The S_3 Flavour Symmetry: Quarks and Leptons

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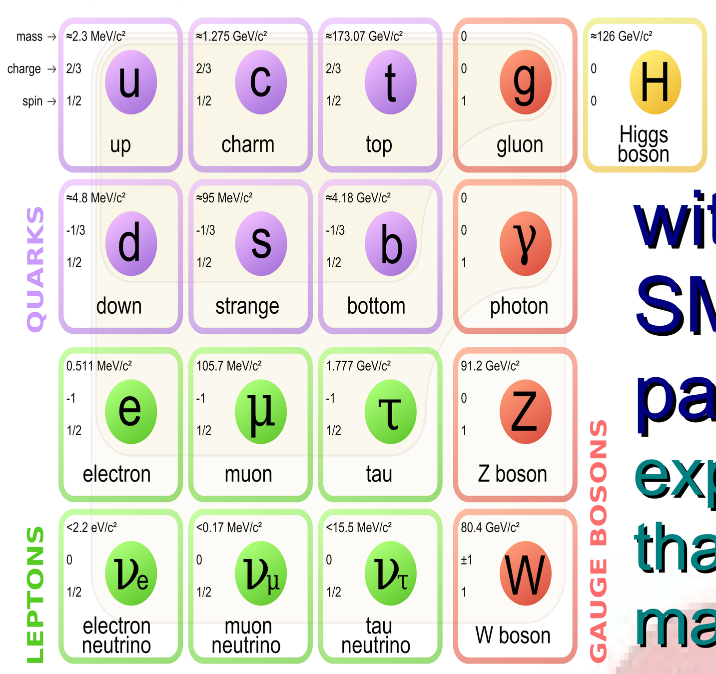
Abstract: In the theoretical framework of a extension of the Standard Model with the S_3 flavour symmetry and three Higgs SU(2) doublets in the scalar sector, we discuss the masses and mixings of Quarks and leptons. The exact expressions for the flavour mixing angles are obtained and the numerical values that these generate are in very good agreement with current experimental data on masses and mixings of quarks and leptons. We also show the branching ratios of some selected flavour changing neutral currents (FCNC) process. (For details see: Fortsch. Phy. 61 (2013) 546-570, Phys. Rev. D 88 (2013) 096004, J. Phys. Conf. Ser. 492 (2014) 012015)

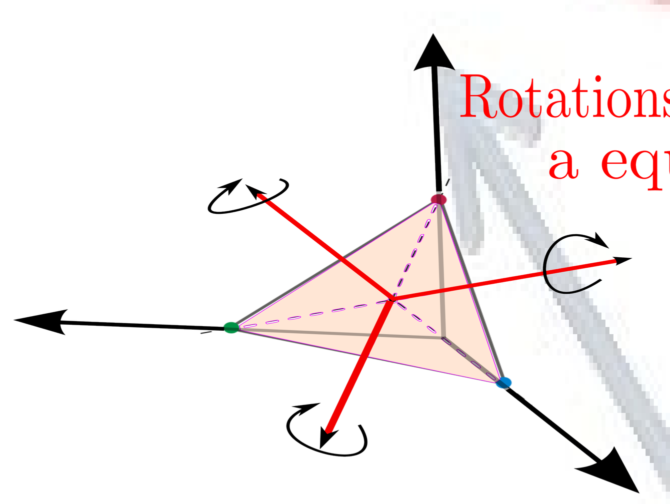
Why we need go beyond Standard Model?



Main cause: **The neutrinos within context of SM are massless particles.** But, the experiments indicate that neutrinos have mass.

The S_3 group of permutations of three objects.

Geometrical representation of S_3



Rotations and translations in a equilateral triangle.

Irreducible representations

$\mathbf{2}$ Doublet

$\mathbf{1}_S$ Symmetric singlet $\mathbf{1}_A$ Antisymmetric singlet

Direct products

$\mathbf{1}_S \otimes \mathbf{1}_S = \mathbf{1}_S$

$\mathbf{1}_S \otimes \mathbf{1}_A = \mathbf{1}_A$

$\mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_S$

$\mathbf{1}_S \otimes \mathbf{2} = \mathbf{2}$

$\mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$

$\mathbf{2} \otimes \mathbf{2} = \mathbf{2} \oplus \mathbf{1}_S \oplus \mathbf{1}_A$

