POWER CORRECTIONS AND EVENT SHAPES
ROADMAP TO PRECISION PHYSICS

Vicent Mateu

IFIC

11 - 04 -2013

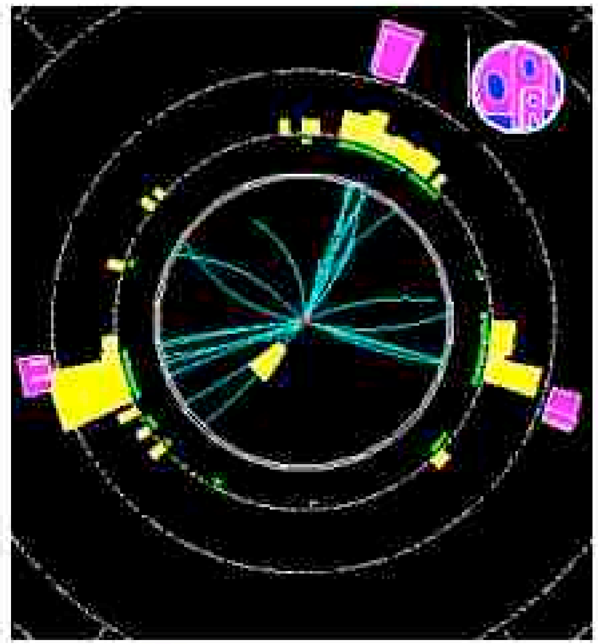
OUTLINE

- Introduction
- Fits to strong coupling constant $\alpha_s(m_Z)$
- Power Corrections
- Hadron Mass effects on Power Corrections
- Anomalous dimension of Power Correction
- Applications for a High-Energy Linear Collider

INTRODUCTION

Event Shapes $e^+ e^- \to \text{jets}$

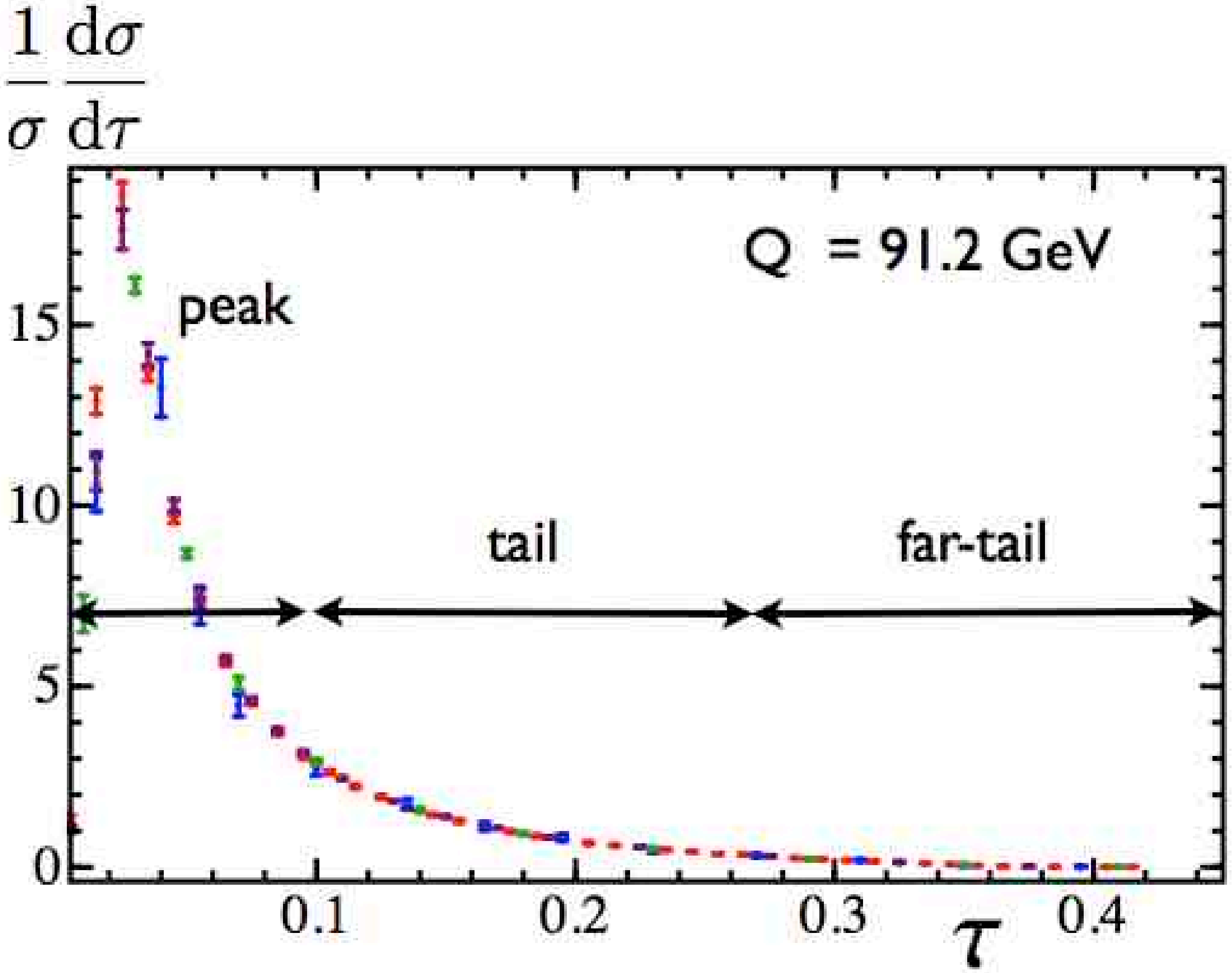
DELPHI 2-jet event



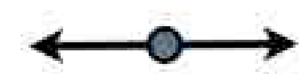
OPAL 3-jet event

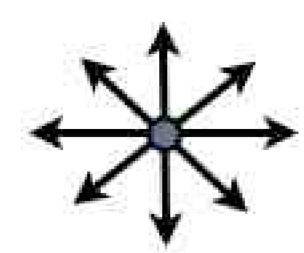
Event shapes characterize in a geometrical way the distribution of hadrons in the final state

They are theoretically **more friendly than a Jet algorithm**



Thrust $\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$

dijet $\tau = 0$ 

spherical $\tau = \frac{1}{2}$ 

Continuous transition from 2-jet to 3-jet, ... multi-jet events

Most common Event shapes

- **Thrust** $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$$ *[E. Farhi]*

- **Angularities** $$\tau_{(a)} = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$ *[Berger, Kucs, Sterman]*

- **Jet Masses** $$\rho_\pm = \frac{1}{Q^2} \Big(\sum_{i \in \pm} p_i \Big)^2$$ *[Clavelli]*
  [Chandramohan Clavelli]

- **Jet Broadening** $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$ *[Catani, Turnock, Webber]*

- **C-parameter** $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$ *[Parisi]*
  [Donoghue, Low, Pi]

- **2-Jettiness** $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$ *[Stewart, Tackmann, Waalewijn]*

Most common Event shapes

- **Thrust** $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|}$$

- **Angularities** $$\tau_{(a)} = \frac{1}{Q} \sum_i E_i\,(\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$

- **Jet Masses** $$\rho_\pm = \frac{1}{Q^2}\Big(\sum_{i\in\pm} p_i\Big)^2$$

- **Jet Broadening** $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$

- **C-parameter** $$C = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

- **2-Jettiness** $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

2-jet event shapes

$e \to 0$

$\updownarrow$

dijet configuration

Most common Event shapes

- Thrust $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|}$$
- Angularities $$\tau_{(a)} = \frac{1}{Q}\sum_i E_i\,(\sin\theta_i)^a(1 - |\cos\theta_i|)^{1-a}$$

Depend on a continuous parameter

- Jet Masses $$\rho_\pm = \frac{1}{Q^2}\Big(\sum_{i\in\pm} p_i\Big)^2$$
- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$
- C-parameter $$C = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j|\sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$
- 2-Jettiness $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

Most common Event shapes

- **Thrust** $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$$

- **Angularities** $$\tau_{(a)} = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a (1 - |\cos\theta_i|)^{1-a}$$

- **Jet Masses** $$\rho_{\pm} = \frac{1}{Q^2} \Big(\sum_{i \in \pm} p_i \Big)^2$$

- **Jet Broadening** $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$ Recoil sensitive

