Matching NLO with parton shower in Monte Carlo scheme

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Outline and motivation

I will talk about **a new method for NLO+PS matching applied to Drell-Yan process.**

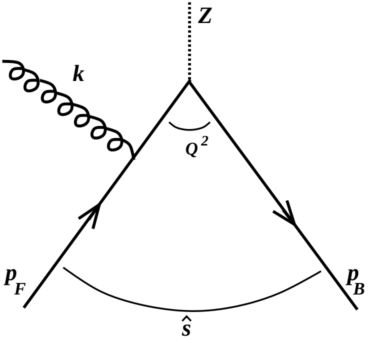
Key ingredients:

- new factorization scheme leading to new MC PDFs
- NLO correction applied to PS via reweighting of MC events

But there are two well established methods, POWHEG and MC@NLO...
...so what is the problem?

- By departing from $\overline{\text{MS}}$, the NLO+PS matching becomes very simple $\rightarrow$ just multiplying by a positive MC weight.
- Possibility of correcting all vertices $\rightarrow$ hence no need for k_T-ordered or truncated showers.
- That alone could have not been worth the effort. But, if it is so simple at NLO+LO PS, you may hope that pushing it to NNLO+NLO PS is possible.

Drell-Yan process ($q\bar{q}$ channel)



$$\hat{s} = (p_F + p_B)^2$$

$$z = \frac{Q^2}{\hat{s}}$$

$$\alpha = \frac{2k \cdot p_B}{\sqrt{\hat{s}}} = \frac{2k^+}{\sqrt{\hat{s}}}$$
$$\beta = \frac{2k \cdot p_F}{\sqrt{\hat{s}}} = \frac{2k^-}{\sqrt{\hat{s}}}$$

$$z = 1 - \alpha - \beta$$
$$k_T^2 = \hat{s}\alpha\beta$$
$$y = \frac{1}{2}\ln\frac{\alpha}{\beta}$$

A generic problem of NLO+PS matching

DY ($q\bar{q}$ channel) cross section at NLO in collinear $\overline{\text{MS}}$ factorization

$$\sigma_{\text{DY}}^{\mathcal{O}(\alpha_s)} \quad = \quad \sigma_{\text{DY}}^{B} \otimes f^{\overline{\text{MS}}}(x_1, \mu^2) \otimes \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}}(z) \otimes f^{\overline{\text{MS}}}(x_2, \mu^2)\,,$$

where

$$C_q^{\overline{\text{MS}}}(z) = C_F \left[4\,(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - 2\frac{1+z^2}{1-z} \ln z + \delta(1-z) \left(\frac{2}{3}\pi^2 - 8 \right) \right].$$

We want to reproduce this with Monte Carlo, in a fully exclusive way.

If we decide to use $\overline{\text{MS}}$ PDFs, we need to generate terms of the type $4\,(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+$ that are technical artefacts of $\overline{\text{MS}}$ scheme.

$\hookrightarrow$ related to oversubtraction from phase space integration in PDFs

This is problematic since those terms correspond to the collinear limit but Monte Carlo lives in 4 dimensions and not in the phase space restricted by $\delta(k_T^2)$.

KrkNLO method [Jadach, Kusina, Płaczek, Skrzypek & Sławińska '13]

- Take a parton shower that covers the (α, β) phase space completely and produces emissions according to approx. matrix element $K \simeq R$.
- Upgrade the real emissions to exact ME by reweighting with R/K.
- Upon integration over transverse d.o.f. this upgraded PS will give an extra term $C_2(z) = \int (R - K)$.
- Redefine PDFs by subtracting the above $C_2(z)$ together with all the z-dependent terms from $\overline{\text{MS}}$ coefficient function. This means transforming PDFs to a new MC factorization scheme.
- Virtual+soft correction, Δ_{S+V}, is just a constant now. Multiply the whole result by $1 + \Delta_{S+V}$ to achieve complete NLO accuracy.

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This has been shown to reproduce exactly the NLO result of fixed order collinear factorization for the case of analytic PS.

Could we implement the method in a popular, general purpose MC?

Practical implementation with CS shower

We used Sherpa with the Catani-Seymour (CS) shower.