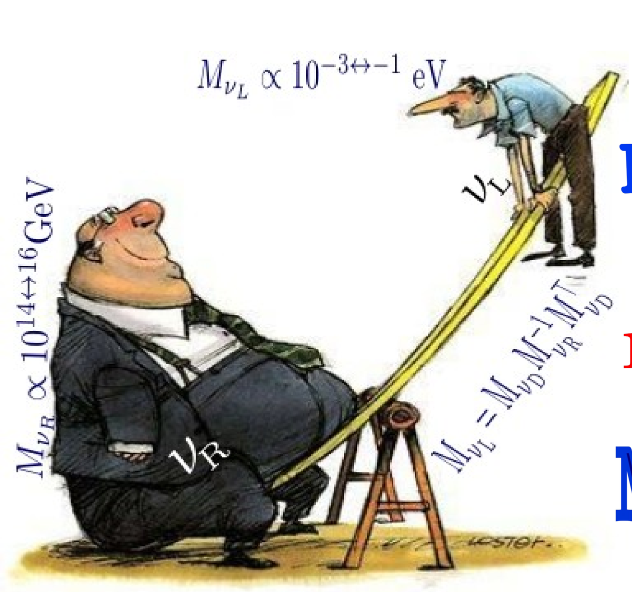
The Dirac fermion mass matrices

$w_1 \equiv \langle 0|H_1|0\rangle \quad w_2 \equiv \langle 0|H_2|0\rangle \quad v_S \equiv \langle 0|H_S|0\rangle$

$$\begin{pmatrix} \sqrt{2}Y_2^f v_S + Y_3^f w_2 & Y_3^f w_1 + \sqrt{2}Y_4^f v_A & \sqrt{2}Y_5^f w_1 \\ Y_3^f w_1 - \sqrt{2}Y_4^f v_A & \sqrt{2}Y_2^f v_S - Y_3^f w_2 & \sqrt{2}Y_5^f w_2 \\ \sqrt{2}Y_6^f w_1 & \sqrt{2}Y_6^f w_2 & 2Y_1^f v_S \end{pmatrix}$$

$f = u, d, l, \nu_D$

The neutrino mass matrix



Seesaw mechanism

$$\mathbf{M}_{\nu_L} = \mathbf{M}_{\nu_D} \mathbf{M}_{\nu_R}^{-1} \mathbf{M}_{\nu_D}^{T}$$

The right handed neutrino matrix

$$\mathbf{M}_{\nu_R} = \mathrm{diag}(M_1, M_2, M_3)$$

The Z_2 symmetry in the lepton sector:

$-$	$+$
$H_S,\ \nu_{3R}$	$H_I,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

The following couplings are forbidden:

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$$

The left handed neutrino mass matrix:

$$\begin{pmatrix} \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\lambda(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\lambda(\mu_2^\nu)^2}{\overline{M}} & \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} & \frac{2(\mu_4^\nu)^2}{\overline{M}} + \frac{(\mu_3^\nu)^2}{M_3} \end{pmatrix}$$

$$\lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \text{ and } \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

Texture zeroes in the fermionic matrices

$$\mathbf{M}_k = \mathbf{Q}_k \mathbf{U}_{\theta_k} \left(\mu_k^0 \mathbb{I}_{3\times3} + \widehat{\mathbf{M}}_k\right) \mathbf{U}_{\theta_k}^\dagger \mathbf{Q}_k^{\dagger(\top)}$$

$k = u, d, l, \nu_L.$

$\mathbf{U}_{\theta_k}$ is a rotation matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \tan\theta = \frac{w_1}{w_2} \text{ for Dirac} \quad \tan\theta = 1 \text{ for Neutrinos}$$

CP violation phases

$$\mathbf{Q} \equiv \mathrm{diag}\left(1, e^{i\phi_{1k}}, e^{i\phi_{2k}}\right)$$

$\widehat{\mathbf{M}}_k$ Matrix with two texture zeroes

$$\begin{pmatrix} 0 & \sqrt{\frac{\widetilde{\sigma}_1^k \widetilde{\sigma}_2^k}{1-\delta_k}} & 0 \\ \sqrt{\frac{\widetilde{\sigma}_1^k \widetilde{\sigma}_2^k}{1-\delta_k}} & \widetilde{\sigma}_1^k - \widetilde{\sigma}_2^k + \delta_k & \sqrt{\frac{\delta_k}{1-\delta_k}\xi_1^k \xi_2^k} \\ 0 & \sqrt{\frac{\delta_k}{1-\delta_k}\xi_1^k \xi_2^k} & 1 - \delta_k \end{pmatrix}$$

$$\xi_1^k \equiv 1 - \widetilde{\sigma}_1^k - \delta_k, \quad \xi_2^k \equiv 1 + \widetilde{\sigma}_2^k - \delta_k,$$

The ratios of shifted masses:

$$\widetilde{\sigma}_1^k = \frac{\widetilde{m}_1^k - \widetilde{\mu}_0^k}{1 - \widetilde{\mu}_0^k} \qquad \widetilde{\sigma}_2^k = \frac{|\widetilde{m}_2^k - \widetilde{\mu}_0^k|}{1 - \widetilde{\mu}_0^k}$$

