The BFKL approach for Mueller-Navelet jets

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Based on results obtained with
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A. Sabio Vera & C. Salas

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Instituto de Física Corpuscular, Valencia

Outline

1. Introductory Concepts
   - Introduction
   - The Mueller-Navelet jets production process
   - Theoretical setup: BFKL and collinear factorization
   - Jet definition
2. Jet Vertex
   - The Impact Factor in the LLA
   - The Impact Factor in the NLA
     - Collinear and QCD coupling counterterms
   - Quark-initiated subprocess
   - Gluon-initiated subprocess
   - Summary
3. Cross section
   - State of the art
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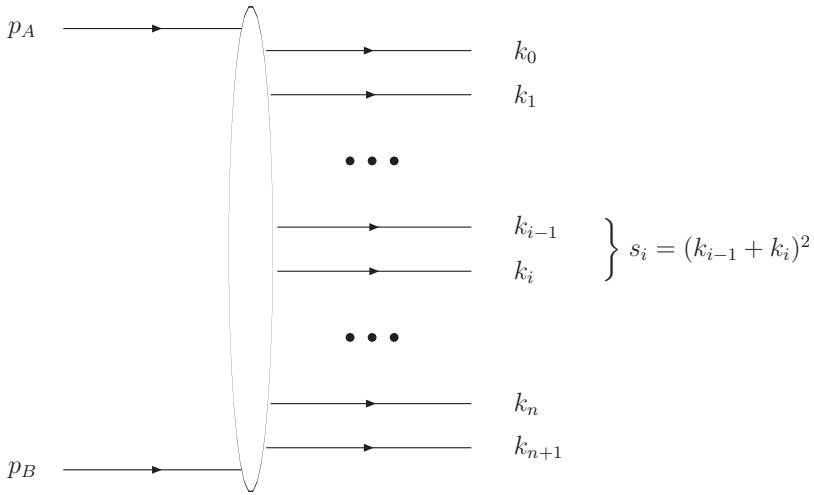
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- $Q^2 \sim \Lambda^2_{QCD} \;\Rightarrow\; \alpha_s\left(Q^2\right) \sim 1 \;\Rightarrow$ Soft processes $\Rightarrow$ non-perturbative theory.

$$\Downarrow$$

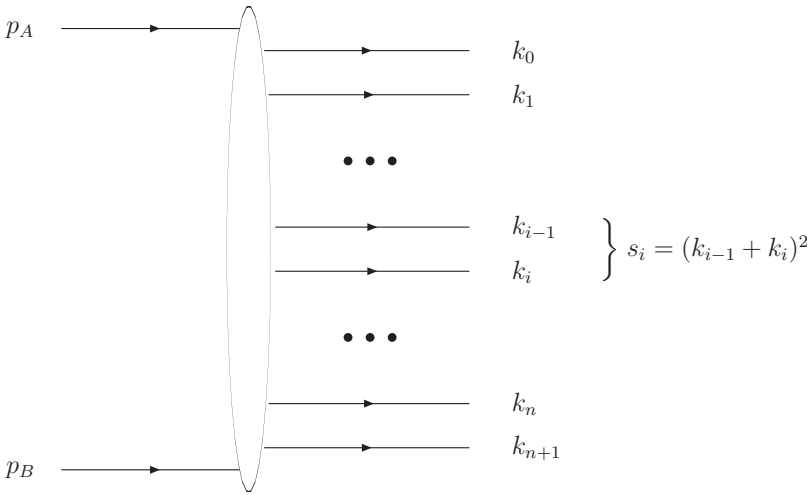
Regge Theory

What happens?



High-Energy Physics

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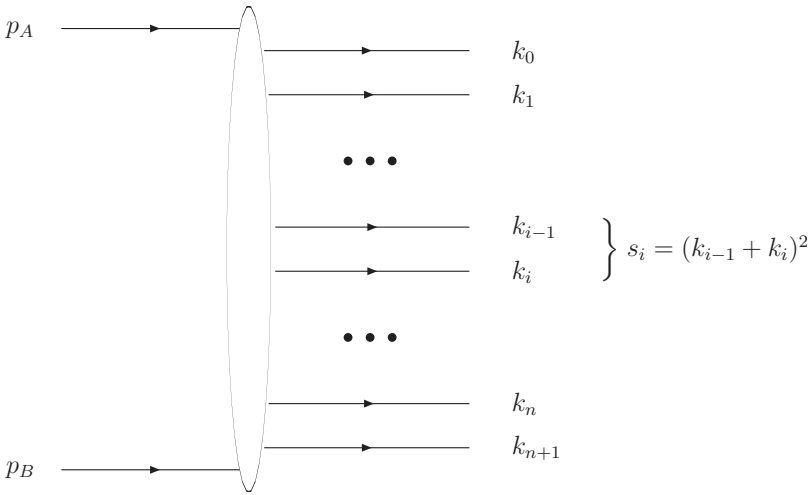
$$\alpha_s \ll 1 \quad \& \quad l \gtrsim 1$$

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Need of resummation

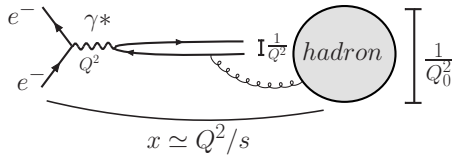
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What happens?



Which kind of log?

$Q^2 >> Q_0^2 \Rightarrow$ perturbative : DIS



$x \simeq Q^2/s$

DGLAP : $x \sim 1 \Rightarrow [\alpha_s \ln(Q^2/\mu_R^2)]^n$

BFKL : $x \to 0 \Rightarrow [\alpha_s \ln(1/x)]^n$

Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP)

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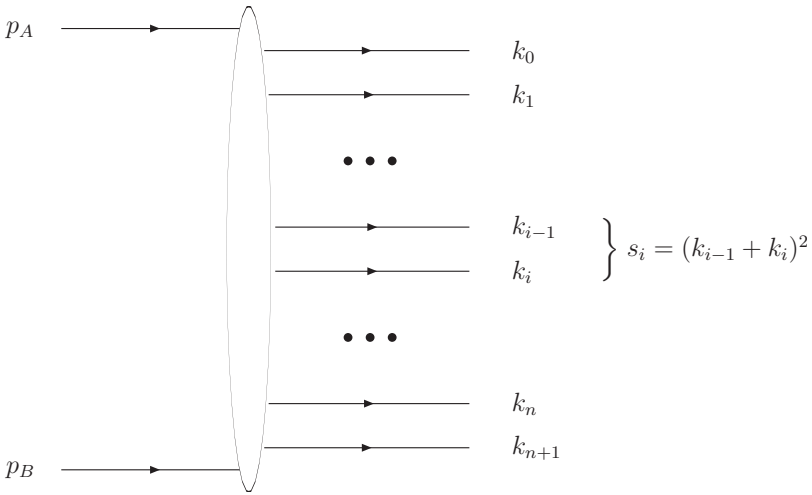
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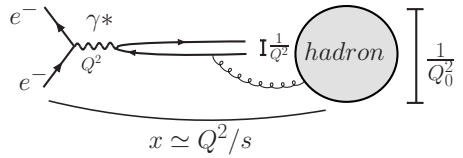
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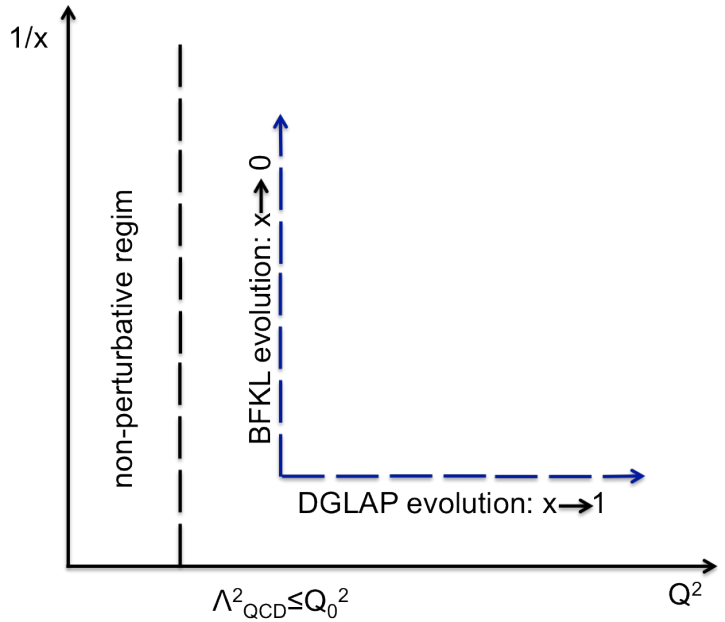
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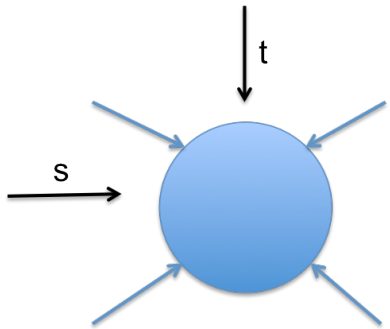
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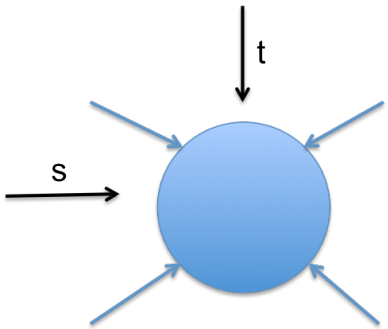
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$s \gg t$, soft processes

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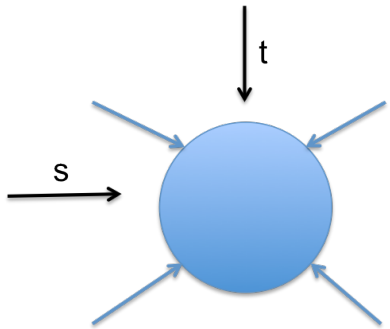
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S matrix postulates

- Unitarity
- Analyticity
- Crossing

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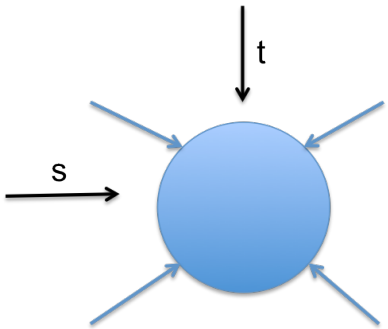
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$$\sigma_{\text{tot}} = \frac{\text{Im}\mathcal{A}(s,0)}{s} \sim s^{j(0)-1} = s^{\omega}$$

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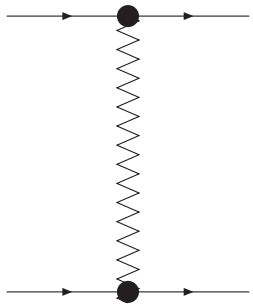
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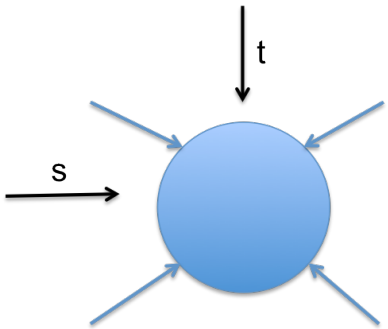
Exchange of "something" with vacuum quantum numbers



