NLO QCD corrections to triple collinear splitting functions

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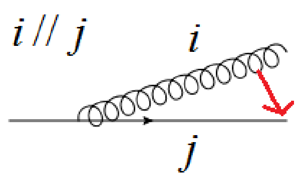
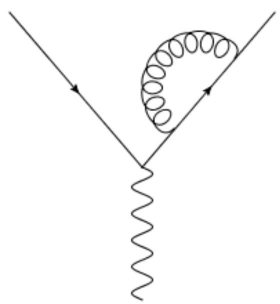
Introduction

- Collinear limit of scattering amplitudes
  - Scattering amplitudes in QCD develop **IR-divergences** when 2 or more particles become **collinear** (i.e. parallel emission)



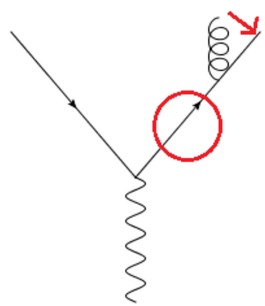
$$k_i \cdot k_j \to 0$$

  - Physical interpretation ⟺ Presence of degenerate states



Virtual corrections

+



Real corrections

= **PHYSICAL RESULT**

Introduction

- Collinear limit of scattering amplitudes
  - Divergent behavior is well known! At amplitude level:

$$\mathcal{M}(p_1,\ldots,p_m,p_{m+1},\ldots,p_n) \sim (1/\sqrt{s})^{m-1} \,\mathrm{mod}\, (\ln^k s)$$

Involves *collinear particles* (2 or more)

  - **Collinear behavior** controlled by **splitting functions,** which are **universal objects** that can be computed in **pQCD**

  - **Splitting functions** dictate the evolution of **PDFs/FFs** through DGLAP equations. Also they are used in **parton shower** generators and in the computation of **hadronic cross-sections at higher orders**

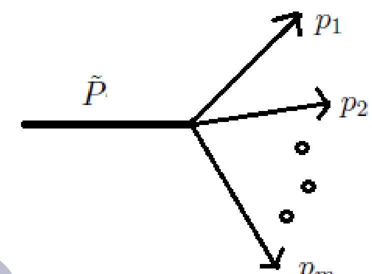
Splitting functions → *PDFs/FFs evolution*; *Parton shower*; *H.O. computations in QCD*

Introduction

- Kinematics and generalities
  - We work in the ***LC-gauge*** (n is the quantization vector) ➡ Keep only physical gluon polarizations ➡ Symplify factorization formulae!!!
  - Use Sudakov parametrization

$$p_i^\mu \;=\; z_i \widetilde{P}^\mu + (k_\perp)_i^\mu - \frac{(k_\perp)_i^2}{z_i} \frac{n^\mu}{2\, n \cdot \widetilde{P}}$$

Approach to collinear limit



$$p_{1,m}^\mu = p_1^\mu + \ldots + p_m^\mu \qquad s_{1,m} = p_{1,m}^2$$

$$\tilde{P}^\mu = p_{1,m}^\mu - \frac{s_{1,m}}{2\, n \cdot \tilde{P}} n^\mu$$

Null-vector associated to the parent parton

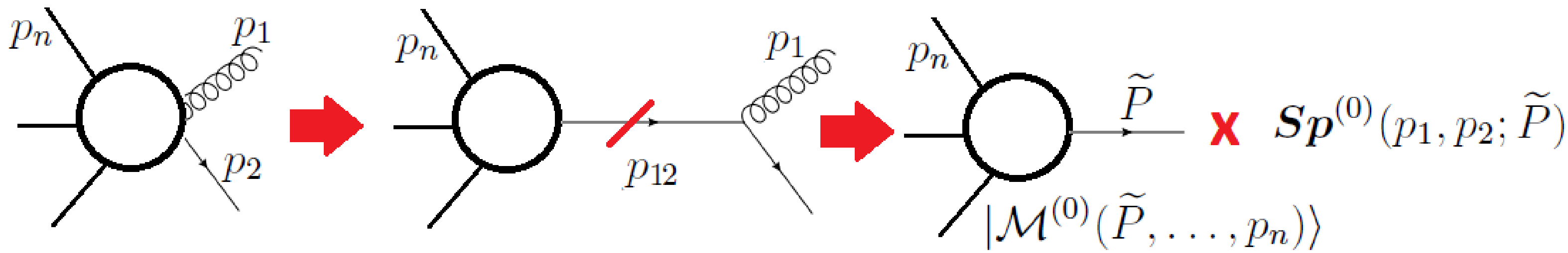
$$s_{ij} \;=\; 2\, p_i \cdot p_j = \frac{-1}{z_i z_j} \left(z_i (k_\perp)_j + z_j (k_\perp)_i\right)^2$$