- The CS shower covers all space of (α, β).
- The evolution variable: $q^2 = (\alpha + \beta)\,\beta\, s$.
- For the $q\bar{q}$ channel:

$$C_2^{\rm CS}(z) = \int (R - K) = \frac{\alpha_s}{2\pi} C_F \left[-2(1-z)\right] .$$

- Quark and anti-quark PDFs are redefined by:
  - subtracting $-\frac{\alpha_s}{2\pi} C_F\,(1-z)$, coming from $\frac{1}{2} C_2^{\rm CS}(z)$
  - absorbing $\frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z}\right]_+$, coming from $\overline{\rm MS}$ coeff. function
- Virtual+soft, $\Delta_{V+S} = \frac{\alpha_s}{2\pi} C_F \left(\frac{4}{3}\pi^2 - \frac{5}{2}\right)$, is applied multiplicatively.
- The hardest real emission is upgraded to ME by reweighting.

Two essential steps

1. **Change the factorization scheme from $\overline{\text{MS}}$ to MC**
   - produce new MC PDFs
   - differences at LO
   - universality: recovering $\overline{\text{MS}}$ NLO result

2. **Reweight parton shower**
   - correct hardest emission by 'real' weight
   - upgrade the cross section/distributions to NLO by multiplicative, constant 'soft+virtual' weight

MC PDFs

Transformation of q and $\bar{q}$ PDFs from $\overline{\mathrm{MS}}$ to MC scheme

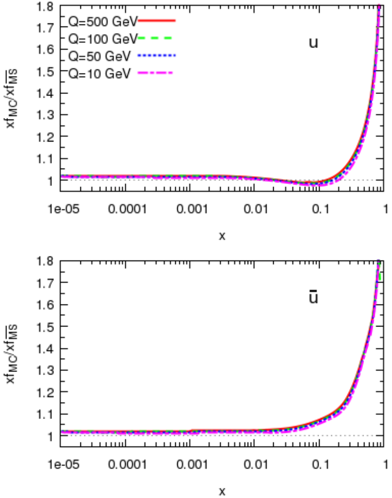
$$q_{\mathrm{MC}}(x, Q^2) = q_{\overline{\mathrm{MS}}}(x, Q^2) + \int_x^1 \frac{dz}{z}\, q_{\overline{\mathrm{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z)$$

$$\Delta C_{2q}(z) = \frac{\alpha_s}{2\pi}\, C_F \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} + 1 - z\right]_+$$

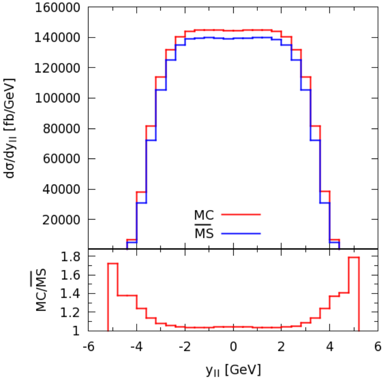
How big is the difference?

MC PDFs

- Ratios with respect to standard $\overline{\text{MS}}$ PDFs for light quarks.



- Effect on LO cross section



$$x_{1,2} = \frac{m_Z}{\sqrt{s}} e^{\pm y_Z}$$

$\overline{\text{MS}}$ scheme vs MC scheme at NLO

$$
\begin{aligned}
\sigma_{\text{tot}}^{\overline{\text{MS}}} &= f_q \otimes (1 + \alpha_s\, C_q^{\overline{\text{MS}}}) \otimes f_{\bar{q}} \\
\sigma_{\text{tot}}^{\text{MC}} &= (f_q + \alpha_s \Delta f_q) \otimes (1 + \alpha_s\, C_q^{\text{MC}}) \otimes (f_{\bar{q}} + \alpha_s \Delta f_{\bar{q}}) \\
&= f_q f_{\bar{q}} + \alpha_s \left(\Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}} \right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s^3)
\end{aligned}
$$

At $\mathcal{O}(\alpha_s)$:

$$C_q^{\overline{\text{MS}}} f_q f_{\bar{q}} = \Delta f_q f_{\bar{q}} + \Delta f_{\bar{q}} f_q + C_q^{\text{MC}} f_q f_{\bar{q}}$$