- **C-parameter** $$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

- **2-Jettiness** $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

Most common Event shapes

- Thrust $$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|}$$

- Angularities $$\tau_{(a)} = \frac{1}{Q}\sum_i E_i\,(\sin\theta_i)^a(1-|\cos\theta_i|)^{1-a}$$

- Jet Masses $$\rho_\pm = \frac{1}{Q^2}\Big(\sum_{i\in\pm} p_i\Big)^2$$

- Jet Broadening $$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$

double sum

- C-parameter $$C = \frac{3}{2}\frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j|\sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

does not require minimization procedure

- 2-Jettiness $$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

Most common Event shapes

• Thrust	$\tau = 1 - \max_{\vec{n}} \frac{\sum_i \lvert \vec{p}_i \cdot \vec{n} \rvert}{\sum \lvert \vec{p}_i \rvert}$	Will show fits for α_s
• Angularities	$\tau_{(a)} = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a (1 - \lvert \cos\theta_i \rvert)^{1-a}$	there is no data
• Jet Masses	$\rho_\pm = \frac{1}{Q^2} \Big(\sum_{i \in \pm} p_i \Big)^2$	α_s fits are work in progress
• Jet Broadening	$B = \frac{\sum_i \lvert \vec{p}_i \times \vec{n} \rvert}{\sum_i \lvert \vec{p}_i \rvert}$	
• C-parameter	$C = \frac{3}{2} \frac{\sum_{i,j} \lvert \vec{p}_i \rvert \lvert \vec{p}_j \rvert \sin^2(\theta_{ij})}{(\sum_i \lvert \vec{p}_i \rvert)^2}$	Will show fits for α_s
• 2-Jettiness	$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i \lvert \vec{p}_i \cdot \vec{n} \rvert}{Q}$	there is no data

FIXED ORDER PREDICTIONS

Fixed order predictions

1-loop: Analytic or one numeric integral

[Ellis, Ross, Terrano 1980]

2-loop: Numerical, delicate virtual-real IR cancellation

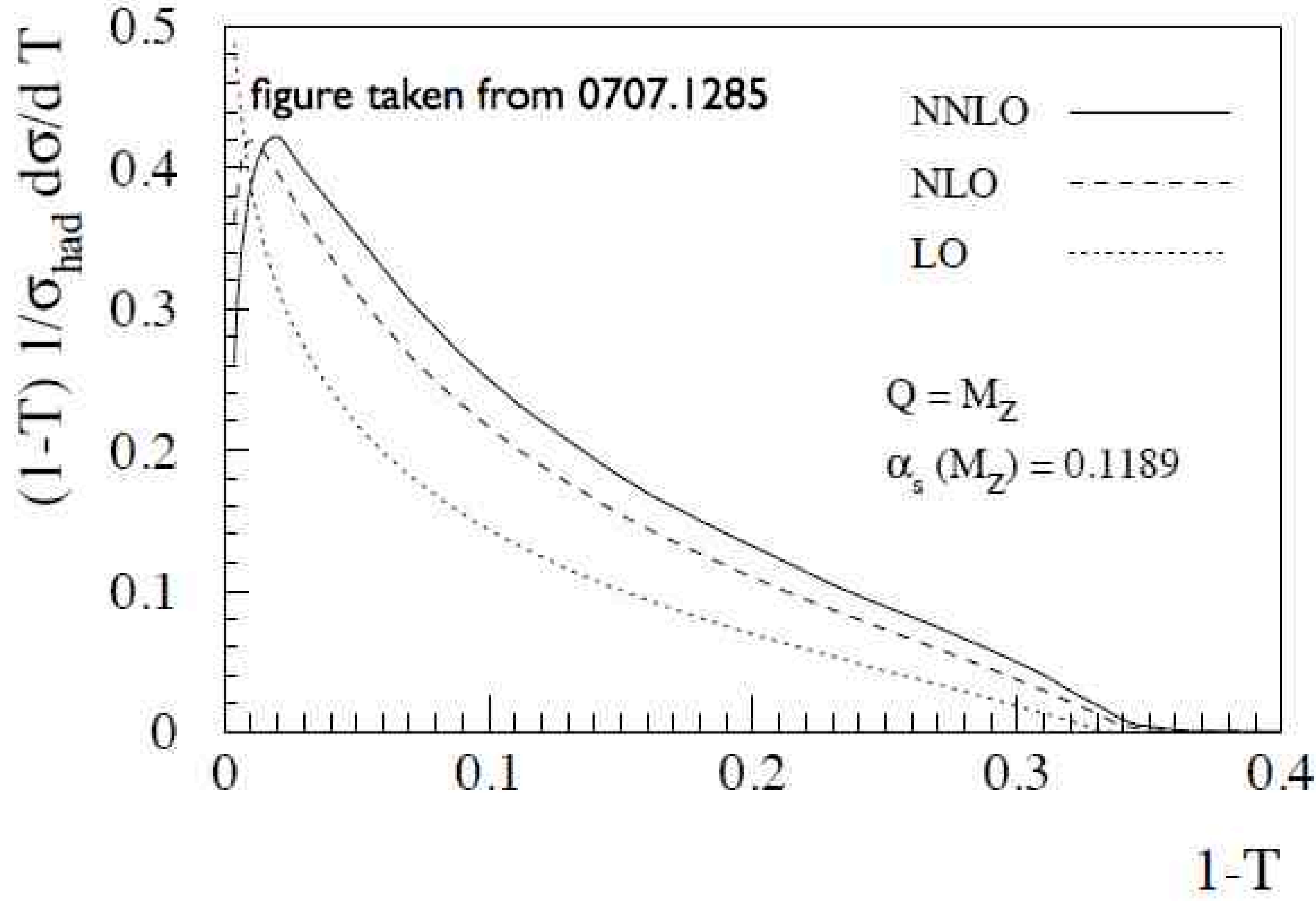
Event [Kunszt, Nason, Marchesini, Webber]

Event 2 [Catani, Seymour 96]

3-loop: Numerical, even more delicate virtual-real IR cancellation

Mercutio [Weinzierl 2008]

EERAD 3 [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007]



RESUMMATION AND FACTORIZATION

Resummation of large logarithms

Event shapes are not inclusive quantities

Incomplete IR cancellation generate large logs at small τ

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\Big(3 + 4\log\tau + \dots\Big)$$

Invalidates perturbative expression for small τ

Resummation of large logarithms

Event shapes are not inclusive quantities

Incomplete IR cancellation generate large logs at small τ

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\Big(3 + 4\log\tau + \ldots\Big)$$

Invalidates perturbative expression for small τ

One has to reorganize the expansion by considering $\alpha_s\,\mathrm{lg}(\tau) \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} \mathrm{d}\tau \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

Resummation of large logarithms

Event shapes are not inclusive quantities

Incomplete IR cancellation generate large logs at small τ

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = -\frac{2\alpha_s}{3\pi}\frac{1}{\tau}\Big(3 + 4\log\tau + \ldots\Big)$$

Invalidates perturbative expression for small τ

One has to reorganize the expansion by considering $\alpha_s \lg(\tau) \sim \mathcal{O}(1)$