[Pomeranchuk (1956)]
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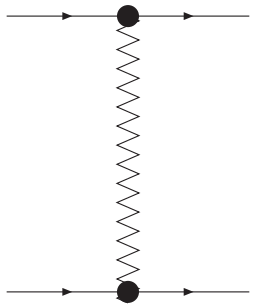
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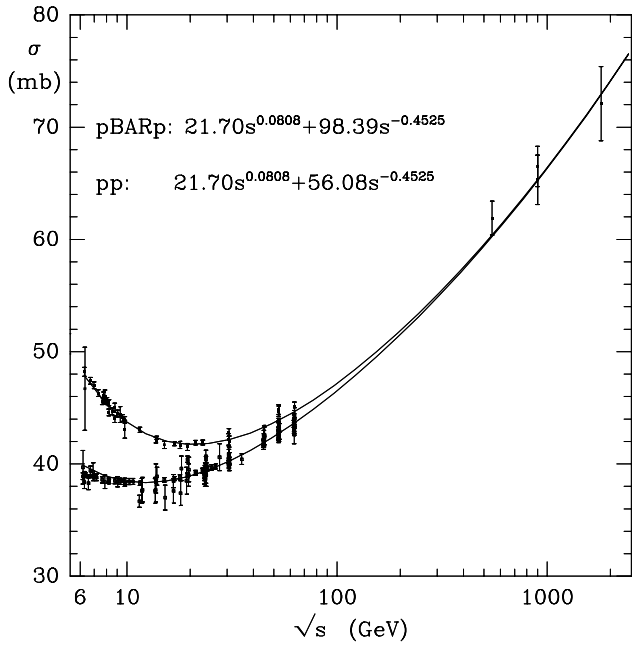
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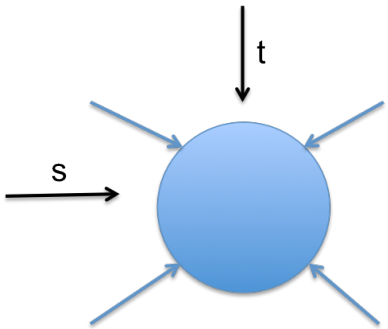
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Experimentally total cross sections rise slowly as s increases



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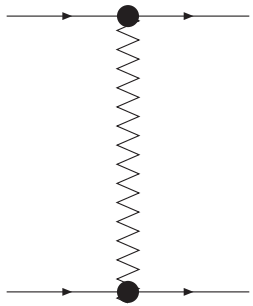
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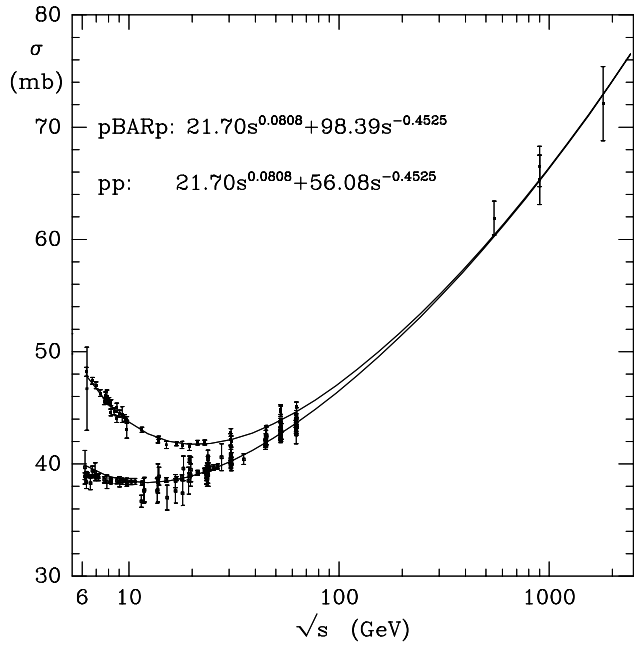
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Hypothesis of a single Reggeon

$$\Downarrow$$

$$j(0) > 1$$

Soft Pomeron:

$$j_P(0) \simeq 1.08 \Rightarrow \omega_p \simeq 0.08$$

[Donnachie and Landshoff (1992)]

QCD in the Regge limit: the BFKL approach

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HERA $\quad \alpha_s\left(Q^2\right) \simeq 0.15 \quad \longrightarrow \quad \omega_{P_{BFKL}} \simeq 0.4 \longrightarrow \quad \omega_{P_{soft}} \neq \omega_{P_{BFKL}}$

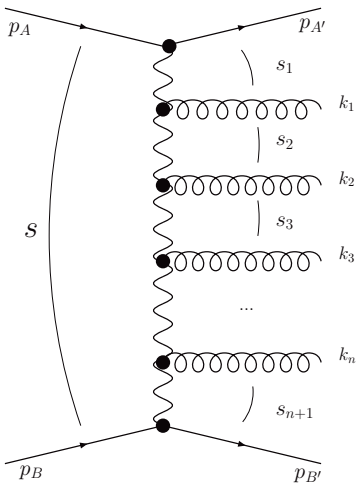
The BFKL approach became popular due to experimental results on **DIS** at **HERA**

Next-to-leading logarithmic approximation

Leading logarithmic approximation (LLA)

$$\left[\alpha_s \ln\left(\frac{s}{s_0}\right)\right]^n$$

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  - $y_1 \gg y_2 \gg y_3 \gg y_4 \gg \ldots$
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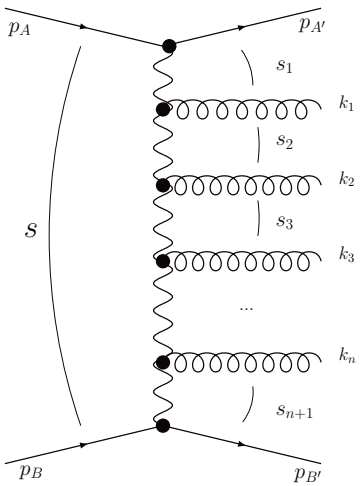


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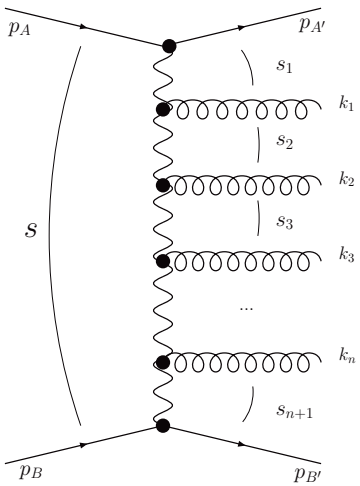
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Next-to-leading logarithmic approximation (NLLA)

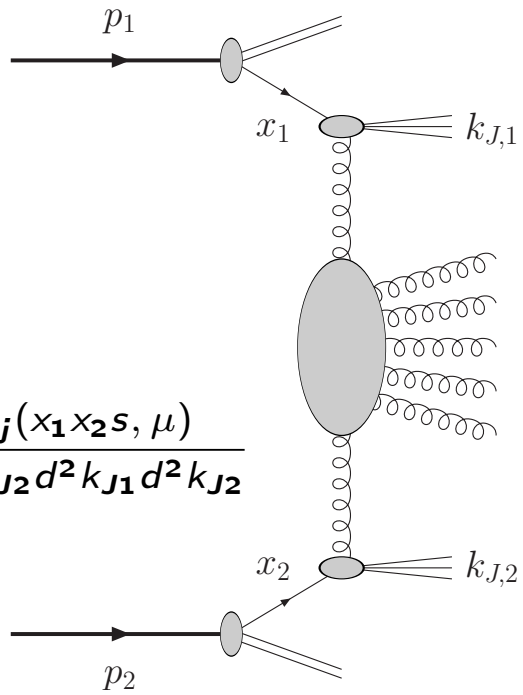
$$\alpha_s \left[\alpha_s \ln\left(\frac{s}{s_0}\right)\right]^{n-1}$$

- Quasi-Multi-Regge-Kinematics (**QMRK**)
  - $y_1 \gg y_2 \simeq y_3 \gg y_4 \gg \ldots$
- Running coupling
- Introduces two scales: μ_R and s_0

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Mueller-Navelet jets



$$\text{proton}(\text{p}_1) + \text{proton}(\text{p}_2) \to \text{jet}_1(\text{k}_1) + \text{jet}_2(\text{k}_2) + \text{X}$$

$$\frac{d\sigma}{dx_{J1}\,dx_{J2}\,d^2k_{J1}\,d^2k_{J2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_{J1}\,dx_{J2}\,d^2k_{J1}\,d^2k_{J2}}$$

Sudakov decomposition:

$$k_{J,1} = x_{J,1} p_1 + \frac{\vec{k}^2_{J,1}}{x_{J,1} s} p_2 + k_{J,1\perp}\,, \quad k^2_{J,1\perp} = -\vec{k}^2_{J,1}\,, \quad s = 2p_1 \cdot p_2$$

$$k_{J,2} = x_{J,2} p_2 + \frac{\vec{k}^2_{J,2}}{x_{J,2} s} p_1 + k_{J,2\perp}\,, \quad k^2_{J,2\perp} = -\vec{k}^2_{J,2}$$

- large jet transverse momenta: $\vec{k}^2_{J,1} \sim \vec{k}^2_{J,2} \gg \Lambda^2_{\rm QCD}$ $\longrightarrow$ perturbative QCD applicable
- large rapidity gap between jets, $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$, $\longrightarrow$ $s = 2p_1 \cdot p_2 \gg \vec{k}^2_{J,1\,2}$

  $\longrightarrow$ BFKL resummation: $\sum_n c_n\, \alpha_s^n \ln^n s + d_n\, \alpha_s^n \ln^{n-1} s$

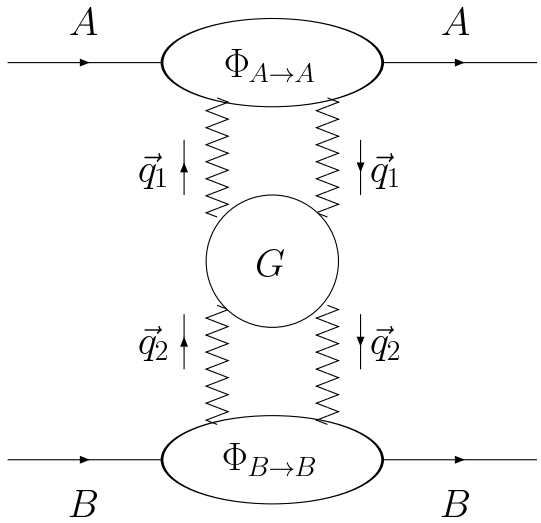
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The BFKL approach

Total cross section $A + B \to X$, via the optical theorem, $\sigma = \frac{\mathrm{Im}_s \mathcal{A}}{s}$

- **Regge limit** ($s \to \infty$)
  $\Rightarrow$ BFKL factorization for $\mathrm{Im}_s \mathcal{A}$:
  convolution of the **Green's function** of two interacting Reggeized gluons and of the **impact factors** of the colliding particles.



- Valid both in
  LLA (resummation of all terms $(\alpha_s \ln s)^n$)
  NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s \mathcal{A} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- The **Green's function** is **process-independent** and is determined through the **BFKL equation**. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\ .$$

Solution of the BFKL equation

- The Green's function obeys the **BFKL equation**

$$\delta^2(\vec{q}_1 - \vec{q}_2) = \omega\, G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_2)$$

- Transverse momentum space definition

$$\hat{\vec{q}}\,|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle\,, \quad \langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2)\,, \quad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A^*(\vec{k})B(\vec{k})$$

- The **kernel operator** $\hat{K}$ is

$$K(\vec{q}_2, \vec{q}_1) = \langle\vec{q}_2|\hat{K}|\vec{q}_1\rangle$$

- Straightforward solution in the transverse momentum space

$$\hat{1} = (\omega - \hat{K})\hat{G}_\omega \quad \longrightarrow \quad \hat{G}_\omega = (\omega - \hat{K})^{-1}\,, \qquad \hat{K} = \bar{\alpha}_s\hat{K}^0 + \bar{\alpha}_s^2\hat{K}^1\,, \quad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

- Solution for the $\hat{G}_\omega$ operator with NLA accuracy

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s\hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s\hat{K}^0)^{-1}\left(\bar{\alpha}_s^2\hat{K}^1\right)(\omega - \bar{\alpha}_s\hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2\hat{K}^1\right)^2\right]$$

- Basis of eigenfunctions of the LO kernel

$$\hat{K}^0|n,\nu\rangle = \chi(n,\nu)|n,\nu\rangle\,, \qquad \chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$\langle \vec{q}\,|n,\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} e^{in\theta}\,, \qquad \cos\theta \equiv q_x$$

$$\langle n',\nu'|n,\nu\rangle = \int \frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu-i\nu'-1} e^{i(n-n')\theta} = \delta(\nu-\nu')\,\delta_{nn'}$$

- The action of the full NLO BFKL kernel on these functions may be expressed as follows:

$$\begin{aligned}\hat{K}|n,\nu\rangle \;&=\; \bar{\alpha}_s(\mu_R)\chi(n,\nu)|n,\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\ln(\mu_R^2)\right)|n,\nu\rangle \\ &+\; \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(n,\nu)\left(i\frac{\partial}{\partial\nu}\right)|n,\nu\rangle\,,\end{aligned}$$

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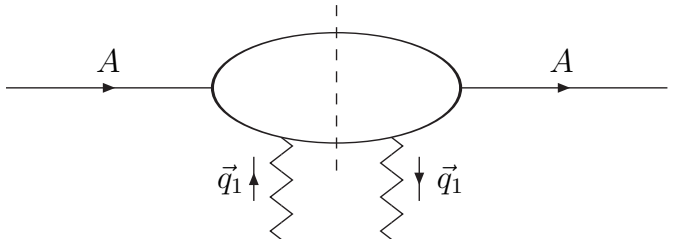
Impact factors

- Projection of the impact factors $\phi_i(\vec{q})$ onto the eigenfunctions of LO BFKL kernel, i.e. the transfer to the (n,ν)-representation

$$c_i(n,\nu) = \int d^2\vec{q}\,\frac{\phi_i(\vec{q})}{\vec{q}^{\,2}}\,\frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} e^{in\phi}\,.$$

Here ϕ is the azimuthal angle of the vector $\vec{q}$ counted from some fixed direction in the transverse space.

