Searching for abelian symmetries in bSM models

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Outline

1. Introduction
2. Finding abelian symmetries
   - General procedure
   - An example: 3HDM with quarks
   - Which symmetries are possible?
3. Non-abelian symmetries
4. Conclusions

Building bSM models

The SM comes with a bunch of free parameters, which it can accommodate but does not predict: quark masses and mixing, neutrino mass matrix, the Higgs boson mass, etc.

Neither does it answer conceptual questions: three fermion generations, smallness of *CP*-violation, origin of the baryon asymmetry, tiny neutrino masses, the dark matter candidates, stabilization of the Higgs boson mass, etc.

Trying to naturally explain (some of) these puzzles leads many physicists to building bSM models based on various new symmetries. These are extra symmetries ("accidental" symmetries) beyond the gauge symmetry group of the SM.

Building bSM models

Main activity, especially within flavor and neutrino physics:
first postulate some symmetry feature, and then add extra fields at will to encode it and generate the desired spontaneous symmetry breaking.

Alternative attitude:
first focus on a certain class of bSM models, and then explore symmetries which are possible with this field content, and what are their phenomenological consequences.

No guarantee that it will eventually succeed in resolving it!

But it can provide extra help and maybe guidance to massive phenomenological model-building activitity.

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Finding symmetries

Suppose you have a lagrangian with certain field content: SM fields + new fields (several Higgs doublets, flavons, heavy right-handed neutrinos, etc.). Within the "alternative attitude", the two main questions are:

- Find out which symmetry groups are possible with this field content;
- Find out how these symmetries can be implemented.

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- Find out how these symmetries can be implemented.

The focus is on exhaustiveness:
once the entire list of possibilities is known, one can explore all physically distinct phenomenological situations,
and guarantee that nothing else is possible with these fields.

Finding symmetries

Usually, one wants to implement discrete symmetry groups G.

Two main steps:

- finding all possible abelian subgroups $A \subset G$;
- constructing all possible non-abelian G knowing the list of A.

Note that usually "abelian groups" = "rephasing symmetry groups"

because we usually search for subgroups of $U(N)$ or $SU(N)$, and every abelian subgroup of $SU(N)$ can be mapped by a basis change to a rephasing subgroup. However, if one looks beyond $SU(N)$, then this equivalence breaks.

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Finding symmetries

I will focus on classifying rephasing symmetries in a generic bSM model.

There exists a simple yet powerful method for determining the rephasing symmetry group of any interaction lagrangian.

It uses a rather simple property of an integer-valued matrices: the Smith normal form.

I will illustrate it with rephasing symmetries of the multi-Higgs-doublet models, but keep in mind that this is a universal method.

First mentioned in *[Petersen, Ratz, Schieren, 2009]* (remnant discrete symmetries in GUT models). Developed much further in: *[IPI, Keus, Vdovin, 2012]* for NHDM scalar sector, and *[IPI, Nishi, 2013]* for NHDM with quarks.

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Rephasing symmetries in NHDM

With N doublets, the total rephasing group if $[U(1)]^N$, with each $U(1)$ generated by individual doublet rephasing $\phi_j \mapsto e^{i\alpha_j}\phi_j$.

Suppose the potential V

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,,$$

has k terms "rephasing-sensitive" terms. Pick up the first term in the potential and see how it changes upon rephasing, for example

$$(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_3) \quad \mapsto \quad e^{i(-2\alpha_1+\alpha_2+\alpha_3)}(\phi_1^\dagger \phi_2)(\phi_1^\dagger \phi_3)\,.$$

Write this phase as $\sum_{j=1}^{N} d_{1j}\alpha_j$, where $d_{1j} = (-2, 1, 1, 0, \ldots, 0)$. Then angles α_j satisfying

$$d_{1j}\alpha_j = 2\pi n_1$$

with any integer n_1 leave this term invariant.

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Repeat it for every other term in V; the i-th term produces d_{ij}.

Rephasing symmetries of the potential are generated by solutions α_j of the following system of equations:

$$d_{ij}\alpha_j = 2\pi n_i \quad \text{with} \quad n_i \in \mathbb{N}.$$

The task is to solve this system for α_j and deduce the symmetry group.