Counting more clear in the exponent of cumulant

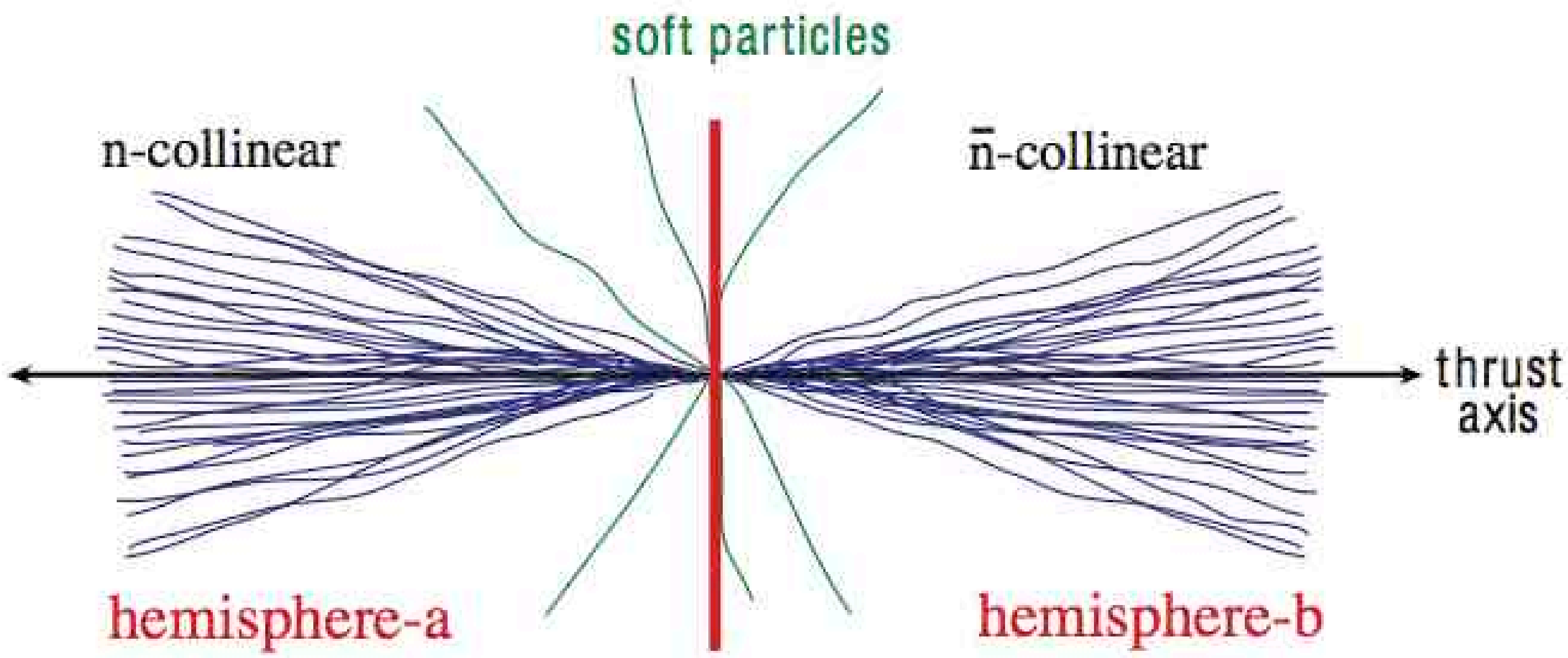
$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} \mathrm{d}\tau \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

$$\log\Sigma(\tau_c) = \log\tau_c \sum_{i=0}(\alpha_s \log\tau_c)^{i+1} + \sum_{i=0}(\alpha_s\log\tau_c)^{i+1} + \alpha_s\sum_{i=0}(\alpha_s\log\tau_c)^{i} + \alpha_s^2\sum_{i=0}(\alpha_s\log\tau_c)^{i}$$

LL NLL NNLL N^{3}LL

+ terms which are not singular as $\tau_c \to 0$

Relevant Scales in the Problem



$$Q^2 = (E_{e^-} + E_{e^+})^2 = \Big(\sum_{i\in a+b} p_i^\mu\Big)^2 \simeq 2\Big(\sum_{i\in a} E_i\Big)\Big(\sum_{i\in b} E_i\Big)$$

$$m_{\rm jet}^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \simeq Q \sum_{i\in a} p_i^+ \sim Q\Lambda_{\rm QCD}$$

$$\mu_{\rm soft}^2 = \Big(\sum_{i\in {\rm soft}} p_i^\mu\Big)^2 \sim \Lambda_{\rm QCD}^2$$

$$Q^2 \gg Q\Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$$

large hierarchy of scales

large logs of ratios

EFT treatment is called for!

Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart]

Designed to study highly energetic particles far off-shell

Can be used for inclusive and exclusive processes

light-cone decomposition of momenta	$p^\mu = n\cdot p\frac{\bar n^\mu}{2} + \bar n\cdot p\frac{n^\mu}{2} + p_\perp^\mu = (p^+, p^-, p_\perp)$ $n^2 = \bar n^2 = 0,\ n\cdot\bar n = 2,\ n\cdot p_\perp = \bar n\cdot p_\perp = 0$

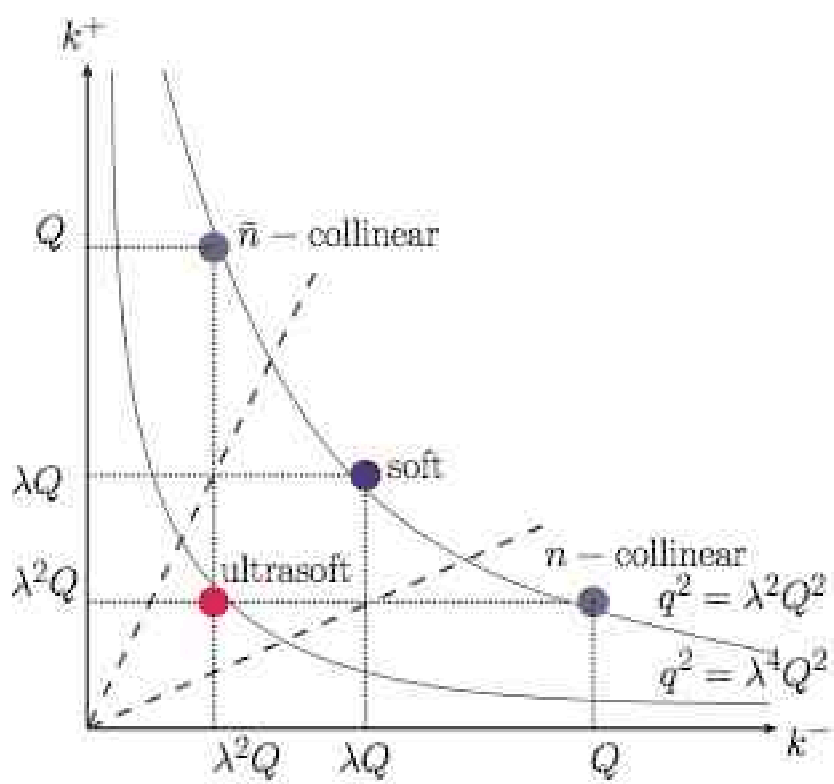
Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart]

Designed to study highly energetic particles far off-shell

Can be used for inclusive and exclusive processes

light-cone decomposition of momenta	$p^\mu = n\cdot p\frac{\bar n^\mu}{2} + \bar n\cdot p\frac{n^\mu}{2} + p_\perp^\mu = (p^+, p^-, p_\perp)$ $n^2 = \bar n^2 = 0,\ n\cdot\bar n = 2,\ n\cdot p_\perp = \bar n\cdot p_\perp = 0$



collinear particles $\begin{cases} \bar{\text{n}}\text{-collinear } Q(1,\lambda^2,\lambda) \\ \text{n-collinear } Q(\lambda^2,1,\lambda) \end{cases}$

soft particles $Q(\lambda,\lambda,\lambda)$ SCET$_{\text{II}}$

ultrasoft particles $Q(\lambda^2,\lambda^2,\lambda^2)$ SCET$_{\text{I}}$

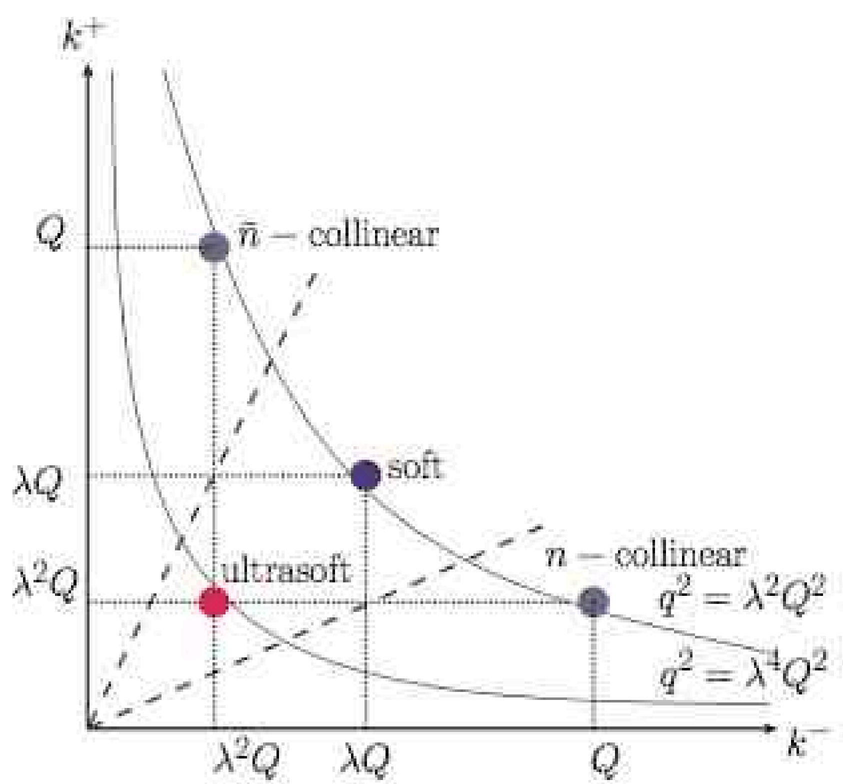
Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart]

