B Meson Decay Constants from QCD Sum Rules

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arXiv:1310.0941 (hep-ph): MJB, J. Bordes, C.A. Dominguez, J. Peñarrocha, K. Schilcher

IFIC Seminar - 24 October 2013

Outline

1. QCD Sum Rules
   - Motivation
   - Two Point Correlator
   - Quark-Hadron Duality
2. B Meson Decay Constants
   - Theoretical Background
   - Perturbative Correlator
   - Non-perturbative Correlator
   - Results

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QCD Lagrangian

$$\mathscr{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \overline{\psi}_q (i\not{D} - m_q)\psi_q$$

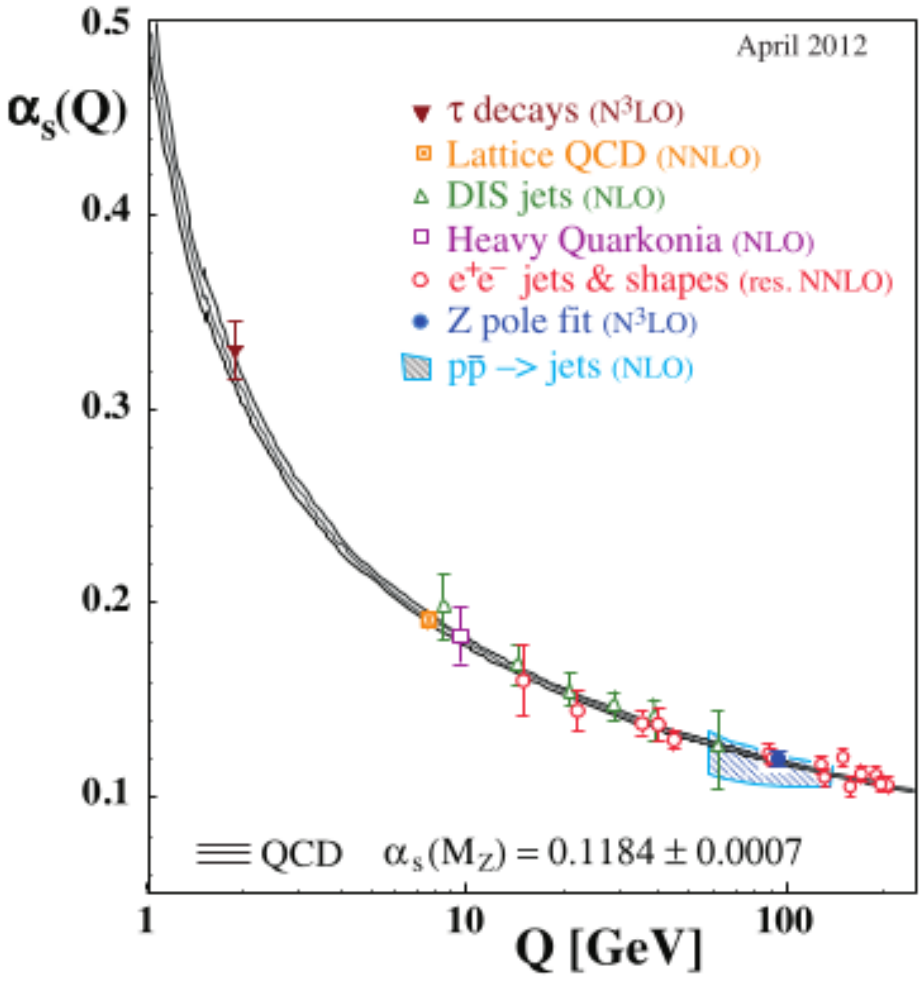
$$\not{D} = \gamma^\mu \partial_\mu - \sqrt{4\pi\alpha_s}\gamma^\mu t^a C^a_\mu$$

QCD Lagrangian

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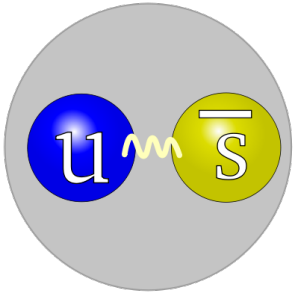
$$\not{D} = \gamma^\mu \partial_\mu - \sqrt{4\pi\alpha_s}\gamma^\mu t^a \mathcal{C}^a_\mu$$

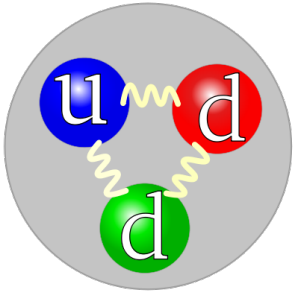
Running of α_s



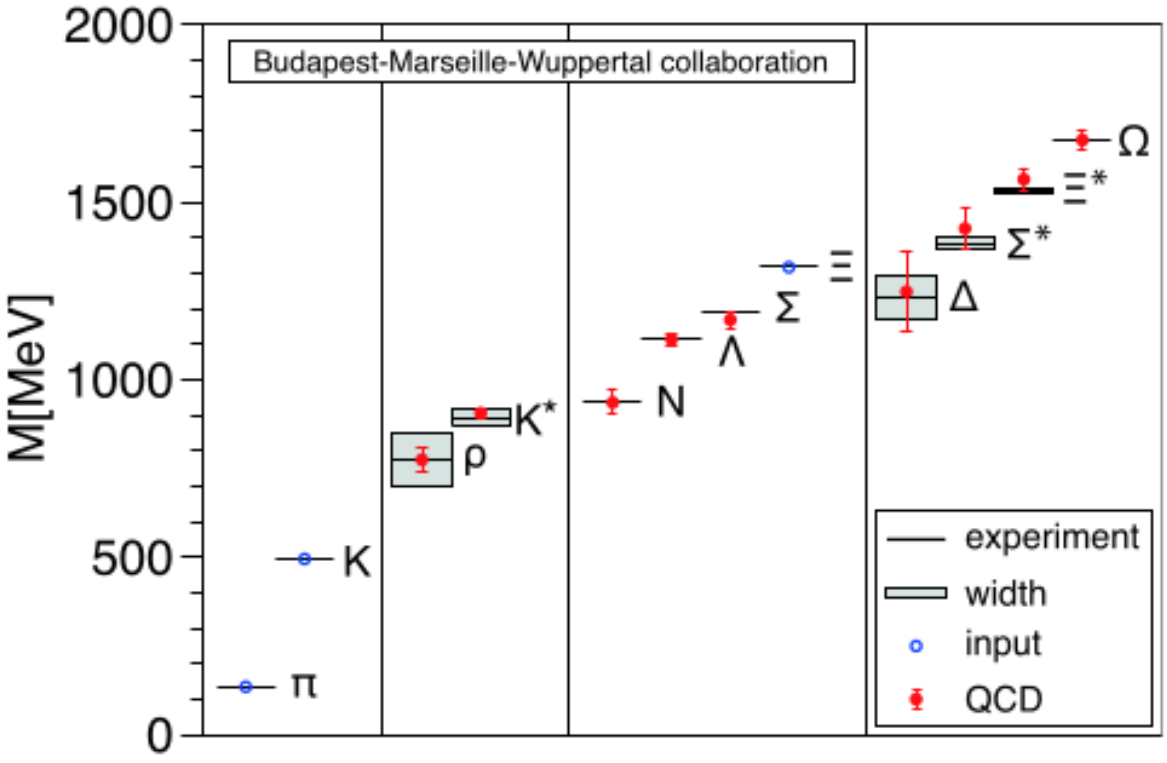
QCD vs. Experimental Data

QCD





Hadronic Parameters



SVZ Method

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QCD AND RESONANCE PHYSICS. THEORETICAL FOUNDATIONS

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QCD AND RESONANCE PHYSICS. APPLICATIONS

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QCD AND RESONANCE PHYSICS: THE ρ-ω MIXING

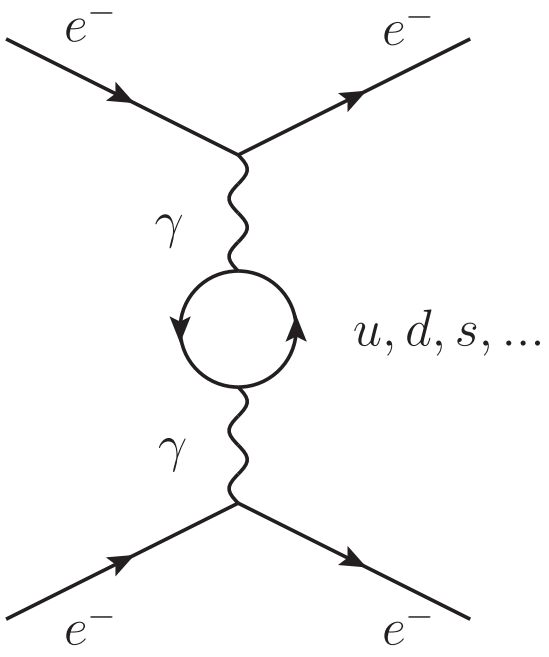
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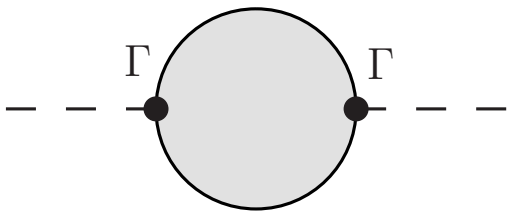
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Electron Scattering



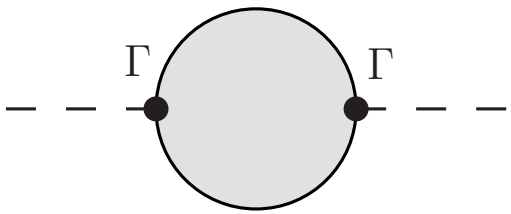
Two Point Correlator



$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle \Omega | T\{j_\mu(x) j_\nu(0)^\dagger\} | \Omega \rangle$$

$$j_\mu = \bar{\psi} \gamma_\mu \psi$$

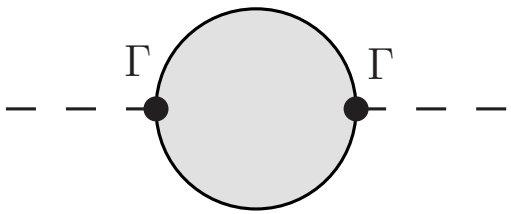
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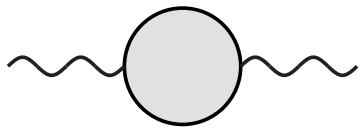
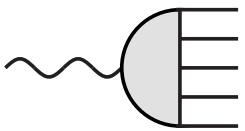
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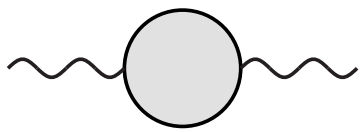
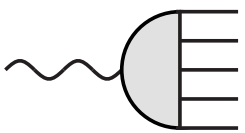
Optical Theorem

$$2\mathrm{Im}$$  $$= \sum_f \int d\Pi_f \left| \right.$$  $$f \left. \right|^2$$

$$\mathrm{Im}[\Pi^{\mathrm{v}}(s)] = \frac{9}{64\pi^2\alpha^2} s\sigma(e^+e^- \to \mathrm{hadrons})$$

