A scenario for critical scalar field collapse in AdS_3

ICHEP2014, Valencia, July 2-9
Parallel Session Formal Theory developments

Alessandro Fabbri
Centro Fermi / IFIC / Univ. Bologna

Based on: G. Clément and A. Fabbri
arXiv:1404.0589 [gr-qc]

July 5, 2014

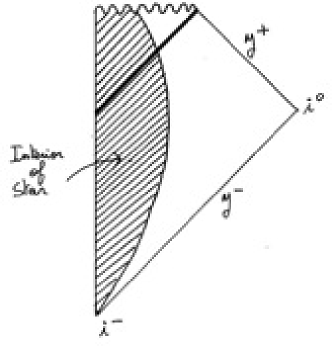
Black Holes in GR

- Schwarzschild (1916) discovered the first black hole solution

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$

- By Birkhoff (1923) theorem it is the only static and spherically symmetric vacuum solution of GR
- Its properties, i.e. the existence of a trapped region $(r < 2M)$ bounded by a horizon $(r = 2M)$ and a singularity $(r = 0)$, are generic properties of BHs even outside spherical symmetric (singularity theorems Hawking and Ellis (1973))
- By the cosmic censorship conjecture, singularities are always hidden inside BHs (Penrose 1969)

Black holes form from gravitational collapse



Gravitational collapse and black hole threshold

There are two types of gravitational collapse:

1. time evolution proceeds towards a static (stationary) solution uniquely characterized by its conserved charges (no-hair theorems Israel (1968), Carter (1971), Hawking (1972), Robinson (1975)); near the threshold we have a universal static (stationary) BH solution with a finite (universal) mass
2. by fine-tuning initial data one can make arbitrary small BHs with universal power-law scaling of the mass and (continuous or discrete) scale-invariant threshold (critical) solution

There is a striking similarity between the latter and critical phase transitions in statistical mechanics (Gundlach '02)

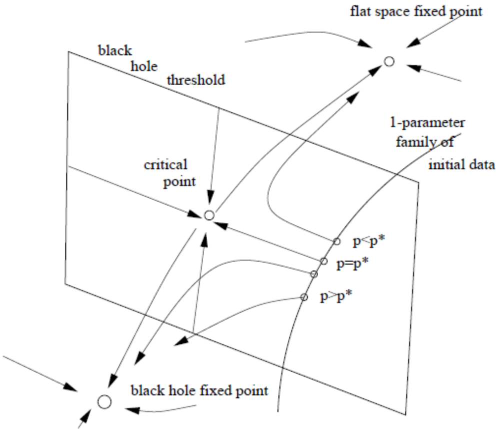
Critical phenomena in gravitational collapse

Choptuik '93

- He studied the collapse of a spherically symmetric massless and minimally coupled scalar field coupled to GR
- He considered 1-parameter (p) families of (regular) initial data: strong data (large p) in which a BH forms, weak data (small p) which disperse and a critical p^* found empirically
- Around the critical point there is a power-law scaling

$$M \simeq C(p - p^*)^\gamma$$

  with a universal exponent $\gamma \simeq 0.374$
- for a finite time in a finite region of space, near-critical $(p \sim p^*)$ data approach the same universal solution


flat space fixed point
black
hole
threshold
1-parameter
family of
initial data
critical
point
p<p*
p=p*
p>p*
black hole fixed point

Critical solution

- Choptuik found that the critical solution is discretely self-similar (DSS), i.e. it is the same provided we rescale space and time according to

$$\phi_*(r,t) = \phi_*(e^{\Delta}r, e^{\Delta}t),$$

  where $\Delta \sim 3.44$ (scale-echoing)
- For perfect fluids the critical solution is continuously self-similar (CSS), i.e. it is invariant under (infinitesimal and finite) rescaling of space and time
- It has a strong singularity
- Analytical approaches consider (CSS or DSS) solutions regular at the center and at the past light cone of the (naked) singularity
- The critical solution is characterized by having only one growing perturbation mode

Gravity in 2+1 dimensions

- Matter affects space-time only globally and not locally (conical singularities) Deser, Jackiw and 't Hooft '84
- A black hole solution exists provided we consider a (negative) cosmological constant ($\Lambda = -\frac{1}{l^2}$) Bañados, Teitelboim and Zanelli '92

$$ds^2 = -(-M + \frac{r^2}{l^2})dt^2 + \frac{dr^2}{(-M + \frac{r^2}{l^2})} + r^2 d\theta^2,$$

$M > 0$ is a black hole, $-1 < M < 0$ a naked (conical) singularity and $M = -1$ (regular) AdS space

Birmingham and Sen '00 have analysed BTZ BH formation in the collision of point-particles and Peleg and Steif '92 gravitational collapse of a dust ring, but not critical solution is involved