Designed to study highly energetic particles far off-shell

Can be used for inclusive and exclusive processes

light-cone decomposition of momenta	$p^\mu = n \cdot p \frac{\bar{n}^\mu}{2} + \bar{n} \cdot p \frac{n^\mu}{2} + p_\perp^\mu = (p^+, p^-, p_\perp)$ $n^2 = \bar{n}^2 = 0,\ n \cdot \bar{n} = 2,\ n \cdot p_\perp = \bar{n} \cdot p_\perp = 0$



collinear particles $\begin{cases} \bar{\text{n}}\text{-collinear } Q(1, \lambda^2, \lambda) \\ \text{n-collinear } Q(\lambda^2, 1, \lambda) \end{cases}$

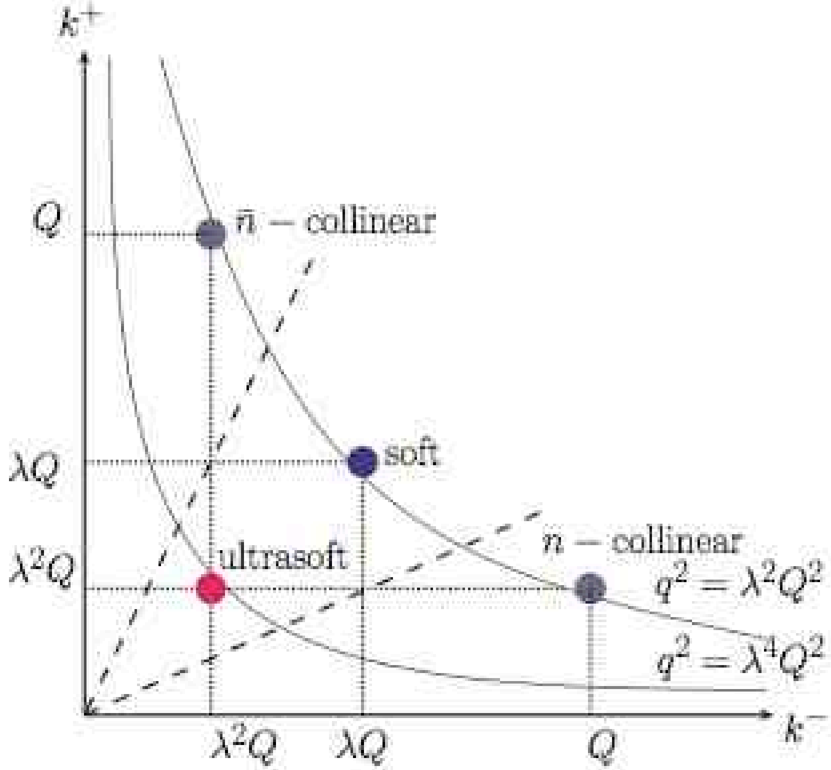
~~soft particles $Q(\lambda, \lambda, \lambda)$ SCET$_{II}$~~

ultrasoft particles $Q(\lambda^2, \lambda^2, \lambda^2)$ SCET$_I$

In physical situations one often only needs soft or usoft not both

Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart]



collinear particles	n̄-collinear	$Q(1, \lambda^2, \lambda)$	
	n-collinear	$Q(\lambda^2, 1, \lambda)$	
~~soft particles~~		~~$Q(\lambda, \lambda, \lambda)$~~	~~SCET_{II}~~
ultrasoft particles		$Q(\lambda^2, \lambda^2, \lambda^2)$	SCET_{I}

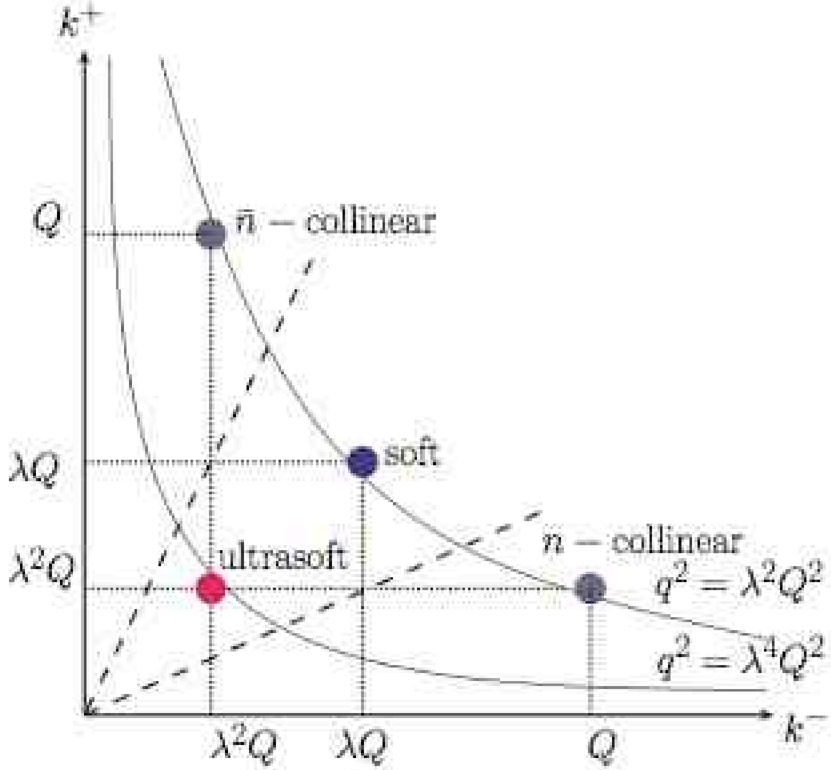
$\mathcal{L}_{\text{us}}$ Same as in full QCD

$p_n + p_{\bar{n}} \longrightarrow$ off-shell mode

no Lagrangian interaction between
two collinear sectors
(just in external operators)

Soft-Collinear Effective Theory (SCET)

[Bauer, Fleming, Luke, Pirjol, Stewart]



collinear particles	n̄-collinear	$Q(1, \lambda^2, \lambda)$	
	n-collinear	$Q(\lambda^2, 1, \lambda)$	
~~soft particles~~		~~$Q(\lambda, \lambda, \lambda)$~~	~~SCET$_{\text{II}}$~~
ultrasoft particles		$Q(\lambda^2, \lambda^2, \lambda^2)$	SCET$_{\text{I}}$

$$p_n + p_{\text{us}} \to p'_n$$
$$p_{\bar{n}} + p_{\text{us}} \to p'_{\bar{n}}$$

ultra soft - collinear interactions in the SCET Lagrangian

$$\mathcal{L}^{(0)} = \bar{\xi}_n \left(i\, n \cdot D_s + g_s\, n \cdot A_n + i\, \not{D}_c^{\perp} W_n \frac{1}{\bar{\mathcal{P}}} W_n^{\dagger} \not{D}_c^{\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n$$

$$i\, D_s^{\mu} = i\, \partial^{\mu} + g_s A_s^{\mu}, \quad i\, D_c^{\mu} = P^{\mu} + g_s A_c^{\mu}$$