$$s = q^2$$

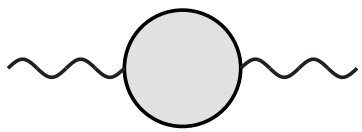
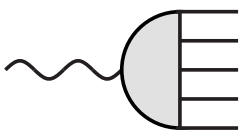
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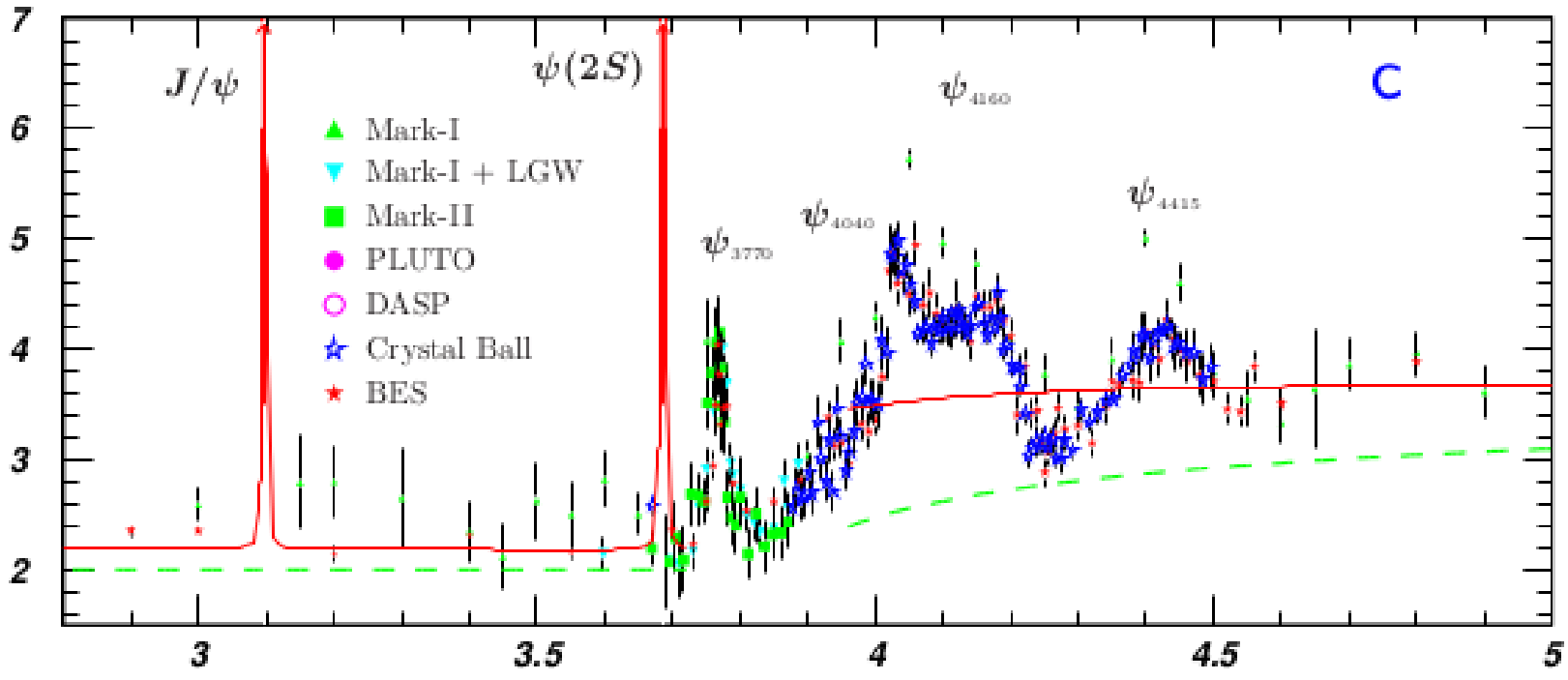
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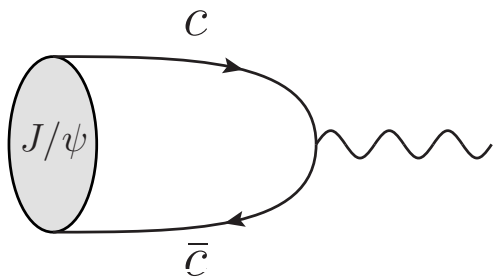
Charmonium Data

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



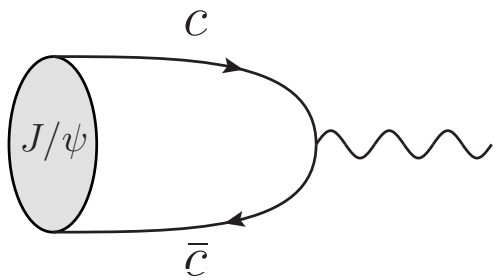
$\sqrt{s}$ GeV

J/ψ Decay Constant



$$\langle \Omega | (ci\gamma_\mu \bar{c})(0) | J/\psi \rangle = f_{J/\psi} m_{J/\psi} \epsilon_\mu$$

J/ψ Decay Constant



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Quark-Hadron Duality

$$\Pi^{\mathrm{QCD}}(s) = \Pi^{\mathrm{Had}}(s)$$

Wilson's Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_n C_{12}^n(x)\mathcal{O}_n(0)$$

$$i\int d^4x e^{iqx} T\{j_\mu(x)j_\nu(0)^\dagger\} = \sum_n C^n(q)\mathcal{O}_n(0)$$

$$\Pi(q^2) = \sum_n C^n(q)\langle\Omega|\mathcal{O}_n(0)|\Omega\rangle$$

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$$\Pi(q^2) = \sum_n \mathcal{C}^n(q)\langle\Omega|\mathcal{O}_n(0)|\Omega\rangle$$

Operators with $d \leq 6$

$$\Pi(q^2) = \sum_n C^n(q) \langle \Omega | \mathcal{O}_n(0) | \Omega \rangle$$

$$\mathcal{O}_0 = I \qquad d = 0$$

$$\mathcal{O}_m = m\bar{q}q \qquad d = 4$$
$$\mathcal{O}_G = G^a_{\mu\nu} G^a_{\mu\nu} \qquad d = 4$$

$$\mathcal{O}_\Gamma = \bar{q}\Gamma_1 q \bar{q}\Gamma_2 q \qquad d = 6$$
$$\mathcal{O}_\sigma = \tilde{m}\bar{q}\sigma_{\mu\nu}\frac{\lambda^a}{2} q G^a_{\mu\nu} \qquad d = 6$$
$$\mathcal{O}_f = f_{abc} G^a_{\mu\nu} G^b_{\nu\gamma} G^c_{\gamma\mu} \qquad d = 6$$

Operators with $d \leq 6$

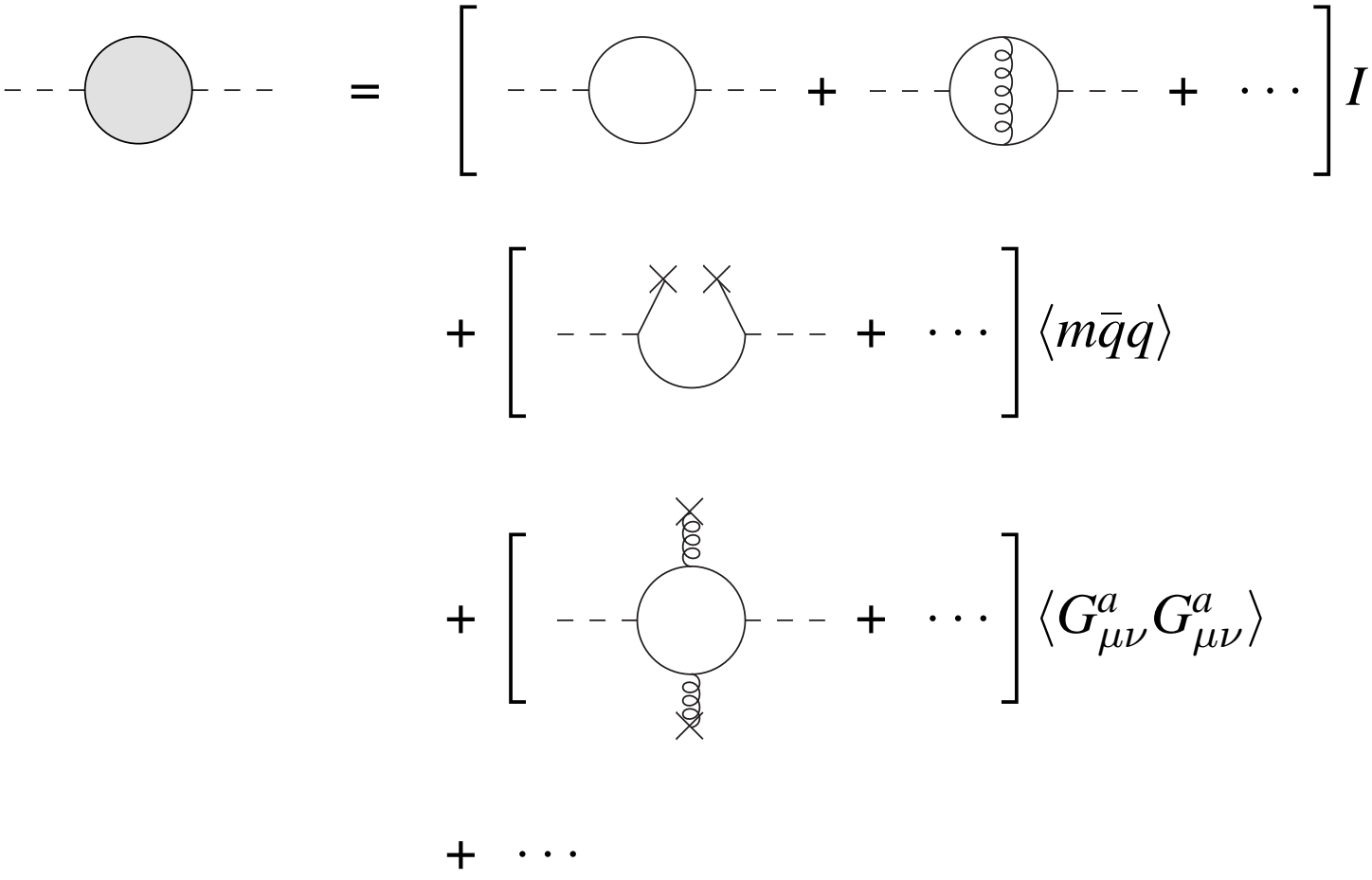
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Calculating $C^n(q)$