Drell-Yan, $q\bar{q}$ channel, $\alpha_s = \alpha_s(m_Z)$, MCFM, MSTW2008LO

$$(336.36 \pm 0.09)\,\text{pb} = \underbrace{25.79\,\text{pb} + 25.79\,\text{pb} + 284.77\,\text{pb}}_{(336.35 \pm 0.09)\,\text{pb}}$$

- Final result is scheme independent up to $\mathcal{O}(\alpha_s)$.
- A nontrivial test as $\Delta f_{q,\bar{q}}$ and C_q come from totally different places. In particular, the former includes convolutions of PDFs.
- Terms $\mathcal{O}(\alpha_s^2) \simeq 16\,\text{pb}$, for this example; $\mathcal{O}(\alpha_s^3) \simeq 0.2\,\text{pb}$.

Catani-Seymour parton shower

The "Sudakov" form factor

$$S(Q^2, \Lambda^2, x) = \int\limits_{\Lambda^2}^{Q^2} \frac{dq^2}{q^2} \int\limits_{z_{\min}(q^2)}^{z_{\max}(q^2)} dz \;\; K(q^2, z, x)\,,$$

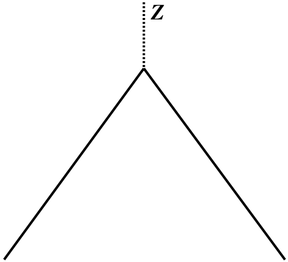
where

$$K(q^2, z, x) = \frac{C_F \alpha_s}{2\pi} \frac{1 + z^2}{1 - z} \; \frac{f(q^2, x/z)/z}{f(q^2, x)}\,.$$

- z, q^2 - internal variables of the shower
- $f(q^2, x)$ - parton distribution functions

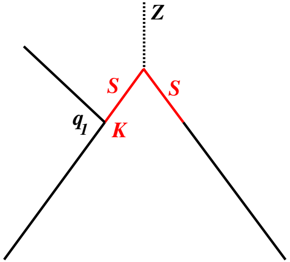
The kernel K is just a CS dipole written in terms of shower's internal variables multiplied by the ratio of PDFs due to backward evolution.

Upgrading to NLO: the hardest emission



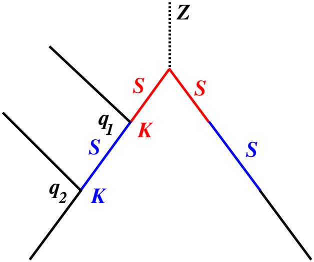
$$\sigma^{\mathrm{LO}} = \sigma_B \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus})$$

Upgrading to NLO: the hardest emission



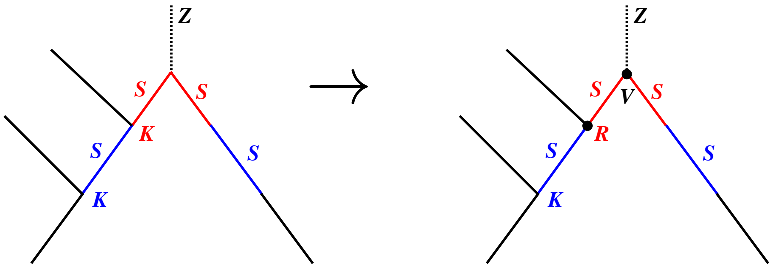
$$\sigma_{1+}^{\rm PS} = \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus)$$
$$\otimes \Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2) + S_\ominus(Q^2, q_1^2) K_\ominus(q_1^2, z_1) S_\oplus(Q^2, q_1^2) \Big\}$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{PS}} \quad = \quad & \sigma_B \otimes D_\oplus(Q^2, x_\oplus) \otimes D_\ominus(Q^2, x_\ominus) \\
& \otimes \Big\{ S_\oplus(Q^2, q_1^2) K_\oplus(q_1^2, z_1) S_\ominus(Q^2, q_1^2) \\
& \quad \otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\} \\
& \quad + S_\ominus(Q^2, q_1^2) \otimes K_\ominus(q_1^2, z_1) \otimes S_\oplus(Q^2, q_1^2) \\
& \quad \otimes \Big\{ S_\oplus(q_2^2, q_1^2) K_\oplus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) + S_\oplus(q_2^2, q_1^2) K_\ominus(q_2^2, z_2) S_\ominus(q_2^2, q_1^2) \Big\} \Big\}
\end{aligned}
$$