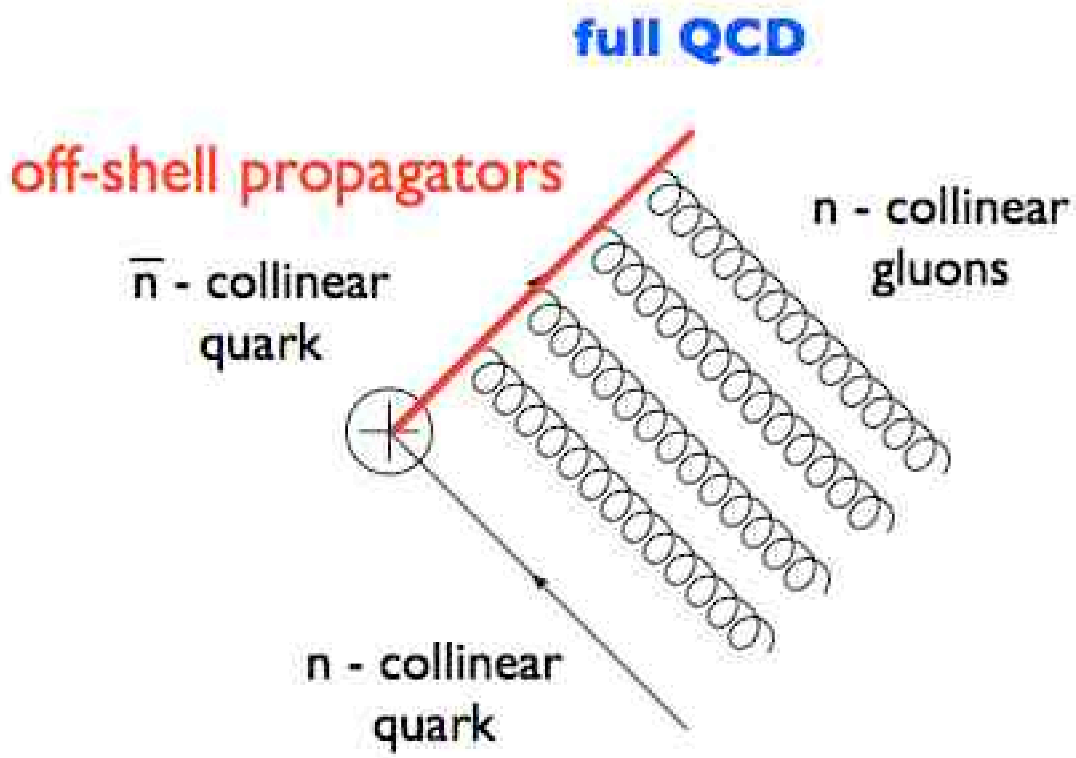
only interaction between collinear and usoft sectors at leading power

Soft-Collinear Factorization

[Bauer, Pirjol, Stewart]

$$A_c^- \sim \mathcal{O}(1), \quad p_c^- \sim \mathcal{O}(1),$$

so they can appear an arbitrary number of times in Wilson coefficients

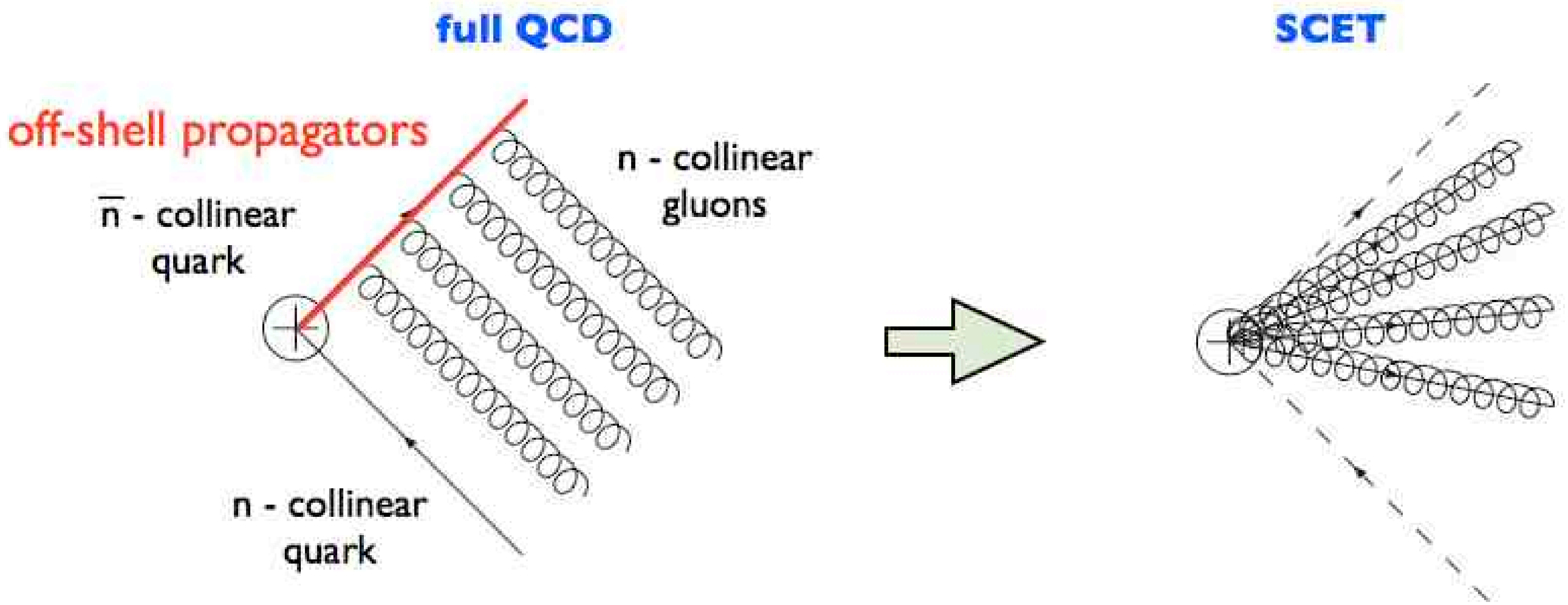


Soft-Collinear Factorization

[Bauer, Pirjol, Stewart]

$A_c^- \sim \mathcal{O}(1), \quad p_c^- \sim \mathcal{O}(1),$

so they can appear an arbitrary number of times in Wilson coefficients

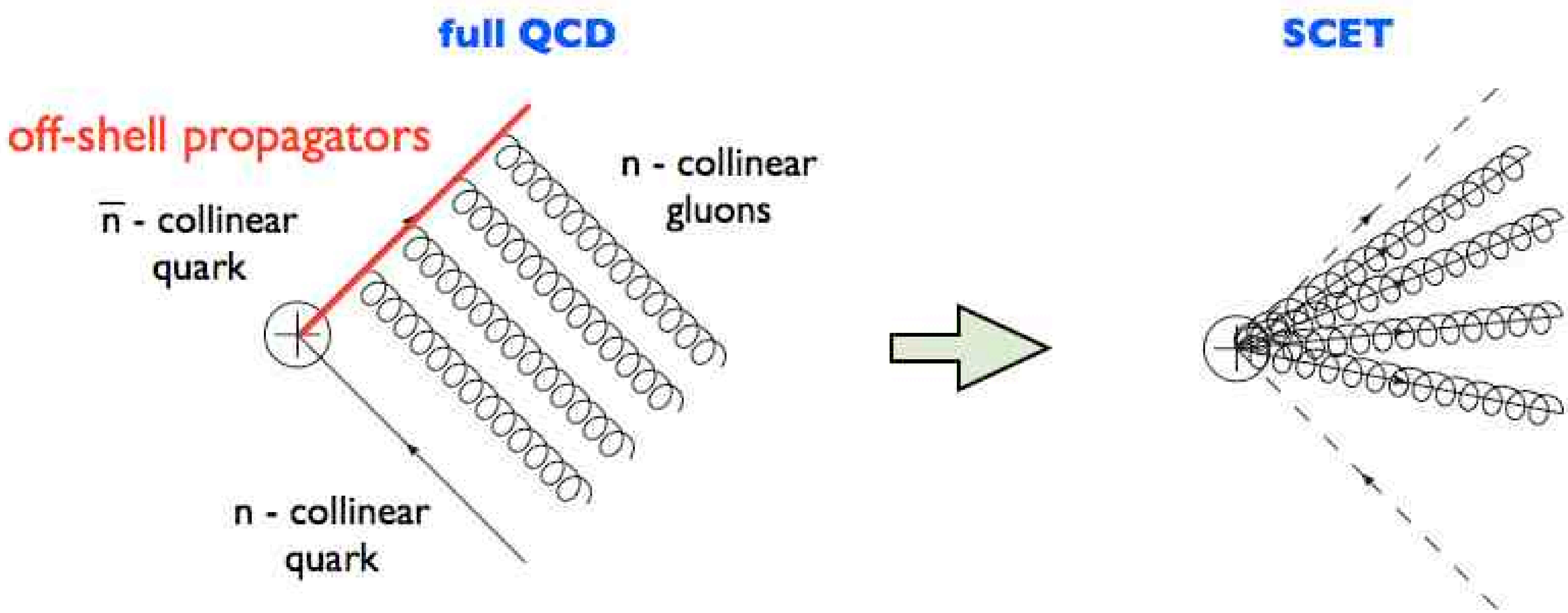


Soft-Collinear Factorization

[Bauer, Pirjol, Stewart]