  - **Multiple scales** in the multiple collinear limit ➡ Increase difficulty!

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Collinear factorization

- Graphical motivation (quark- started double splittings)



$$\frac{\imath(\not{p}_1 + \not{p}_2)}{s_{12}} \approx \frac{\imath \tilde{\not{P}}}{s_{12}} + \mathcal{O}(s_{12}^0) = \sum_{\text{phys.pol.}} \frac{\imath u_\sigma(\tilde{P})}{s_{12}} \bar{u}_\sigma(\tilde{P})$$

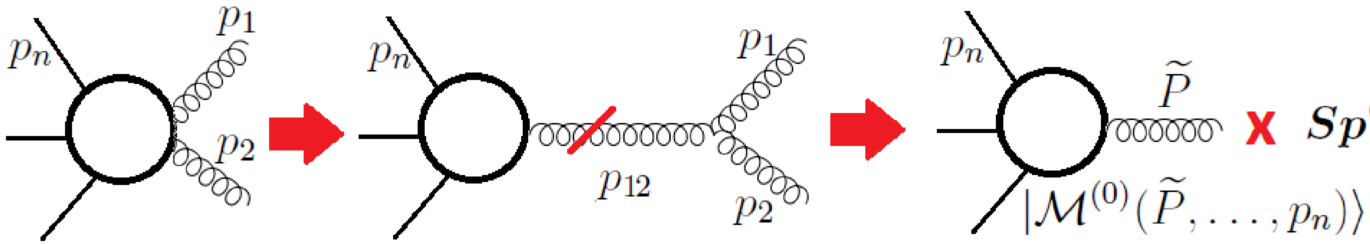
Intermediate particle propagator in the collinear limit (dominant contribution)

- Important remarks:
  - *Splitting functions and matrices are computed using a* ***on-shell massless parent*** *particle, but* ***off-shell*** *kinematics*

$$\boldsymbol{Sp}_{q \to a_1 a_2} = \frac{1}{s_{12}} \left| \left(\mathcal{A}_{\text{amp}}\right)_{q,a_1,a_2} (p_1, p_2; -p_{12}) \right\rangle u(\tilde{P})$$

Collinear factorization

- Graphical motivation (gluon- started double splittings)



$\times\ \boldsymbol{Sp}^{(0)}(p_1,p_2;\widetilde{P})$

$$\frac{i d_{\mu\nu}(p_1+p_2)}{s_{12}} \approx \frac{i d_{\mu\nu}(\tilde{P})}{s_{12}} + \mathcal{O}(s_{12}^0) = \sum_{\text{phys.pol.}} \epsilon^*_\mu(\tilde{P},\sigma)\frac{i\epsilon_\nu(\tilde{P},\sigma)}{s_{12}}$$

LCG propagator properties!!!!

Intermediate particle propagator in the collinear limit (dominant contribution)

- Important remarks:
  - *Splitting functions and matrices are computed using a* ***on-shell massless parent*** *particle, but* ***off-shell*** *kinematics*

$$\boldsymbol{Sp}_{g\to a_1 a_2} = \frac{1}{s_{12}}\left|(\mathcal{A}_{\text{amp}})^{\mu}_{g,a_1,a_2}(p_1,p_2;-p_{12})\right\rangle \epsilon_\mu(\tilde{P})$$

Collinear factorization

- We introduce the *splitting matrices* in color+spin space; they describe the divergent behavior of scattering amplitudes in the collinear limit
  - Tree-level factorization

$$|\mathcal{M}^{(0)}\rangle \simeq \boldsymbol{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{\mathcal{M}}^{(0)}\rangle$$