- **Impact factors** are **process-dependent**;



only very few have been calculated in the NLA:

- colliding partons

  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

- $\gamma^* \longrightarrow V$, with $V = \rho^0, \omega, \phi$, forward case

  [D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

- forward jet production

  [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. M., A. Papa, A. Perri (2012)]
  (small-cone approximation) [D.Yu. Ivanov, A. Papa (2012)]

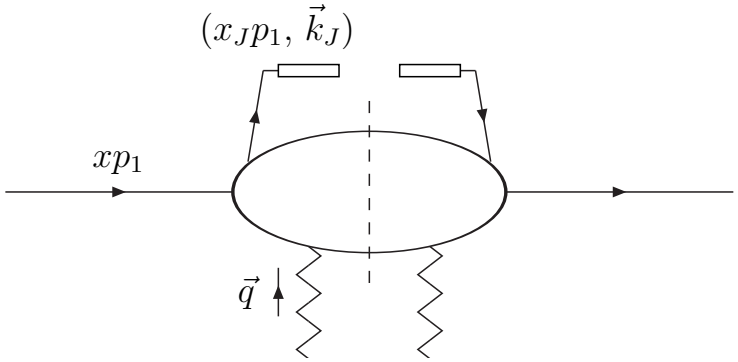
- forward identified hadron production

  [D.Yu. Ivanov, A. Papa (2012)]

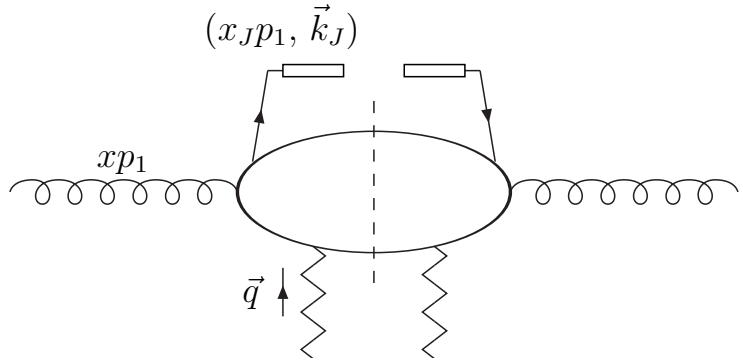
- $\gamma^* \longrightarrow \gamma^*$

  [J. Bartels *et al.* (2001) $\rightarrow$]
  [I. Balitsky, G.A. Chirilli (2011)-(2013)]

- **Step 1**: "open" one of the integrations over the phase space of the intermediate state to allow one parton to form a jet

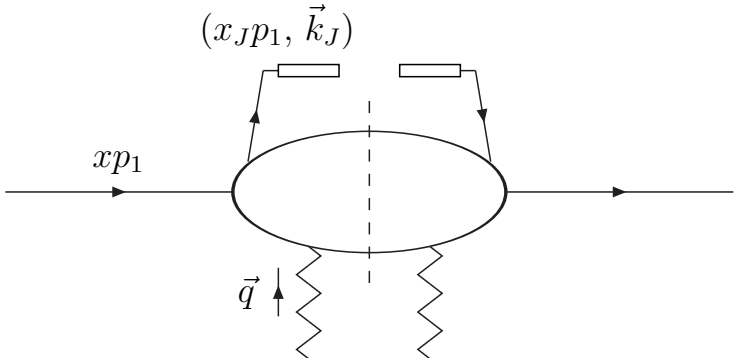


quark vertex

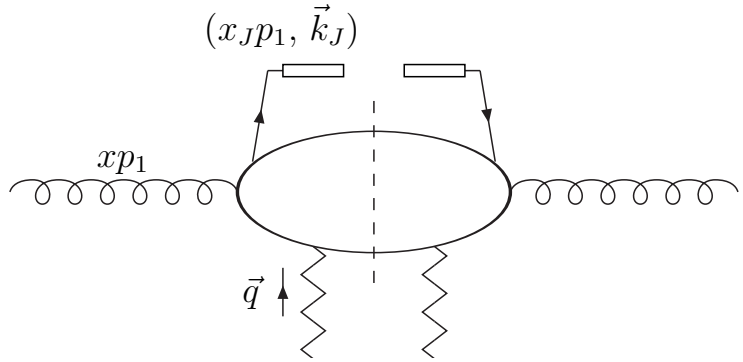


gluon vertex

- **Step 1**: "open" one of the integrations over the phase space of the intermediate state to allow one parton to form a jet



quark vertex

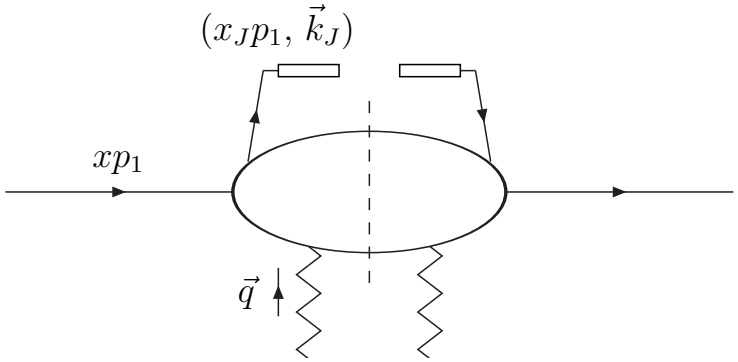


gluon vertex

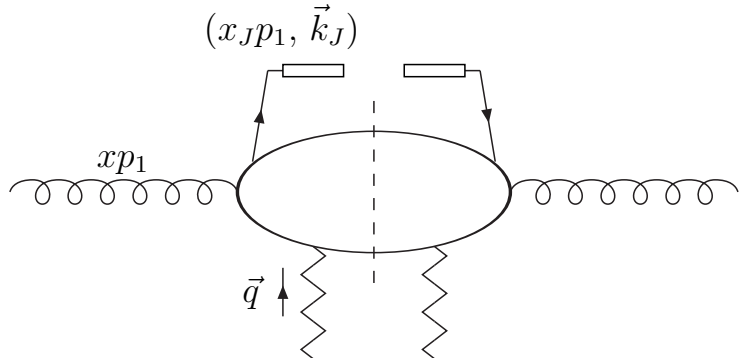
- **Step 2**: take the convolution with the PDFs and the jet function

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes S_J \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes S_J$$

- **Step 1**: "open" one of the integrations over the phase space of the intermediate state to allow one parton to form a jet



quark vertex



gluon vertex

- **Step 2**: take the convolution with the PDFs and the jet function

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes S_J \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes S_J$$

- **Step 3**: project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer to the (ν, n)-representation

$$\Phi(\nu, n) = \int d^2\vec{q}\, \frac{\Phi(\vec{q})}{\vec{q}^{\,2}} \frac{1}{\pi\sqrt{2}} \left(\vec{q}^{\,2}\right)^{\gamma - \frac{n}{2}} \left(\vec{q}\cdot\vec{l}\,\right)^n , \qquad \gamma = i\nu - \frac{1}{2} , \qquad \vec{l}^{\,2} = 0$$

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   - The Impact Factor in the NLA
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Jet definition

LO: one-particle final state.

The kinematics of the produced parton *a* is completely fixed by the jet kinematics.

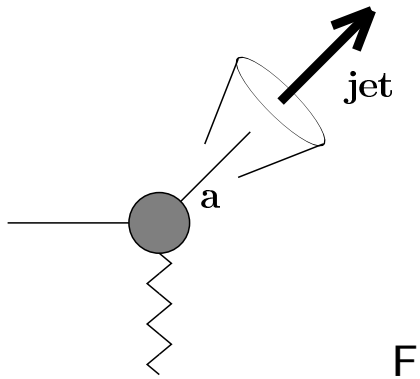


Fig. 1

NLO, virtual corrections: one-particle final state (see Fig. 1).

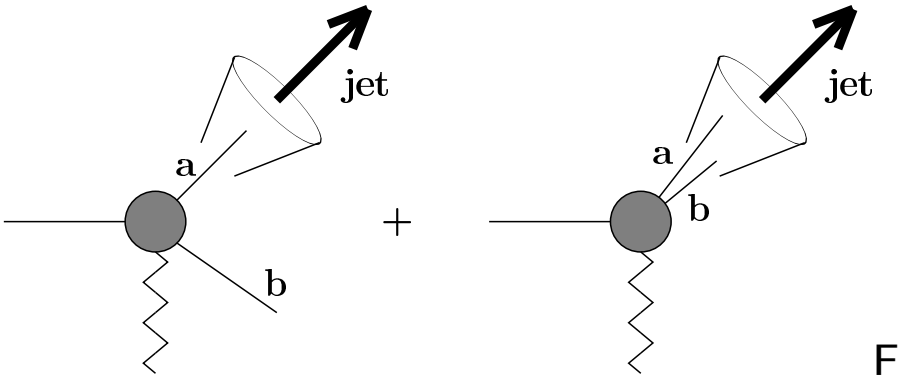


Fig. 2

NLO, real corrections: two-particle final state (see Fig. 2).

Outline

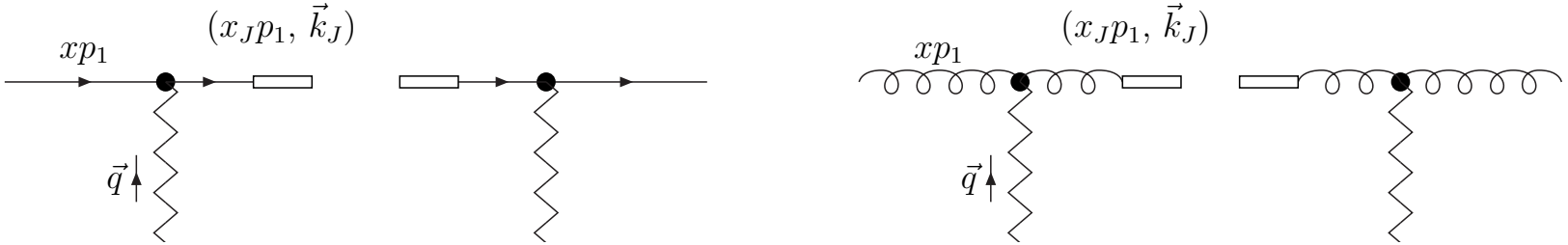
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The Impact Factor in the LLA

The starting point is given by "inclusive" LO parton impact factors:

$$\Phi_q = g^2 \frac{\sqrt{N^2-1}}{2N}\,, \qquad \Phi_g = \frac{C_A}{C_F}\Phi_q$$

- Step 1: "open" the integration over the one-particle final state, i.e. introduce a delta function



- Step 2: take the convolution with the PDFs and the jet function ($D = 4 + 2\varepsilon$)

$$\frac{d\Phi_J^{(0)}(\vec{q})}{dJ} = \Phi_q \int_0^1 dx \int d^{D-2}\vec{k}\,\delta^{(D-2)}\left(\vec{k}-\vec{q}\right) S_J^{(2)}(\vec{q};\, x)$$

$$\times \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right)$$

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Collinear and QCD coupling counterterms

