LIGHT NEUTRINO MASS SPECTRUM WITH ONE OR TWO RIGHT-HANDED SINGLET FERMIONS ADDED

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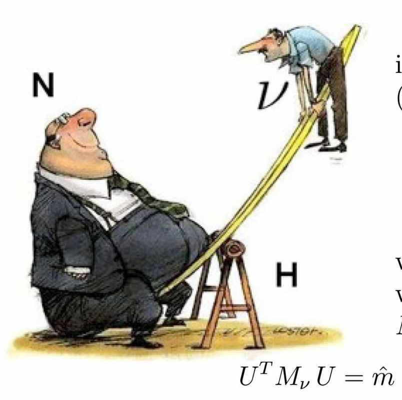
1 Introduction

Neutrino oscillation experiments showed that neutrinos have tiny but non-zero masses. The seesaw mechanism is the most fruitful explanation of the light neutrino masses and mixings, which connects the tiny neutrino masses with heavy right-handed neutrino masses. After spontaneous symmetry breaking of the Standard Model gauge group one obtains a $(n_L+n_R)\times(n_L+n_R)$ Majorana mass matrix M_ν for the neutrinos. The mixing between the n_R right-handed singlet fermions and the neutral parts of the n_L lepton doublets gives masses to the neutrinos which are of the size expected from neutrino oscillations.

The diagonalization of the mass matrix gives rise to a split spectrum consisting of heavy and light states of neutrinos given by $U^T M_\nu U = \mathrm{diag}(m_{n_L}^{\mathrm{light}}, m_{n_R}^{\mathrm{heavy}})$. We analyse two cases of the minimal extension of the Standard Model when one or two right-handed fields are added to the three left-handed fields. A second Higgs doublet (two Higgs doublet model – 2HDM) is included in our model.

We calculate the one-loop radiative corrections to the mass parameters which produce mass terms for the neutral leptons. In both cases we numerically analyse light neutrino masses as functions of the heavy neutrino masses. Parameters of the model are varied to find light neutrino masses that are compatible with experimental data of solar $\Delta m^2_\odot$ and atmospheric $\Delta m^2_{\mathrm{atm}}$ neutrino mass differences for normal and inverted hierarchy. We choose values for the parameters of the tree-level by numerical scans, where we look for the best agreement between computed and experimental neutrino oscillation angles. Different mixing angles between the Higgs fields give different mass spectra of light neutrinos and different distributions of neutral Higgs masses.

2 Description Of The Model



The mass terms for the neutrinos can be written in a compact form as a mass term with an $(n_L+n_R)\times(n_L+n_R)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & \hat{M}_R \end{pmatrix}, \tag{1}$$

where M_D is a $n_L \times n_R$ Dirac neutrino mass matrix, while the hat indicates that $\hat{M}_R$ is a diagonal matrix. M_ν can be diagonalized as

$$U^T M_\nu U = \hat{m} = \mathrm{diag}\left(m_1, m_2, \ldots, m_{n_L+n_R}\right), \tag{2}$$

where the m_i are real and non-negative. In order to implement the seesaw mechanism [1] we assume that the elements of M_D are of order m_D and those of M_R are of order m_R, with $m_D \ll m_R$. Then, the neutrino masses m_i with $i = 1, 2, \ldots, n_L$ are of order m_D^2/m_R, while those with $i = n_L+1, \ldots, n_L+n_R$ are of order m_R.

In the standard seesaw, one-loop corrections to the mass matrix, i.e. the self energies, are determined by the neutrino interactions with the Z boson, the neutral Goldstone boson G^0, and the Higgs boson h^0. Each diagram contains a divergent piece but when summing up the three contributions the result turns out to be finite.

Once the one-loop corrections are taken into account the neutral fermion mass matrix is given by

$$M_\nu^{(1)} = \begin{pmatrix} \delta M_L & M_D^T + \delta M_D^T \\ M_D + \delta M_D & \hat{M}_R + \delta M_R \end{pmatrix} \approx \begin{pmatrix} \delta M_L & M_D^T \\ M_D & \hat{M}_R \end{pmatrix} \tag{3}$$

where the $0_{3\times3}$ matrix appearing at tree level (1) is replaced by the contribution δM_L. This correction is a symmetric matrix, it has the largest influence as compared to other corrections.