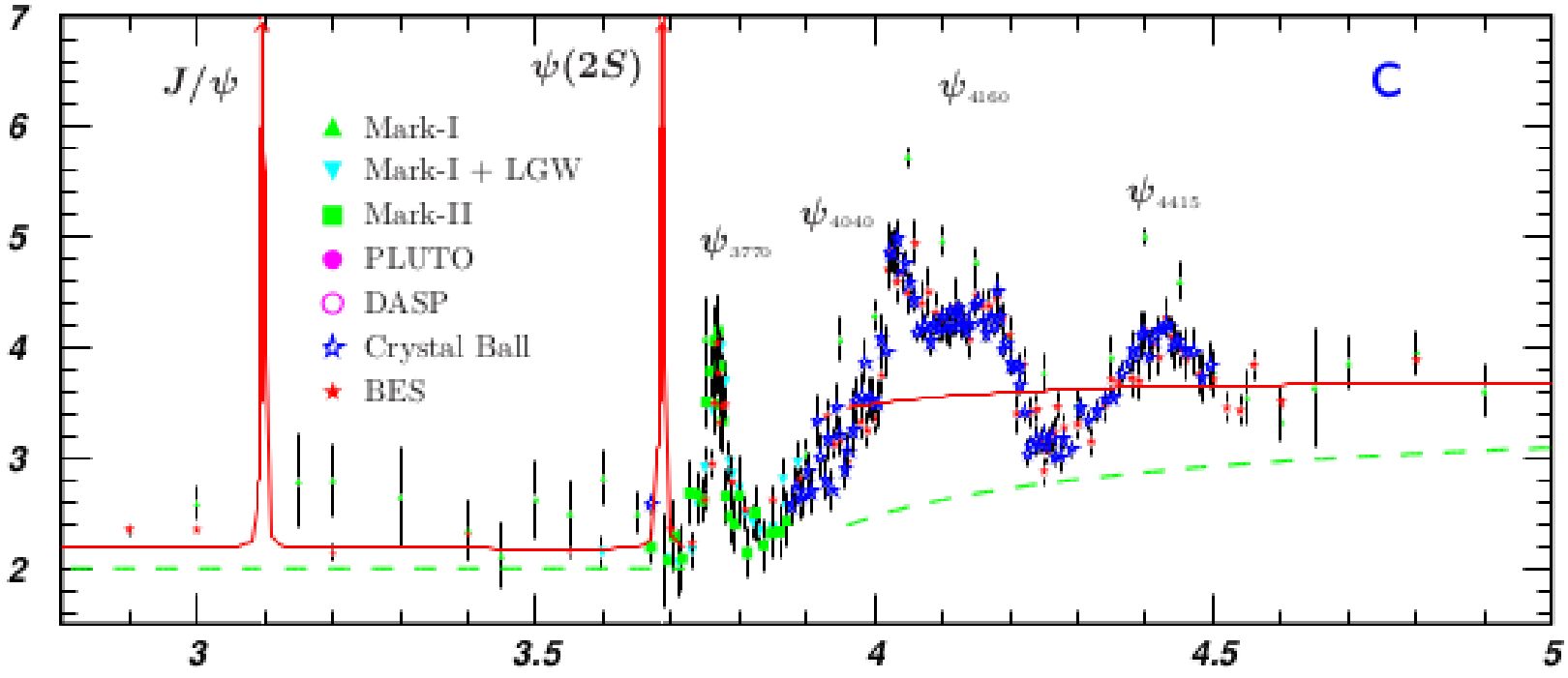


Quark-Hadron Duality

$$\Pi^{\mathrm{QCD}}(s) = \Pi^{\mathrm{Had}}(s)$$

Phenomenological Side

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



$$\rho(s) = \frac{1}{\pi}\text{Im}\Pi(s)$$
$$\approx f_{J/\psi}\delta(s - m^2_{J/\psi}) + f_{\psi(2S)}\delta(s - m^2_{\psi(2S)}) + \rho^{\text{cont}}(s)\theta(s - s_{\text{cont}})$$

Quark-Hadron Duality

$$\Pi^{\mathrm{QCD}}(s) = \Pi^{\mathrm{Had}}(s)$$

$$\| \qquad \cup$$

$$i\,\mathrm{Im}\Pi^{\mathrm{Had}}(s)$$

$$\| \qquad \|$$

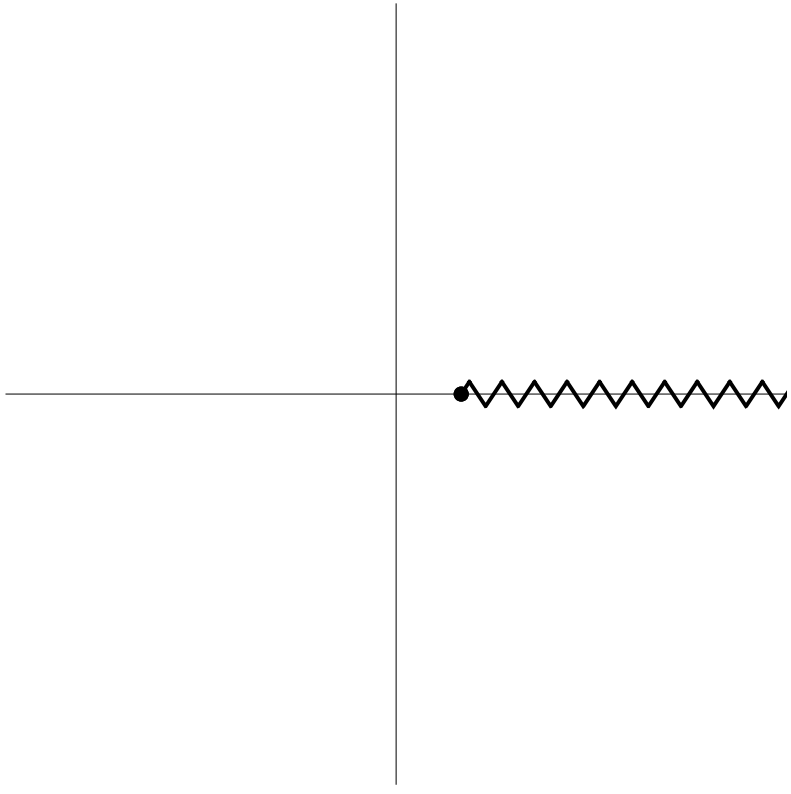
$$\sum_n C^n(s)\langle\Omega|\mathcal{O}_n(0)|\Omega\rangle \qquad i\pi \sum_H f_H\,\delta(s - m_H^2) + \mathrm{cont}$$

Valid at high s where α_s is small and low d operators dominate

Valid at low s where there is good experimental data

Complex Structure of $\Pi(s)$

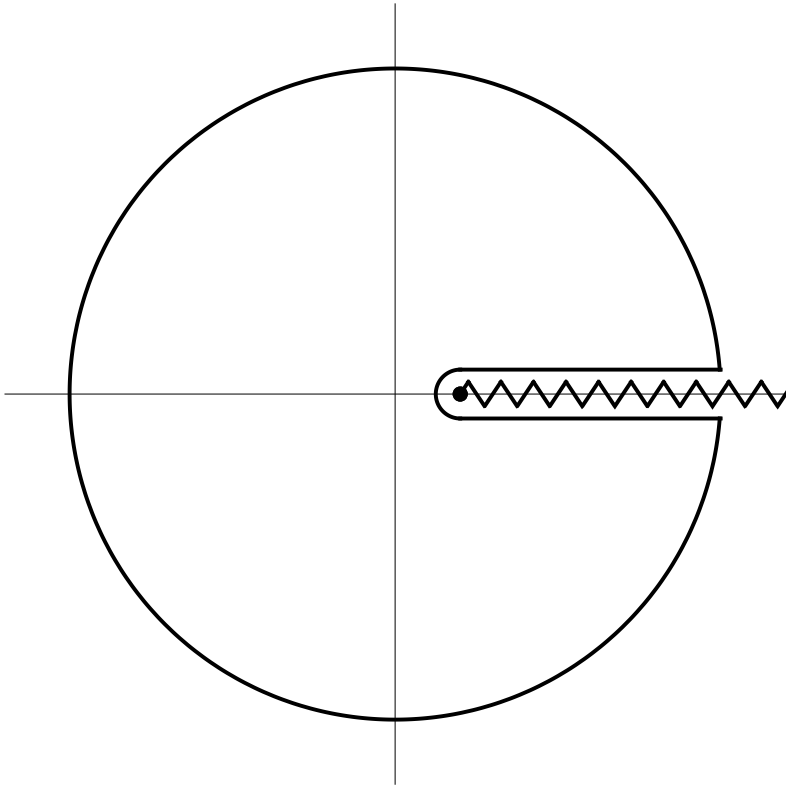
$\Pi(s)$



Cauchy Integral Theorem

$$\Pi(s)$$

$$\frac{1}{2\pi i}\oint_{\Gamma}\Pi(s)ds = 0$$

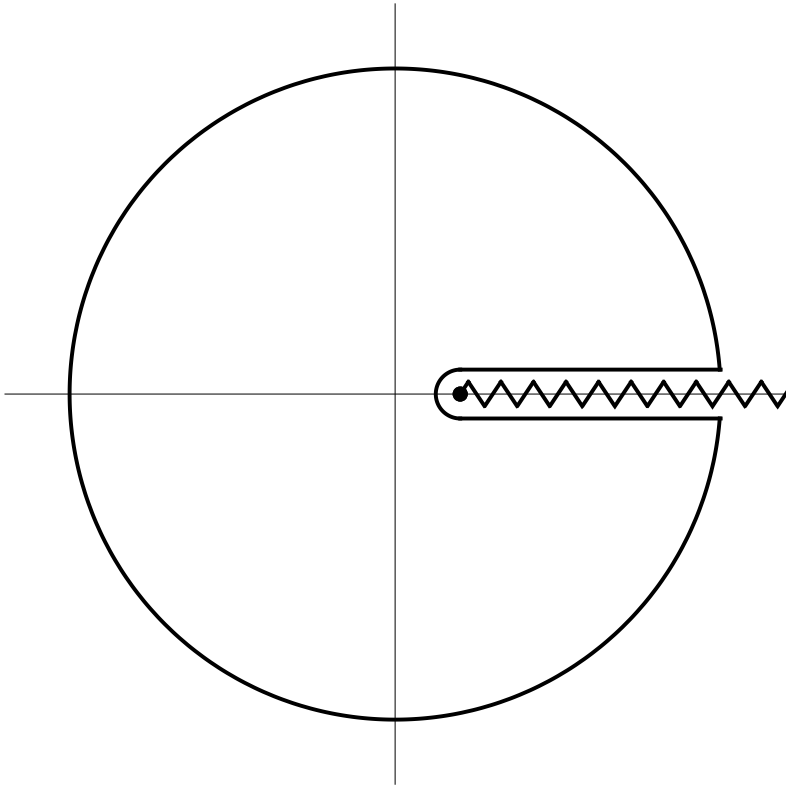


Cauchy Integral Theorem

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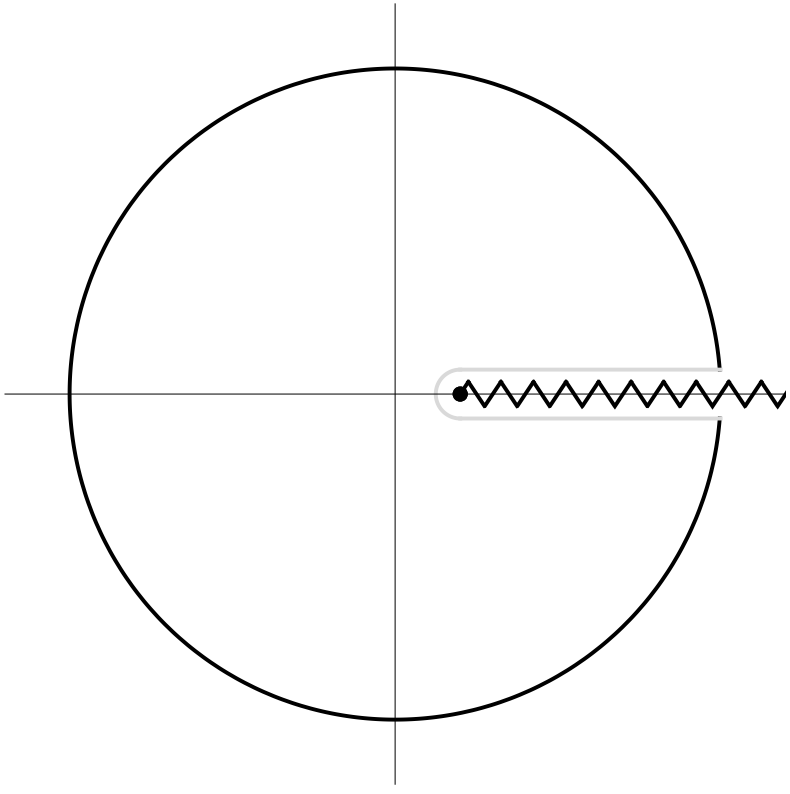
$$\frac{1}{2\pi i}\oint_{\Gamma}\Pi(s)ds = 0$$