$\widetilde{m}_{1,2}^k$ are the ratios of fermion masses

The flavour mixing matrices :

$$\mathrm{V}_{CKM} = \mathrm{O}_u^\top \mathrm{P}^{(u-d)} \mathrm{O}_d \text{ and } \mathrm{V}_{lep} = \mathrm{O}_l^\top \mathrm{P}^{(\nu-l)} \mathrm{O}_\nu$$

$$\begin{pmatrix} O_{11}^k & -O_{12}^k & O_{13}^k \\ O_{21}^k & O_{22}^k & O_{23}^k \\ -O_{31}^k & -O_{32}^k & O_{33}^k \end{pmatrix}$$

$$O_{11}^k = \sqrt{\frac{\widetilde{\sigma}_1^k \delta_k \xi_{k2}}{\mathcal{D}_{k1}}}, \quad O_{12}^k = \sqrt{\frac{\widetilde{\sigma}_2^k \delta_k \xi_{k1}}{\mathcal{D}_{k2}}}, \quad O_{13}^k = \sqrt{\frac{\xi_{k1}\xi_{k2}}{\mathcal{D}_{k3}}},$$

$$O_{21}^k = \sqrt{\frac{\widetilde{\sigma}_1 (1-\delta_k)\xi_{k1}}{\mathcal{D}_{k1}}}, \quad O_{22}^k = \sqrt{\frac{\widetilde{\sigma}_2 (1-\delta_k)\xi_{k2}}{\mathcal{D}_{k2}}}, \quad O_{23}^k = \sqrt{\frac{\delta_k (1-\delta_k)}{\mathcal{D}_{k3}}},$$

$$O_{31}^k = \sqrt{\frac{\widetilde{\sigma}_2^k \xi_{k1}}{\mathcal{D}_{k1}}}, \quad O_{32}^k = \sqrt{\frac{\widetilde{\sigma}_1^k \xi_{k2}}{\mathcal{D}_{k2}}}, \quad O_{33}^k = \sqrt{\frac{\widetilde{\sigma}_1^k \widetilde{\sigma}_2^k \delta_k}{\mathcal{D}_{k3}}},$$

Numerical Results

Quarks

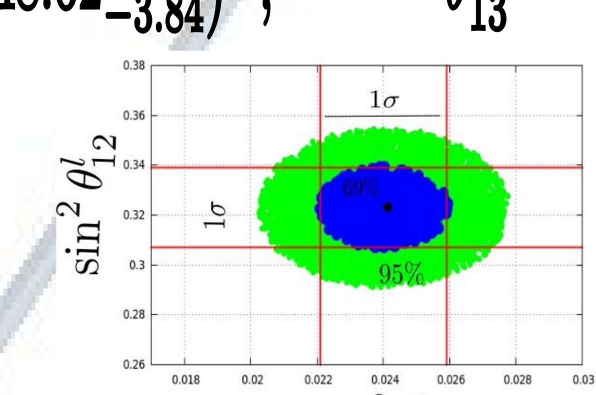
Parameter	Central value	χ^2	Values with restricted precision	χ^2
Fit using $\lvert V_{cd}\rvert$ and the 2013 values of the parameters with $\widetilde{m}_s^{th}$				
$\widetilde{\sigma}_u(M_Z)$	1.63792×10^{-6}		$(1.64 \pm 0.16) \times 10^{-6}$	
$\widetilde{\sigma}_c(M_Z)$	3.58675×10^{-3}		$(3.59 \pm 0.78) \times 10^{-3}$	
$\widetilde{\sigma}_d(M_Z)$	1.21487×10^{-3}		$(1.21 \pm 0.88) \times 10^{-3}$	
$\widetilde{\sigma}_s(M_Z)$	1.89366×10^{-2}		$(1.90 \pm 0.81) \times 10^{-3}$	
δ_u	9.98871×10^{-2}		$(1.00 \pm 0.5) \times 10^{-2}$	
δ_d	5.37158×10^{-2}		$(5.37 \pm 7.19 \times 10^{-2}$	
$\cos\phi_2$	7.82382×10^{-1}		$(7.82 \pm 9.35) \times 10^{-1}$	
		3.1×10^{-2}		4

Leptons

$$m_{\nu_2} = \left(6.24^{+3.74}_{-1.46}\right) \times 10^{-2}, \quad m_{\nu_1} = \left(6.18^{+3.77}_{-1.48}\right) \times 10^{-2},$$

$$m_{\nu_3} = \left(3.65^{+5.12}_{-3.45}\right) \times 10^{-2}, \quad \theta_{12}^{\ell th} = \left(34.64^{+1.89}_{-1.98}\right)^\circ,$$

$$\theta_{23}^{\ell th} = \left(48.62^{+3.91}_{-3.84}\right)^\circ, \quad \theta_{13}^{\ell th} = \left(8.93^{+0.65}_{-0.75}\right)^\circ.$$



Flavour changing neutral currents

FCNC processes	Theoretical BR	Experimental upper bound BR
$\tau \to 3\mu$	8.43×10^{-14}	2×10^{-7}
$\tau \to \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-7}
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}

IFIC — Institut de Física Corpuscular

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