$A_c^- \sim \mathcal{O}(1), \quad p_c^- \sim \mathcal{O}(1),$

so they can appear an arbitrary number of times in Wilson coefficients



$$q \to W_n \xi_n \equiv \chi_n \quad \text{Jet field}$$

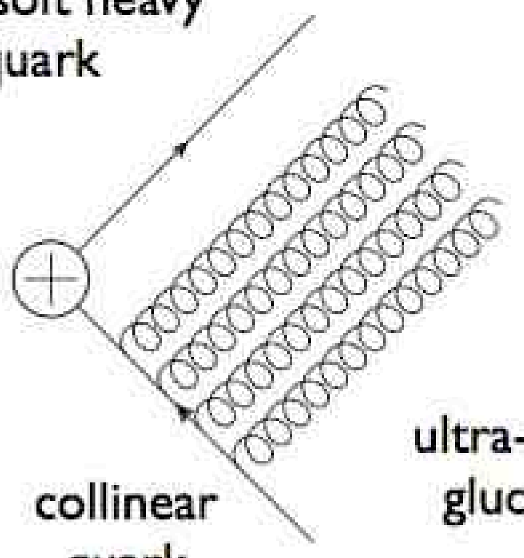
$$W_n(x) = \mathrm{P} \exp \left[i\, g_s \int_{-\infty}^{x} \mathrm{d}s\, \bar{n} \cdot A_n(s\, \bar{n}) \right]$$

Gauge invariance ensures that the ξ_n appears with a collinear Wilson line

Soft-Collinear Factorization

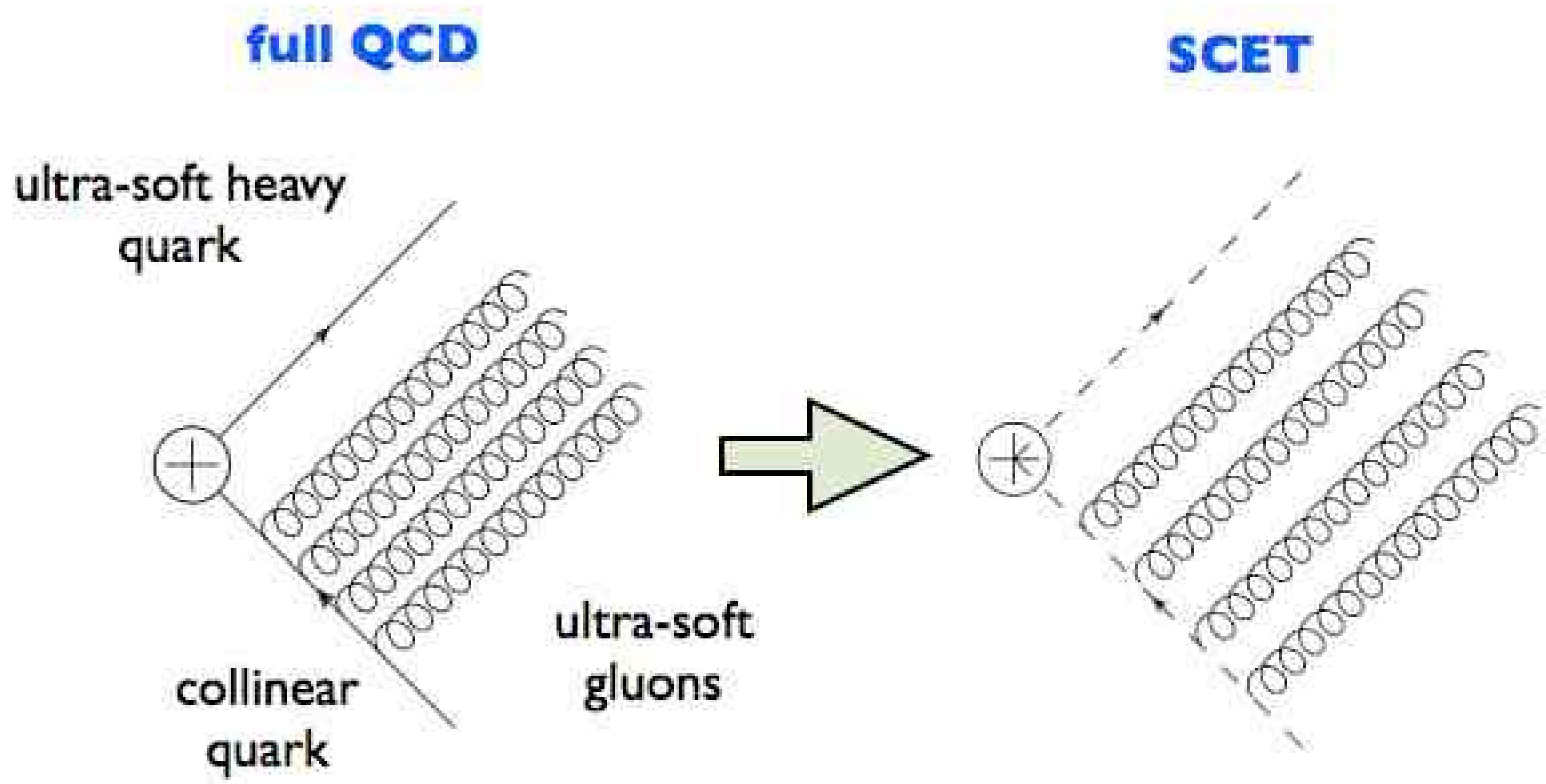
[Bauer, Pirjol, Stewart]

full QCD



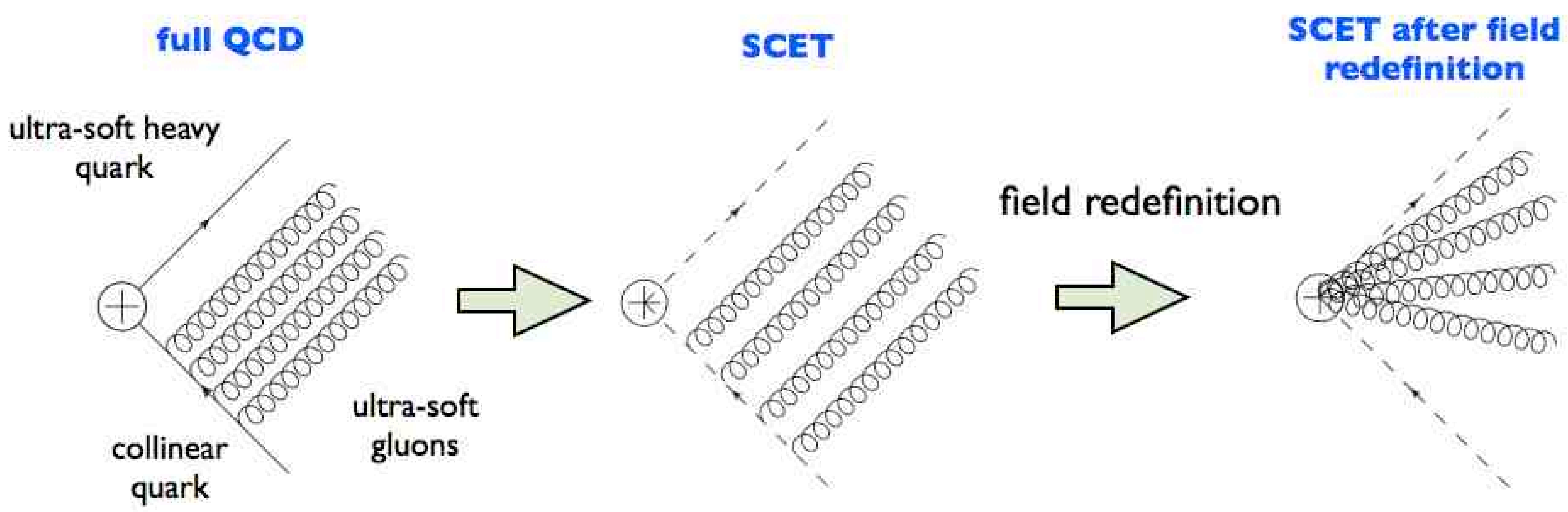
Soft-Collinear Factorization

[Bauer, Pirjol, Stewart]



Soft-Collinear Factorization

[Bauer, Pirjol, Stewart]



