Effective Spectral Functions for Scattering on Nuclear Targets

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Ref: A. Bodek, M. E. Christy, B. Coopersmith (to be published EPJC 2014) arXiv:1405.0583v1
The problem

Spectral Functions are functions that describe the momentum distributions and removal energy of nucleons bound in a nucleus.

Neutrino MC generators use spectral functions to model Fermi motion for Quasielastic (and inelastic) lepton scattering from nuclear targets.

The problem is that MC generators which use spectral functions do not describe the measured energy spectrum of final state leptons QE electron nucleus scattering.

This is because spectral functions do not account for final state interactions, which change the amplitude and therefore change the momentum of the final state leptons.
The superscaling formalism correctly describes the energy of the final state leptons for the longitudinal QE scattering cross sections.

Superscaling functions are taken from data on the longitudinal cross sections from QE electron scattering. So they fit the data by construction.

The final state lepton energy spectrum derived from the superscaling function includes both initial state and final state interaction effects.

As shown by the superscaling curve, final state interactions remove strength from the peak and increase the tails.
How do electron scattering experiments model scattering from nuclear targets (in order to do radiative corrections)

- Superscaling is used to describe the longitudinal part of the QE cross section

- Superscaling does not fully describe the transverse cross QE section because of the additional contribution from multi-nucleon effects such as meson exchange currents. These Multinucleon effects are added separately.

- Pauli blocking suppression is included at low $Q^2$

- Resonance and Inelastic free nucleon cross sections are smeared by Fermi motion (using a function which is motivated by superscaling), and the modified further by the EMC effect
Why can’t we use a similar approach in neutrino scattering?

- The superscaling formalism calculates cross sections for QE scattering. However, it has not been implemented in the form of a MC generator. Neutrino MC generators need to generate

- Although superscaling can describe the spectrum of the final state leptons in QE scattering it does not provide information on the composition of the hadronic final state (except for its total energy).

- In addition to QE scattering, neutrino MC generators need to also account for Fermi motion and nuclear effects in the resonance and DIS regions.

- Therefore, neutrino MC generators use spectral functions, which can be applied to any process. However, theoretically speaking superscaling cannot be derived from spectral functions.
Our solution

- STEP 1: Construct an **Effective Spectral Function** that closely yields the same final state lepton distribution in QE scattering as superscaling. We do this by using a functional form to parametrize the Benhar-Fantoni spectral function, and then varying the parameters to find the best parameters that reproduce superscaling for QE scattering at various values of $Q^2$.

- STEP 2: Account for the **enhancement in the transverse cross section** in QE scattering that originates from two body currents by modifying the magnetic form factors of the proton and neutron for nucleons which are bound in a nucleons (such as to reproduce the observed enhancement in electron scattering).

- Include Pauli suppression at low $Q^2$.

- Use the same Effective Spectral Function to account for Fermi motion in the resonance and DIS region.
Fig. 2. Nucleon momentum distributions in a $^{12}$C nucleus for several spectral functions. The blue line is the momentum distribution for the *effective spectral function* described in this paper (color online).
Modeling Removal energy for the 1p1h process (V=1)

Spectator nucleus is on shell: Energy momentum conservation yields the energy of the off shell interacting nucleon.

\[ E_n(1p1h) = M_A - \sqrt{V k^2 + (M_{A-1}^*)^2} \]
\[ \approx M_n - \Delta - \frac{V k^2}{2M_{A-1}^*} \]

**Fig. 1.** Top: Scattering from an off-shell bound neutron of momentum \( P_i = k \) in a nucleus of mass \( A \). The on-shell recoil \((A - 1)^* \) (spectator) nucleus has a momentum \( P_{A-1}^* = P_s = -k \). This process is referred to as the 1p1h process (one proton one hole). Bottom: The 1p1h process including final state interaction with the spectator nucleons.
Modeling Removal energy for the 2p2h process (2 nucleon correlations) V=1

A single spectator nucleon is on shell: Energy momentum conservation yields the energy of the off shell interacting nucleon.

\[ E_n(2p2h) = (M_p + M_n) - 2\Delta - \sqrt{V k^2 + M_p^2} \]

We use the same momentum distribution for the 1p1h and 2p2h process.

We fit for the fraction of 1p1 and 2p2 processes (we get about 20%).
Small deviations for $Q^2 > 1.5$ GeV$^2$
In order for the effective spectral function to work at for $Q^2 < 0.3 \text{ GeV}^2$, we need make a small modification to the removal energy at low $Q^2$ using the parameter $V(Q^2)$. 
We also apply Pauli suppression at low $Q^2$
Step 2: Add Transverse Enhancement.


Top: Ratio of integrated transverse QE cross section in electron scattering to the sum of the transverse cross section for free nucleons.

\[ R_T = 1 + A Q^2 e^{-Q^2/B} \]

\[ G_{Mp}^{nuclear}(Q^2) = G_{Mp}(Q^2) \times \sqrt{1 + A Q^2 e^{-Q^2/B}} \]

\[ G_{Mn}^{nuclear}(Q^2) = G_{Mn}(Q^2) \times \sqrt{1 + A Q^2 e^{-Q^2/B}}. \]

Bottom: Prediction for the \( Q^2 \) dependence ratio of the neutrino cross sections to free nucleon cross sections using the transverse enhancement model.
Summary

By construction. Neutrino MC generators which use our effective effective spectral function, with free nucleon electric form factors, and modified magnetic form factors for bound nucleons (Transverse Enhancement for QE scattering) with Pauli suppression are automatically in agreement with electron scattering (vector) cross sections (for both $d\sigma/dQ^2$ and $d\sigma/d\nu$).

It only took a few days to implement in the NEUT and GENIE neutrino MC generators (since generators are already constructed to use spectral functions).

Electron scattering cross sections only provide information on the vector QE cross sections. However, we note that if we include TE and use the free nucleon axial form factor, we get agreement with the QE $Q^2$ distributions as measured in the MINERvA neutrino experiment (thus any enhancement in the axial cross sections is small).

In our paper we provide parametrizations for the effective spectral functions for all nuclei starting with deuterium (D) up to lead (Pb).

Additional details can be found in A. Bodek, M. E. Christy, B. Coopersmith (to be published EPJC 2014) arXiv:1405.0583v1
Additional Slides
We use the effective spectral function to get a better Fermi smearing function for inelastic scattering in the resonance region and DIS continuum.
\[
E_n(1p1h) = M_A - \sqrt{Vk^2 + (M_{A-1}^*)^2}
\]
\[
\approx M_n - \Delta - \frac{Vk^2}{2M_{A-1}^*}
\]

\[
E_n(2p2h) = (M_p + M_n) - 2\Delta - \sqrt{Vk^2 + M_p^2}
\]

**Fig. 6.** Comparison of energy for on-shell and off-shell bound neutrons in $^{12}$C. The on-shell energy is $E_n = \sqrt{k^2 + M_n^2}$. The off-shell energy is shown for both the 1p1h ($E_n = M_n - \Delta - \frac{Vk^2}{2M_{A-1}^*}$) and 2p2h process ($E_n = (M_p + M_n) - 2\Delta - \sqrt{Vk^2 + M_p^2}$), where $(M_p + M_n)$ and $\Delta$ is the effective binding energy ($V \approx 1$ for $Q^2 > 0.3$ GeV$^2$, as shown in Fig. 7) (color online).