  - One-loop level factorization

$$|\mathcal{M}^{(1)}\rangle \simeq \boldsymbol{Sp}^{(1)}(p_1,\ldots,p_m;\widetilde{P};p_{m+1},\ldots,p_n)\,|\overline{\mathcal{M}}^{(0)}\rangle + \boldsymbol{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\,|\overline{\mathcal{M}}^{(1)}\rangle$$

Splitting matrix at LO

Splitting matrix at NLO

with the reduced scattering amplitude $\overline{\mathcal{M}} = \mathcal{M}(\widetilde{P},p_{m+1},\ldots,p_n)$

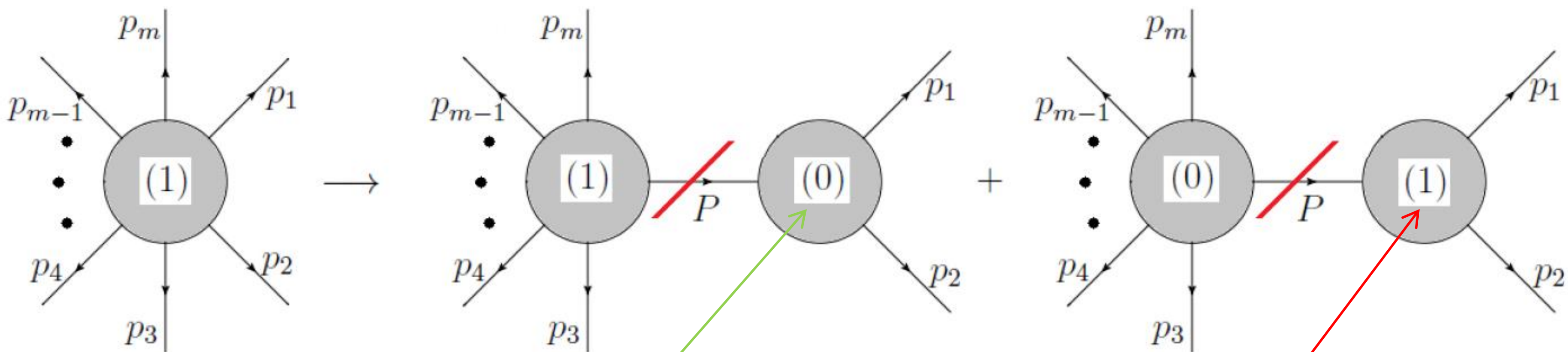
- Some remarks:
  - *It is important to note that these expressions only consider the **most divergent contributions** in the collinear limit*

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Collinear factorization

- Graphical motivation (one-loop double collinear)

Only dominant contributions in the collinear limit



Splitting matrix at LO

Splitting matrix at NLO

$$
\begin{aligned}
|\mathcal{M}^{(1)}\rangle \;\simeq\;& \boldsymbol{Sp}^{(1)}(p_1,\ldots,p_m;\widetilde{P};p_{m+1},\ldots,p_n)\;|\overline{\mathcal{M}}^{(0)}\rangle \\
+\;& \boldsymbol{Sp}^{(0)}(p_1,\ldots,p_m;\widetilde{P})\;|\overline{\mathcal{M}}^{(1)}\rangle
\end{aligned}
$$

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Splitting functions at NLO

- Factorization in color space
  - General structure of one-loop splitting matrices (1->2)

$$\boldsymbol{Sp}^{(1)}(p_1,p_2;\widetilde{P};\underbrace{p_3,\ldots,p_n}) = \boldsymbol{Sp}_H^{(1)}(p_1,p_2;\widetilde{P}) + \boldsymbol{I}_C(p_1,p_2;p_3,\ldots,p_n)\,\boldsymbol{Sp}^{(0)}(p_1,p_2;\widetilde{P})$$

One-loop splitting matrix — Finite contribution — Singular contribution — Tree-level splitting matrix

  - More details:
    - **Sp$_H$** contains only rational functions of the momenta and only depends on collinear particles.
    - **I$_C$** contains trascendental functions and can depend of *non-collinear* particles (through colour correlations). This contribution introduces a violation of *strict* collinear factorization.
    - This structure is verified also in the multiple collinear limit.