The expression for one-loop corrections is given by [2]

$$\begin{aligned} \delta M_L &= \sum_b \frac{1}{32\pi^2} \Delta_b^T U_R^* \hat{m} \left(\frac{\hat{m}^2}{m^2_{H_b^0}} - \mathbb{1}\right)^{-1} \ln\left(\frac{\hat{m}^2}{m^2_{H_b^0}}\right) U_R^\dagger \Delta_b \\ &+ \frac{3g^2}{64\pi^2 m_W^2} M_D^T U_R^* \hat{m} \left(\frac{\hat{m}^2}{m_Z^2} - \mathbb{1}\right)^{-1} \ln\left(\frac{\hat{m}^2}{m_Z^2}\right) U_R^\dagger M_D, \end{aligned} \tag{4}$$

with $\Delta_b = \sum_k b_k \Delta_k$, where b are two-dimensional complex unit vectors and the sum $\sum_b$ runs over all neutral physical Higgses H_b^0.

Neutral Higgses are characterized by the unit b vectors which hold special orthogonality conditions. One set of b solutions could be parametrised in the following way

$$b_Z = \begin{pmatrix} i \\ 0 \end{pmatrix}, \quad b_1 = \begin{pmatrix} c_{12}c_{13} \\ -s_{12} - i c_{12}s_{13} \end{pmatrix}, \quad b_2 = \begin{pmatrix} s_{12}c_{13} \\ c_{12} - i s_{12}s_{13} \end{pmatrix}, \quad b_3 = \begin{pmatrix} s_{13} \\ i c_{13} \end{pmatrix} \tag{5}$$

where $c_{ij} \equiv \cos(\alpha_{ij})$ and $s_{ij} \equiv \sin(\alpha_{ij})$. These components of b vectors coincide with functions of the invariant mixing angles α_{12} and α_{13}, which are entries of the orthogonal diagonalization matrix, that diagonalizes the neutral Higgs squared-mass matrix [3]. For calculations one can choose a convention where $-\frac{\pi}{2} \leqslant \alpha_{12}, \alpha_{13} < \frac{\pi}{2}$. The numerical analysis of the neutrino mass spectrum was performed considering the conditions where CP-invariant Higgs potential can arise in the 2HDM under one of the the six cases listed in Table 1.

	I	II	III	IV	V	VI
	$\alpha_{12}=0$	$\alpha_{13}=0$	$\alpha_{12}=-\frac{\pi}{2}$	$\alpha_{12}=0$, $\alpha_{13}=0$	$\alpha_{12}=-\frac{\pi}{2}$, $\alpha_{13}=0$	$\alpha_{13}=-\frac{\pi}{2}$
b_1	$\begin{pmatrix} c_{13} \\ -is_{13} \end{pmatrix}$	$\begin{pmatrix} c_{12} \\ -s_{12} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ ie^{i\alpha_{12}} \end{pmatrix}$
b_2	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} s_{12} \\ c_{12} \end{pmatrix}$	$\begin{pmatrix} -c_{13} \\ is_{13} \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ e^{i\alpha_{12}} \end{pmatrix}$
b_3	$\begin{pmatrix} s_{13} \\ ic_{13} \end{pmatrix}$	$\begin{pmatrix} 0 \\ i \end{pmatrix}$	$\begin{pmatrix} s_{13} \\ ic_{13} \end{pmatrix}$	$\begin{pmatrix} 0 \\ i \end{pmatrix}$	$\begin{pmatrix} 0 \\ i \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Table 1: Basis-independent conditions for a CP-conserving 2HDM scalar potential and vacuum [3]. Higgs mixing angles α_{ij} are defined with respect to the mass-ordering $m_{H_1^0} \leq m_{H_2^0} \leq m_{H_3^0}$.

3 Case $n_R = 1$

First we consider the minimal extension of the standard model by adding only one right-handed field ν_R to the three left-handed fields contained in ν_L.