BPS field redefinition

$$\xi_n \to Y_n \xi_n^{(0)}, \qquad W_n \to Y_n W_n^{(0)} Y_n^\dagger$$

$$Y_n(x) = \mathrm{P}\exp\left[\int_{-\infty}^{x} \mathrm{d}s\, n\cdot A_{\mathrm{us}}(s\,\bar{n})\right] \quad \mathcal{L}^{(0)} = \bar{\xi}_n^{(0)}\left(i\,n\cdot\partial_s + g_s\,n\cdot A_n^{(0)} + i\,\not{D}_c^\perp W_n^{(0)}\frac{1}{\bar{\mathcal{P}}}W_n^{(0)\dagger}\not{D}_c^\perp\right)\frac{\not{\bar{n}}}{2}\xi_n^{(0)}$$

usoft gluons completely decouple at leading order

ultrasoft Wilson line

moves usoft-collinear interactions from the Lagrangian to the current

Factorization theorem for event shapes

(Will focus on SCET but CSS is equivalent)

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{QCD}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

[Berger, Kuks, Sterman]

- H_Q: Universal Wilson Coefficient
- J_e: Jet function
- S_e: Soft function
- $\mathcal{O}(\dots)$: Nonsingular terms, power corrections

$H_Q \times J_e$: Calculable in perturbation theory

S_e: Perturbative and nonperturbative components

$$S_e(e) = \langle 0 | \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

Soft Wilson lines; $Q\hat{e}$: event shape operator

Leading power correction comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

[Korchemsky, Sterman, Tafat]

$\hat{S}_e$: perturbative

F_e: nonperturbative & *[Korchemsky & Sterman]*
perturbative *[VM, Thaler, Stewart]*

$\Rightarrow$

$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} \otimes F_e$$

Tree level OPE for nonperturbative corrections

$$S_e(e) = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$ *[Lee & Sterman]*

For $e \gg \frac{\Lambda_{\mathrm{QCD}}}{Q}$

$$\delta(\ell - Q\hat{e}) \simeq \delta(\ell) - \delta'(\ell) Q\hat{e} + \dots$$

Correct up to $\mathcal{O}(\alpha_s)$

Shape function can be expanded in the tail

$$F_e(\ell) \simeq \delta(\ell) - \Omega_1 \delta'(\ell)$$

$$\Omega_1 = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q\hat{e} Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

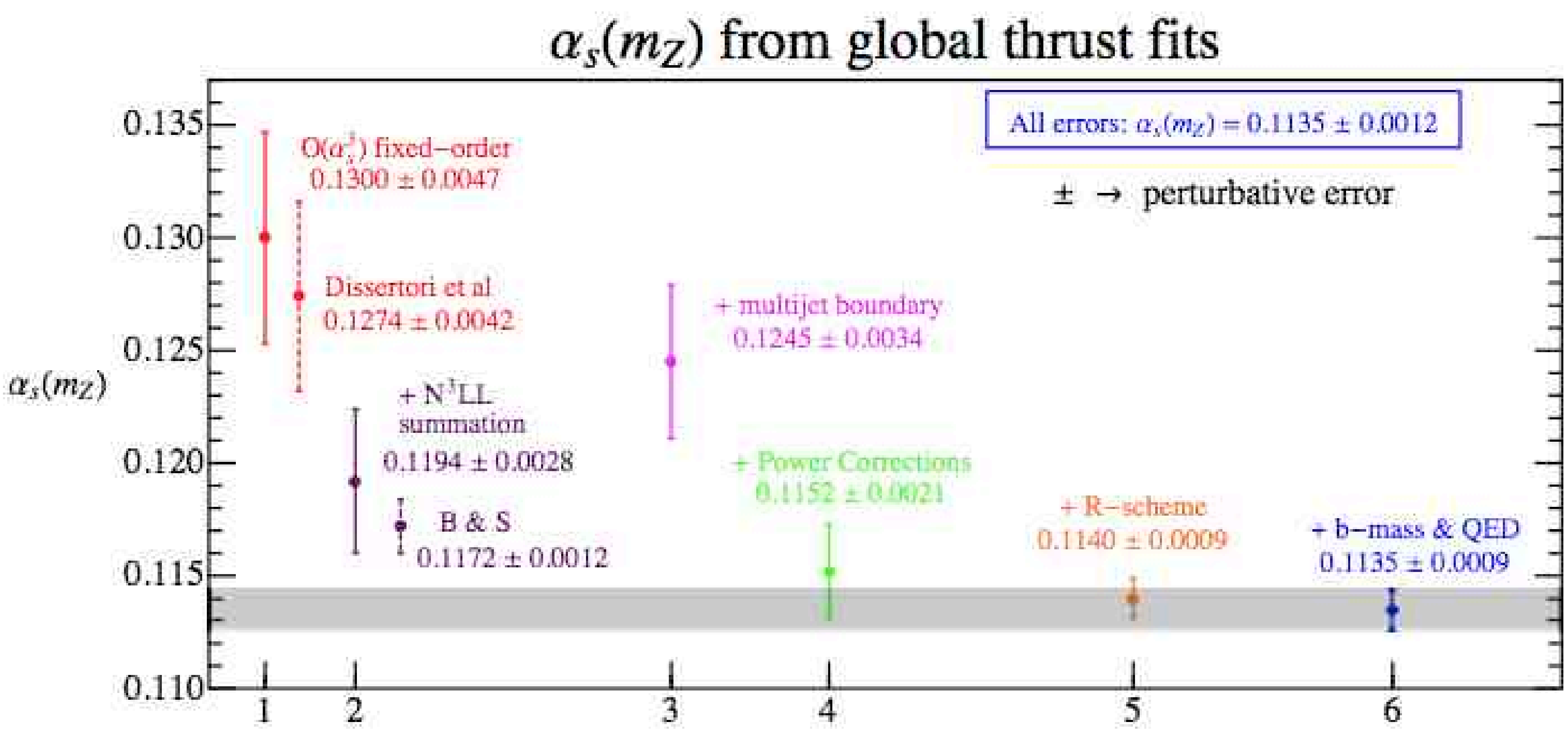
$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} - \frac{\Omega_1}{Q} \frac{\mathrm{d}}{\mathrm{d}e} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e} \simeq \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}e}\left(e - \frac{\Omega_1}{Q}\right) + \mathcal{O}\left[\left(\frac{\Lambda_{\mathrm{QCD}}}{Q\, e}\right)^2\right]$$

Leading nonperturbative correction in the tail is a shift of the distribution

POWER CORRECTIONS FOR EVENT SHAPES

Why are Power Corrections so important?

- Event shapes have been extensively used to determine $\alpha_s(m_Z)$
- Power Corrections play an essential role in that determination
- Also important effects in Jet Substructure — *[Boost 2012 proceedings]* *[Feige, Schwartz, Stewart, Thaler 2012]*
- Important in hadronization and underlying event at the LHC

