2}\right]$$

A partial cross-check

Quite recently NLO corrections to the coefficient of λ_2 have been investigated in the massless limit $m_c \to 0$

- T. Mannel, A. A. Pivovarov and D. Rosenthal, (2014)

This paper reports the analytical result for the total rate:

$$\Gamma_{B\to X_c e\nu} = \Gamma_0 \left[1 + \ldots - \frac{\alpha_S}{\pi}\left(C_A\left(\frac{31}{18} - \frac{\pi^2}{9}\right) + C_F\left(\frac{43}{144} - \frac{19\pi^2}{36}\right)\right)\frac{\mu_G^2}{2m_b^2}\right]$$

In our own work we calculate the differential decay rate analytically, then perform the phase space integration numerically, so the comparison is not so straightforward.

Anyway, running our numerical integration for the rate with a very small mass m_c we find a compatible result.

Conclusions

We have analytically computed the differential rate for λ_1

- AA, T. Ewerth, P. Gambino and S. Nandi, (2013)

We have analytically computed the differential rate for λ_2

- AA, P. Gambino and S. Nandi, (2014)

All is now ready to systematically compute rate and moments for different values of mass m_c and lepton energy cut Λ_{cut}, and then perform numerical fits as described by Dr. K. J. Healey.