For this case we use the parametrization $\Delta_i = \left(\sqrt{2}\, m_D/v\right) \vec{a}_i^T$, where $\vec{a}_1^T = (0,0,1)$ and $\vec{a}_2^T = \left(0, n, e^{i\phi}\sqrt{1-n^2}\right)$. Diagonalization of the symmetric mass matrix M_ν (1) in block form is

$$U^T M_\nu U = U^T \begin{pmatrix} 0_{3\times3} & m_D \vec{a}_1 \\ m_D \vec{a}_1^T & \hat{M}_R \end{pmatrix} U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{6}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined analytically by finding eigenvalues of the hermitian matrix $M_\nu M_\nu^\dagger$. These eigenvalues are the squares of the masses of the neutrinos $\hat{M}_l = \mathrm{diag}(0,0,m_l)$ and $\hat{M}_h = m_h$. Solutions $m_D^2 = m_h m_l$ and $m_R^2 = (m_h - m_l)^2 \approx m_h^2$ correspond to the seesaw mechanism.

It is possible to estimate masses of the light neutrinos from experimental data of solar and atmospheric neutrino oscillations [4] assuming that the lightest have some concrete value m_{l_3}. Performing numerical scan by value m_{l_3} we choose parameters for the tree level for which agree most with experimental oscillation data, see Fig. 1.

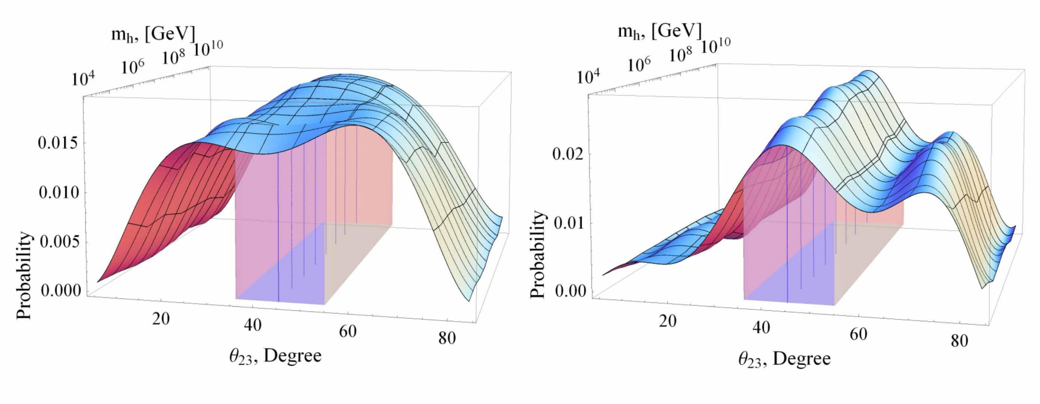


Figure 1: 3D histograms of θ_{23} oscillation angles. The oscillation angles are introduced in this model by parametrizing U_{loop} and including the MNSP mixing matrix for 4-dimensional case. For $n_R = 1$ case only $\theta_{\mathrm{atm}} \neq 0$ because the third light neutrino is massless. The left plot represents the case III of Table 1, the right plot represents case I. The filled boxes in the plots indicates the experimental boundaries, the blue vertical lines denote the experimental middle value of $\theta_{23} = 45°$.

For the calculation of radiative corrections we use the orthogonal complex vectors b listed in Table 1. Diagonalization of the mass matrix after calculation of one-loop corrections is performed with a unitary matrix $U_{\mathrm{loop}} = U_{\mathrm{egv}} U_\varphi(\varphi_1, \varphi_2, \varphi_3)$, where U_{egv} is an eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$ and U_φ is a phase matrix. The second light neutrino obtains its mass from radiative corrections. The third light neutrino remains massless.

The numerical analysis shows that we can reach the allowed neutrino mass ranges for a heavy singlet with the mass close to 10^4 GeV when the angle of oscillations θ_{atm} fixed to the experimental 3σ range, see Fig. 2.

The free parameters n, ϕ, $m_{H_2^0}$ and $m_{H_3^0}$ are restricted by the parametrization used and by oscillation data. Figure 3 illustrates the allowed values of Higgs masses for different values of the heavy singlet. The values of Higgs masses spread to two separated sets. These different sets generate appropriate values of the parameters n and ϕ, see figure 4.

