Heavy flavour precision physics from $N_f = 2 + 1 + 1$ lattice simulations

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Outline

- **tmLQCD with** $2 + 1 + 1$ dynamical flavours
- **charm** sector
  - Determination of $m_c$ and $m_c/m_s$
  - Accurate determination of pseudoscalar meson decay constants $f_{Ds}$, $f_{Ds}/f_D$ and $f_D$
- **bottom** sector: Ratio Method + tmLQCD (relativistic quarks)
  - computation of $m_b$ with heavy quark systematic uncertainties under control
  - non-perturbative determination of the quark mass ratio $m_b/m_c$
  - On going analysis for $f_{B(s)}$, $B$-bag parameters and $\xi$
    (for $B$-physics computations with $N_f = 2$ tmLQCD see ETMC, Carrasco et al. JHEP 03(2014)016.)
- Lattice ME are required for CKM unitarity tests.
- Higgs decays sensitive to $m_b$ (less to $m_c$):
  $$\Gamma(H \rightarrow q\bar{q}) \propto m_H m_q^2(m_H), \quad q \equiv b, c$$
ETMC – $N_f = 2 + 1 + 1$ action

- Glue: Iwasaki \[\text{[Iwasaki NPB 1985]}\]

- $N_f = 2$ - light MtmQCD Dirac operator
  \[D_\ell = D_W + m_{\text{crit}} + i\mu_\ell \gamma_5 \tau^3\] \[\text{[Frezzotti, Rossi, JHEP 2004]}\]

  * $\mu_\ell$ bare light twisted mass; $\tau^3$ acts on flavour doublet

- $N_f = 1 + 1$ twisted mass strange/charm doublet
  \[D_h = D_W + m_{\text{crit}} + i\mu_\sigma \tau^1 + \mu_\delta \tau^3\] \[\text{[Frezzotti, Rossi, NP Proc Suppl 2004]}\]

  - Only $O(a^2)$ discretisation errors on physical quantities
    - Multiplicative light quark mass renormalization
    - $m_{c,s} = \frac{1}{Z_P}(\mu_\sigma \pm \frac{Z_P}{Z_S}\mu_\delta)$
    - No need for RC in pseudoscalar decay constant (PCAC)

- Breaking of parity and isospin – recovered as $a \to 0$

- $(N_f = 1 + 1)$ – mixing of strange and charm in the unitary setup
Osterwalder-Seiler (flavour diagonal) action in the valence sector –
multiplicative quark mass renormalisation - no s/c mixing.

Mixed action setup – unitary up to $O(a^2)$ [Frezzotti, Rossi, JHEP 2004]

- 3 latt. spacings, $a \in [0.06, 0.09]$ fm
- $M_{ps} \in [210, 440]$ MeV
- $M_{ps}L \in [3.1, 5.4]$
- $L \in [2, 3]$ fm
- scale setting determination from $f_\pi$
- non-perturbative computation of RCs (RI-MOM)
Simulation details for $N_f = 2 + 1 + 1$ ensembles

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$V/a^4$</th>
<th>$a\mu_{\text{sea}} = a\mu_\ell$</th>
<th>$a\mu_\sigma$</th>
<th>$a\mu_\delta$</th>
<th>$N_{\text{cfg}}$</th>
<th>$a\mu_s$</th>
<th>$a\mu_c - a\mu_h$</th>
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<td>1.96</td>
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<td>4.32</td>
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<tr>
<td>2.10</td>
<td>2.97</td>
<td>211</td>
<td>3.19</td>
</tr>
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**ETMC, Baron et al.** *JHEP 2010*
**ETMC, Baron et al.** *Comp.Phys.Com. 2011*
**ETMC, Carrasco et al. 2014*
charm sector
**$m_c$**

- Determine $m_c$ using either $M_D^{\text{expt}} = (M_D^+ + M_D^0)^{\text{expt}} / 2$ or $M_{Ds}^{\text{expt}}$ as input

- $m_{ud}(\overline{\text{MS}}, 2 \text{ GeV}) = 3.70(17) \text{ MeV}, \ m_s(\overline{\text{MS}}, 2 \text{ GeV}) = 99.6(4.1) \text{ MeV}$

  [ETMC, Carrasco et al. 1403.4504]

  using inputs $M_{\pi^0}^{\text{expt}} = 134.98 \text{ MeV}, \ M_K^{\text{expt}} = 494.2(4) \text{ MeV}$, respectively

  (pure QCD corrected $M_K^{\text{expt}}$ for leading strong and EM isospin breaking effects - FLAG 2013).