$$\frac{1}{2\pi i}\oint_{\Gamma}P(s)\Pi(s)ds = 0$$



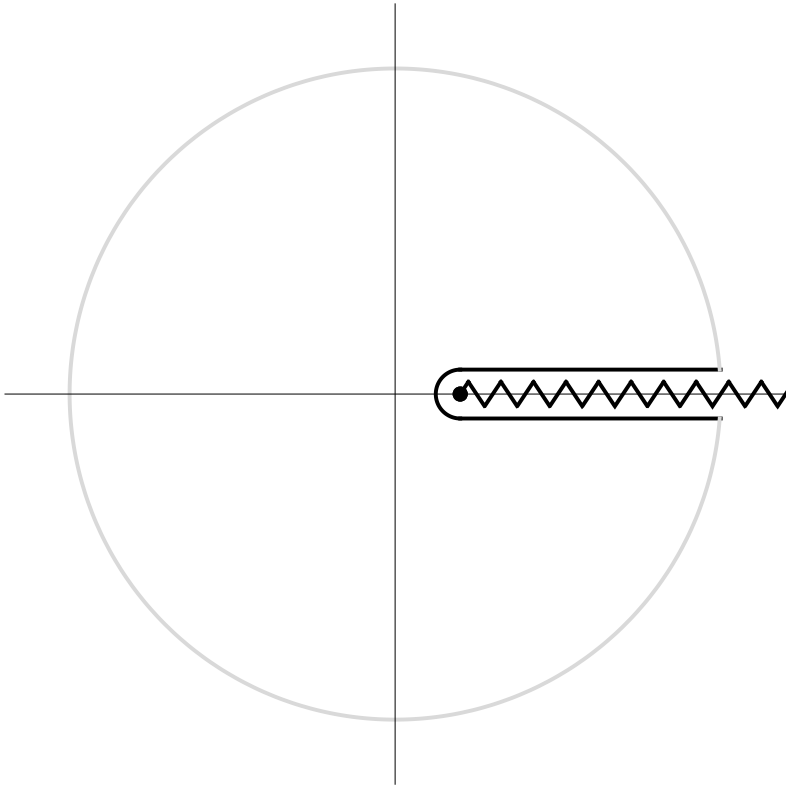
Cauchy Integral Theorem

$$\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\mathrm{QCD}}(s)ds$$



Schwarz Reflection Principle

$$\frac{1}{\pi}\int_{s_{\text{pole}}}^{s_0} \text{Im}[P(s)\Pi^{\text{Had}}(s)]ds$$



Master Equation

$$\frac{1}{\pi}\int_{s_{\text{pole}}}^{s_0} \text{Im}[P(s)\Pi^{\text{Had}}(s)]ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

Integration Kernel

$$P_n(s) = a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n$$

$$\int_{s_{\text{cont}}}^{s_0} s^k P_n(s) ds = 0, \qquad \forall k \in 0, \ldots, n-1$$

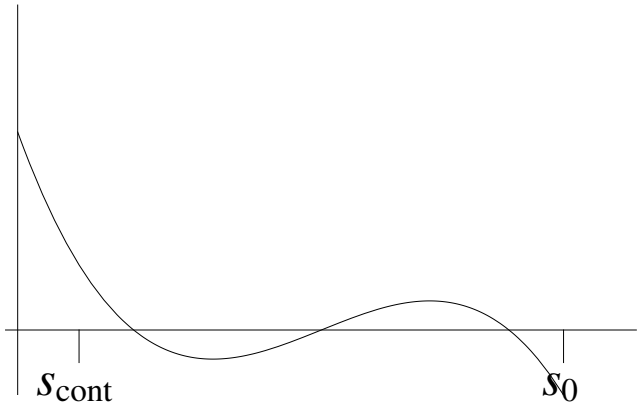
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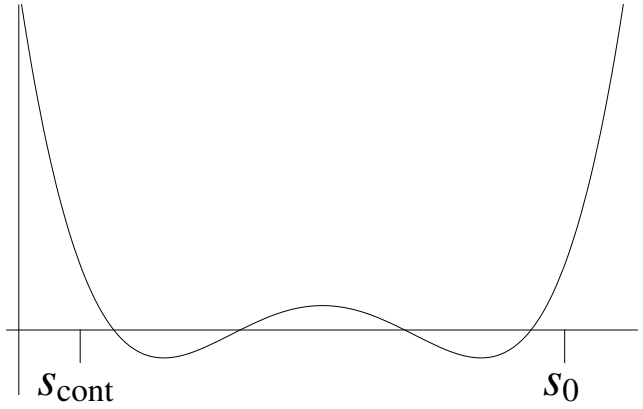
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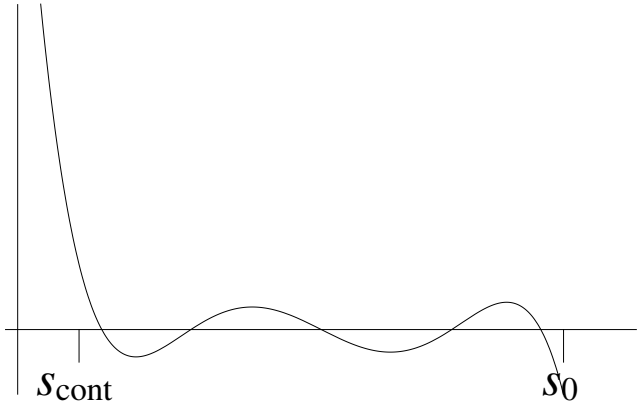
$P_3(s)$



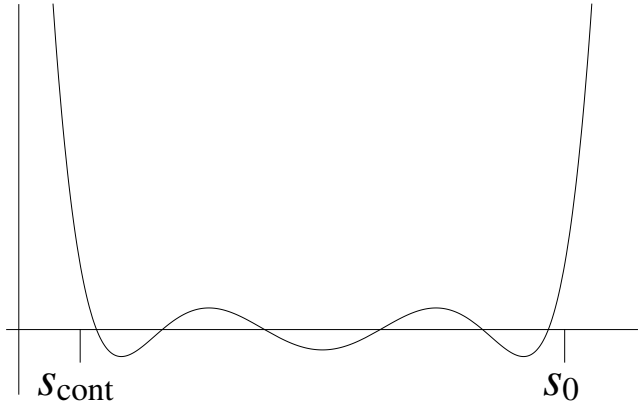
$P_4(s)$



$P_5(s)$



$P_6(s)$



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Pseudoscalar Correlator

$$\langle \Omega | (m_b + m_q)(\bar{q} i\gamma_5 b)(0) | B_q \rangle = f_{B_q} M_{B_q}^2$$

$$\Pi(q^2) = i \int dx e^{iqx} \langle \Omega | T\{ j_5(x) j_5(0)^\dagger \} | \Omega \rangle$$

$$j_5(x) = (m_b + m_q) : \bar{q}(x) i\gamma_5 b(x) :$$

Pseudoscalar Correlator

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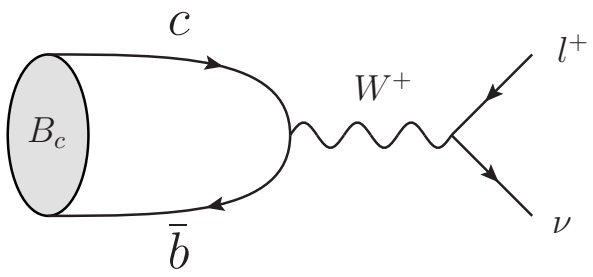
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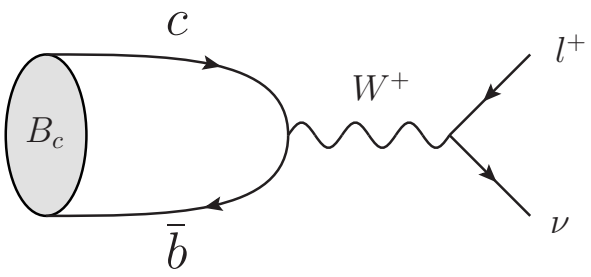
$$j_5(x) = (m_b + m_q) : \bar{q}(x) i \gamma_5 b(x) :$$

Purely Leptonic B_c Decay



$$\Gamma(B_c \to l^+\nu) = \frac{G_F^2}{8\pi} f_{B_c}^2 m_l^2 M_{B_c} \left(1 - \frac{m_l^2}{M_{B_c}^2}\right)^2 |V_{bc}|^2$$

Purely Leptonic B_c Decay



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Master Equation

$$\frac{1}{\pi}\int_{s_{\text{pole}}}^{s_0}\text{Im}[P(s)\Pi^{\text{Had}}(s)]ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

$$\frac{1}{\pi}\text{Im}[P(s)\Pi^{\text{Had}}(s)] = M_{B_q}^4 f_{B_q}^2 \delta(s - M_{B_q}^2) + \theta(s - s_{\text{pole}}) \cdot (\text{any } \mathcal{O}(s^{(n-1)}) \text{ polynomial})$$

$$M_{B_q}^4 f_{B_q}^2 P_n(M_{B_q}^2) = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

Master Equation

$$\frac{1}{\pi}\int_{s_{\text{pole}}}^{s_0} \text{Im}[P(s)\Pi^{\text{Had}}(s)]ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

$$\frac{1}{\pi}\text{Im}[P(s)\Pi^{\text{Had}}(s)] = M_{B_q}^4 f_{B_q}^2 \delta(s - M_{B_q}^2) + \theta(s - s_{\text{pole}}) \cdot (\text{any } \mathcal{O}(s^{(n-1)}) \text{ polynomial})$$

$$M_{B_q}^4 f_{B_q}^2 P_n(M_{B_q}^2) = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