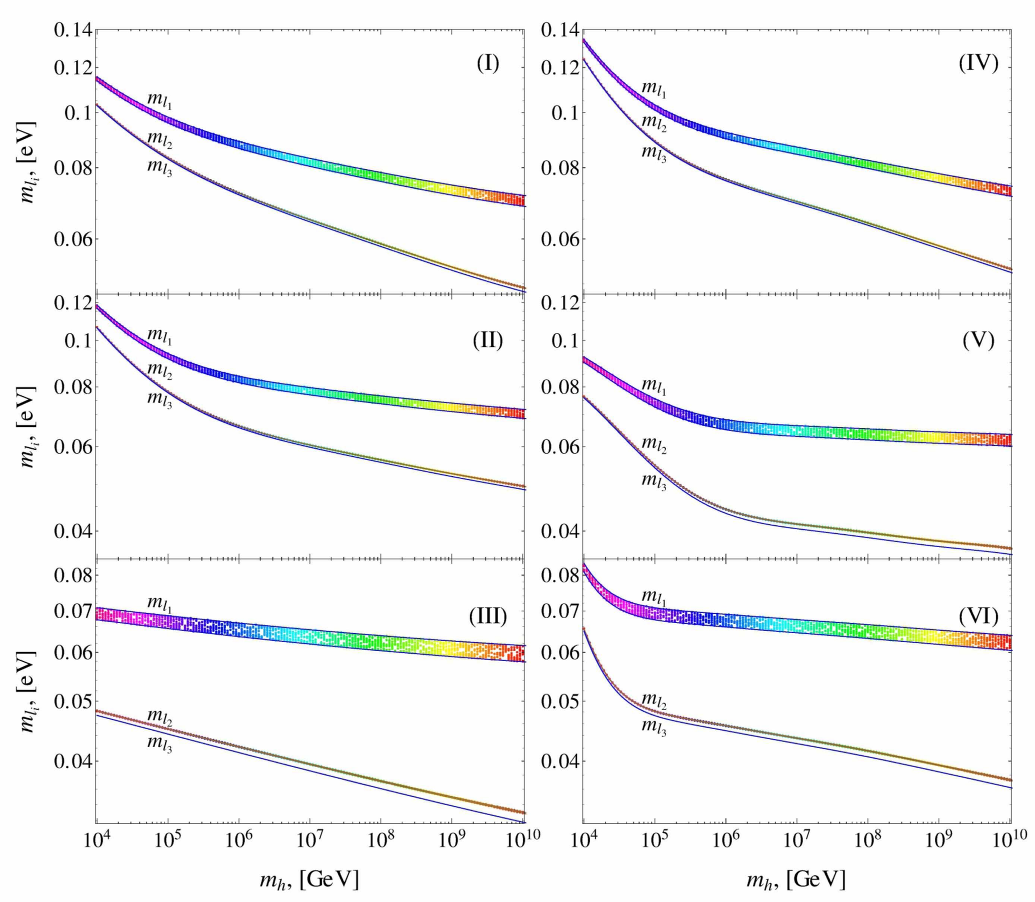


Figure 2: Calculated masses of two light neutrinos as a function of the heavy neutrino mass m_h for 6 different cases of Table 1. In the plots the blue solid line represents chosen value of m_{l_3} used for the calculations of the tree level masses. The colors in the bound of m_{l_1} values corresponds to the mass of m_h, given in the bottom of the plots. This color coding is used also for Figures 3 and 4.