A 4HDM example:

$$V = V_{\text{reph.inv.}} + \lambda_1(\phi_4^\dagger\phi_1)(\phi_3^\dagger\phi_1) + \lambda_2(\phi_4^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_3(\phi_4^\dagger\phi_3)(\phi_2^\dagger\phi_3) + \text{h.c.}$$

gives

$$d_{ij} = \begin{pmatrix} 2 & 0 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ 0 & -1 & 2 & -1 \end{pmatrix}.$$

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The matrix d_{ij} always has integer entries. By certain elementary steps

- permutation of rows or columns,
- sign flips of rows or columns,
- adding a column/row to another column/row

it can be diagonalized:

$$d_{ij} \quad \mapsto \quad \begin{pmatrix} d_1 & & & & & & \\ & d_2 & & & & & \\ & & \ddots & & & & \\ & & & d_r & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \quad \cdots \end{pmatrix}, \quad r = \operatorname{rank} d_{ij}$$

with all d_i positive and such that d_i divides d_{i+1}.

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This is known as the Smith Normal Form (SNF) of d_{ij}.
It exists and is unique for any integer-valued matrix.

Rephasing symmetries in NHDM

The key observation: elementary steps do not change the set of solutions.

The equations become decoupled; each $d_i\tilde{\alpha}_i = 2\pi\tilde{n}_i$ has solutions $\tilde{\alpha}_i = 2\pi\tilde{n}_i/d_i$, which generate the group $\mathbb{Z}_{d_i}$.

The rephasing group of the potential is therefore

$$A = \mathbb{Z}_{d_1} \times \mathbb{Z}_{d_2} \times \cdots \times \mathbb{Z}_{d_r} \times [U(1)]^{N-r} .$$

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gives

$$d_{ij} \mapsto \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 7 & 0 \end{pmatrix} , \quad A = \mathbb{Z}_7 \times U(1) .$$

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3HDM with quarks

Consider the following 3HDM with a rephasing-invariant Higgs potential and with the following Yukawa interactions with quarks:

$$-\mathcal{L}_Y = \Gamma^{(j_\phi)}_{j_L j_d} \bar{Q}_{L j_L} \phi_{j_\phi} d_{R j_d} + \Delta^{(j_\phi)}_{j_L j_u} \bar{Q}_{L j_L} \tilde{\phi}_{j_\phi} u_{R j_u} + h.c.$$

We order the 12 fields as $(\phi_{j_\phi};\ Q_{L j_L};\ d_{R j_d};\ u_{R j_u})$, where $j_\phi,\ j_L,\ j_d,\ j_u = 1, 2, 3$, and we choose the following textures:

$$\Gamma^{(1)} = \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & 0 & 0 \end{pmatrix}, \quad \Gamma^{(3)} = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix},$$

$$\Delta^{(1)} = \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix}, \quad \Delta^{(2)} = \begin{pmatrix} 0 & * & 0 \\ 0 & 0 & 0 \\ * & 0 & 0 \end{pmatrix}, \quad \Delta^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix}.$$

There are 12 Yukawa terms; 6 with d_R's and 6 with u_R's.

3HDM with quarks

Each Yukawa term produces a row d_{ij} with entries ± 1 or 0.

For example, the term with $\Gamma^{(1)}_{13}$ is $\bar{Q}_{L1}\phi_1 d_{R3}$, and its row d_{ij} is

$$(\overbrace{1,\,0,\,0}^{\phi}\,|\,\overbrace{-1,\,0,\,0}^{Q_L}\,|\,\overbrace{0,\,0,\,1}^{d_R}\,|\,\overbrace{0,\,0,\,0}^{u_R}),$$

and the term with $\Delta^{(2)}_{31}$ is $\bar{Q}_{L3}\tilde{\phi}_2 u_{R1}$, and its row d_{ij} is

$$(0\;-1,\,0\,|\,0,\,0,\,-1\,|\,0,\,0,\,0\,|\,1,\,0,\,0\,).$$

3HDM with quarks

The entire matrix d_{ij} is a 12×12 matrix:

$$d_{ij} = \left(\begin{array}{ccc|ccc|ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right).$$

3HDM with quarks

Its Smith normal form is

$$\mathrm{diag}(1,\,1,\,1,\,1,\,1,\,1,\,1,\,1,\,1,\,5,\,0,\,0)\,.$$

The symmetry group is

$$G = \mathbb{Z}_5 \times U(1)_Y \times U(1)_B\,.$$

The $\mathbb{Z}_5$-charges of the fields are:

$$q_{\mathbb{Z}_5} = (0,\,2,\,4\,|\,2,\,1,\,0\,|\,3,\,1,\,2\,|\,2,\,4,\,0)\,.$$

Beyond case-by-case checks

In principle, the SNF technique gives the rephasing symmetry group for any interaction lagrangian. Suppose we want to know all possible abelian symmetry groups for a given field content.

Naively, we would need to write down all possible interaction terms, then consider all possible combinations of these terms, for each combination construct d_{ij}, and then find its SNF. Is it feasible?

Example: scalar sector of 3HDM: 3 doublets, 36 rephasing sensitive terms, 2^{36} possible combinations of these interactions terms! But do we really need to check all these zillions of combinations?