Upgrading to NLO: the hardest emission



$$
\begin{aligned}
\sigma_{2+}^{\mathrm{NLO+PS}} \quad = \quad & \sigma_B \, (1+V) \otimes D_{\oplus}(Q^2, x_{\oplus}) \otimes D_{\ominus}(Q^2, x_{\ominus}) \\
& \otimes \Big\{ S_{\oplus}(Q^2, q_1^2) K_{\oplus}(q_1^2, z_1) S_{\ominus}(Q^2, q_1^2) \, R_{\oplus}(q_1^2, z_1) / K_{\oplus}(q_1^2, z_1) \\
& \quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \\
& \quad + S_{\ominus}(Q^2, q_1^2) \otimes K_{\ominus}(q_1^2, z_1) \otimes S_{\oplus}(Q^2, q_1^2) \, R_{\ominus}(q_1^2, z_1) / K_{\ominus}(q_1^2, z_1) \\
& \quad \otimes \Big\{ S_{\oplus}(q_2^2, q_1^2) K_{\oplus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) + S_{\oplus}(q_2^2, q_1^2) K_{\ominus}(q_2^2, z_2) S_{\ominus}(q_2^2, q_1^2) \Big\} \Big\}
\end{aligned}
$$

The MC weight for CS shower

Real part:

$$W_R^{q\bar{q}}(\alpha, \beta) = 1 - \frac{2\alpha\beta}{1 + (1 - \alpha - \beta)^2},$$

where $\alpha = \frac{2k \cdot p_B}{\sqrt{\hat{s}}}$ and $\beta = \frac{2k \cdot p_F}{\sqrt{\hat{s}}}$.

Virtual + soft:

$$W_{V+S}^{q\bar{q}} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2}\right].$$

- **Real weight is a simple function of kinematic variables.**
  One can compute it on the fly, inside an MC, or outside, using information from event record.
- **Virtual+soft weight is a constant.**
  No need to generate strictly collinear contributions (like $d\Sigma^{c\pm}$ terms in MC@NLO).

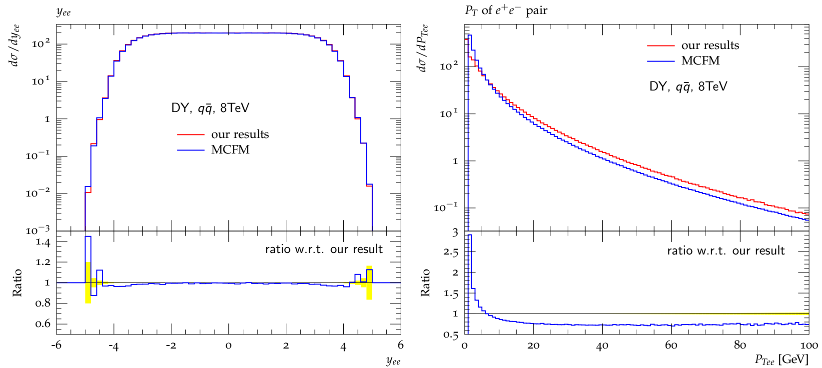
Matched results: total cross section

Total cross section for DY, $q\bar{q}$ channel, 8 TeV

	σ_{tot} [pb]
MCFM ($\overline{\text{MS}}$ PDFs)	1344.1 ± 0.1
MCFM (MC PDFs)	1361.6 ± 0.3
PS (MC PDFs)	1044.1 ± 0.9
PS+real (MC PDFs)	1031.1 ± 0.9
KrkNLO [PS+NLO (MC PDFs)]	1355.9 ± 0.8

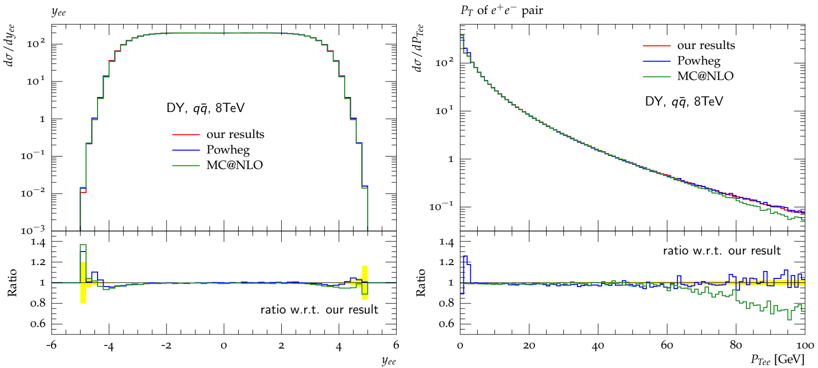
- The difference between fully corrected PS+NLO is at the level of 0.8% w.r.t. MCFM in $\overline{\text{MS}}$ scheme and 0.4% w.r.t. to MCFM in MC scheme.

