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THE HIGGS COINCIDENCE PROBLEM: why $m_H^2 = m_Z m_t$?

E. Torrente-Lujan(*), J.J. Fernandez-Melgarejo *U. Murcia. ((*)etl@um.es)*

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SUMMARY. The ratios ρ_t, ρ_{Wt}

We present a phenomenological and theoretical evaluation of the mass ratios (using up-to-date experimental values [1], 1σ errors)

$$\rho_t^{(exp)} = \frac{m_Z m_t}{m_H^2} = 1.0022 \pm 0.008,$$
$$\rho_{Wt}^{(exp)} = \frac{m_W + m_t}{2m_H} = 1.007 \pm 0.006$$

It is tempting to think that such values,so close to one, are not a mere coincidence but, on naturalness grounds, a signal of some more deeper symmetry.

ρ_t can be seen as the ratio of the highest massive representatives of the spin $(0, 1/2, 1)$ SM particles

$$m_{s=1} m_{s=1/2} / m_{s=0}^2 \simeq 1.$$

Taking at face value these relations we can write any two mass ratios as a function of, exclusively, θ_W (at 1% or better precision):

$$\frac{m_i}{m_j} \simeq f_{ij}(\theta_W), \quad i, j = W, Z, H, t.$$

For example:

$$\frac{m_W}{m_Z} \simeq \cos\theta_W,$$
$$\frac{m_H}{m_Z} \simeq 1 + \sqrt{2}\sin^2\frac{\theta_W}{2},$$
$$\frac{m_H}{m_t}\cos\theta_W \simeq 1 - \sqrt{2}\sin^2\frac{\theta_W}{2}.$$

In the limit $\cos\theta_W \to 1$ and $m_Z \to m_W \to m_t \to m_H$. Such relations could be interpreted as a hint for a role of the $SU(2)_c$ custodial symmetry, together with other unknown mechanism.

We review the theoretical situation of these quantities in the SM and beyond. We show how in the SM these relations are rather stable under RGE pointing out to some underlying UV symmetry. In the SM such a ratios hint for a non-casual relation of the type $\lambda \simeq \kappa\left(g^2 + g'^2\right)$ with $\kappa \simeq 1 + o(g/g_t)$.

Without a symmetry at hand to explain then in the SM, it arises a Higgs mass coincidence problem:

why $\rho_t, \rho_{Wt} \sim 1$?,

can we find a mechanism that naturally gives $m_H^2 = m_Z m_t$, $2m_H = m_W + m_t$?.

In the SM:

In the SM, at tree level:

$$\rho_t^{SM} = \frac{1}{4\sqrt{2}\cos\theta_W} \frac{g g_t}{\lambda}.$$

The SM Higgs selfcoupling λ is non determined. Assuming that *both* expressions $\rho_t, \rho_{Wt} \simeq 1$ are not a coincidence ($c \sim o(1)$)

$$\lambda \simeq c\sqrt{g^2 + g'^2} g_t, \quad \lambda \simeq c^2 (g + g_t)^2$$

or, assuming $g_t >> g$ (with $\kappa \simeq 1 + o(g/g_t)$)

$$\lambda \simeq \kappa\left(g^2 + g'^2\right)$$

At one loop:

$$\rho_t = \rho_t^0\left(1 + c_1\lambda - c_2 g_t^2 - c_s g_s^2\right).$$

$\delta_t^{QCD} \sim 5\%$ is the most correction. Both $\delta_t^w, \delta_\lambda$, are of opposite sign and $\sim 1\%$.

SM Renormalization group equations. We consider first a reduced system of one-loop RGEs for λ, g_t. (All the other couplings are considered very small or not running at all)

$$\frac{dg_t^2}{dt} = \frac{9}{16\pi^2} g_t^4, \qquad (1)$$
$$\frac{d\lambda}{dt} = \frac{6}{16\pi^2}\left(4\lambda^2 + 2\lambda g_t^2 - g_t^4\right). \qquad (2)$$

(with $t = \log(\mu/\Lambda)$, expressions valid for $\mu >> m_t, m_H$, or for $\Lambda \to \infty$). With $R = \frac{\lambda}{g_t^2}$ the RGE equations become decoupled with nested solutions, $g_t = g_t(\mu)$,$R = R(g_t)$,$\rho_t = \rho_t(R)$. we have (with $f(R) = 8R^2 + R - 2$)

$$g_t^2 \frac{dR}{dg_t^2} = \frac{1}{3} f(R), \qquad (3)$$
$$\frac{d\rho_t}{dR} = -\frac{3\rho_t}{2f(R)}\left(1 + \frac{2f(R)}{3R}\right). \qquad (4)$$

Eqs. (1,3,4) can be solved explicitly, in particular

$$\rho_t = k\left(\frac{R_0 - R}{R_1 + R}\right)^{R_0 - R_1} R^2,$$

where R_0, R_1 are the fixed points of Eq.(3), $f(R_{0,1}) = 0$. For a light m_H and large m_t , R is small. At low scales $R^{exp} \sim 10^{-1}$:

$$R(g_t) = R_c - \frac{4}{3}\log g_t,$$
$$\rho_t \sim kR^2 \sim (R_c - \frac{4}{3}\log g_t)^2 \sim kR_c^2 \sim \rho_t^0.$$

At large energies ($\mu >> m_t$, as long as $R > 0$ or $\lambda > 0$), $\rho_t(\mu)$ keeps approximately constant, only sligtly decreasing with $\log g_t$.

A reduced Higgs-top-strong system where the λ, g_t, g_s are non-vanishing has been also considered. Similar results are obtained.

In summary in the SM, at this level, the relations $\rho_t, \rho_{Wt} \sim 1$ are rather stable under RGE pointing out to some underlying UV symmetry.

Beyond SM, further discussion

We expect new physics that cuts off the divergent top, gauge and higgs loop contributions to m_H at $\lesssim 10$ TeV. Many different possibilities have been well explored, they usually include, more or less ad-hoc, new particles with properties tightly associated to those of the SM. Some of these possibilities are[2]: a) The new particles are just the, softly broken, SUSY, superpartners with couplings and Yukawas strongly dictated by supersymmetry and the soft breaking itself. b) The Higgs is a composite resonance, or c) The "Little" Higgs is a pseudo-Nambu-Goldstone boson with respect a "softly" broken approximate global symmetry. This scalar sector is accompanied by some new particles belonging to enlarged multiplets together with the SM particles.

It is a general feature that, in all or most of these models, the λ parameter, and then m_H, is related to g, g', g_t in a more or less explicit way. The reason is clear [2], the new one-loop which are proportional to the couplings of the SM gauge sector have to match and cancel the top and the other cuadratic loops.

In [2] we review the situation in the MSSM and Littlest Higgs scenarios. Let us mention here the "Littlest" Higgs scenario. Here the usual Higgs doublet is the lightest of a set of pseudo goldstone bosons in an non-linear sigma model including in its gauge group different $SU(2) \times U(1)$ factors. The product group is broken to the diagonal, identified as the SM electroweak gauge group. g_t generates a negative mass squared triggering EW SSB. New particles are added, in particular heavy top partners, which cancel the one loop quadratically divergent corrections. The λ parameter naturally arises as

$$\lambda \sim o(g^2, g_t^2).$$

Particular scenarios can apparently be tuned so that either g or g_t dominate and $m_H \sim m_Z$ or $m_H \sim m_t$ as extreme cases. Approximate accidental global symmetries related to the Little Higgs scenario could play a role in the understanding of the ρ_t ratio, as the global custodial $SU(2)_c$ symmetry [2] plays for the ρ ratio.

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