The collinear singularities which will arise in the NLA calculation are to be removed by the **renormalization of PDFs and QCD charge**:

$$f_q(x) = f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{qq}(z) f_q\left(\frac{x}{z},\mu_F\right) + P_{qg}(z) f_g\left(\frac{x}{z},\mu_F\right)\right]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\left[P_{gq}(z) f_q\left(\frac{x}{z},\mu_F\right) + P_{gg}(z) f_g\left(\frac{x}{z},\mu_F\right)\right]$$

$$\alpha_s = \alpha_s(\mu_R)\left[1 + \frac{\alpha_s(\mu_R)}{4\pi}\left(\frac{11 C_A}{3} - \frac{2 N_F}{3}\right)\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\right]$$

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\epsilon)}{\epsilon(4\pi)^\epsilon}$$

- **Collinear counterterm**:

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta \int_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

- **QCD renormalization counterterm**:

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{charge c.t}}}{dJ} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6} - \frac{N_F}{3}\right)\Phi_q^{(0)}$$

$$\times\int_0^1 dx\left(\frac{C_A}{C_F}f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right) S_J^{(2)}(\vec{q};x)$$

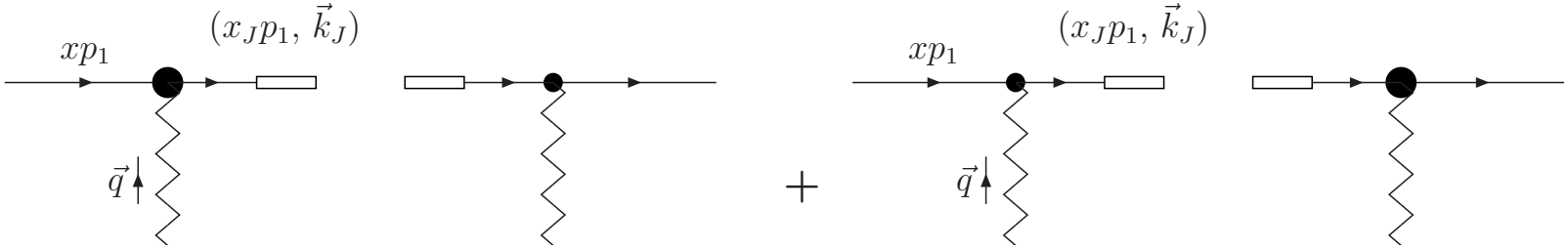
In the following

$$\frac{d\Phi_J^{(1)}(\vec{q}\,)}{dJ} \equiv \frac{\alpha_s}{2\pi}\Phi_q^{(0)}\,\mathbf{V}(\vec{\mathbf{q}}\,)$$

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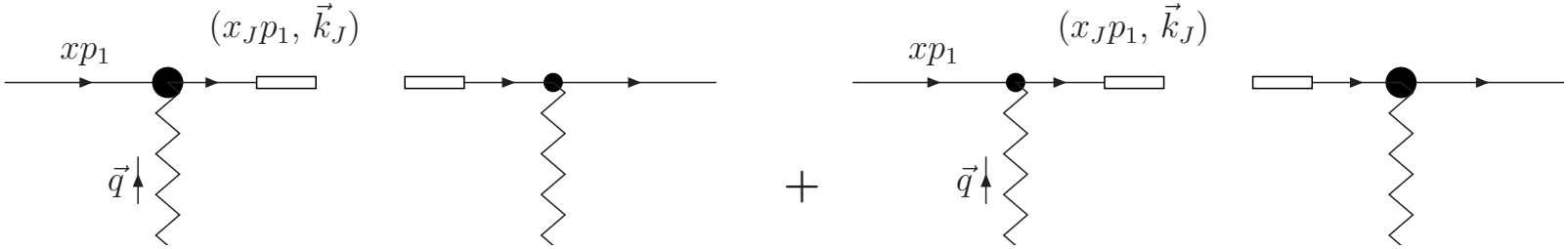
Virtual corrections



$$V_q^{(V)}(\vec{q}\,) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^{\varepsilon}}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\,(\vec{q}^{\,2})^{\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\, S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\frac{2}{\varepsilon}-3\right)+\left(\frac{11}{6}C_A-\frac{N_F}{3}\right)+C_A \ln\frac{s_0}{\vec{q}^{\,2}}\right]+\text{finite terms}$$

Virtual corrections



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QCD renormalization counterterm

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{charge c.t}}}{dJ} = \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}}+\ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6}-\frac{N_F}{3}\right)\Phi_q^{(0)}$$

$$\times\int_0^1 dx\left(\frac{C_A}{C_F}f_g(x)+\sum_{a=q,\bar{q}} f_a(x)\right) S_J^{(2)}(\vec{q};x)$$

"Real" corrections: quark-gluon intermediate state

Starting point

$$V_q^{(R)}(\vec{q}) = \frac{1}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 d\beta\, \mathcal{P}_{gq}(\varepsilon,\beta) \frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2\left(\vec{k}-\beta\vec{q}\right)^2}$$

$$\times\left\{C_F\beta^2\left(\vec{q}-\vec{k}\right)^2 + C_A(1-\beta)\,\vec{k}\cdot(\vec{k}-\beta\vec{q}\,)\right\} S_J^{(3)}\left(\vec{q}-\vec{k},\vec{k},x\beta;x\right)$$

β, $1-\beta$ and $\vec{k}$, $\vec{q}-\vec{k}$: relative longitudinal and transverse momenta of the gluon, quark

- **term proportional to C_F**

$$V_q^{(R)(C_F)}(\vec{q}) = C_F \frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\left[\left(\frac{2}{\varepsilon}-3\right) S_J^{(2)}(\vec{q}\,;x)\right.$$

$$\left. + \int_0^1 d\beta\, P_{qq}(\beta)\, S_J^{(2)}(\vec{q};x\beta)\right] + \text{finite terms}$$

"Real" corrections: quark-gluon intermediate state

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$$V_q^{(R)}(\vec{q}) = \frac{1}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 d\beta\, \mathcal{P}_{gq}(\varepsilon,\beta) \frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2\left(\vec{k}-\beta\vec{q}\right)^2}$$
$$\times\left\{C_F\beta^2\left(\vec{q}-\vec{k}\right)^2 + C_A(1-\beta)\,\vec{k}\cdot(\vec{k}-\beta\vec{q}\,)\right\} S_J^{(3)}\left(\vec{q}-\vec{k},\vec{k},x\beta;x\right)$$

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$$\left. + \int_0^1 d\beta\, P_{qq}(\beta)\, S_J^{(2)}(\vec{q};x\beta)\right] + \text{finite terms}$$

Virtual corrections

$$V_q^{(V)}(\vec{q}\,) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\,2}\right)^\varepsilon \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\, S_J^{(2)}(\vec{q}\,;x)$$
$$\times\left[C_F\left(\frac{2}{\varepsilon}-3\right) + \left(\frac{11}{6}C_A - \frac{N_F}{3}\right) + C_A \ln\frac{s_0}{\vec{q}^{\,2}}\right] + \text{finite terms}$$

Collinear counterterm

$$\frac{d\Phi_J(\vec{q}\,)|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta \int_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

Collinear counterterm

$$\frac{d\Phi_J(\vec{q})|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta \int_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

- **term proportional to C_A**

$$V_q^{(R)(C_A)}(\vec{q}) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\left[\int_0^1 d\beta\, \frac{\Gamma[1-\varepsilon]}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\frac{P_{gq}(\beta)}{C_F}\, S_J^{(2)}(\vec{q};x\beta)\right.$$

$$\left. + \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\,\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}\, S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

Collinear counterterm

$$\frac{d\Phi_J(\vec{q})|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta \int_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

- **term proportional to C_A**

$$V_q^{(R)(C_A)}(\vec{q}) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\left[\int_0^1 d\beta\,\frac{\Gamma[1-\varepsilon]}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\frac{P_{gq}(\beta)}{C_F}S_J^{(2)}(\vec{q};x\beta)\right.$$

$$\left. + \int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel

Collinear counterterm

$$\frac{d\Phi_J(\vec{q})|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta \int_0^1 dx\, S_J^{(2)}(\vec{q}\,;\beta x)$$

$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x) + P_{qg}(\beta)f_g(x)\right) + \frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x) + P_{gq}(\beta)\sum_{a=q,\bar{q}} f_a(x)\right)\right]$$

- **term proportional to C_A**

$$V_q^{(R)(C_A)}(\vec{q}) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)\left[\int_0^1 d\beta\,\frac{\Gamma[1-\varepsilon]}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\frac{P_{gq}(\beta)}{C_F}S_J^{(2)}(\vec{q};x\beta)\right.$$

$$\left. + \int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\,2}}{\vec{k}^{\,2}\left(\vec{q}-\vec{k}\right)^2}\ln\frac{s_0}{\left(|\vec{k}|+|\vec{q}-\vec{k}|\right)^2}S_J^{(2)}\left(\vec{q}-\vec{k};x\right)\right] + \text{finite terms}$$

After the **projection** onto the eigenfunctions of the LO BFKL kernel

Virtual corrections

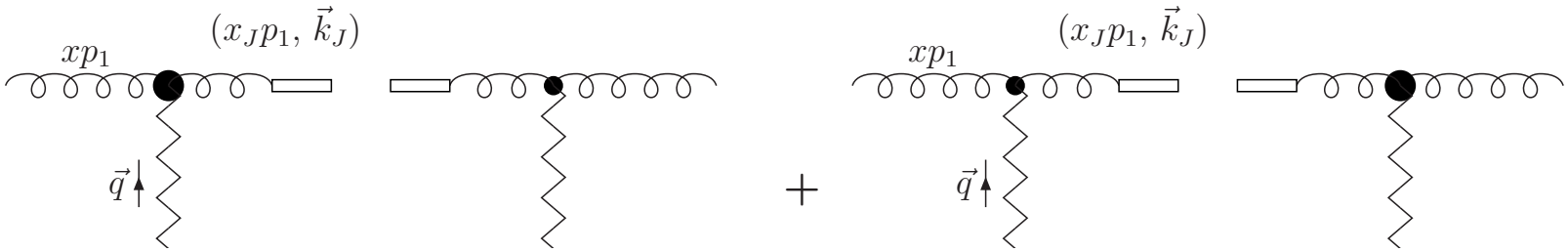
$$V_q^{(V)}(\vec{q}) = -\frac{1}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\vec{q}^{\,2}\right)^\varepsilon\int_0^1 dx\sum_{a=q,\bar{q}} f_a(x)\,S_J^{(2)}(\vec{q}\,;x)$$

$$\times\left[C_F\left(\cancel{\frac{2}{\varepsilon}} - \cancel{3}\right) + \left(\cancel{\frac{11}{6}}C_A - \cancel{\frac{N_F}{3}}\right) + C_A\ln\frac{s_0}{\vec{q}^{\,2}}\right] + \text{finite terms}$$

Outline

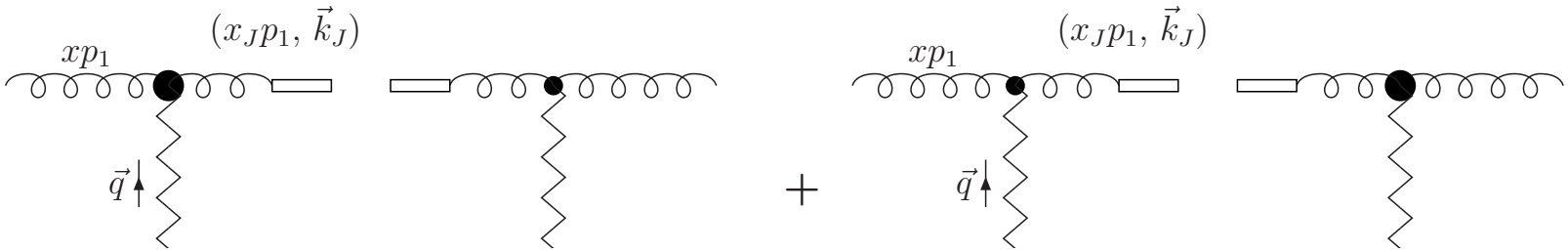
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Virtual corrections



$$V_g^{(V)}(\vec{q}) = -\frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^{\varepsilon}}\,\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\,(\vec{q}^{\,2})^{\varepsilon}\int_0^1 dx\,\frac{C_A}{C_F}\,f_g(x)\,S_J^{(2)}(\vec{q}\,;x)$$
$$\times\left[\left(\frac{N_F}{3}-\frac{11}{6}C_A\right)+C_A\left(\ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)+\frac{2}{\varepsilon}\right)\right]+\text{finite terms}$$