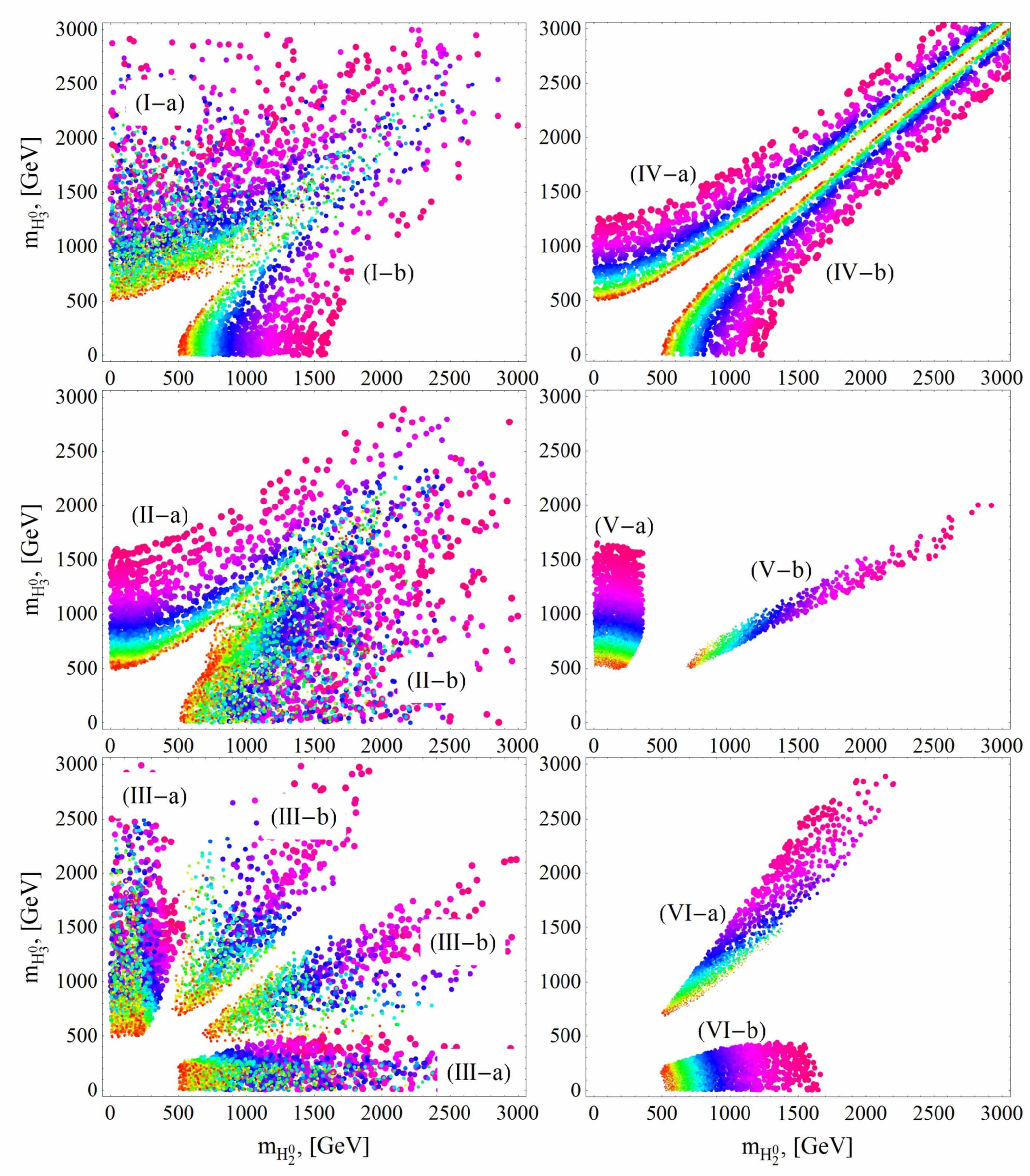


Figure 3: The values of the free parameters $m_{H_2^0}$ and $m_{H_3^0}$ as functions of the heaviest right-handed neutrino mass m_h for the 6 different cases of Table 1. The meaning of colors is given in Figure 2. The mass of the SM Higgs boson is fixed to $m_{H_1^0} = 125$ GeV.