- Use scaling variables either $r_0/a$ or (fictitious ps-mass) $M_{c'/s'}$ computed at fixed values of strange- and charm-like quark masses $\rightarrow$ control of discretisation effects.

- fit ansatz: $M_{h\ell/s} = P_0 + P_1 m_\ell (+P_2 m_\ell^2) + P_3 a^2$

- tune $m_h \rightarrow m_c$ such as after CL & phys. pion mass extrapolation $M_{h\ell/s} \rightarrow M_{D(s)}^{\text{expt}}$.

- $m_c(\overline{\text{MS}}, m_c) = 1.348(13)_{\text{stat+fit}}(2)_{\text{ch}}(6)_{\text{discr}}(34)_{\text{scale}}(20)_{\text{RC-syst}} \text{ GeV} = 1.348(42) \text{ GeV}$

[ETMC, Carrasco et al. 1403.4504]
\[ \frac{m_c}{m_s} \]

- Control systematics of \( \frac{m_c}{m_s} \) using

\[ R(m_c, m_s, m_l, a^2) = \frac{m_s}{m_c} \frac{(M_{\eta_c} - M_{D_s})(2M_{D_s} - M_{\eta_c})}{2M_K^2 - M_\pi^2} \] (→ leading term independent of \( m_c \) and \( m_s \))

- \( M_{\eta_c} \equiv (\bar{c}c) \) PS-meson; on the lattice compute (fermionic) connected diagrams; disconnected contribution negligible [HPQCD, Davies et al. PRL (2010)].

- Extrapolate to CL & phys. point → \( R^{\text{extrap}} \)

- \( \frac{m_c}{m_s} = \left[ \frac{(M_{\eta_c} - M_{D_s})(2M_{D_s} - M_{\eta_c})}{2M_K^2 - M_\pi^2} \right]^{\text{expt}} \frac{1}{R^{\text{extrap}}} = 11.62(16) \] [error ∼ 1.4%]

[ETMC, Carrasco et al. 1403.4504]
Leptonic Decays: $D_{(s)} \rightarrow \ell \nu$

$$\Gamma(D_{(s)} \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} m_\ell^2 M_{D_{(s)}} \left(1 - \frac{m_\ell^2}{M_{D_{(s)}}^2}\right)^2 f_{D_{(s)}}^2 |V_{cd_{(s)}}|^2$$

Knowledge of $f_{D_{(s)}} \Rightarrow$ necessary to extract $|V_{cd_{(s)}}|$

ETMC computation:
- smeared-smeared and smeared-local correlators $\rightarrow$ plateau at earlier time separation
- determination of ps-meson decay constant (PCAC)
  $$f_{D_{(s)}} = \frac{\mu_c + \mu_{d_{(s)}}}{M_{D_{(s)}} \sinh(M_{D_{(s)}})} \langle 0 | P_5 | D_{(s)} \rangle$$
- $\sinh(M_{D_{(s)}})$ from discretised axial current derivative $\rightarrow$ reduces discretisation errors
- Interpolation at $s$ and $c$ quark mass
- Scaling analysis for $f_{cs}/M_{cs}$
- Simultaneous CL & chiral fit

$$(f_{cs}/M_{cs}) \times M_{Ds}^{\text{expt}} = c_0 + c_1 \mu_{\ell} \left( + c_2 \mu_{\ell}^2 \right) + D a^2$$

- In CL & physical point

$$(f_{cs}/M_{cs}) \times M_{Ds}^{\text{expt}} \rightarrow f_{Ds}$$

$\Rightarrow$ No scale setting uncertainty

$\Rightarrow$ small discretisation effects

$f_{Ds} = 248.5 (1.0)_{\text{stat}} (2.9)_{(m_c,m_s)} (1.2)_{\text{ch}} (1.8)_{\text{discr}} (0.2)_{V} (0.3)_{M_{Ds-expt}} \text{ MeV}$

$f_{Ds} = 248.5 (3.8) \text{ MeV} \ [\text{error } \sim 1.5\%] \ [\text{Preliminary!}]$

(From scaling analysis in terms of $r_0$ we get fully compatible result: $f_{Ds} = 251(9) \text{ MeV}$)
Lattice results obtained in the CL.