Virtual corrections



$$V_g^{(V)}(\vec{q}) = -\frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^{\varepsilon}}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\,(\vec{q}^{\,2})^{\varepsilon}\int_0^1 dx\,\frac{C_A}{C_F}\,f_g(x)\,S_J^{(2)}(\vec{q}\,;x)$$
$$\times\left[\left(\frac{N_F}{3}-\frac{11}{6}C_A\right)+C_A\left(\ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)+\frac{2}{\varepsilon}\right)\right]+\text{finite terms}$$

QCD renormalization counterterm

$$\frac{d\Phi_J(\vec{q})|_{\text{charge c.t}}}{dJ}=\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}}+\ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6}-\frac{N_F}{3}\right)\Phi_q^{(0)}$$
$$\times\int_0^1 dx\left(\frac{C_A}{C_F}f_g(x)+\sum_{a=q,\bar{q}}\!\!\!\!\!\!\diagup\,f_a(x)\right)S_J^{(2)}(\vec{q};x)$$

"Real" corrections

Starting point ($T_R = 1/2$)

- **quark-antiquark intermediate state**:

$$V_g^{(R_{q\bar{q}})}(\vec{q}\,) = \frac{N_F}{(4\pi)^\varepsilon} \int_0^1 dx\, \frac{C_A}{C_F}\, f_g(x) \int_0^1 d\beta \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\, \frac{\vec{q}^{\,2}}{\vec{k}^{\,2}(\vec{k}-\vec{q}\,)^2} \times T_R \left(1 - \frac{2\beta\,(1-\beta)}{1+\varepsilon}\right)$$

$$\times \left[\frac{C_F}{C_A} + \frac{\beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})}{(\vec{k}-\beta\vec{q}\,)^2}\right] S_J^{(3)}(\vec{q}-\vec{k}, \vec{k}, x\beta; x)\ ,$$

β, $1-\beta$ and $\vec{k}$, $\vec{q}-\vec{k}$: relative longitudinal and transverse momenta of the quark, antiquark

- **two-gluon intermediate state**:

$$V_g^{(R_{gg})}(\vec{q}\,) = \frac{C_A}{(4\pi)^\varepsilon} \int_0^1 dx\, \frac{C_A}{C_F}\, f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^{1-\beta_0} d\beta \frac{\vec{q}^{\,2}\ \mathcal{P}_{gg}(\beta)}{(\vec{k}-\beta\vec{q}\,)^2\vec{k}^2(\vec{k}-\vec{q})^2}$$

$$\times \left\{\beta^2(\vec{k}-\vec{q}\,)^2 + (1-\beta)^2\vec{k}^2 - \beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})\right\} S_J^{(3)}(\vec{q}-\vec{k}, \vec{k}, x\beta; x)$$

where $\mathcal{P}_{gg}(\beta) = P(\beta) + P(1-\beta)$, with $P(\beta) = \left(\frac{1}{\beta} + \frac{\beta}{2}\right)(1-\beta)$

β, $1-\beta$ and $\vec{k}$, $\vec{q}-\vec{k}$: relative longitudinal and transverse momenta of the two gluons

How the divergences disappear

QCD ren. counterterm

Collinear counterterm

Virtual corrections

Real corrections

How the divergences disappear

QCD ren. counterterm

Collinear counterterm

Virtual corrections

Real corrections

$$\propto \frac{1}{\varepsilon} C_A \ \ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)$$

How the divergences disappear

QCD ren. counterterm

Collinear counterterm

Virtual corrections

Real corrections

$$\propto \frac{1}{\varepsilon} C_A \ \ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)$$

After the projection onto the eigenfunctions of the LO BFKL kernel

[F. Caporale, D.Yu. Ivanov, B. M., A. Papa, A. Perri, (2012)]

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Summary I

- We have recalculated, completely within the original BFKL approach at the next-to-leading order, the jet vertex relevant for the production of Mueller-Navelet jets in proton collisions.
- It has been shown that the result is free of divergences.
  An essential role has been played by the projection onto LO BFKL eigenfunctions.
- The energy scale s_0 remains untouched and need not be fixed at any definite value.
- Fixing the s_0 scale at the value suggested by J. Bartels, D. Colferai, G.P. Vacca (2003), we find a complete agreement before the projection.

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State of the art

Observables of interest:

- Differential cross section
- Moments of the azimuthal decorrelation of the jets

State of the art

Observables of interest:

- Differential cross section
- Moments of the azimuthal decorrelation of the jets

So far...

- Convolution of the NLLA Green's function with the LO jet vertices

  [A. Sabio Vera (2006)]
  [A. Sabio Vera, F. Schwennsen (2007)]
  [C. Marquet, C. Royon (2007)]

- Full NLO calculation

  [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]
  [B. Ducloué, L. Szymanowski, S. Wallon (2012)]

Necessary Ingredients

- Small Cone Approximation (**SCA**): small jet cone aperture in the rapidity-azimuthal angle plane. [D.Yu. Ivanov, A. Papa (2012)]
  - Easily implementable in numerical calculations.
  - Particularly suitable for a semi-analytical cross-check of the numerical approaches which treat the cone size exactly.
- Principle of Minimal Sensitivity (**PMS**): optimized method. [P.M. Stevenson (1981)]

Cross section

Moments of the azimuthal decorrelations

$$\langle \cos\left[n\left(\phi_{J_1} - \phi_{J_2} - \pi\right)\right]\rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}, \qquad \mathcal{R}_{m,n} = \frac{\langle \cos\left(m\Delta\phi\right)\rangle}{\langle \cos\left(n\Delta\phi\right)\rangle}$$

$$\mathcal{C}_n = \int_0^{2\pi} d\phi_{J_1} \int_0^{2\pi} d\phi_{J_2} \cos\left[n\left(\phi_{J_1} - \phi_{J_2} - \pi\right)\right] \frac{d\sigma_n}{dJ_1\, dJ_2}$$

$$\text{with } dJ_i = dx_{J_i} dk_{J_i}$$

and

$$\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)k^{(1)}(n,\nu)} \alpha_s^2\left(\mu_R\right) c_1\left(n,\nu\right) c_2\left(n,\nu\right) \times \left[1 + \bar{\alpha}_s\left(\mu_R\right)\left(\frac{c_1^{(1)}\left(n,\nu\right)}{c_1\left(n,\nu\right)} + \frac{c_2^{(1)}\left(n,\nu\right)}{c_2\left(n,\nu\right)}\right)\right]$$

with

$$k^{(1)}\left(n,\nu\right) = \bar{\chi}\left(n,\nu\right) + \frac{\beta_0}{8N_c}\chi\left(n,\nu\right)\left(-\chi\left(n,\nu\right) + \frac{10}{3} + \imath\frac{d}{d\nu}\ln\left(\frac{c_1\left(n,\nu\right)}{c_2\left(n,\nu\right)}\right) + 2\ln\left(\mu_R^2\right)\right)$$

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Kinematical settings

- In order to compare our predictions with the **forthcoming LHC data** we fix

$$R = 0.5$$

$$\sqrt{s} = 14 \text{ TeV} \qquad \text{and} \qquad \sqrt{s} = 7 \text{ TeV}$$

- Following a CMS study [CMS Collaboration, S. Cerci, D. d'Enterria (2009)], we restrict the rapidities of the jets to the region

$$3 \leq |y_J| \leq 5 \,, \qquad \text{with steps} \qquad \Delta y_J = 0.5$$

- Symmetric case:
  - $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV $\longrightarrow$ comparison with previous results.
    [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]
    [B. Ducloué, L. Szymanowski, S. Wallon (2013)]
  - $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 20$ GeV $\longrightarrow$ more undetected gluons radiated in the final state.

Our analysis

- We study the cross section and the azimuthal correlations versus the relative rapidity $Y = y_{J_1} - y_{J_2}$.
- Unavoidable terms beyond NLA depend on μ_R and s_0, whose numerical impact can be non-negligible $\Longrightarrow$ **optimization needed!**

Our analysis

- We study the cross section and the azimuthal correlations versus the relative rapidity $Y = y_{J_1} - y_{J_2}$.
- Unavoidable terms beyond NLA depend on μ_R and s_0, whose numerical impact can be non-negligible $\Longrightarrow$ **optimization needed!**

PMS

We take as optimal choices for μ_R and s_0 those values for which the physical observable under exam exhibits the minimal sensitivity to changes of both these scales. [P.M. Stevenson (1981)]

The complete resummation of the perturbative series would not depend on μ_R and s_0
$\Longrightarrow$ PMS is supposed to mimic the effect of the most relevant unknown subleading terms.

$$Y_0 = \ln \frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} = 0, 1, 2, ... \qquad \mu_R = N\sqrt{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \qquad N = 1, 2, 3, ...$$

$\Longrightarrow$ For each fixed value of Y we look for a stationary point in the $N - Y_0$ plane.

Outline

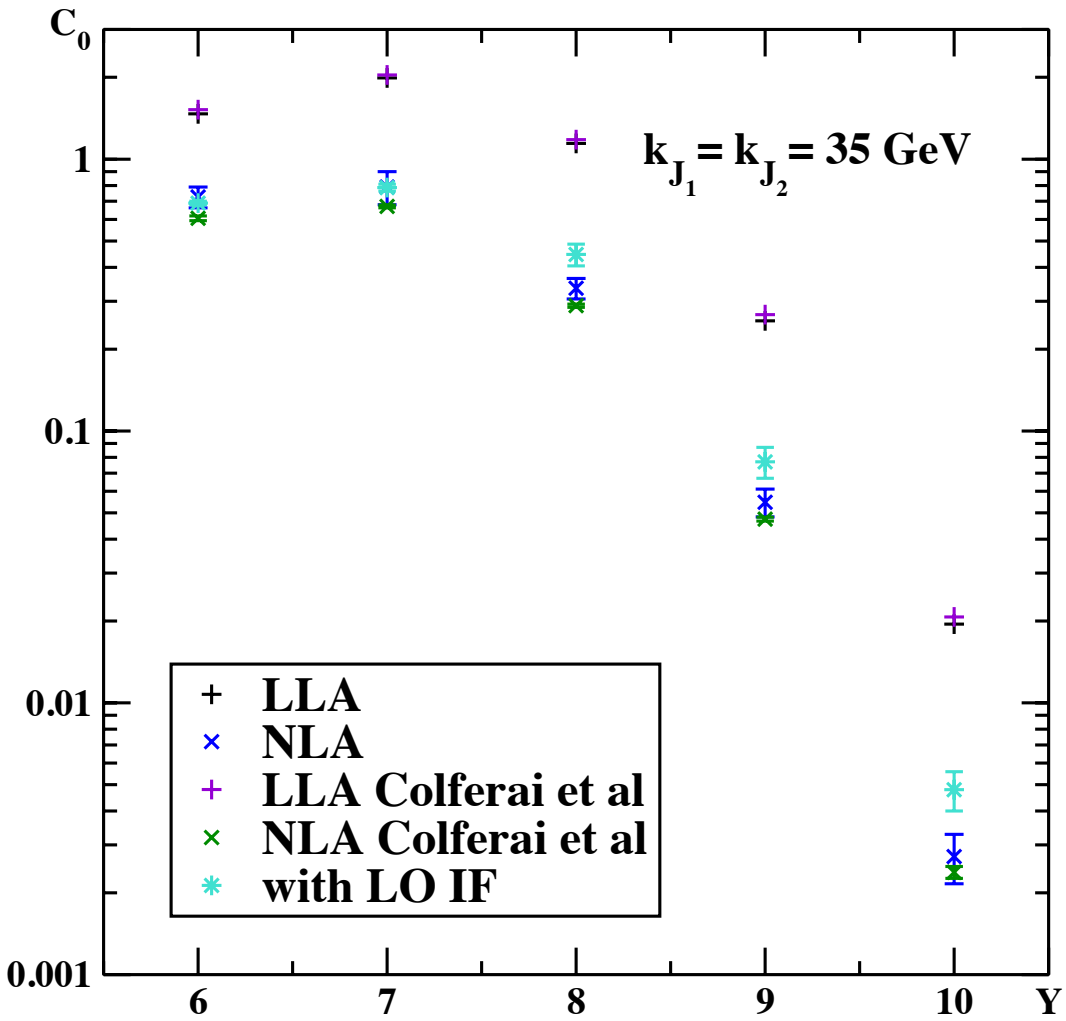
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Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV

Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	1	2
7	2	2
8	2	2
9	3	2
10	4	2

Values of the parameters corresponding to our "optimal" values.