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Splitting functions at NLO

- Divergent behavior of splittings at one-loop
  - Divergent structure of one-loop splitting matrices (1->2)
    - Time-like region (TL): $s_{ij}>0$ for all particles
    - Space-like region (SL): $s_{ij}<0$ for some i,j

$$
\begin{aligned}
\boldsymbol{I}_C(p_1,p_2;p_3,\ldots,p_n) &= g_S^2\, c_\Gamma \left(\frac{-s_{12}-i0}{\mu^2}\right)^{-\epsilon} \\
&\times \left\{ \frac{1}{\epsilon^2}\left(C_{12}-C_1-C_2\right) + \frac{1}{\epsilon}\left(\gamma_{12}-\gamma_1-\gamma_2+b_0\right) \right. \\
&\left. + \frac{2}{\epsilon}\sum_{j=3}^{n} \boldsymbol{T}_j\cdot\left[\boldsymbol{T}_1\, f(\epsilon;z_1-i0s_{j1}) + \boldsymbol{T}_2\, f(\epsilon;z_2-i0s_{j2})\right]\right\}
\end{aligned}
$$

Diagonal in color space

Color correlations

Sum over all partons

Explicit correlation with non-collinear particles in SL region

$$
\begin{aligned}
f(\epsilon;1/x) &\equiv \frac{1}{\epsilon}\left[\,{}_2F_1(1,-\epsilon;1-\epsilon;1-x)-1\right] \\
&= \ln x - \epsilon\left[\mathrm{Li}_2(1-x) + \sum_{k=1}^{+\infty}\epsilon^k\,\mathrm{Li}_{k+2}(1-x)\right]
\end{aligned}
$$

Presence of transcendental functions!

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Splitting functions at NLO

- Divergent behavior of multiple-collinear splitting amplitudes
  - Extension for QCD+QED in TL kinematics (**strict factorization**)

$$\boldsymbol{Sp}^{(1)\,\mathrm{div.}}(p_1,\dots,p_m) = c_\Gamma g_s^2 \left(\frac{-s_{1\dots m}-\imath 0}{\mu^2}\right)^{-\epsilon} \left\{ \frac{1}{\epsilon^2} \sum_{i,j=1(i\neq j)}^{\bar{m}} \boldsymbol{T}_i\cdot\boldsymbol{T}_j \left(\frac{-s_{ij}-\imath 0}{-s_{1\dots m}-\imath 0}\right)^{-\epsilon} + \frac{1}{\epsilon^2}\sum_{i,j=1}^{\bar{m}} \boldsymbol{T}_i\cdot\boldsymbol{T}_j \left(2-(z_i)^{-\epsilon}-(z_j)^{-\epsilon}\right) - \frac{1}{\epsilon}\left(\sum_{i=1}^{\bar{m}}\left(\gamma_i - \epsilon\tilde{\gamma}_i^{\mathrm{R.S.}}\right) - \left(\gamma_a - \epsilon\tilde{\gamma}_a^{\mathrm{R.S.}}\right) - \frac{\bar{m}-1}{2}\left(\beta_0 - \epsilon\tilde{\beta}_0^{\mathrm{R.S.}}\right)\right)\right\} \times \boldsymbol{Sp}^{(0)}(p_1,\dots,p_m)$$

Multiple scales

For the triple collinear limit….

$$x_i = \frac{-s_{jk}-\imath 0}{-s_{123}-\imath 0}$$

Scheme-dependence (up to ϵ^0)

  - Useful to test our results (**only the divergent structure**…)
  - Transition rules well known

(For more details see: Catani, de Florian and Rodrigo, arXiv:1112.4405 [hep-ph])