Weak dependence on the number of dynamical flavours within the present accuracy.

PDG estimate makes use of $|V_{cs}| \simeq |V_{ud}| - |V_{cb}|^2/2$, $|V_{ud}| = 0.97425(22)$, $|V_{cb}| = 0.04$. 

<table>
<thead>
<tr>
<th>$f_{D_s}$ [MeV]</th>
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<td>230 240 250 260 270 280</td>
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<th>$N_f = 2 + 1 + 1$</th>
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<td>FNAL/MILC '13</td>
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<td>HPQCD '12</td>
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<th>$N_f = 2$</th>
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<td>ALPHA '13</td>
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SU(3) breaking ratio: cancellation of many systematics; sensitive to chiral extrapolation

Interpolation at $s$ and $c$

Define $\frac{(f_{cs}/f_{c\ell})(f_{\ell\ell}/f_{s\ell})}{(f_{K}/f_{\pi})^{phen}}$ → large cancellation of SU(2) (HM)ChPT logs


fit ansätze:

$* a^{(1)} + b^{(1)} \bar{\mu}_\ell + D^{(1)} a^2$

$* a^{(2)} \left[ 1 + b^{(2)} \bar{\mu}_\ell + \left[ 3(1+\delta^2) - \frac{5}{4} \right] \frac{2B_0 \bar{\mu}_\ell}{(4\pi f_0)^2} \log \left( \frac{2B_0 \bar{\mu}_\ell}{(4\pi f_0)^2} \right) \right] + D^{(2)} a^2$

$(f_{K}/f_{\pi})^{FLAG} = 1.194(5)$ (aver. over $N_f=2+1$ and 2+1+1 results) [FLAG 2013]

$B_0, f_0$ from [ETMC Carrasco et al. 2014]

@ CL & phys. point, fit results are compatible within one $\sigma_{stat+fit}$

$\Rightarrow$ small discretisation effects

$f_{Ds}/f_D = 1.186 (9)_{stat+fit} (6)_{ch} (8)_{disc} (5)_{v} (5)_{f_K/f_{\pi}}$

$f_{Ds}/f_D = 1.186 (15)$ [error $\sim 1.3\%$] [Preliminary]
\( f_{Ds}/f_D: \) lattice results

<table>
<thead>
<tr>
<th>( N_f = 2 ) + ( 1 ) + ( 1 )</th>
<th>( f_{Ds}/f_D )</th>
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</table>

★ Lattice results obtained in the CL.
★ PDG estimate for \( f_D \) makes use of |\( V_{cd} \) | \( \simeq \) |\( V_{us} \) | (− tiny correction)
\[ f_D = f_{Ds}/(f_{Ds}/f_D) \]

\[ = 209.5(1.9)_{\text{stat}} + (2.7)_{\text{fit}} (1.5)_{\text{ch}} (2.0)_{\text{disc}} (0.8)_{V} (0.8)(f_K/f_\pi, M_{Ds}) \text{ MeV} \]

\[ f_D = 209.5(4.3) \text{ MeV} \text{ [error } \sim 2.1\%\text{]} \] [Preliminary!]

- Lattice results obtained in the CL.
- PDG estimate for \( f_D \) makes use of \( |V_{cd}| \approx |V_{us}| \) (− tiny correction)
bottom sector
ETMC - Ratio Method: B-physics lattice computations

- We use correlators with relativistic quarks
- We consider ratios of heavy-light \((h, \ell/s)\) observables that have an exact and known static limit
- HQET-inspired interpolation of \((h, \ell/s)\) ratios to the bottom-region from the charm region and the static limit
- charm region computations have small discretisation errors
- Ratios computed for pairs of nearby heavy quark masses show smooth chiral and continuum limit behaviour. Heavy quark systematics are suppressed
- Determine \(b\)-observables through well-controlled ratio chain

Ratio Method

- For any observable \( Q_{h\ell/s} \equiv M_{h\ell/s}, f_{h\ell/s}, \ldots \) consider the HQET scaling quantity \( \Phi_Q = Q_{h\ell/s} \times (\mu_h^{\text{pole}})^\alpha \)