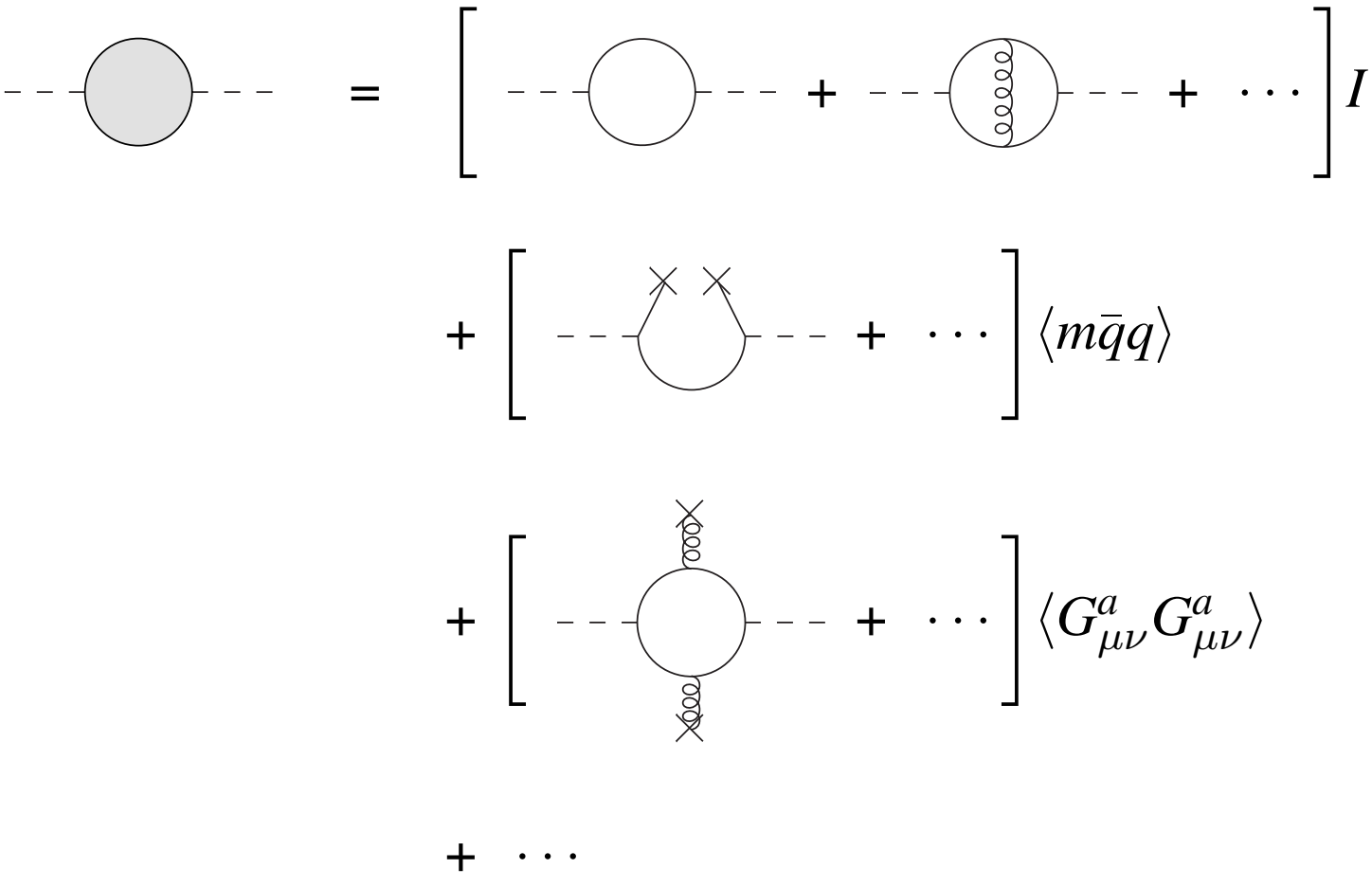
Master Equation

$$\frac{1}{\pi}\int_{s_{\text{pole}}}^{s_0} \text{Im}[P(s)\Pi^{\text{Had}}(s)]ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

$$\frac{1}{\pi}\text{Im}[P(s)\Pi^{\text{Had}}(s)] = M_{B_q}^4 f_{B_q}^2 \delta(s - M_{B_q}^2) + \theta(s - s_{\text{pole}}) \cdot (\text{any } \mathcal{O}(s^{(n-1)}) \text{ polynomial})$$

$$M_{B_q}^4 f_{B_q}^2 P_n(M_{B_q}^2) = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\Pi^{\text{QCD}}(s)ds$$

QCD Correlator



Outline

1 QCD Sum Rules
- Motivation
- Two Point Correlator
- Quark-Hadron Duality

2 B Meson Decay Constants
- Theoretical Background
- Perturbative Correlator
- Non-perturbative Correlator
- Results

Perturbative Contribution

$$\Pi^{\text{pert}}(s) = \Pi^{\text{pert}}_{m_q=0}(s) + \Pi^{\text{pert}}_{\text{mass cor}}(s)$$

$$\Pi^{\text{pert}}_{m_q=0}(s) = \Pi^{(0)}_{m_q=0}(s) + \Pi^{(1)}_{m_q=0}(s)\left(\frac{\alpha_s(\mu)}{\pi}\right) + \Pi^{(2)}_{m_q=0}(s)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$

Perturbative Contribution

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Perturbative Contribution

$$\Pi^{\text{pert}}_{m_q=0}(s) = \Pi^{(0)}_{m_q=0}(s) + \Pi^{(1)}_{m_q=0}(s)\left(\frac{\alpha_s(\mu)}{\pi}\right) + \Pi^{(2)}_{m_q=0}(s)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$

$$\Pi^{(0)}_{m_q=0}(s) = \frac{3}{16\pi^2}(\widehat{m}_b + \widehat{m}_q)^2 s\left\{3 + 4L_z - 8L_z\frac{1}{z} + (-3+4L_z)\frac{1}{z^2} + \right.$$
$$\left. + \frac{2}{3}\frac{1}{z^3} + \frac{1}{6}\frac{1}{z^4} + \frac{1}{15}\frac{1}{z^5} + \frac{1}{30}\frac{1}{z^6} + \frac{2}{105}\frac{1}{z^7}\right\}$$

$$z = s/\widehat{m}_b^2, \qquad L_z = -(\ln(-z))/2$$

Perturbative Contribution

$$\Pi^{\text{pert}}_{m_q=0}(s) = \Pi^{(0)}_{m_q=0}(s) + \Pi^{(1)}_{m_q=0}(s)\left(\frac{\alpha_s(\mu)}{\pi}\right) + \Pi^{(2)}_{m_q=0}(s)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$

$$\Pi^{(0)}_{m_q=0}(s) = \frac{3}{16\pi^2}(\widehat{m}_b + \widehat{m}_q)^2 s\Bigg\{3 + 4L_z - 8L_z\frac{1}{z} + (-3+4L_z)\frac{1}{z^2} + \\ + \frac{2}{3}\frac{1}{z^3} + \frac{1}{6}\frac{1}{z^4} + \frac{1}{15}\frac{1}{z^5} + \frac{1}{30}\frac{1}{z^6} + \frac{2}{105}\frac{1}{z^7}\Bigg\}$$

$$z = s/\widehat{m}_b^2, \qquad L_z = -(\ln(-z))/2$$

$$\Pi^{\text{pert}}_{m_q=0}(s) = (3\cdot(3+(19\cdot\alpha_s\cdot cf)/4+(4783\cdot\alpha_s^2\cdot ca\cdot cf)/288+(1961\cdot\alpha_s^2\cdot cf^2)/288-(46\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/9-(95\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/18+2/(105\cdot z^7)+(532267\cdot\alpha_s\cdot cf)/(9261000\cdot z^7)-(471073727\cdot\alpha_s^2\cdot ca\cdot cf)/(92610000\cdot z^7)+(339666207683\cdot\alpha_s^2\cdot cf^2)/(32672808000\cdot z^7)-(74992868899\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(1296540000\cdot z^7)+(218460799\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(3889620000\cdot z^7)+1/(30\cdot z^6)+(10243\cdot\alpha_s\cdot cf)/(108000\cdot z^6)+(16344943\cdot\alpha_s^2\cdot ca\cdot cf)/(5184000\cdot z^6)-(2411369\cdot\alpha_s^2\cdot cf^2)/(405000\cdot z^6)-(29545141\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(1620000\cdot z^6)+(54839\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(540000\cdot z^6)+1/(15\cdot z^5)+(3521\cdot\alpha_s\cdot cf)/(18000\cdot z^5)-(15978853\cdot\alpha_s^2\cdot ca\cdot cf)/(4320000\cdot z^5)+(175044221\cdot\alpha_s^2\cdot cf^2)/(19440000\cdot z^5)-(34816841\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(4860000\cdot z^5)+(1217977\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(4860000\cdot z^5)+1/(6\cdot z^4)+(263\cdot\alpha_s\cdot cf)/(288\cdot z^4)+(420179\cdot\alpha_s^2\cdot ca\cdot cf)/(15552\cdot z^4)-(1271839\cdot\alpha_s^2\cdot cf^2)/(20736\cdot z^4)-(34093\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(3888\cdot z^4)+(6755\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(3888\cdot z^4)+2/(3\cdot z^3)-(107\cdot\alpha_s\cdot cf)/(18\cdot z^3)-(16153\cdot\alpha_s^2\cdot ca\cdot cf)/(972\cdot z^3)+(137243\cdot\alpha_s^2\cdot cf^2)/(3888\cdot z^3)+(3685\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(486\cdot z^3)-(191\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(486\cdot z^3)-3/z^2+(8\cdot\alpha_s\cdot cf)/z^2+(5239\cdot\alpha_s^2\cdot ca\cdot cf)/(288\cdot z^2)-(237\cdot\alpha_s^2\cdot cf^2)/(8\cdot z^2)+(1639\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(72\cdot z^2)-(341\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(72\cdot z^2)-(7\cdot\alpha_s\cdot cf)/z-(821\cdot\alpha_s^2\cdot ca\cdot cf)/(72\cdot z)+(3\cdot\alpha_s^2\cdot cf^2)/(8\cdot z)-(191\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr)/(18\cdot z)+(61\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr)/(18\cdot z)+\alpha_s\cdot cf\cdot z2+(193\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/24-(1235\cdot\alpha_s^2\cdot cf^2\cdot z2)/72-10\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2+(23\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/6+(16\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/(105\cdot z^7)-(4\cdot\alpha_s^2\cdot cf^2\cdot z2)/(7\cdot z^7)-(32\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/(105\cdot z^7)+(16\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/(105\cdot z^7)+(7\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/(30\cdot z^6)-(7\cdot\alpha_s^2\cdot cf^2\cdot z2)/(8\cdot z^6)-(7\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/(15\cdot z^6)+(7\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/(30\cdot z^6)+(2\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/(5\cdot z^5)-(3\cdot\alpha_s^2\cdot cf^2\cdot z2)/(2\cdot z^5)-(4\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/(5\cdot z^5)+(2\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/(5\cdot z^5)+(5\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/(6\cdot$$

$$z^4)-(25\cdot\alpha_s^2\cdot cf^2\cdot z2)/(8\cdot z^4)-(5\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/(3\cdot z^4)+(5\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/(6\cdot z^4)+(8\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/(3\cdot z^3)-(10\cdot\alpha_s^2\cdot cf^2\cdot z2)/z^3-(16\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/(3\cdot z^3)+(8\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/(3\cdot z^3)-(7\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/z^2+(105\cdot\alpha_s^2\cdot cf^2\cdot z2)/(4\cdot z^2)+(14\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/z^2-(7\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/z^2-(4\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2)/z+(15\cdot\alpha_s^2\cdot cf^2\cdot z2)/z+(8\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2)/z-(4\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2)/z-(427\cdot\alpha_s^2\cdot ca\cdot cf\cdot z4)/32+(363\cdot\alpha_s^2\cdot cf^2\cdot z4)/16-(25\cdot\alpha_s^2\cdot ca\cdot cf\cdot z4)/(2\cdot z^2)+(25\cdot\alpha_s^2\cdot cf^2\cdot z4)/z^2+(25\cdot\alpha_s^2\cdot ca\cdot cf\cdot z4)/z-(50\cdot\alpha_s^2\cdot cf^2\cdot z4)/z-(39\cdot Sqrt[3]\cdot\alpha_s^2\cdot ca\cdot cf\cdot Im[PolyLog[2,E^(I/3)\cdot Pi)]])/8+(39\cdot Sqrt[3]\cdot\alpha_s^2\cdot cf^2\cdot Im[PolyLog[2,E^(I/3)\cdot Pi)]])/4-9\cdot Sqrt[3]\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Im[PolyLog[2,E^(I/3)\cdot Pi)]]-(3\cdot\alpha_s^2\cdot ca\cdot cf\cdot Im[PolyLog[2,E^(I/3)\cdot Pi)]]^2)/4+(3\cdot\alpha_s^2\cdot cf^2\cdot Im[PolyLog[2,E^(I/3)\cdot Pi)]]^2)/2-15\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]+30\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]-(16\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/(35\cdot z^7)+(32\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/(35\cdot z^7)-(7\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/(10\cdot z^6)+(7\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/(5\cdot z^6)-(6\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/(5\cdot z^5)+(12\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/(5\cdot z^5)-(5\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/(2\cdot z^4)+(5\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/z^4-(8\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/z^3+(16\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/z^3+(21\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/z^2-(42\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/z^2+(12\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2])/z-(24\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2])/z-4\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]^2+8\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]^2-(4\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]^2)/z^2+(8\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]^2)/z^2+(8\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]^2)/z-(16\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]^2)/z+(2\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[2]^4)/3-(4\cdot\alpha_s^2\cdot cf^2\cdot Log[2]^4)/3+(2\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[2]^4)/(3\cdot z^2)-(4\cdot\alpha_s^2\cdot cf^2\cdot Log[2]^4)/(3\cdot z^2)-(4\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[2]^4)/(3\cdot z)+(8\cdot\alpha_s^2\cdot cf^2\cdot Log[2]^4)/(3\cdot z)-2\cdot Log[-z]-(9\cdot\alpha_s\cdot cf\cdot Log[-z])/2-(49\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/3-(109\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/16+(13\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/6+(31\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/6-(743\cdot\alpha_s\cdot cf\cdot Log[-z])/(22050\cdot z^7)+(3758939963\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(277830000\cdot z^7)-(215940050209\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/(7779240000\cdot z^7)-(378496261\cdot\alpha_s^2\cdot$$