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No!
The matrix d_{ij} itself offers the answer.

Beyond case-by-case checks

The point is that d_{ij} typically has rows of very simple structure.

Scalar sector in NHDM:

$$(2, -2, 0, 0, \dots), \quad (2, -1, -1, 0, \dots), \quad (1, 1, -1, -1, 0, \dots),$$

up to permutations.

Quarks Yukawa interactions with Higgs doublets:

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up to permutations.

Usually, one can find all possible values of $|\det d_{ij}|$.

Beyond case-by-case checks

But $|\det d_{ij}|$ remains unchanged after we bring it to the SNF: $|\det d_{ij}| = \prod_j d_j$. The procedure is then the following:

- get rid of all "automatic" $U(1)$'s;
- using the structure of d_{ij}, find all values of $|\det d_{ij}| = |A|$;
- if the resulting $|A|$ is a number whose prime decomposition involves only first powers of primes, then the group A is uniquely determinted;
- if the prime decomposition of $|A|$ involves higher powers of primes, then ambiguity exists; only in this case one needs to explicitly find SNF.

This analysis can be often done manually, without computer-algebra assistance!

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Beyond case-by-case checks

For example,

- if $|A| = 5$, then the group A must be $\mathbb{Z}_5$;
- if $|A| = 30$, then the group A must be $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5$;
- if $|A| = 4$, then the group A can be either $\mathbb{Z}_2 \times \mathbb{Z}_2$ or $\mathbb{Z}_4$. One needs to check whether SNF is $(\ldots, 1, 2, 2)$ or $(\ldots, 1, 1, 4)$.

Additionally, it is usually easy to obtain the general upper bound on $|A|$.

- scalar sector of NHDM: $|A| \leq 2^{N-1}$ for any N;
- NHDM with quarks: $|A| \leq (N+1)^2/3$ for any N.

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What seemed to be a hugely complicated computer-assisted calculation, turns into an easy arithmetical exercise!

Reconstructing non-abelian symmetry groups

Suppose we have a bSM model with a given field content. We can now find all abelian symmetry groups A it can have. The big question is how can one reconstruct all possible non-abelian symmetry groups?

Unfortunately, no universal recipe exists (yet). But in certain cases, pure group theory can lead to very strong results.

The basic idea:

- G can have abelian subgroups only from the list of A we have found,
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3HDM example

Example: non-abelian discrete symmetries in the scalar sector of 3HDM *[IPI, Vdovin, 2013]*:

- From the list of abelian groups, we get only two prime factors: 2 and 3. Therefore, $|G| = 2^a 3^b$.
- By applying Burnside's theorem, we know that G is a solvable group.
- Solvable groups have a normal abelian subgroup. Even more, we proved that in 3HDM, G always contains a normal abelian self-centralizing subgroup A.
- Then, the structure of the group G is immediately reconstructed: $G \subset A \rtimes Aut(A)$. What remains is to check explicitly very few cases.

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Results:

$$\mathbb{Z}_2\,,\quad \mathbb{Z}_3\,,\quad \mathbb{Z}_4\,,\quad \mathbb{Z}_2\times\mathbb{Z}_2\,,\quad S_3\,,\quad D_4\,,\quad A_4\,,\quad S_4\,,\quad \Delta(54)\,,\quad \Sigma(36\phi)\,.$$

We also investigated generalized *CP*-symmetries and an intricate interplay between G and explicit/spontaneous/geometric *CP*-violation in the Higgs sector.

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This list is complete:

imposing any other finite Higgs-family symmetry group on the 3HDM scalar sector will lead to a potential symmetric under a continuous group.

We also investigated generalized *CP*-symmetries and an intricate interplay between G and explicit/spontaneous/geometric *CP*-violation in the Higgs sector.

3HDM example

Results:

$$\mathbb{Z}_2\,,\quad \mathbb{Z}_3\,,\quad \mathbb{Z}_4\,,\quad \mathbb{Z}_2\times\mathbb{Z}_2\,,\quad S_3\,,\quad D_4\,,\quad A_4\,,\quad S_4\,,\quad \Delta(54)\,,\quad \Sigma(36\phi)\,.$$

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Conclusions

- Searching for mathematical tools capable of producing rigorous results on symmetries in bSM models is a very viable and promising direction of research. It complements the straightforward model-building, providing new insights and guidance.
- The Smith normal form technique is a powerful method to attack abelian symmetries in any model, with potentially important consequences.
- The problem of classification of allowed discrete symmetry groups in NHDM, either in the scalar sector only or in the entire lagrangian, is gradually advancing. However many more issues need to be solved!