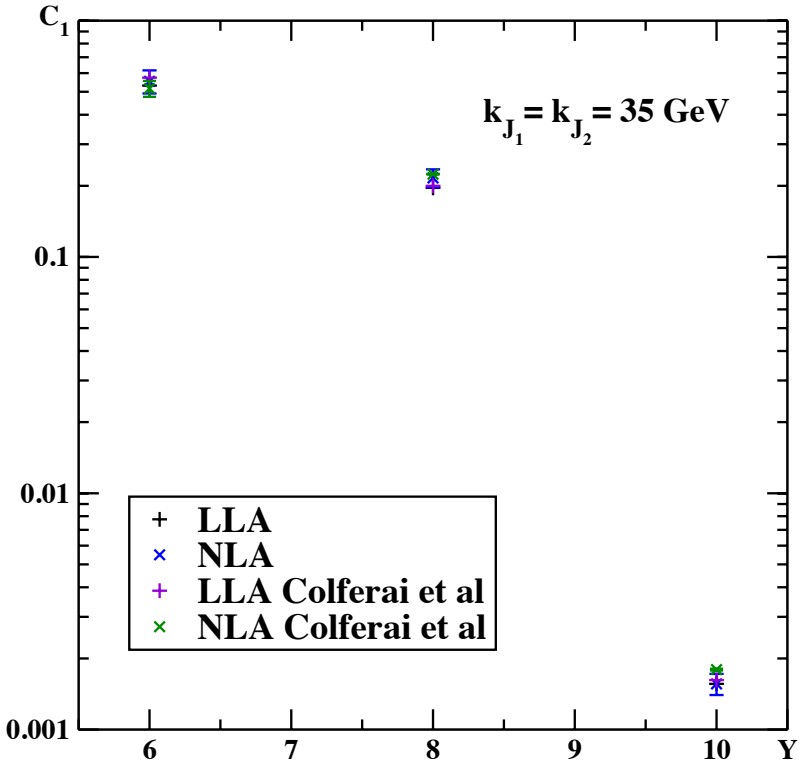
LLA: $\mu_R^2 = s_0 = |\vec{k}_{J_1}||\vec{k}_{J_2}|$

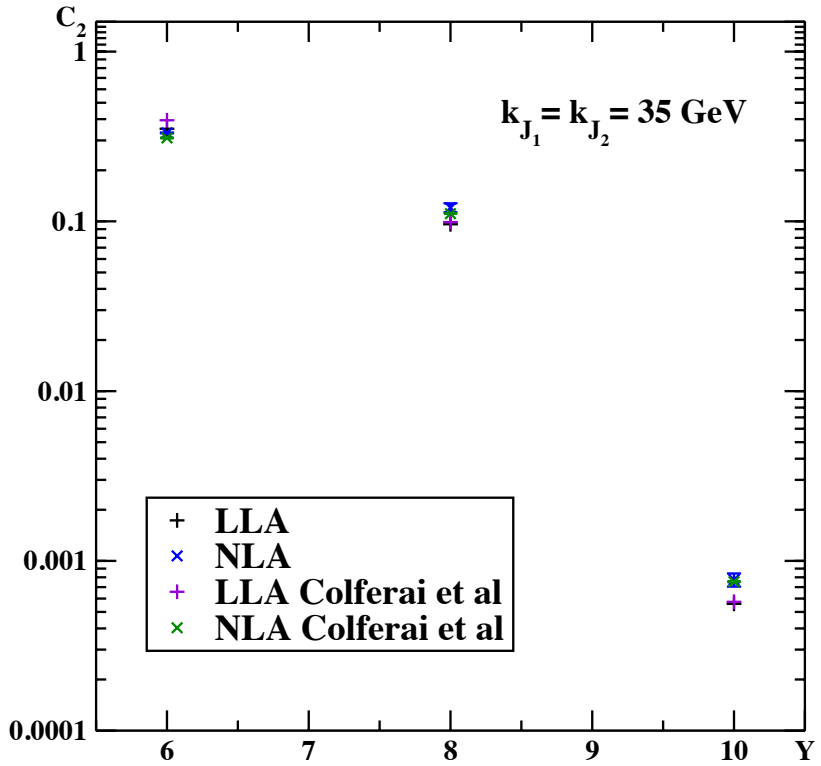


- We have found wide regions of stability.
- The uncertainty comes from the resolution of our grid in the N–Y_0 plane.
- Our predictions are compatible with previous calculations $\Longrightarrow$ reliability of the SCA.

[F. Caporale, D. Yu. Ivanov, B. M., A. Papa, (2013)]

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV





Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	1	2
8	2	2
10	3	2

Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	0	1.5
8	2	2.5
10	4	4

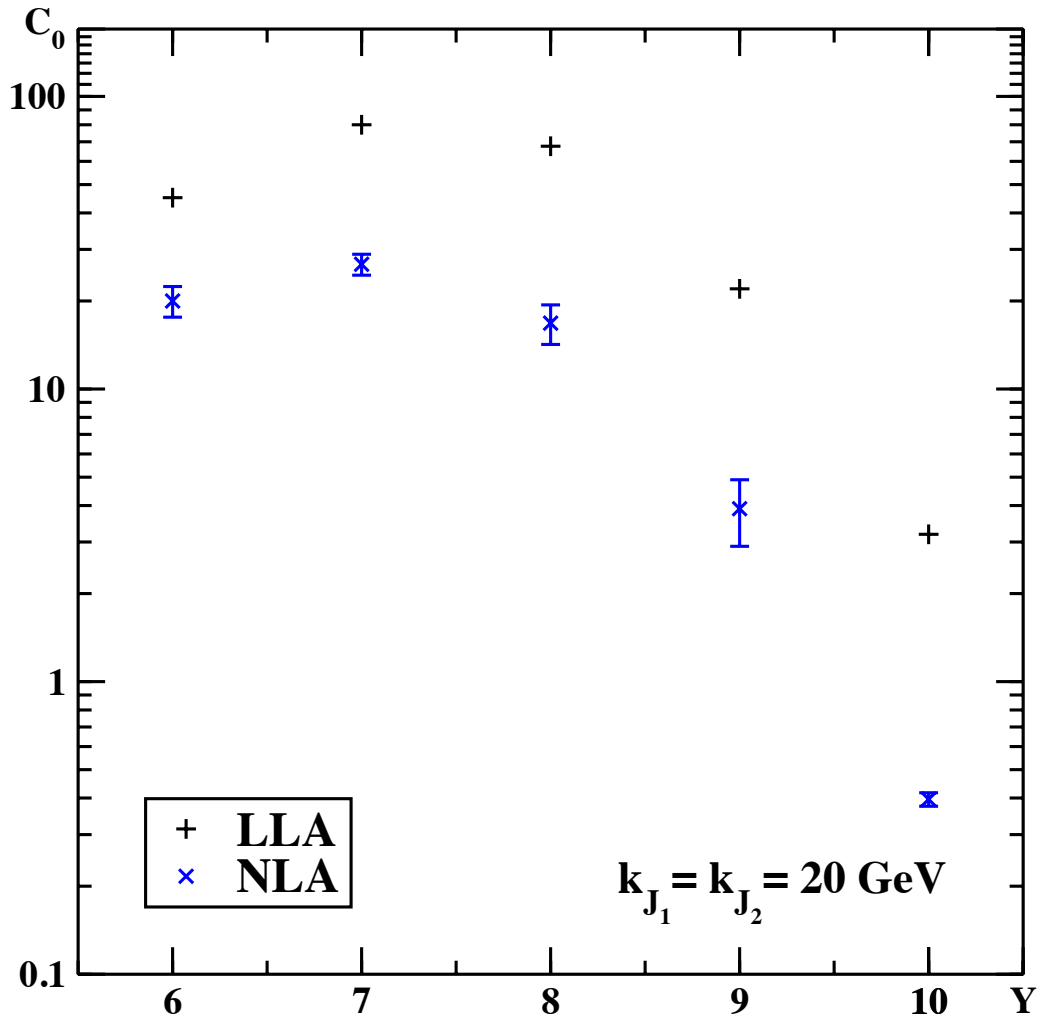
- It is possible to calculate $\langle\cos\phi\rangle$ and $\langle\cos(2\phi)\rangle$.
- Our predictions are compatible with previous calculations $\Longrightarrow$ reliability of the SCA.

[F. Caporale, D. Yu. Ivanov, B. M., A. Papa, (2013)]

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 20$ GeV

Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	1	1
7	1	1
8	2	1
9	3	1
10	3	2

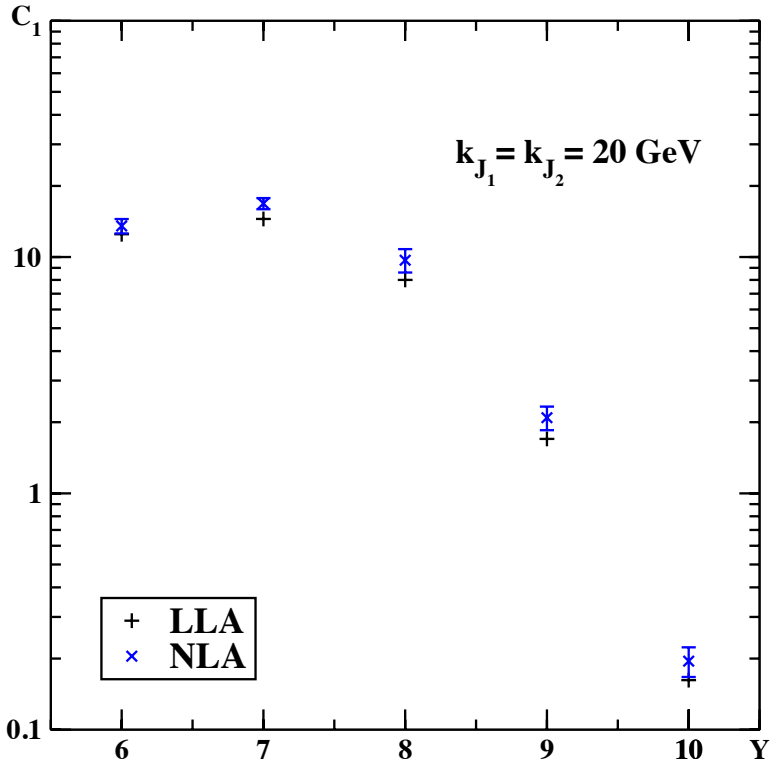
- We have found wide regions of stability.

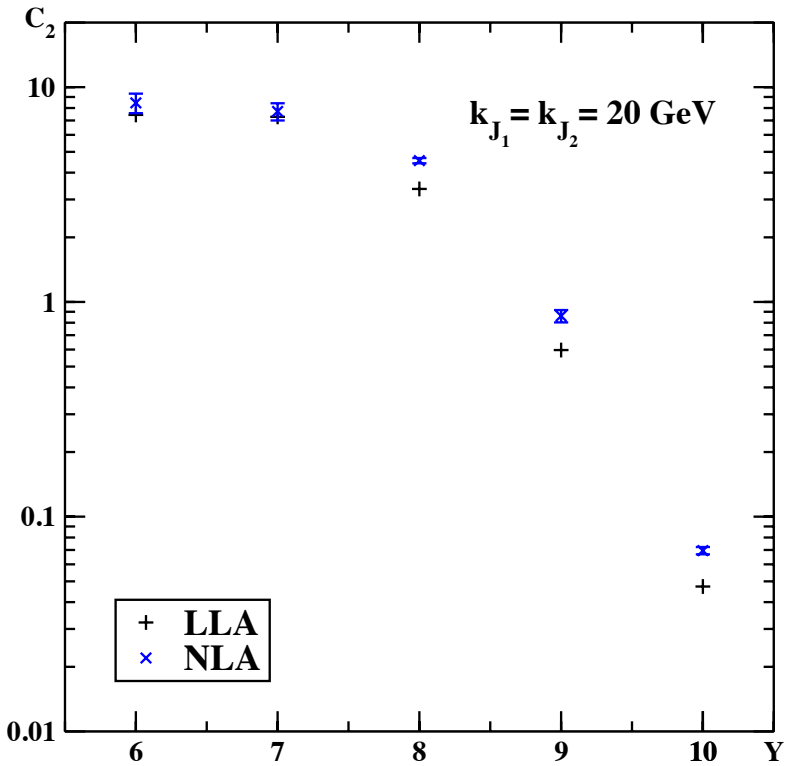


- Smaller transverse momentum $\Longrightarrow$ more undetected gluons $\Longrightarrow$ possible discrimination between BFKL and fixed-order approach.