Triple collinear splittings

- Motivation and objectives
  - We are interested in computing **NLO QCD corrections to triple-collinear** splitting functions in QCD+QED (i.e. allowing the presence of QED photons)
  - **Polarized splittings** allow a complete description of the collinear limit (gluon-mediated processes) ➡ **Spin correlations!**
  - Required for hadronic cross-sections computations at NNNLO accuracy (subtraction of collinear divergences)
  - Tree-level results already known for all possible QCD channels

Catani, Grazzini ‘98

(de Florian, Rodrigo and GS, in preparation)

Triple collinear splittings

- Motivation and objectives
  - Only partial results available at NLO

$$\langle \hat{P}^{(1)\,(\mathrm{an.})}_{q_1\bar{Q}_2Q_3} \rangle = \frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{(4\pi)^{-\epsilon}\,\Gamma(1-2\epsilon)}\, C_F\, T_R\, \frac{N_c^2-4}{4N_c} \left(\frac{-s_{123}-i0}{\mu^2}\right)^{-\epsilon}$$
$$\times \left(\left\{ \frac{1}{\epsilon^2} \left[\left(\left(\frac{s_{23}}{s_{123}}\right)^{-\epsilon} + 1 \right) \left(\left(\frac{z_2}{z_2+z_3}\right)^{-\epsilon} - \left(\frac{z_3}{z_2+z_3}\right)^{-\epsilon} + \left(\frac{s_{13}}{s_{123}}\right)^{-\epsilon} - \left(\frac{s_{12}}{s_{123}}\right)^{-\epsilon} \right) \right.\right.\right.$$
$$\left.\left. + \left(\frac{s_{12}}{s_{123}}\right)^{-\epsilon} \left(\frac{z_3}{z_2+z_3}\right)^{-\epsilon} - \left(\frac{s_{13}}{s_{123}}\right)^{-\epsilon} \left(\frac{z_2}{z_2+z_3}\right)^{-\epsilon} \right] \right\} \langle \hat{P}^{(0)}_{q_1\bar{Q}_2Q_3} \rangle / (C_F T_R)$$
$$+ \left\{ \left(\hat{a} - \frac{s_{13}}{s_{123}-s_{13}}\,\hat{b} \right) \ln\left(\frac{s_{13}}{s_{123}}\right) + \left(\frac{s_{23}}{s_{123}-s_{23}}\,\hat{a} - \hat{b} \right) \ln\left(\frac{s_{23}}{s_{123}}\right) \right.$$
$$+ \frac{s_{123}\, s_{12}}{s_{12}+s_{13}-z_1\, s_{123}} \left(\frac{z_2\,\hat{a} - z_1\,\hat{b}}{s_{12}(1-z_1)} + \frac{\hat{a}+\hat{b}}{s_{123}} \right) \ln\left(\frac{s_{12}\, z_2}{s_{13}\, z_3}\right) \ln\left(\frac{s_{23}}{s_{123}(z_2+z_3)}\right)$$
$$+ \frac{(2s_{12}+s_{23})\,\hat{a} + (s_{12}-s_{13})\,\hat{b}}{s_{12}}$$
$$\left.\left. \times \left[\ln\left(\frac{s_{13}}{s_{123}}\right) \ln\left(\frac{s_{23}}{s_{123}}\right) + \mathrm{Li}_2\left(1-\frac{s_{13}}{s_{123}}\right) + \mathrm{Li}_2\left(1-\frac{s_{23}}{s_{123}}\right) - \frac{\pi^2}{6} \right] - (2 \leftrightarrow 3) \right\} + \mathcal{O}(\epsilon) \right)$$
$$+ \ \text{complex conjugate}\,,$$

$$\hat{a} = \frac{s_{123}}{s_{23}} \left(\frac{z_1\, s_{13}}{s_{123}} + \frac{z_1(z_1\, s_{23} - z_2\, s_{13} - z_3\, s_{12})}{2(z_2+z_3)\, s_{12}} + \frac{z_1\, s_{23} + z_2\, s_{13} - z_3\, s_{12}}{2s_{123}} \right)$$
$$\hat{b} = \frac{s_{123}}{s_{23}} \left(\frac{z_2\, s_{23}}{s_{123}} + \frac{z_2(z_1\, s_{23} - z_2\, s_{13} + z_3\, s_{12})}{2(z_2+z_3)\, s_{12}} + \frac{z_1\, s_{23} + z_2\, s_{13} - z_3\, s_{12}}{2s_{123}} \right)$$