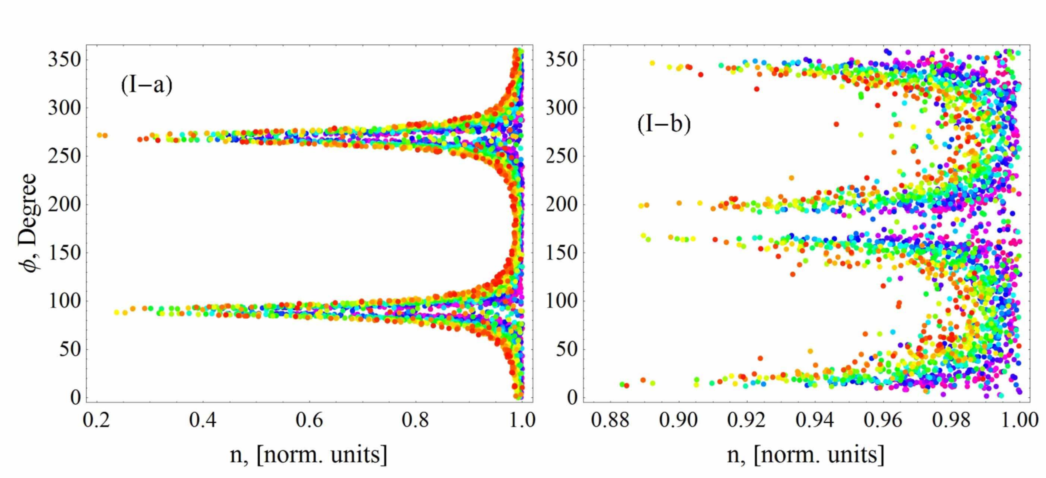


Figure 4: The values of the free parameters n and ϕ as functions of the heaviest right-handed neutrino mass m_h for the case I. The meaning of colors is given in Figure 2. Different sets of Higgs masses correspond to appropriate values of the parameters n and ϕ.

4 Case $n_R = 2$

When we add two singlet fields ν_R to the three left-handed fields ν_L, the radiative corrections give masses to all three light neutrinos.

We parametrize $\Delta_i = \frac{\sqrt{2}}{v}\left(m_{D_2}\vec{a}_i^T, m_{D_1}\vec{b}_i^T\right)^T$ with $|\vec{a}| = 1$, and $|\vec{b}| = 1$. Diagonalizing the symmetric mass matrix M_ν (1) in block form we write:

$$U^T M_\nu U = U^T \begin{pmatrix} 0_{3\times3} & m_{D_2}\vec{a}\ m_{D_1}\vec{b} \\ m_{D_2}\vec{a}^T & \\ m_{D_1}\vec{b}^T & \hat{M}_R \end{pmatrix} U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{7}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined by the seesaw mechanism: $m_{D_i}^2 \approx m_{h_i} m_{l_i}$ and $m_{R_i}^2 \approx m_{h_i}^2$, $i = 1, 2$. We use $m_1 > m_2 > m_3$ ordering of masses. At tree level the third light neutrino is massless.

In numerical calculations the model parameters as well as the derived masses of the light neutrinos are obtained in several steps. First, the diagonal mass matrix for the tree level is constructed. The masses of the other two light neutrinos are estimated from experimental data on solar and atmospheric neutrino oscillations by choosing a value m_{l_3} which agrees with experimental oscillation data, see Fig. 5. The masses of the heavy neutrinos are input parameters. This diagonal tree level matrix is used to constrain the parameters that enter the tree-level mass matrix M_ν and its diagonalization matrix. Then the diagonalization matrix is used to evaluate one-loop corrections to the mass matrix. Diagonalization of the corrected mass matrix yields masses for the three light neutrinos. If the calculated mass difference is compatible with the experimental oscillations data, the parameter set is kept. Otherwise, another set of parameters is generated. Figure 6 illustrates the obtained results for cases IV and VI. Both normal and inverted neutrino mass orderings are considered.

Figure 7 illustrates the allowed values of Higgs masses for different values of the heavy singlet. The values of Higgs masses spread to two separated sets as for the $n_R = 1$ case.