[F. Caporale, D. Yu. Ivanov, B. M., A. Papa, (2013)]

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 20$ GeV





Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	0	1
7	0.5	1
8	1	1
9	1	1
10	1	1

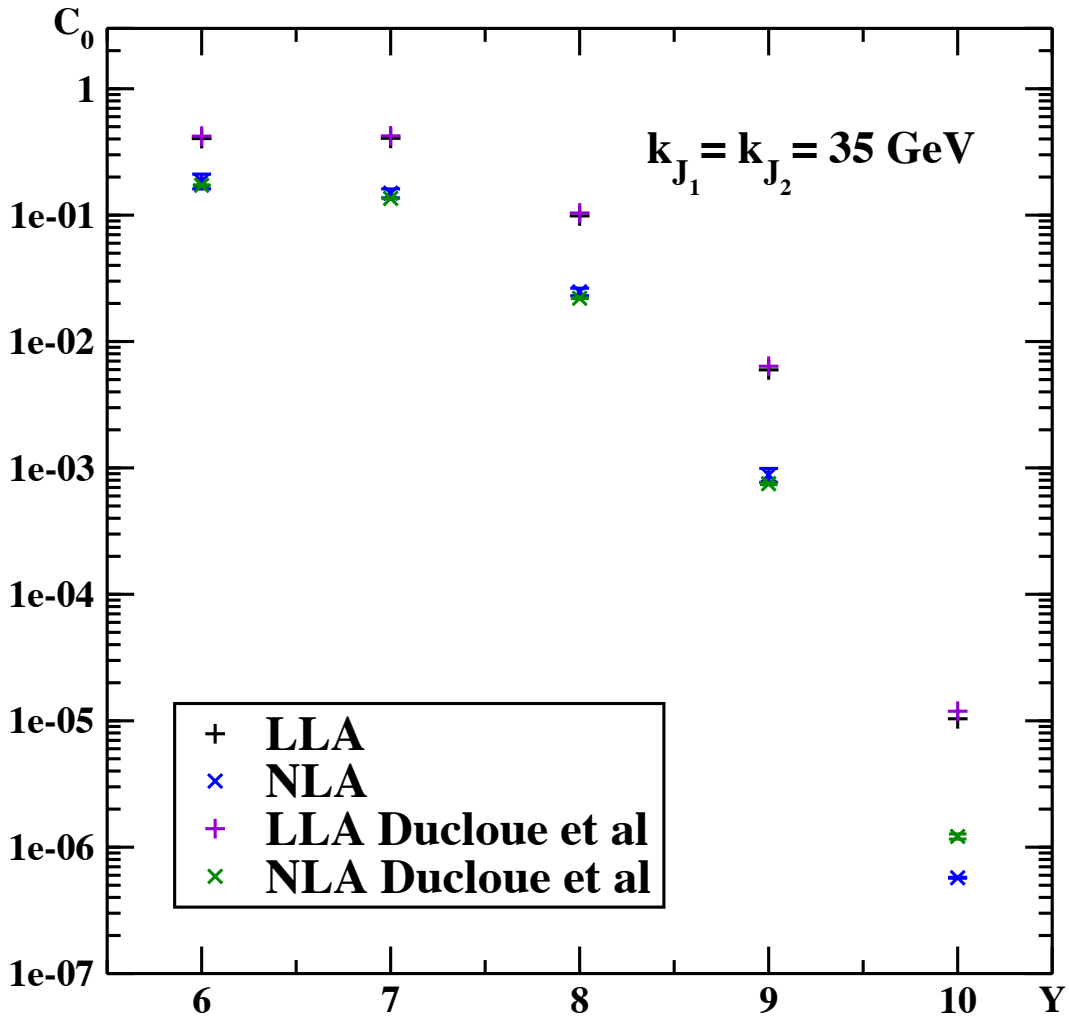
Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	1	3
7	0.5	1
8	2.5	4
9	3	5
10	3.5	3

[F. Caporale, D. Yu. Ivanov, B. M., A. Papa, (2013)]

Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV, $\sqrt{s} = 7$ TeV

Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	2	2
7	2	3
8	3	3
9	4	3
10	5.5	7

- We have found wide regions of stability.
- With our approach we reproduced the same results of previous calculations.



[F. Caporale, D. Yu. Ivanov, B. M., A. Papa, (2013)]

Asymmetric case

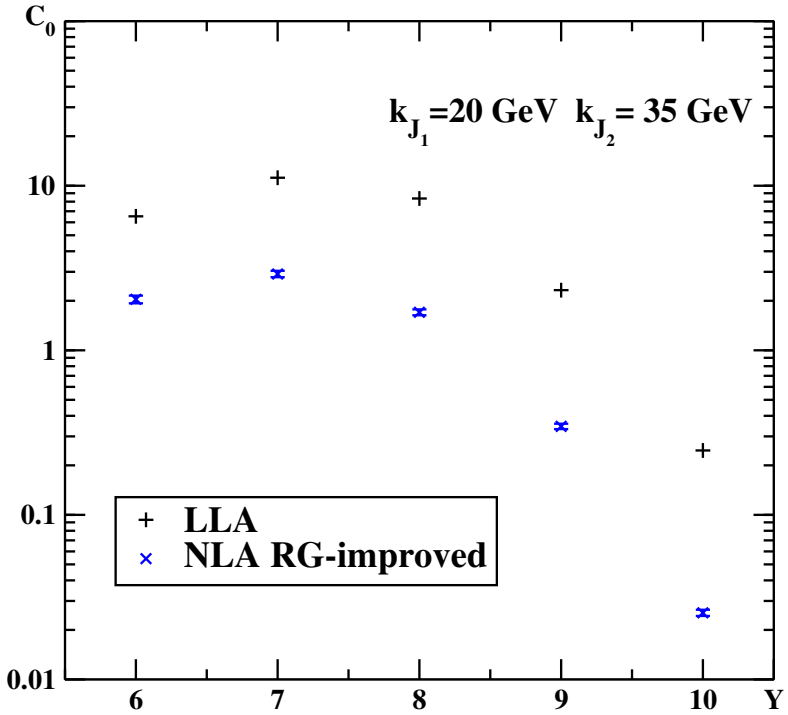
- Possible discrimination between BFKL and DGLAP
- Kinematical region: $\vec{k}_{J_1}^2 \ll \vec{k}_{J_2}^2 \Rightarrow$ Collinear improvement
- Modification of the BFKL kernel

$$\begin{aligned}\hat{K}'|n,\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(n,\nu)|n,\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\ln(\mu_R^2)\right)|n,\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(n,\nu)\left(i\frac{\partial}{\partial\nu}\right)|n,\nu\rangle + \chi_{RG}(n,\nu)\,|n,\nu\rangle\end{aligned}$$

with $\chi_{RG}(n,\nu)$ [A. Sabio Vera (2005)-(2006)], [A. Sabio Vera, F. Schwennsen (2007)]

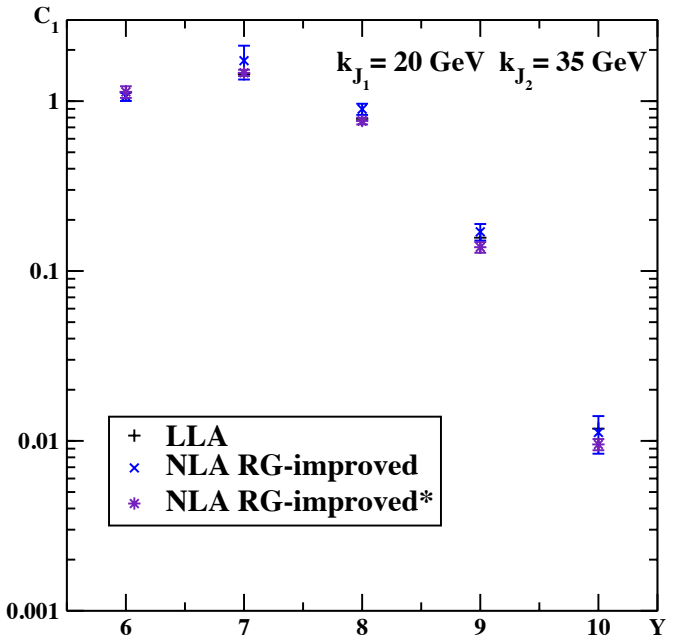
- the eigenvalue of the collinear contributions
- $\mathcal{O}\left(\alpha_s^3\right)$

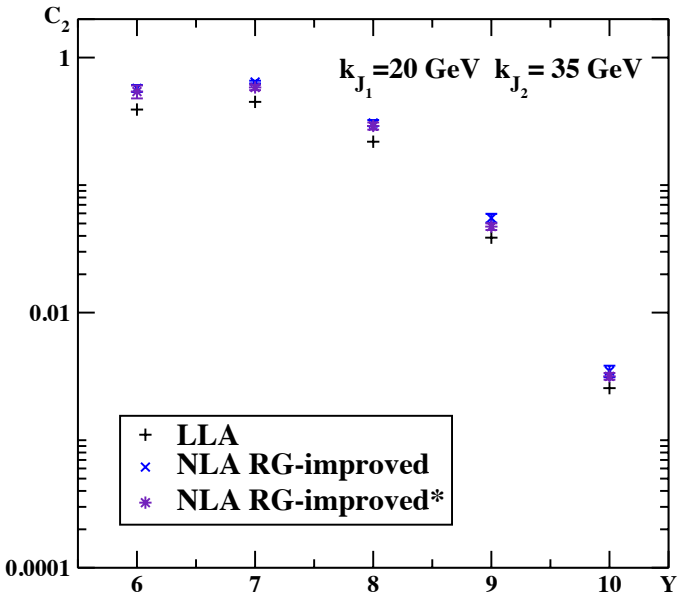
Results: $|\vec{k}_{J_1}| = 20$ GeV, $|\vec{k}_{J_2}| = 35$ GeV



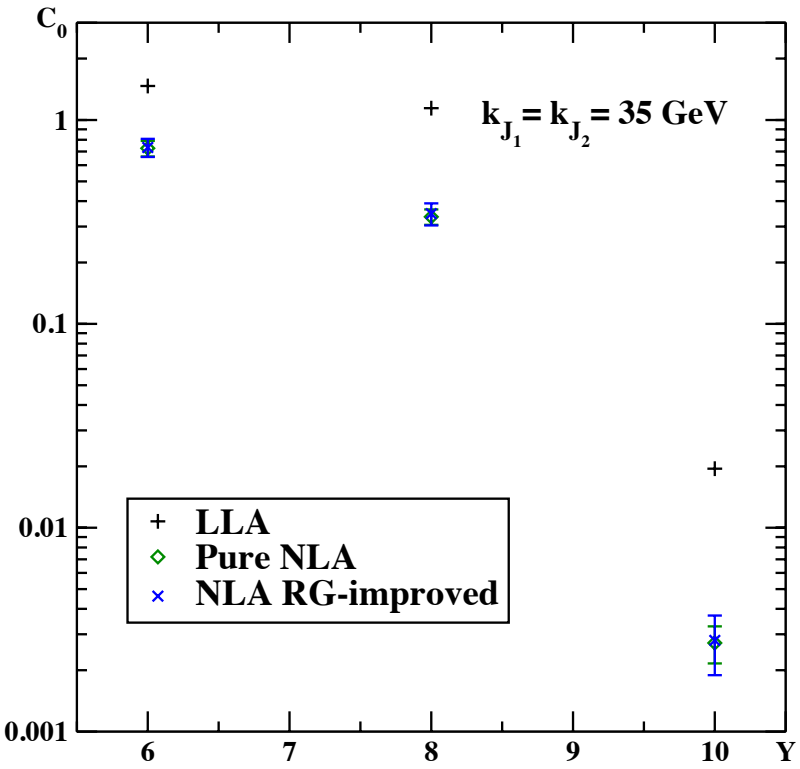
Y	Y_0	$\mu_R/\sqrt{\lvert\vec{k}_{J_1}\rvert\lvert\vec{k}_{J_2}\rvert}$
6	2	3
7	1	2.5
8	2	2.5
9	1.5	3
10	2.5	3