Critical scalar field collapse in AdS_3

Choptuik and Pretorius '00

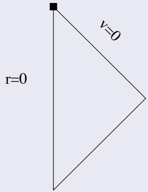
- They considered the circularly symmetric collapse of a massless and minimally coupled scalar field
- They considered families of initial data with length scale $r_0 \sim 0.32l$ (i.e. the effects of the cosmological constant are suppressed by a factor 0.1) and tuned to the threshold of black hole formation on the initial implosion
- They find CSS critical behaviour and power-law scaling (of the maximum value of the Ricci scalar and of the mass scaling) $(p - p^*)^{2\gamma}$, with $\gamma \sim 1.20 \pm 0.05$
- Independently, Husain and Olivier '01 found $\gamma \sim 0.81$

Analytical approach to critical scalar field collapse in AdS_3

Garfinkle '01

- He found a 1-parameter (n) family of CSS solutions to the $\Lambda = 0$ equations regular at the center, to reproduce the critical solution close to the singularity
- In appropriate double-null coordinates the metric reads

$$ds^2 = -A(v^n + (-u)^n)^{4-\frac{2}{n}} du dv - \frac{1}{4}(v^{2n} + (-u)^{2n})^2 d\theta^2$$



- Regularity of the solutions at the past light-cone of the singularity requires that n is a positive integer
- The solution with $n = 4$ agrees well with the numerical data

Perturbing the Garfinkle solution

- Garfinkle and Gundlach '02 considered the linear perturbation analysis of the Garfinkle solutions
- Being the relevant time parameter $\tau = -ln(-u)^{2n}$ ($\tau = +\infty$ corresponds to the singularity), the perturbations are expanded in modes $e^{k\tau} = (-u)^{-2nk}$ which grow when $Re(k) > 0$
- k is related to scaling in near-critical collapse: a quantity Q with dimension (length)^s will scale as $|p - p_*|^{\frac{s}{k}}$, i.e. $\gamma = \frac{1}{k}$
- They imposed regularity condition at the center ($g_{\theta\theta} = 0$) and smoothness at $v = 0$: they found that the $n = 4$ solution has 3 unstable modes, while $n = 2$ has only one unstable mode with $k = \frac{3}{4}$, giving $\gamma = \frac{4}{3}$
- The analysis was extended to $O(\Lambda)$ by Cavaglià, Clément and Fabbri '04

An alternative scenario

- A point probably overlooked in Choptuik and Pretorius' analysis is that introduction of a point particle (a conical singularity) left unchanged (up to a phase shift in proper time related to the mass of the particle) the critical solution
- This suggests that the critical solution, instead of having a regular center, might have no center at all (as $M = 0$ BTZ vacuum)
- Moreover, in the Garfinkle solutions $v = 0$ is an apparent horizon: the critical solution must be something else outside the past light cone and it is not clear what are the correct conditions to be imposed on the perturbations along this surface

Separable solutions

- Baier, Stricker, Taanila '14 derived, from a self-similar ansatz, a class of solutions conformal to the 3d Minkowski cylinder (i.e. with no center)
- Such solutions belong to a class of separable solutions (Clément and Fabbri'02)

$$ds^2 = F^2(T)[-dT^2 + dR^2 + G^2(R)d\theta^2], \ \phi = \phi(T)$$

  and are characterized by two parameters α and b.
- When b (the scalar field strength) vanishes we recover the BTZ solutions with $\alpha = -M$
- BST incorrectly pointed out that $b = 0$ identified the critical solution: this is not possible since the critical solution has a strong singularity (Clément and Fabbri '14)

Dynamical solutions ($b \neq 0$)

- Unlike Garfinkle' solutions we have exact solutions for $\Lambda \neq 0$. The $\alpha < 0$ solutions are black-hole like, while those for $\alpha > 0$ have a center (regular if $\alpha = 1$): $\alpha = 0$ is a candidate critical solution
- For $\Lambda = 0$ they take the form

$$ds^2 = b^2 \sinh^2(T)[-dT^2 + dR^2 + (e^R - \frac{\alpha}{4} e^{-R})^2 d\theta^2], \; \phi = \sqrt{2} \ln \tanh(-\frac{T}{2})$$

  where we see that the center ($\alpha > 0$) is sent to $R \to -\infty$ when $\alpha = 0$.
- They have a singularity at $T = 0$, and when $\alpha = 0$ (unlike the Garfinkle solutions) the apparent horizon is at an infinite geodesic distance
- The subcritical ($\alpha > 0$) solutions near the singularity are in qualitative agreement with numerical data, and (for $\alpha = 1$) reproduce $n = 1$ Garfinkle solution

Open questions

- The linear perturbation analysis indicates there is only one unstable growing mode with $k = 2$, giving $\gamma = \frac{1}{2}$ which disagrees with the value(s) from the numerical analysis
- The subcritical $(\alpha = 1)$ solution agrees with $n = 1$ Garfinkle solution, but the numerical data are best fit for $n = 4$.. is there a family of solutions which reproduces, near the singularity, the $n = 4$ solution?
- are these new dynamical black-hole type solutions relevant for the discussed instability of AdS_3 (Bizon and Jalmuzna '13)?