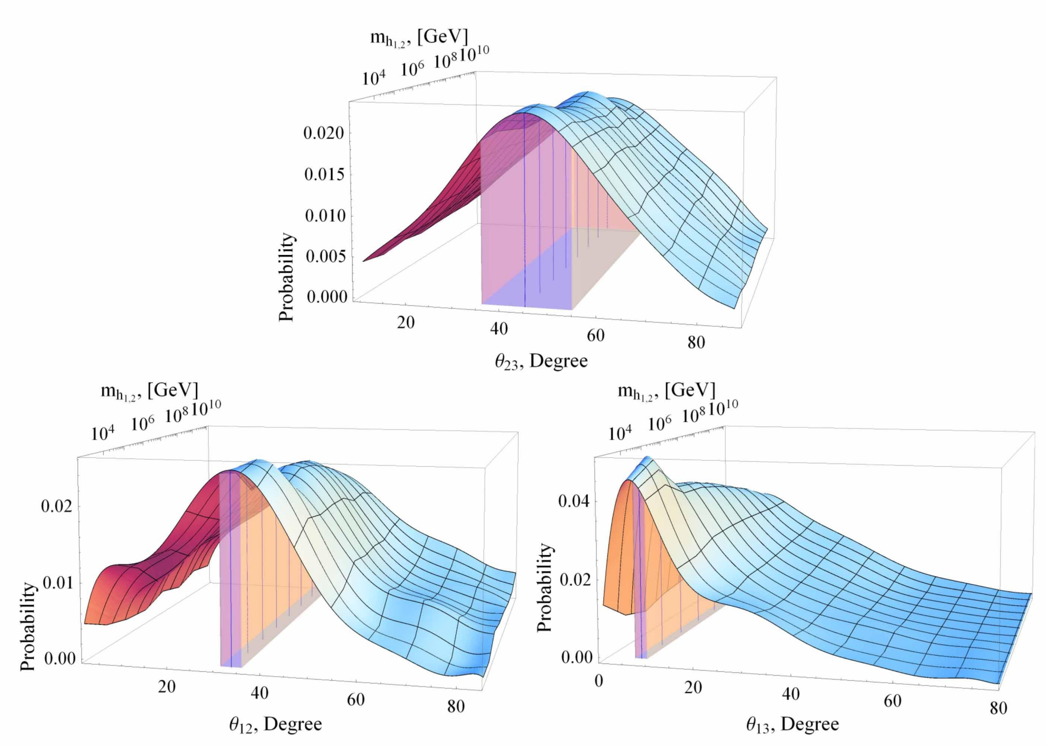


Figure 5: The 3D histograms of oscillation angles for case VI and inverted hierarchy. Filled boxes in plots denote the experimental boundaries, the blue vertical lines denote experimental middle values.

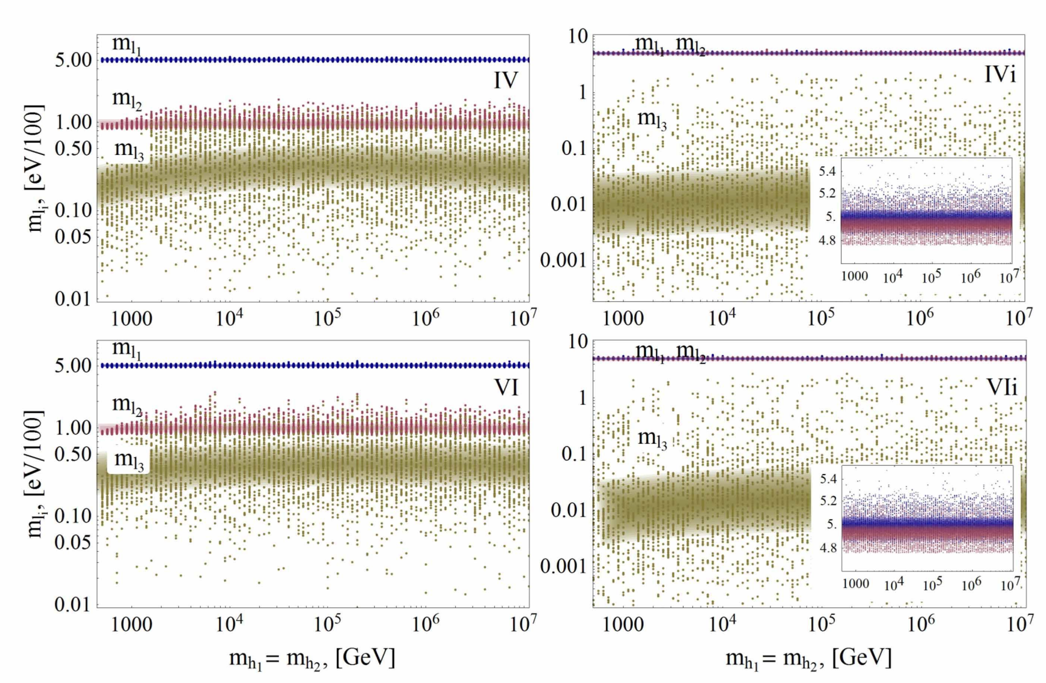


Figure 6: The masses m_{l_i} of the light neutrinos as functions of the heaviest right-handed neutrino mass $m_{h_{1,2}}$, for the cases IV and VI. The plots on the left represents the normal hierarchy, and the plots on the right represents the inverted hierarchy of the light neutrinos. The wide solid lines indicate the area of the most frequent values of the scatter data. The nearly degenerate masses m_{l_1} and m_{l_2} are shown separately in the lower right plots for the cases of the inverted hierarchy.