- observing that \( \lim_{\mu_h^{\text{pole}} \to \infty} \left( Q_{h\ell/s} \times (\mu_h^{\text{pole}})^\alpha \right) = \text{constant} \)

  e.g. \( \alpha = -1, 1/2 \) for \( M_{h\ell/s}, f_{h\ell/s} \), respectively

- construct ratios at nearby \( \lambda \mu_h \) (\( \lambda\mu_h^{n-1} \))

  \[
y_Q(\lambda; \bar{\mu}_\ell/s, a) = \frac{\Phi_Q(\lambda; \bar{\mu}_\ell/s, a)}{\Phi_Q(\lambda^{n-1}; \bar{\mu}_\ell/s, a)} \frac{C_Q(\lambda)}{C_Q(\lambda^{n-1})}
\]

  \( C_Q(\lambda) \) are relevant in the “1/\( \bar{\mu}_h \)” interpolation and include possible anomalous dimension in HQET and \( \rho \) factors, \( \mu_h^{\text{pole}} = \rho(\lambda; \mu_h, \mu^*) \bar{\mu}_h(\mu^*) \)

  (with \( \mu_h \leftarrow \overline{\text{MS}} \) scheme)

- \( y_Q(\bar{\mu}, \lambda; \bar{\mu}_\ell/s, a = 0) \xrightarrow{\bar{\mu}_h \to \infty} 1 + O(1/\log \mu_h) \) (residual and small)
Consider \( Q_h \equiv \frac{M_{hs}}{(M_{hl})^\gamma} \)

HQET asymptotic behaviour
\[
\lim_{\mu_h^{\text{pole}} \to \infty} \left( \frac{M_{hs}/(M_{hl})^\gamma}{(\mu_h^{\text{pole}})^{1-\gamma}} \right) = \text{const.}
\]

For a sequence of heavy quark masses \( (\mu_h^{(1)}, \mu_h^{(2)}, \ldots, \mu_h^{(N)}) \) with fixed ratio
\[
\mu_h^{(n)} = \lambda \mu_h^{(n-1)}
\]

Form ratios
\[
y_Q(\mu_h^{(n)}, \lambda, \mu, \mu_s, a) \equiv \frac{Q_h(\mu_h^{(n)}, \mu, \mu_s, a)}{Q_h(\mu_h^{(n-1)}, \mu, \mu_s, a)} \cdot \left( \frac{\mu_h^{(n)} \rho(\mu_h^{(n)}, \mu^*)}{\mu_h^{(n-1)} \rho(\mu_h^{(n-1)}, \mu^*)} \right)^{(\gamma-1)}
\]

For each \( n = 2, \ldots, N \), extrapolate \( y_Q \) to the CL and phys. pion mass
**b-mass computation**

- Cutoff effects increase (naturally) with $n$, still under control;
simultaneous chiral and CL fits are smooth ($\chi^2 \lesssim 1$)

![Graph 1](image1)

- **CL - phys. point**

  - $\beta = 2.10$
  - $\beta = 1.95$
  - $\beta = 1.90$

![Graph 2](image2)

- **CL - phys. point**

  - $\beta = 2.10$
  - $\beta = 1.95$
  - $\beta = 1.90$

![Graph 3](image3)

- **CL - phys. point**

  - $\beta = 2.10$
  - $\beta = 1.95$
  - $\beta = 1.90$

- **exactly known static value**

- **fit ansatz** $y_Q(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$
  (inspired by HQET)

- **curvature denotes a large $1/\bar{\mu}_h^2$ contribution to ratios $y_Q$**

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Heavy flavour precision physics from $N_f = 2 + 1 + 1$ simulation
**b-mass computation**

- use the chain equation
  \[ y_Q(\mu_h^{(2)}) y_Q(\mu_h^{(3)}) \cdots y_Q(\mu_h^{(K+1)}) = \lambda^K(\gamma-1) \frac{Q_h(\mu_h^{(K+1)})}{Q_h(\mu_h^{(1)})} \left( \frac{\rho(\mu_h^{(K+1)}, \mu^*)}{\rho(\mu_h^{(1)}, \mu^*)} \right)^{\gamma-1} \]

- Evaluate the (lhs) with \( y_Q(\mu_h^{(n)}) \) set to the best fit values

- \( Q_h(\mu_h^{(1)}) \) accurately computable in the charm region.