$$cf\cdot nh\cdot tr\cdot Log[-z])/(6945750\cdot z^7)+(160228\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(1157625\cdot z^7)-(13\cdot\alpha_s\cdot cf\cdot Log[-z])/(225\cdot z^6)+(10567931\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(1296000\cdot z^6)-(5650789\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/(324000\cdot z^6)-(3698791\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/(324000\cdot z^6)+(26531\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(108000\cdot z^6)-(47\cdot\alpha_s\cdot cf\cdot Log[-z])/(450\cdot z^5)+(650497\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(324000\cdot z^5)-(17774837\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/(3240000\cdot z^5)-(752611\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/(162000\cdot z^5)+(85273\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(162000\cdot z^5)-(\alpha_s\cdot cf\cdot Log[-z])/(9\cdot z^4)+(42247\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(10368\cdot z^4)-(139555\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/(5184\cdot z^4)-(8347\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/(2592\cdot z^4)+(4981\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(2592\cdot z^4)-(64\cdot\alpha_s\cdot cf\cdot Log[-z])/(9\cdot z^3)-(9395\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(648\cdot z^3)+(2351\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/(54\cdot z^3)+(913\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/(81\cdot z^3)+(59\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(81\cdot z^3)-(2\cdot Log[-z])/z^2+(12\cdot\alpha_s\cdot cf\cdot Log[-z])/z^2+(405\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(16\cdot z^2)-(299\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/(8\cdot z^2)-(3\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/(4\cdot z^2)-(27\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(4\cdot z^2)+(4\cdot Log[-z])/z+(77\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z])/(12\cdot z)+(21\cdot\alpha_s^2\cdot cf^2\cdot Log[-z])/z-(7\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z])/(3\cdot z)-(7\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z])/(3\cdot z)-2\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[-z]+(15\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[-z])/2+4\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2\cdot Log[-z]-2\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2\cdot Log[-z]-(6\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[-z])/z^2+(45\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[-z])/(2\cdot z^2)+(12\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2\cdot Log[-z])/z^2-(6\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2\cdot Log[-z])/z^2+(8\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[-z])/z-(30\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[-z])/z-(16\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot z2\cdot Log[-z])/z+(8\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot z2\cdot Log[-z])/z+6\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]\cdot Log[-z]-12\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]\cdot Log[-z]+(18\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]\cdot Log[-z])/z^2-(36\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]\cdot Log[-z])/z^2-(24\cdot\alpha_s^2\cdot ca\cdot cf\cdot z2\cdot Log[2]\cdot Log[-z])/z+(48\cdot\alpha_s^2\cdot cf^2\cdot z2\cdot Log[2]\cdot Log[-z])/z+(3\cdot\alpha_s\cdot cf\cdot Log[-z]^2)/2+(71\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/12+(57\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/16-(11\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot$$

$$Log[-z]^2)/6-(11\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/6-(\alpha_s\cdot cf\cdot Log[-z]^2)/(105\cdot z^7)-(6917\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(22050\cdot z^7)+(18459061\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/(37044000\cdot z^7)+(491947\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(14700\cdot z^7)+(5693\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(132300\cdot z^7)-(\alpha_s\cdot cf\cdot Log[-z]^2)/(60\cdot z^6)+(1979\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(11520\cdot z^6)-(78107\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/(144000\cdot z^6)+(34153\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(3600\cdot z^6)+(719\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(10800\cdot z^6)-(\alpha_s\cdot cf\cdot Log[-z]^2)/(30\cdot z^5)-(47\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(160\cdot z^5)+(63049\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/(216000\cdot z^5)+(3683\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(1800\cdot z^5)+(649\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(5400\cdot z^5)-(\alpha_s\cdot cf\cdot Log[-z]^2)/(12\cdot z^4)-(6847\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(1728\cdot z^4)+(2719\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/(1728\cdot z^4)+(217\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(144\cdot z^4)+(143\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(432\cdot z^4)-(4\cdot\alpha_s\cdot cf\cdot Log[-z]^2)/(3\cdot z^3)+(209\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(108\cdot z^3)+(24\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/z^3+(31\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(9\cdot z^3)-(34\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(27\cdot z^3)+(6\cdot\alpha_s\cdot cf\cdot Log[-z]^2)/z^2+(509\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(48\cdot z^2)-(201\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/(8\cdot z^2)-(13\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(12\cdot z^2)-(31\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(12\cdot z^2)-(6\cdot\alpha_s\cdot cf\cdot Log[-z]^2)/z-(185\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^2)/(12\cdot z)-(3\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^2)/(4\cdot z)+(13\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^2)/(3\cdot z)+(13\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^2)/(3\cdot z)-(11\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^3)/24-(3\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^3)/4+(\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^3)/6+(\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^3)/6-(12451\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^3)/(18900\cdot z^7)+(17681\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^3)/(13230\cdot z^7)-(1964\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^3)/(945\cdot z^7)-(2\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^3)/(945\cdot z^7)-(2833\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^3)/(4320\cdot z^6)+(7247\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^3)/(5400\cdot z^6)-(191\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^3)/(135\cdot z^6)-(\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^3)/(270\cdot z^6)-(427\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]^3)/(1080\cdot z^5)+(757\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]^3)/(900\cdot z^5)-(34\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]^3)/(45\cdot z^5)-(\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]^3)/(135\cdot$$

$$z^5) - (193 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Log[-z]^3)/(216 \cdot z^4) + (575 \cdot \alpha_s^2 \cdot cf^2 \cdot Log[-z]^3)/(432 \cdot z^4) - (\alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Log[-z]^3)/(9 \cdot z^4) - (\alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Log[-z]^3)/(54 \cdot z^4) + (31 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Log[-z]^3)/(27 \cdot z^3) + (112 \cdot \alpha_s^2 \cdot cf^2 \cdot Log[-z]^3)/(27 \cdot z^3) - (8 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Log[-z]^3)/(27 \cdot z^3) - (55 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Log[-z]^3)/(24 \cdot z^2) - (39 \cdot \alpha_s^2 \cdot cf^2 \cdot Log[-z]^3)/(4 \cdot z^2) + (5 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Log[-z]^3)/(6 \cdot z^2) + (5 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Log[-z]^3)/(6 \cdot z^2) + (11 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Log[-z]^3)/(6 \cdot z) + (6 \cdot \alpha_s^2 \cdot cf^2 \cdot Log[-z]^3)/z - (2 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Log[-z]^3)/(3 \cdot z) - (2 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Log[-z]^3)/(3 \cdot z) + 16 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot PolyLog[4, 1/2] - 32 \cdot \alpha_s^2 \cdot cf^2 \cdot PolyLog[4, 1/2] + (16 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot PolyLog[4, 1/2])/z^2 - (32 \cdot \alpha_s^2 \cdot cf^2 \cdot PolyLog[4, 1/2])/z^2 - (32 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot PolyLog[4, 1/2])/z + (64 \cdot \alpha_s^2 \cdot cf^2 \cdot PolyLog[4, 1/2])/z - 6 \cdot \alpha_s \cdot cf \cdot Zeta[3] - (5 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/2 - (631 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/12 + (64 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/3 + 6 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3] - (591 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/(490 \cdot z^7) + (1261 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/(588 \cdot z^7) - (312 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(35 \cdot z^7) + (4 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(105 \cdot z^7) - (3939 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/(800 \cdot z^6) + (11261 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/(1200 \cdot z^6) - (74 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(15 \cdot z^6) + (\alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(15 \cdot z^6) + (118 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/(75 \cdot z^5) - (809 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/(200 \cdot z^5) - (44 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(45 \cdot z^5) + (2 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(15 \cdot z^5) - (725 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/(48 \cdot z^4) + (113 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/(4 \cdot z^4) + (26 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(9 \cdot z^4) + (\alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(3 \cdot z^4) + (8 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/z^3 - (41 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/z^3 + (14 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(9 \cdot z^3) + (10 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(3 \cdot z^3) - (6 \cdot \alpha_s \cdot cf \cdot Zeta[3])/z^2 + (17 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/(24 \cdot z^2) + (51 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/(2 \cdot z^2) - (20 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(3 \cdot z^2) - (2 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(3 \cdot z^2) + (12 \cdot \alpha_s \cdot cf \cdot Zeta[3])/z + (41 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Zeta[3])/(6 \cdot z) + (137 \cdot \alpha_s^2 \cdot cf^2 \cdot Zeta[3])/(2 \cdot z) + (46 \cdot \alpha_s^2 \cdot cf \cdot nh \cdot tr \cdot Zeta[3])/(3 \cdot z) - (26 \cdot \alpha_s^2 \cdot cf \cdot nl \cdot tr \cdot Zeta[3])/(3 \cdot z) + (25 \cdot \alpha_s^2 \cdot ca \cdot cf \cdot Log[-z] \cdot Zeta[3])/4 + (15 \cdot \alpha_s^2 \cdot cf^2 \cdot Log[-z] \cdot Zeta[3])/2 - 2 \cdot \alpha_s^2 \cdot$$

$$cf\cdot nh\cdot tr\cdot Log[-z]\cdot Zeta[3]-2\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]\cdot Zeta[3]-(2\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/(35\cdot z^7)+(2\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/(35\cdot z^7)-(\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/(10\cdot z^6)+(\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/(10\cdot z^6)-(\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/(5\cdot z^5)+(\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/(5\cdot z^5)-(\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/(2\cdot z^4)+(\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/(2\cdot z^4)-(8\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/z^3+(8\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/z^3+(25\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/(4\cdot z^2)+(69\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/(2\cdot z^2)-(2\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]\cdot Zeta[3])/z^2-(2\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]\cdot Zeta[3])/z^2-(11\cdot\alpha_s^2\cdot ca\cdot cf\cdot Log[-z]\cdot Zeta[3])/z-(36\cdot\alpha_s^2\cdot cf^2\cdot Log[-z]\cdot Zeta[3])/z+(4\cdot\alpha_s^2\cdot cf\cdot nh\cdot tr\cdot Log[-z]\cdot Zeta[3])/z+(4\cdot\alpha_s^2\cdot cf\cdot nl\cdot tr\cdot Log[-z]\cdot Zeta[3])/z-(5\cdot\alpha_s^2\cdot ca\cdot cf\cdot Zeta[5])/2+15\cdot\alpha_s^2\cdot cf^2\cdot Zeta[5]-(15\cdot\alpha_s^2\cdot ca\cdot cf\cdot Zeta[5])/(2\cdot z^2)+(5\cdot\alpha_s^2\cdot cf^2\cdot Zeta[5])/z^2+(5\cdot\alpha_s^2\cdot ca\cdot cf\cdot Zeta[5])/z-(50\cdot^2\cdot cf^2\cdot Zeta[5])/z))/(16\cdot Pi^2))$$