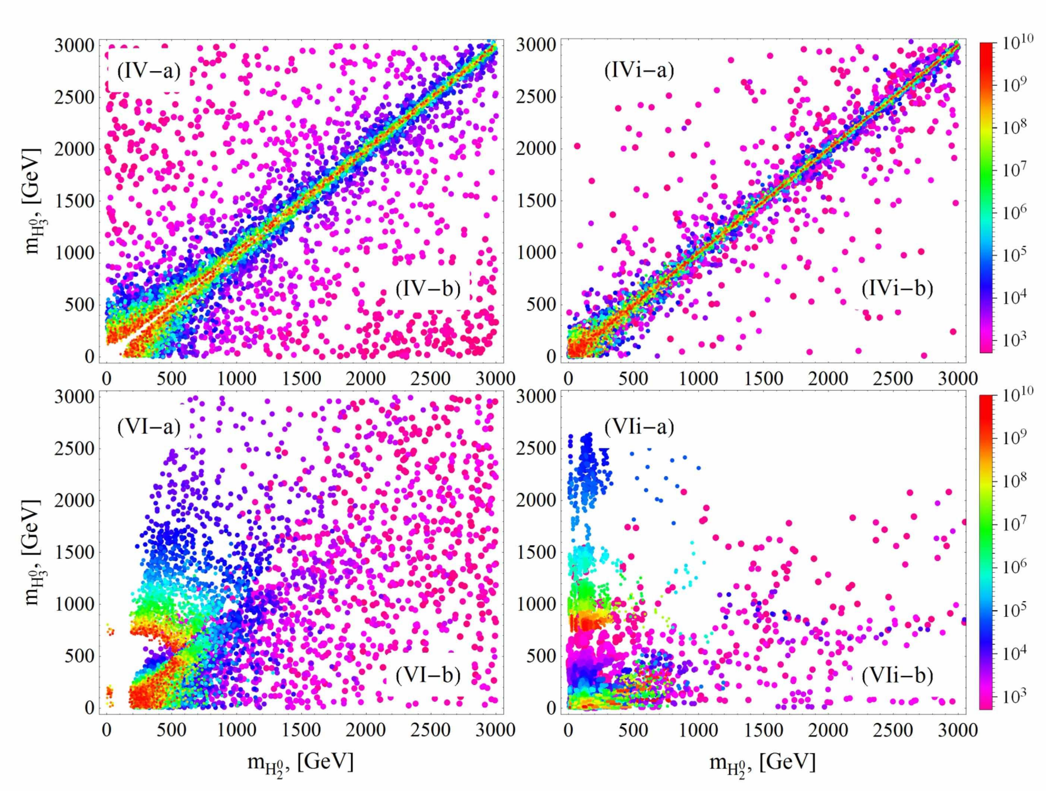


Figure 7: The values of the free parameters $m_{H_2^0}$ and $m_{H_3^0}$ as functions of the heaviest right-handed neutrino mass $m_{h_{1,2}}$ for the cases IV and VI. The plots on the left represents the normal hierarchy, and the plots on the right represents the inverted hierarchy of the light neutrinos.

5 Conclusions

1. For the case $n_R = 1$ we can match the differences of the calculated light neutrino masses to $\Delta m^2_\odot$ and $\Delta m^2_{\mathrm{atm}}$ with the mass of a heavy singlet larger than 10^4 GeV. The parametrization used for this case and restrictions from the neutrino oscillation data limit the values of free parameters. Only normal ordering of neutrino masses is possible.
2. In the case $n_R = 2$ we obtain three non vanishing masses of light neutrinos for normal and inverted hierarchies. The numerical analysis shows that the values of parameters and Higgs masses depend on the choice of the set of b vectors and the heavy neutrinos masses. The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.

6 References

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