Antisymmetric contribution to *q->qQQbar* AP kernel at NLO (Catani, de Florian, Rodrigo; Phys.Lett. B586, 323-331,2004)

  - Presence of **different scales** ➡ **Difficult computation!**
  - Start to work with QCD+QED and splittings with photons (simpler color structure)

Triple collinear splittings

- Computation strategy: general case
  - Use Feynman diagram approach
  - Work at squared-amplitude level , in TL region and in CDR ➡ Easier calculation!
  - Use IBP and other techniques to simplify loop-integrals in the LC-gauge
  - Expand the result up to $O(\varepsilon^0)$ ➡ Transcendentality

Divergent contribution (also includes $O(\varepsilon^0)$ terms)

$$\langle \hat{P}^{(1)}_{a_1\cdots a_m}\rangle = \left(\frac{s_{1\ldots m}}{2\,\mu^{2\epsilon}}\right)^{m-1} \left(\overline{\boldsymbol{Sp}^{(1)}_{a_1\ldots a_m} \left(\boldsymbol{Sp}^{(0)}_{a_1\ldots a_m}\right)^{\dagger}} + \text{h.c.}\right)$$

$$= I^{(1)}_{a_1\cdots a_m}(p_1,\ldots,p_m;\tilde{P})\,\langle \hat{P}^{(0)}_{a_1\cdots a_m}\rangle + R^{(1)}_{a_1\cdots a_m} + \text{h.c.}$$

Finite remainder

$$R^{(1)}_{a_1\cdots a_m} = \left(\frac{s_{1\ldots m}}{2\,\mu^{2\epsilon}}\right)^{m-1} \overline{\boldsymbol{Sp}^{(1)\,\text{fin.}}_{a_1\ldots a_m} \left(\boldsymbol{Sp}^{(0)}_{a_1\ldots a_m}\right)^{\dagger}}$$

$$= c^{a_1\cdots a_m}_{\text{Factor}} \left[C_0 + \sum_{i=1}^{2} \sum_{j\in\mathcal{F}_i} C^i_j F^i_j(\{s_{kl}, z_k\}) + \mathcal{O}(\epsilon)\right]$$

Set of functions of transcendentality *i*

(de Florian, Rodrigo and GS, in preparation)

Triple collinear splittings

- Computation strategy: polarized splittings (gluon started)
  - Definition:

$$P^{\mu\nu}_{P\to a_1\ldots a_m} \equiv \left(\boldsymbol{Sp}_{\mu}\ (p_1\ldots p_m;\widetilde{P})\right)^{\dagger} \boldsymbol{Sp}_{\nu}\ (p_1\ldots p_m;\widetilde{P})$$

Amputated splittings

$$\mathbf{P}_{P\to a_1\ldots a_m}(\lambda,\lambda') = (\epsilon^{\mu}(\widetilde{P},\lambda))^{*}\epsilon^{\nu}(\widetilde{P},\lambda')P^{\mu\nu}_{P\to a_1\ldots a_m}$$

Physical polarizations

  - Write a **tensor basis** to decompose all the possible structures (combinations of external momenta, n and the metric tensor)

$n^{\mu}\quad \{p_i^{\mu}\}_{i\in C}\quad (\eta^{D})^{\mu\nu}$ → 17 rank-2 tensor structures for 1->3 processes

  - Tensor integrals → Use **reduction techniques**!
  - Project over non-vanishing structures:

Physical on-shell gluon states $\epsilon(\widetilde{P})\cdot\widetilde{P}=0$, $\epsilon(\widetilde{P})\cdot n=0$ →

$$P^{\mu\nu}_{P\to a_1\ldots a_3} = \sum_{j=0}^{3} A_j^{\rm sym} f_j^{\mu\nu} + A^{\rm asym} f_{12}^{\mu\nu}$$

$$f_1^{\mu\nu}=\eta^{\mu\nu}, f_2^{\mu\nu}=2\frac{p_1^{\mu}p_1^{\nu}}{s_{123}}, f_3^{\mu\nu}=\frac{p_1^{\mu}p_2^{\nu}+p_1^{\nu}p_2^{\mu}}{s_{123}}, f_4^{\mu\nu}=2\frac{p_2^{\mu}p_2^{\nu}}{s_{123}}$$ Symmetric