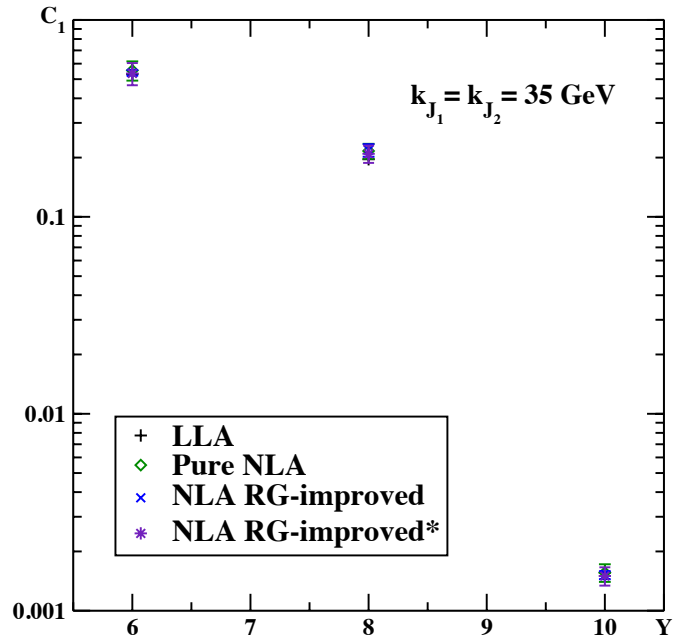
[F. Caporale, B. M., A. Sabio Vera, C. Salas, (2013)]

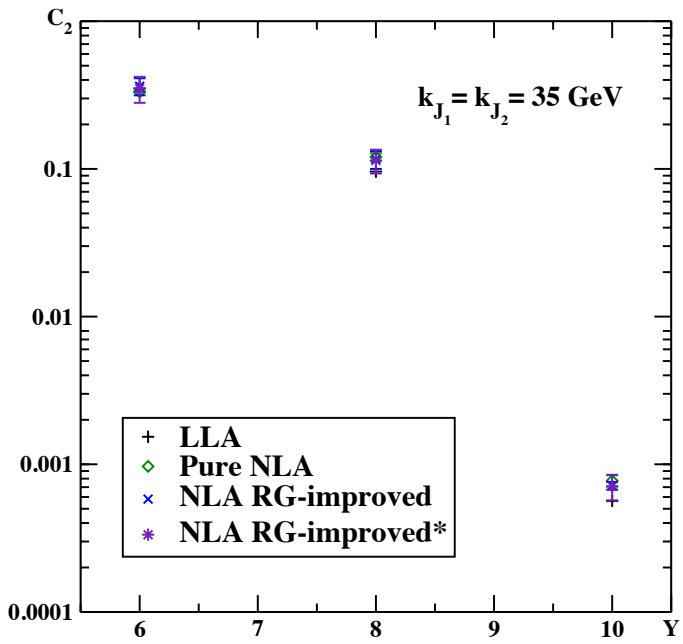




Results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV







Small shift towards naturalness less evident than in previous work on vector mesons [F. Caporale, A. Papa, A. Sabio Vera (2008)]

[F. Caporale, B. M., A. Sabio Vera, C. Salas, (2013)]

Numerical results: $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV

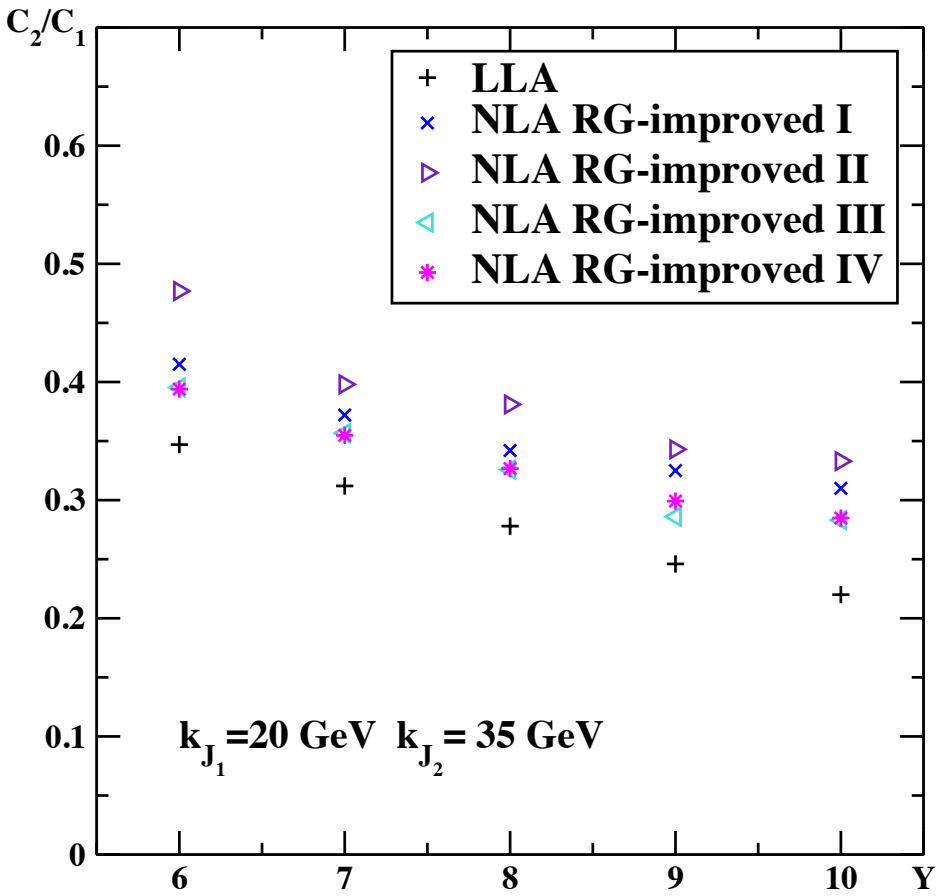
Y	$C_0^{(\mathrm{NLA})}$	Y_0	n_R	$C_0^{(\mathrm{RG-improved-NLA})}$	Y_0	n_R
6	0.726(64)	1	2	0.733(75)	1	2
8	0.335(29)	2	2	0.347(43)	2	2
10	0.00272(56)	4	2	0.00280(91)	3	2

Y	$C_1^{(\mathrm{NLA})}$	Y_0	n_R	$C_1^{(\mathrm{RG-improved-NLA})}$	Y_0	n_R
6	0.554(62)	1	2	0.539(17)	0	1
8	0.216(19)	2	2	0.218(16)	1	2
10	0.00156(16)	3	2	0.001516(71)	2	2

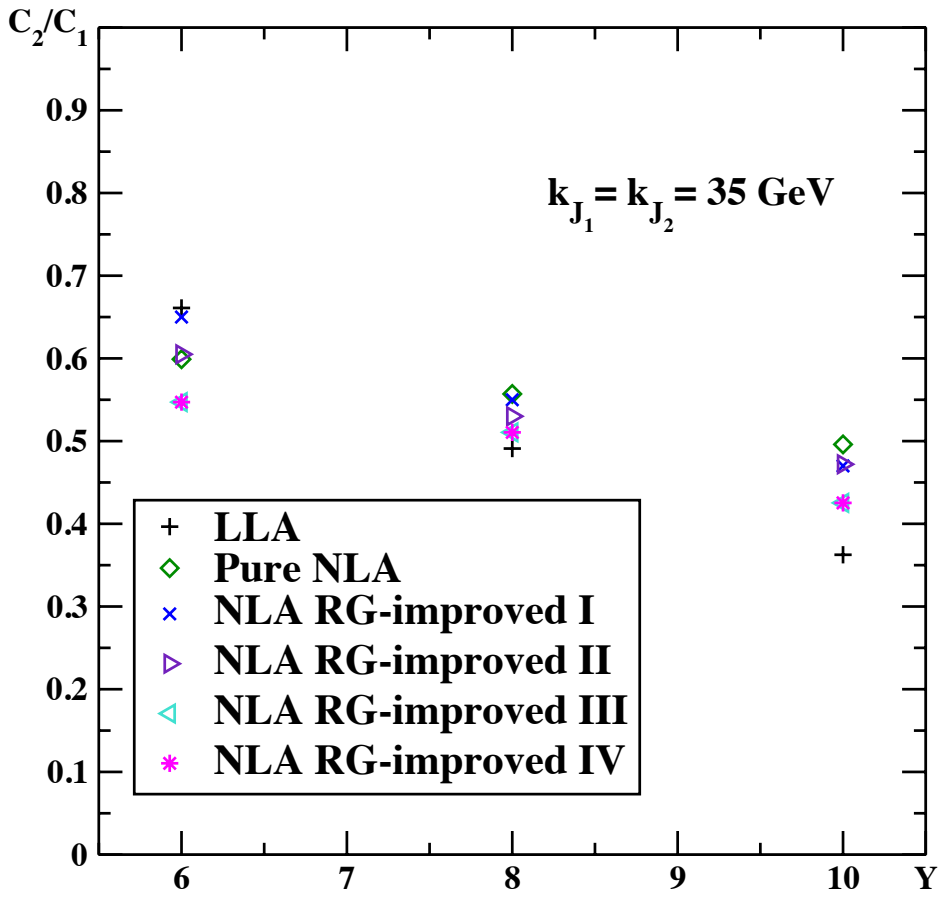
Y	$C_2^{(\mathrm{NLA})}$	Y_0	n_R	$C_2^{(\mathrm{RG-improved-NLA})}$	Y_0	n_R
6	0.3320(18)	0	1.5	0.326(15)	0	1
8	0.1203(74)	2	2.5	0.116(16)	2	3
10	0.000774(69)	4	4	0.000716(43)	2	2

[F. Caporale, B. M., A. Sabio Vera, C. Salas, (2013)]

The ideal BFKL observables: C_2/C_1



$|\vec{k}_{J_1}| = 20$ GeV, $|\vec{k}_{J_2}| = 35$ GeV



$|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV

[F. Caporale, B. M., A. Sabio Vera, C. Salas, (2013)]

Outline

1. Introductory Concepts
   - Introduction
   - The Mueller-Navelet jets production process
   - Theoretical setup: BFKL and collinear factorization
   - Jet definition
2. Jet Vertex
   - The Impact Factor in the LLA
   - The Impact Factor in the NLA
     - Collinear and QCD coupling counterterms
   - Quark-initiated subprocess
   - Gluon-initiated subprocess
   - Summary
3. Cross section
   - State of the art
   - Our analysis
   - Some results
   - Summary

Summary II

- We have studied the Y-dependence of the differential cross section and of the azimuthal decorrelation of the Mueller-Navelet jets in full NLO BFKL using the jet vertex in the small cone approximation.

- We did not fix the energy scale and the argument of the running coupling
  $\Longrightarrow$ we used an optimization method.

- Our results for $|\vec{k}_{J_1}| = |\vec{k}_{J_2}| = 35$ GeV with $\sqrt{s} = 14$ TeV and $\sqrt{s} = 7$ TeV are compatible with those performed by D. Colferai *et al.* and B. Ducloué *et al.*, respectively.
  $\Longrightarrow$ Confirmation of their results.
  $\Longrightarrow$ Reliability of the small cone approximation.

- We considered also smaller transverse momenta w.r.t. D. Colferai *et al.* $\longrightarrow$ we expect more discrimination between BFKL and fixed-order approach.

- In asymmetric case we do not find a stability region $\Longrightarrow$ Collinear improvement $\Longrightarrow$ stability region & lower values for the optimal scales (symmetric case).

THANK YOU!