Perturbative Contribution

$$\Pi^{\text{pert}}(s) = \Pi^{\text{pert}}_{m_q=0}(s) + \Pi^{\text{pert}}_{\text{mass cor}}(s)$$

$$\text{Im}\left[\Pi^{\text{pert}}_{\text{mass cor}}(s)\right] = \text{Im}\left[\Pi^{(0)}_{\text{mass cor}}(s)\right] + \text{Im}\left[\Pi^{(1)}_{\text{mass cor}}(s)\right]\left(\frac{\alpha_s(\mu)}{\pi}\right)$$

$$\text{Im}\left[\Pi^{(0)\text{pert}}_{\text{mass cor}}(s)\right] = \frac{3}{8\pi^2}(m_b+m_q)^2\left\{2(1-x)m_b m_q - 2m_q^2 + \right.$$

$$\left. - 2\frac{(1+x)}{(1-x)}\frac{m_b m_q^3}{s} + \frac{(1-2x-x^2)}{(1-x)^2}\frac{m_q^4}{s}\right\}$$

$$x = m_b^2/s$$

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$$-\frac{1}{2\pi i}\oint_{|s|=s_0} P_n(s)\Pi^{\text{pert}}_{\text{mass cor}}(s)ds = \frac{1}{\pi}\int_{(m_b+m_q)^2}^{s_0} \text{Im}[P_n(s)\Pi^{\text{pert}}_{\text{mass cor}}(s)]ds$$

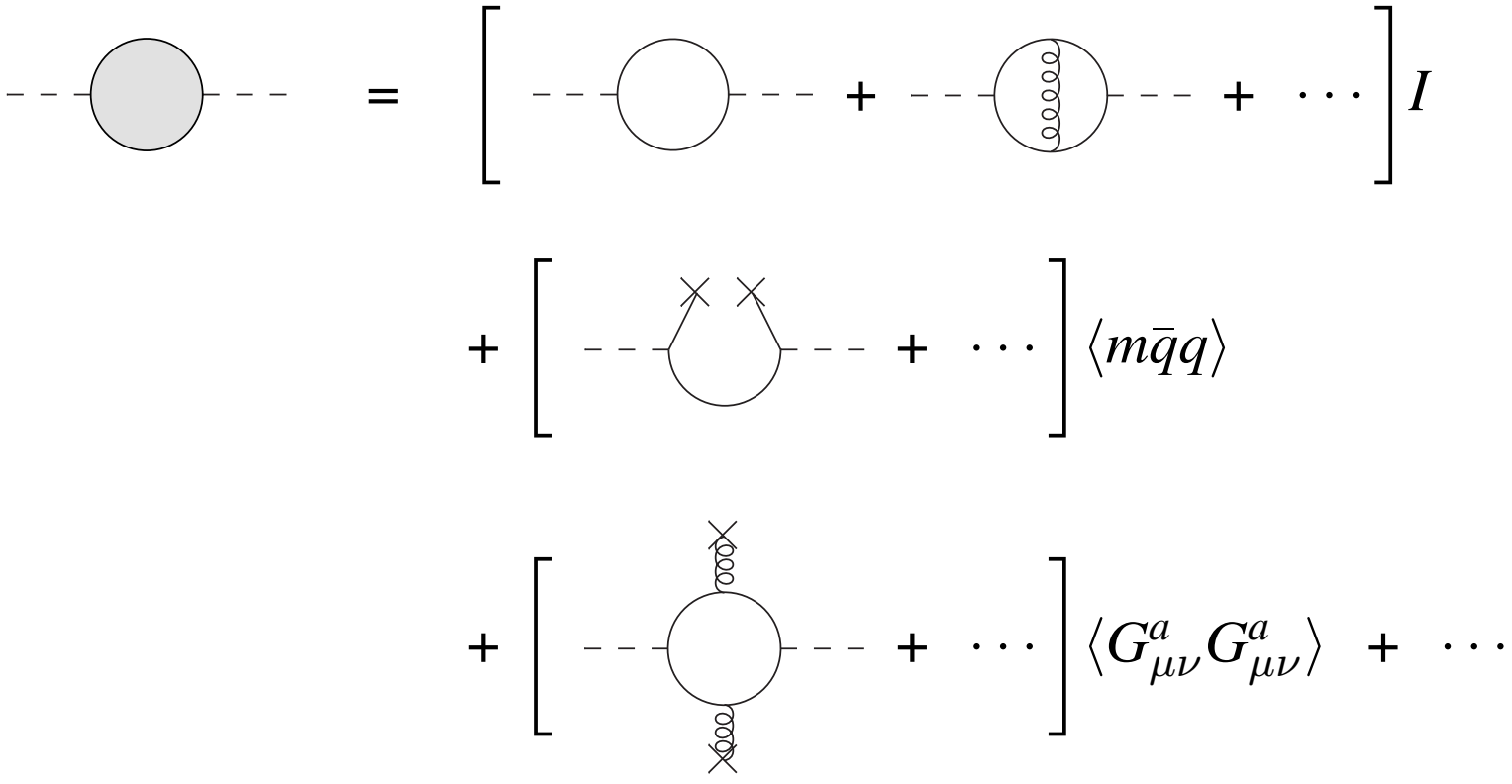
Outline

1 QCD Sum Rules
- Motivation
- Two Point Correlator
- Quark-Hadron Duality

2 B Meson Decay Constants
- Theoretical Background
- Perturbative Correlator
- Non-perturbative Correlator
- Results

Non-perturbative Contribution

$$\Pi^{\text{QCD}}(s) = \Pi^{\text{pert}}(s) + \Pi^{\text{non-pert}}(s)$$



Non-perturbative Contribution

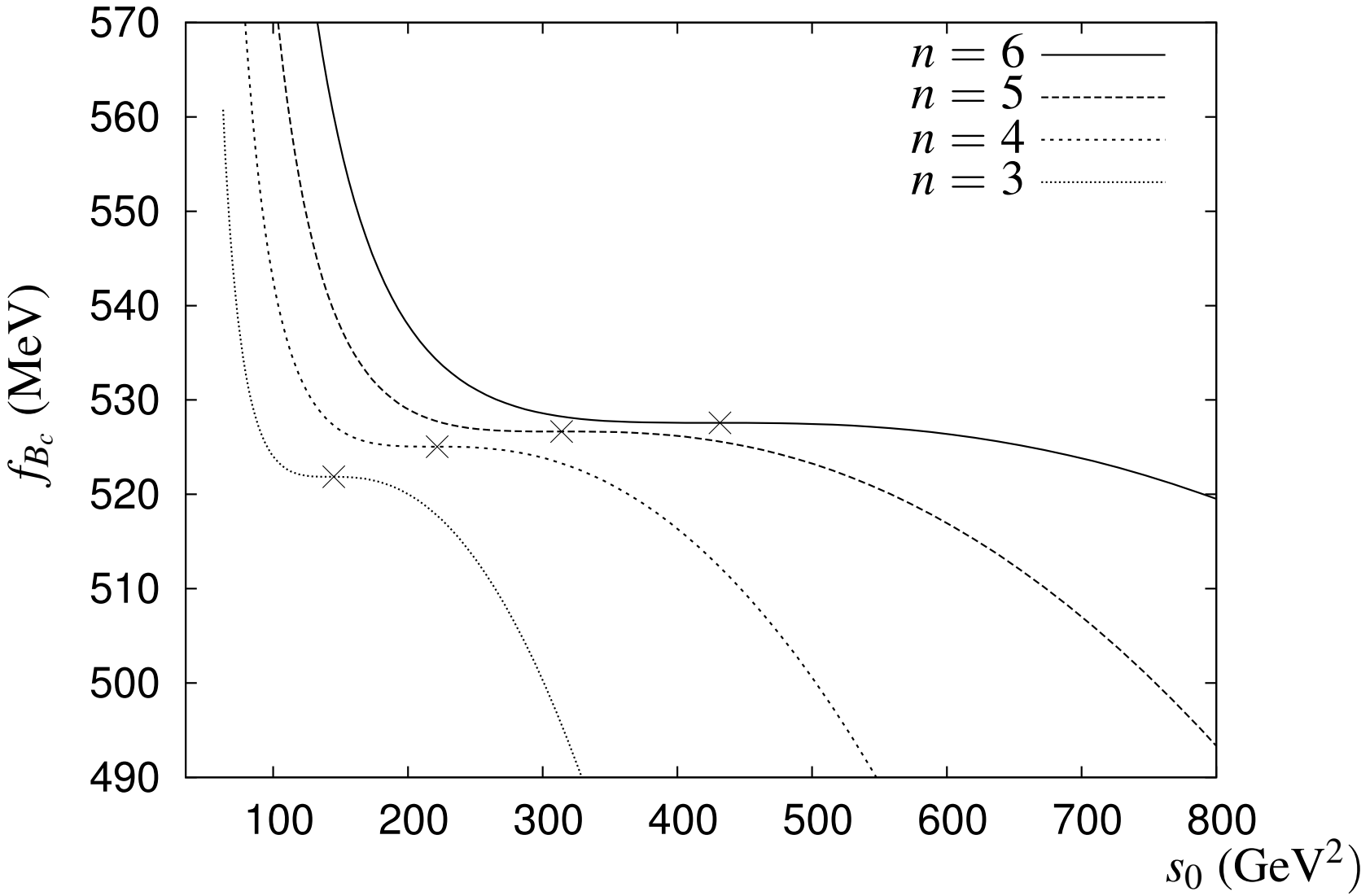
$$\Pi^{\text{QCD}}(s) = \Pi^{\text{pert}}(s) + \Pi^{\text{non-pert}}(s)$$

$$\begin{aligned}\Pi^{\text{non-pert}}(s) =& (\widehat{m}_b + \widehat{m}_q)^2 \left\{ \frac{\widehat{m}_b \langle \bar{q}q \rangle}{s - \widehat{m}_b^2} \left(1 + 2\frac{\alpha_s}{\pi}\right) - \frac{1}{12} \frac{1}{s - \widehat{m}_b^2} \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle + \right. \\ & - \frac{\widehat{m}_b \langle \bar{q}\sigma Gq \rangle}{2} \left[\frac{1}{(s - \widehat{m}_b^2)^2} + \frac{\widehat{m}_b^2}{(s - \widehat{m}_b^2)^3} \right] + \\ & \left. - \frac{8\pi\alpha_s \langle \bar{q}q \rangle^2}{27} \left[\frac{2}{(s - \widehat{m}_b^2)^2} + \frac{\widehat{m}_b^2}{(s - \widehat{m}_b^2)^3} - \frac{\widehat{m}_b^4}{(s - \widehat{m}_b^2)^4} \right] \right\}\end{aligned}$$