- Adjust \((\lambda, \mu_h^{(1)})\) such that \(K\) integer and \(Q_h(\mu_h^{(K+1)}) \equiv Q^{\text{expt}}_B\)
  \[ \rightarrow \text{Obtain } b\text{-quark mass: } \mu_b = \lambda^K(\mu_h^{(1)}) \]

- **strong** cancellations of perturbative factors \(\rho\)'s in the ratios:
  \[ \rightarrow \text{subpercent dependence on } \rho\text{'s PT-order} \]

- No need to tune exponent \(\gamma\). A large range of choices \(\gamma \in [0, 1)\) provides fully compatible final results, while allowing control on
  (a) discretisation effects
  (b) HQET-inspired fit of ratios vs. \(1/\mu_h\)
\( m_b(\overline{\text{MS}}, m_b) = 4.26(7)_{\text{stat} + \text{fit}} (4)_{\text{ ratios}} (2)_{\text{ch}} (3)_{\text{discr}} (3)_{\rho - PT} (11)_{\text{scale}} (6)_{\text{RC - syst}} \) GeV = 4.26(16) GeV

- Dominant sources of ETMC uncertainty from scale setting and quark mass RC systematics.

- We use ratio method and set the \textit{triggering} quark mass \( \mu_h^{(1)} = m_c \), we get full non-perturbative determination of (scheme and scale independent) quark mass ratio

\[
\frac{m_b}{m_c} = 4.40(8) \\
(m_b/m_c = 4.40(6)_{\text{stat} + \text{fit}} (3)_{\text{ ratios}} (2)_{\text{ch}} (3)_{\text{discr}})
\]
Other $B$-observables

- Use scaling analysis and employ ratio method on observables $(f_{hs}/M_{hs}) \times M_{Bs}^{\text{expt}} \rightarrow \text{small discr. effects and no scale setting uncertainty.}$
- Ongoing analysis for computation of $f_{Bs}$, $f_{Bs}/f_{B}$, $f_{B}$ and $\xi$.
- ETMC 2013 results (preliminary) from $N_f = 2 + 1 + 1$ simulations:

$$
\begin{align*}
    f_{Bs} &= 235(9) \text{ MeV} \\
    f_{B} &= 196(9) \text{ MeV} \\
    f_{Bs}/f_{B} &= 1.201(25)
\end{align*}
$$

[ETMC, Carrasco et al. PoS LAT2013]
Summary

- ETMC has a rich program of precision computations in the charm and bottom sector using simulations with $N_f = 2 + 1 + 1$ dynamical flavours.
- Accurate determinations for quark mass ratios $m_c/m_s$ and $m_b/m_c$ (total uncertainty $\sim 1.5 - 1.8\%$).
- Accurate calculation for $m_b$ using the Ratio Method. Total uncertainty on $m_c$ and $m_b$ results mainly due to scale setting uncertainty. Heavy quark systematics in $m_b$ calculation well under control.
- Precision computation for $D$-sector pseudoscalar decay constants (uncertainties $\sim 1.3 - 2.1\%$).
- On going analysis for improving precision of $B$-sector pseudoscalar decay constants. Determination of $\xi$.

- Lattice QCD has definitely entered the era of precision results aiming at systematically improving control of systematic uncertainties.
- The consistency among heavy flavour lattice results obtained from much different lattice QCD formulations and methods (by a few groups) is very reassuring.
Summary of (preliminary) Results from ETMC

<table>
<thead>
<tr>
<th>$m_c(\overline{\text{MS}}, m_c)$ (GeV)</th>
<th>$m_c/m_s$</th>
<th>$f_{D_s}$ (MeV)</th>
<th>$f_D$ (MeV)</th>
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Thank you for your attention!
extra slides
using fit ansatz: $M_{h\ell} = P_0 + P_1 m_\ell + P_2 m^2_\ell + P_3 a^2$
$f_{Ds}$ analysis using scale $r_0$

$f_{Ds} = 251(9)$ MeV
Improved plateaux

\[ \beta = 2.10 \]

\[ M_{\text{eff}}(x_0) \]

\[ x_0/a \]

Valencia - ICHEP 2014, July 2-9

Petros Dimopoulos

Heavy flavour precision physics from \( N_f = 2 + 1 + 1 \) simulations