Matched results: distributions (vs fixed order)



- Excellent reproduction of y_Z distribution at NLO.
- As expected, p_T distribution suppressed at low p_T due to Sudakov.
- Virtual correction spread over a range of p_T.

Matched results: distributions (vs matched results)



- y_Z and p_T distributions very close to POWHEG
  (difference at low p_T due to slightly different evolution variable)
- y_Z very close to MC@NLO, same for low and intermediate p_T
  (differences for the tail of p_T distributions come from mixed $\mathcal{O}(\alpha_s^2)$ terms)

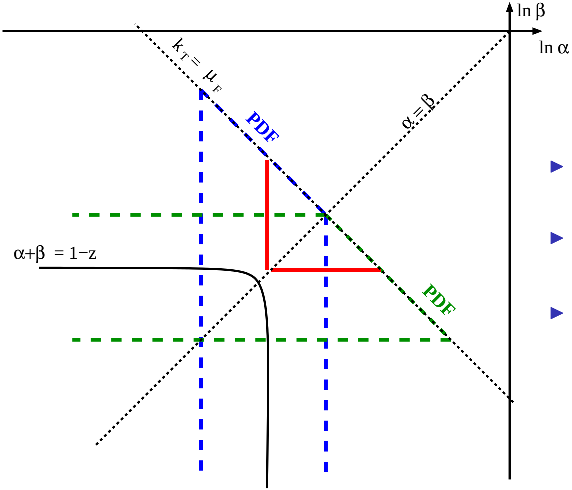
Conclusions

- I have discussed a method of NLO+PS matching:
  - Real emissions are corrected by simple reweighting.
  - Collinear terms are dealt with by putting them to PDFs. This amounts to change of factorization scheme from $\overline{\text{MS}}$ to MC.
  - Virtual correction is just a constant and does not depend on Born kinematics.
- The method has been implemented on top of Catani-Seymour shower in Sherpa event generator.
- It has been validated against fixed order NLO for Drell-Yan process in $q\bar{q}$ channel.
- First comparisons to MC@NLO and POWHEG.

Near future: qg channel (hence full DY), correction of n emissions, public code.

BACKUP SLIDES

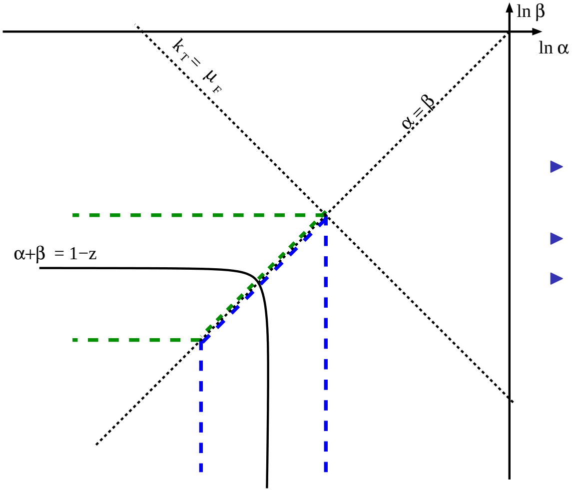
Origin of $4\frac{\ln(1-z)}{1-z}$ in $\overline{\text{MS}}$



- Integration extends up to a fixed $k_T = \mu_F$.
- For one PDF we get $2\frac{\ln(1-z)}{1-z}$
- Combining two PDFs leads to overcounting by $4\frac{\ln(1-z)}{1-z}$

Could we reorganize phase space integration to remove the oversubtraction?

Alternative factorization scheme



- Integration in angle rather than k_T.
- No overcounting.
- This is equivalent to saying that the $4\frac{\ln(1-z)}{1-z}$ term gets absorbed into PDFs.