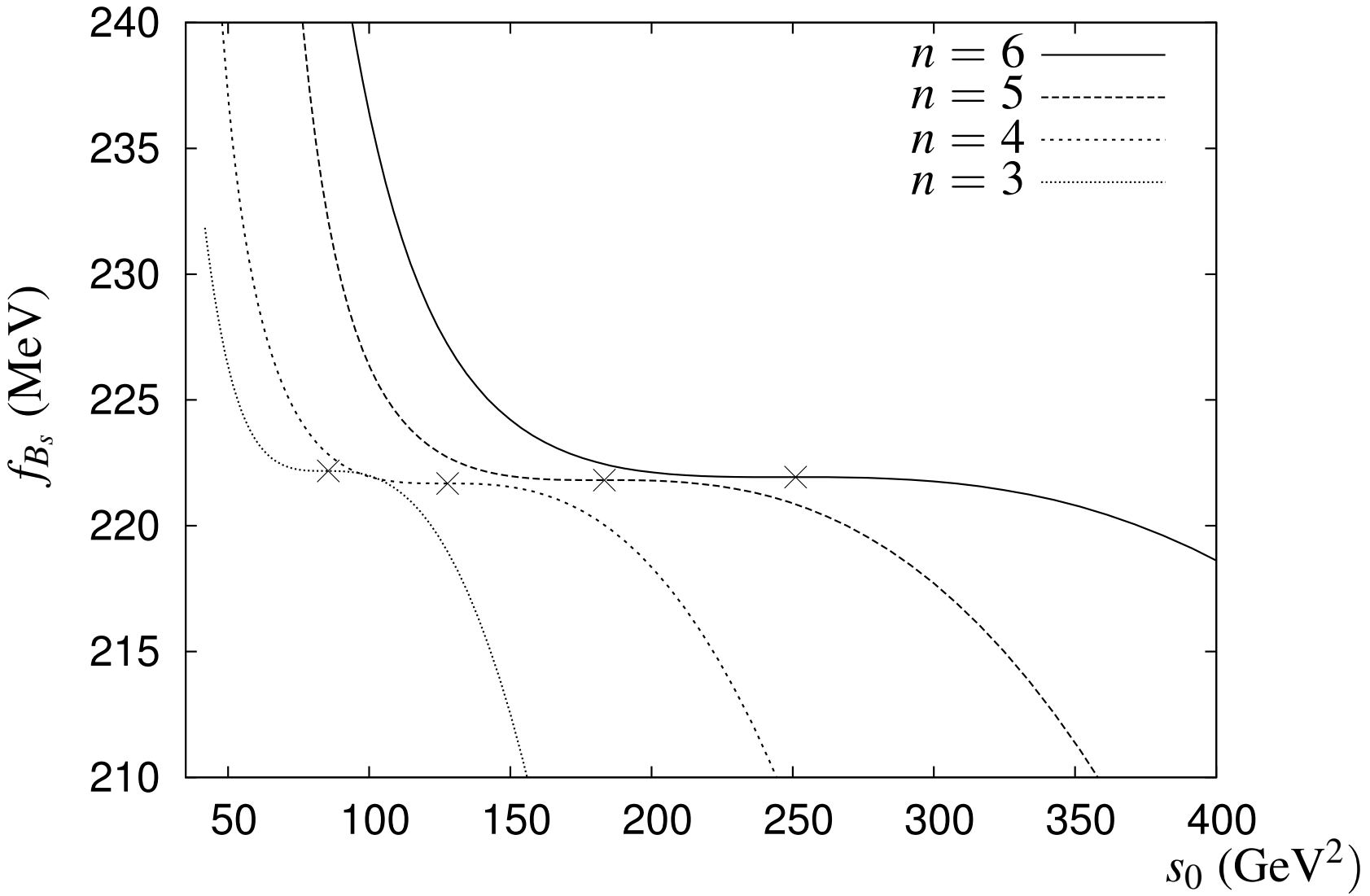
Outline

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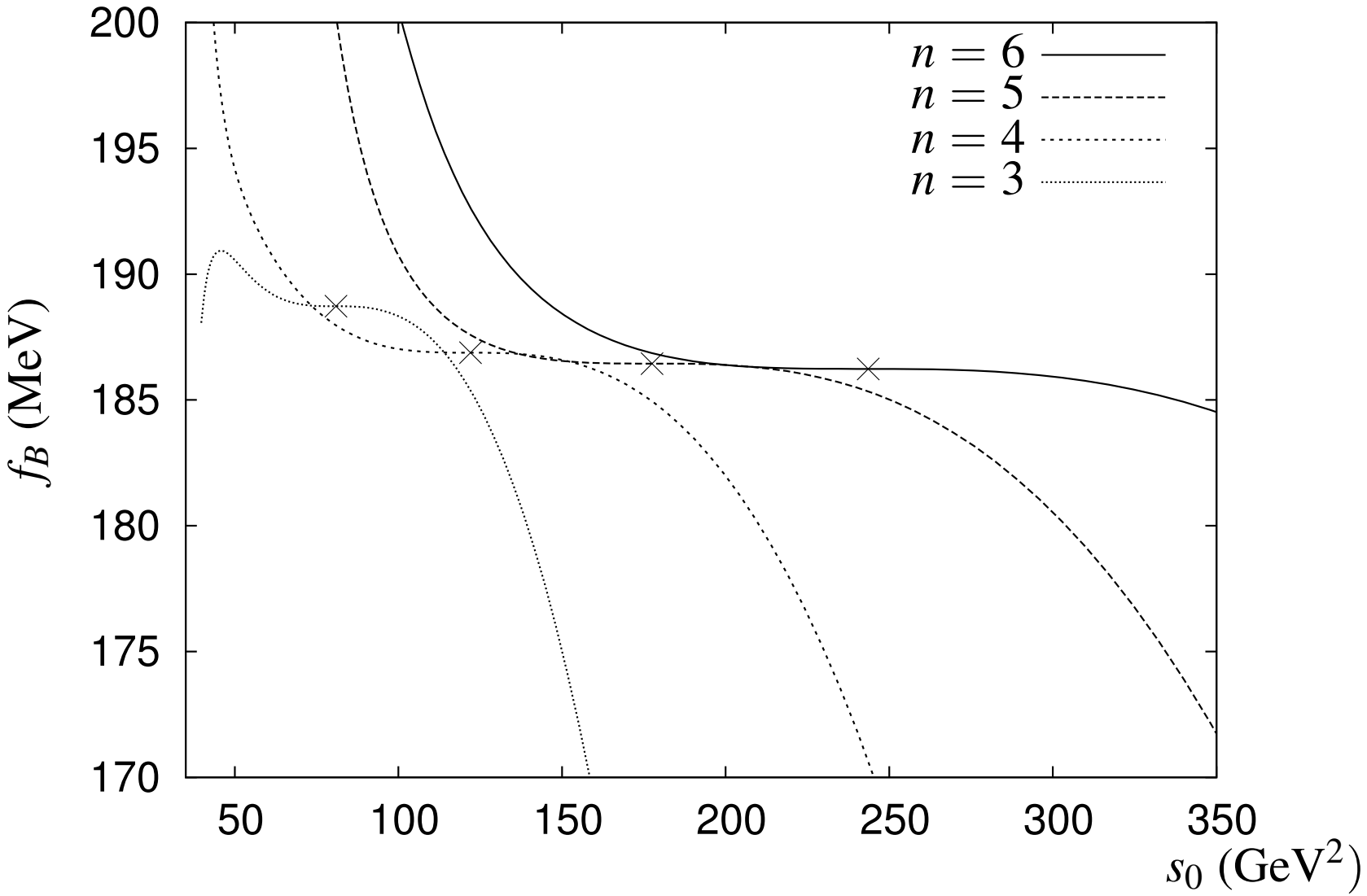
Results: f_{B_c}


570
560
550
540
530
520
510
500
490
100
200
300
400
500
600
700
800
f_{B_c} (MeV)
s_0 (GeV^2)
$n = 6$
$n = 5$
$n = 4$
$n = 3$

Results: f_{B_s}


240
235
230
225
220
215
210
f_{B_s} (MeV)
50
100
150
200
250
300
350
400
s_0 (GeV^2)
$n = 6$
$n = 5$
$n = 4$
$n = 3$

Results: f_B


200
195
190
185
180
175
170
f_B (MeV)
50
100
150
200
250
300
350
s_0 (GeV^2)
$n = 6$
$n = 5$
$n = 4$
$n = 3$

Results

$$f_B = 186 \pm 11 \pm 9 \text{ MeV}$$

$$f_{B_s} = 222 \pm 11 \pm 4 \text{ MeV}$$

$$f_{B_c} = 528 \pm 9 \pm 14 \text{ MeV}$$

$$\frac{f_{B_s}}{f_B} = 1.19 \pm 0.02 \pm 0.04$$

Results

$$f_B = 186 \pm 11 \pm 9 \text{ MeV}$$

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$$f_{B_c} = 528 \pm 9 \pm 14 \text{ MeV}$$

$$\frac{f_{B_s}}{f_B} = 1.19 \pm 0.02 \pm 0.04$$

Other Estimates

f_B, f_{B_s}	(QCD Sum Rules)	Lucha *et al.*	2011
		Gelhausen *et al.*	2013
		Narison	2013
		Wang	2013
	(Lattice)	Bazarov *et al.*	2012
		McNeile *et al.*	2012
		Na *et al.*	2012
		Dowdall *et al.*	2013
f_{B_c}	(Lattice)	McNeile *et al.*	2012

Other Estimates

f_B	$186 \pm 11 \pm 9$ MeV	$189 - 207$ MeV	(QCD Sum Rules)
		$186 - 197$ MeV	(Lattice)
f_{B_s}	$222 \pm 11 \pm 4$ MeV	$216 - 242$ MeV	(QCD Sum Rules)
		$224 - 242$ MeV	(Lattice)
f_{B_c}	$528 \pm 9 \pm 14$ MeV	427 ± 6 MeV	(Lattice)

Experimental: Belle obtain

$f_B|V_{ub}| = 0.74 \pm 0.08 \pm 0.05$ MeV $\Rightarrow$ $f_B = 185 \pm 35$ MeV

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Summary

- Reviewed the finite energy QCD sum rules method
- Using Legendre Polynomials
- Calculated B Meson decay constants

- Outlook
  - Calculate f_{B_c} using a different methodology, perhaps using the Borel transform
  - Apply to more systems - light quark masses, vector mesons, ...

Error Estimates

Input	Value	Δf_B (MeV)	Δf_{B_s} (MeV)	Δf_{B_c} (MeV)
$O(\alpha_s^2)$	twice $O(\alpha_s^2)$ ignoring $O(\alpha_s^2)$	∓ 7	∓ 4	± 13
$\alpha_s(M_Z)$	0.1184 ± 0.0007	∓ 0.5	∓ 0.3	± 0.6
$m_b(\overline{MS})$	4.18 ± 0.03 GeV	$^{-11}_{+10}$	∓ 10	∓ 9
$m_s(2\text{ GeV})$	0.095 ± 0.005 GeV	ø	± 0.9	ø
$m_c(\overline{MS})$	1.275 ± 0.025 GeV	ø	ø	± 1.3
$\langle\bar{q}q\rangle(2\text{ GeV})$	$(-267 \pm 5)^3$ MeV^3	∓ 2.1	∓ 1.1	-
$\langle\alpha_s G^2/\pi\rangle$	0.024 ± 0.012 GeV^4	$^{+0.1}_{-0.5}$	± 0.3	± 0.2
R_{sq}	0.8 ± 0.2	ø	$^{+3.2}_{-3.4}$	ø
m_0^2	0.8 ± 0.2 GeV^2	∓ 0.1	-	-
M_B	5279.58 ± 0.17 MeV	-	ø	ø
M_{B_s}	5366.77 ± 0.24 MeV	ø	-	ø
M_{B_c}	6.277 ± 0.006 GeV	ø	ø	± 1.7