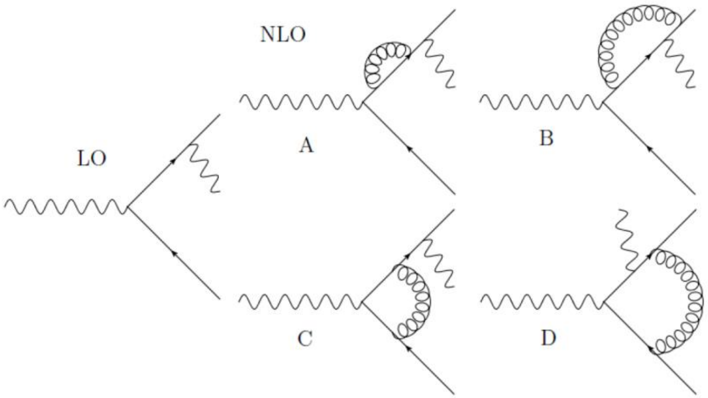
$$f_{12}^{\mu\nu}=\frac{p_1^{\mu}p_2^{\nu}-p_1^{\nu}p_2^{\mu}}{s_{123}}$$ Antisymmetric

  - Extract the divergent contribution proportional to $\mathbf{I}_{\mathrm{C}}$ (analogous to unpolarized computation)

(de Florian, Rodrigo and GS, in preparation)

Triple collinear splittings

- Status and some (compact…) results:
  - Splitting functions with photons computed at NLO!
  - Photon started splittings are very simple: $\gamma \to q\bar{q}\gamma$



$$\langle \hat{P}^{(1)\,\text{fin.}}_{\gamma\to q_1\bar{q}_2\gamma_3} \rangle = C_F C_A e^4 e_q^4 g_S^2 \left[C^{(0)} + \sum_{i=1}^{2} C_i^{(1)} F_i^{(1)} + C_1^{(2)} F_1^{(2)} + (1 \leftrightarrow 2) \right]$$

Weight 1: $F_1^{(1)} = \log(x_1)\ ,\ F_2^{(1)} = \log(x_3)$

Weight 2: $F_1^{(2)} = \mathcal{R}(x_1, x_3)$

$$C_1^{(2)} = \frac{2\left(x_2\left(x_3\Delta^{0,1}_{3,2} + \Delta^{3,0}_{0,3}(\Delta^{0,1}_{2,3} + z_1) + x_2^3 + 2x_2x_3z_1\right) + (\Delta^{0,1}_{3,2})^2\right)}{x_1 x_2^3} - \frac{4\Delta^{1,0}_{0,1}(x_3 - z_1)}{x_1 x_2}$$

$$\mathcal{R}(x_1, x_2) = \frac{\pi^2}{6} - \log(x_1)\log(x_2) - \text{Li}_2(1 - x_1) - \text{Li}_2(1 - x_2)$$

This function appears in all the triple splittings at NLO ⟺ Scalar boxes!!!

  - Recursive construction of splittings, **replacing photons by gluons** (increasing the complexity of possible color structures)

$$x_i = \frac{-s_{jk} - \imath 0}{-s_{123} - \imath 0}$$

(de Florian, Rodrigo and GS, in preparation)

Conclusions and perspectives

- Splitting amplitudes describe collinear factorization properties and are process-independent quantities (except, maybe, in some kinematical configurations).
- Polarized splitting functions allow to keep spin-correlations (full description of the collinear limit).
- Splitting functions containing at least one photon computed at NLO (in QCD), in the triple collinear limit (*to be published soon*…)

- Outlook:
  - Extend the results for more collinear particles beyond tree-level (recursion relations?)
  - Violation of strict collinear factorization?
  - Develop more efficient techniques to compute splittings (lengthy intermediate steps, but compact results....)

THANK YOU!