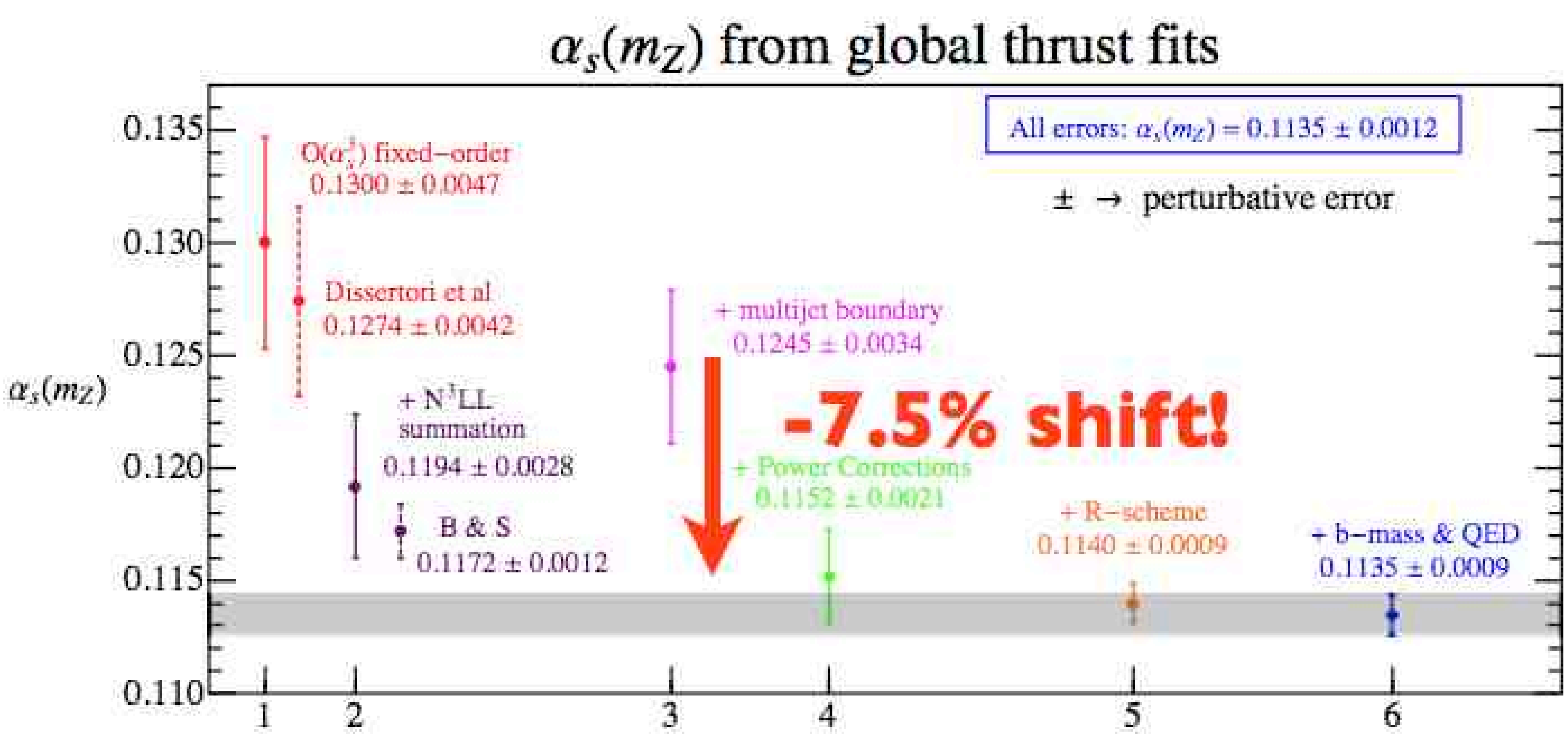


[Abbate, Fickinger, Hoang, VM, Stewart]

arXiv:1006.3080
arXiv:1204.5746

Why are Power Corrections so important?

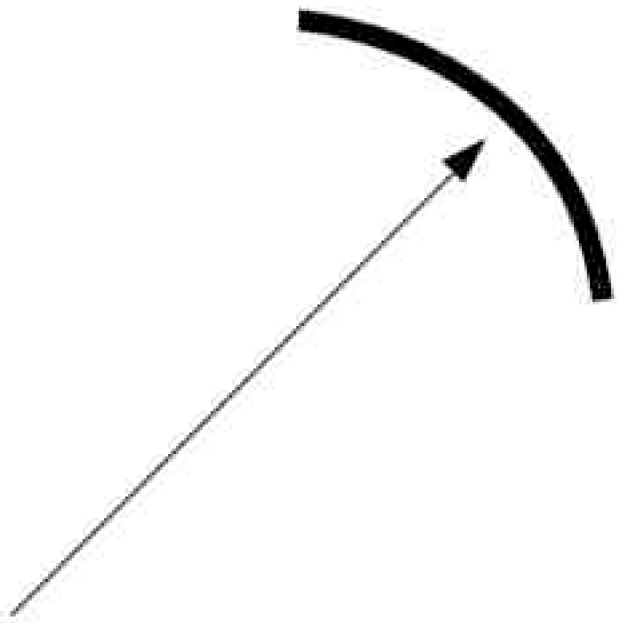
- Event shapes have been extensively used to determine $\alpha_s(m_Z)$
- Power Corrections play an essential role in that determination
- Also important effects in Jet Substructure *[Boost 2012 proceedings]* *[Feige, Schwartz, Stewart, Thaler 2012]*
- Important in hadronization and underlying event at the LHC



[Abbate, Fickinger, Hoang, VM, Stewart]

arXiv:1006.3080
arXiv:1204.5746

Hadron masses and Schemes



What can be measured when a particle hits the detector?

Ideally we would like **energy and momentum separately measured**, but that is not always possible.

If a **particle** is **not identified**, mass is not known, no information on magnitude of momentum.

One can assume all particles are pions [default scheme]

Alternatively one can use only energy and directions [E scheme] $|\vec{p}| \to E$

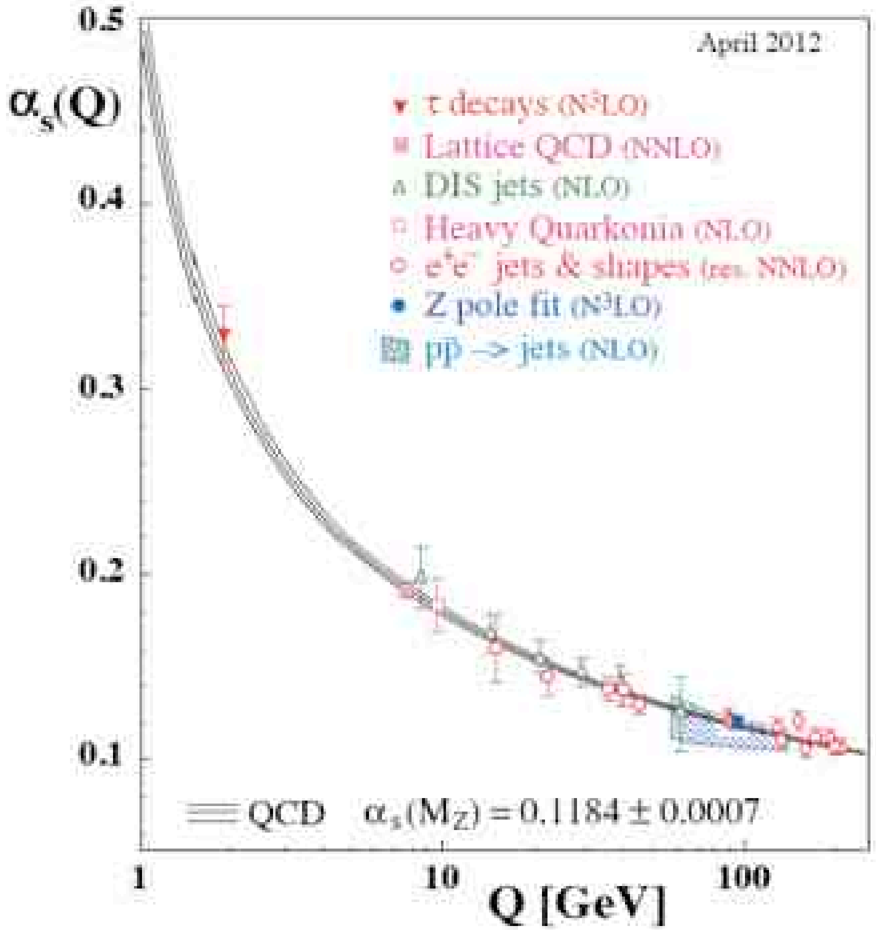
Finally one can use only momenta and directions [P scheme] $E \to |\vec{p}|$

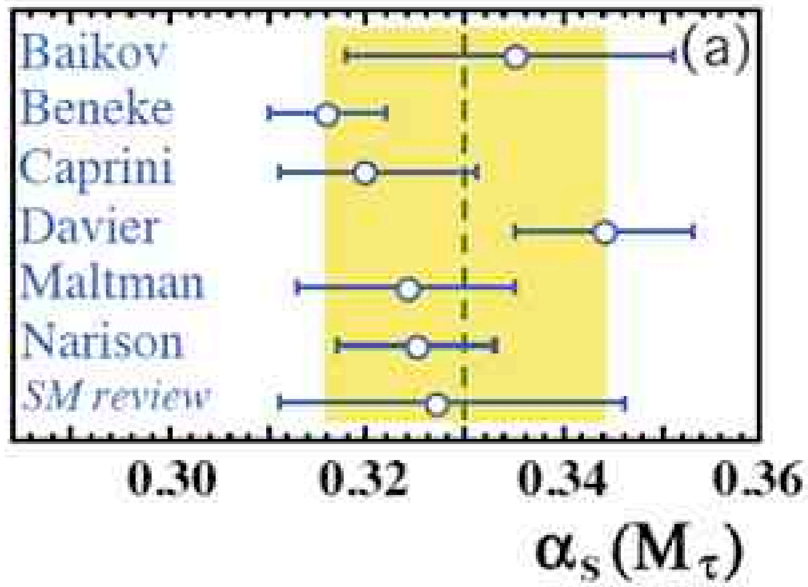
These considerations are irrelevant in perturbation theory, but have important consequences for power corrections!

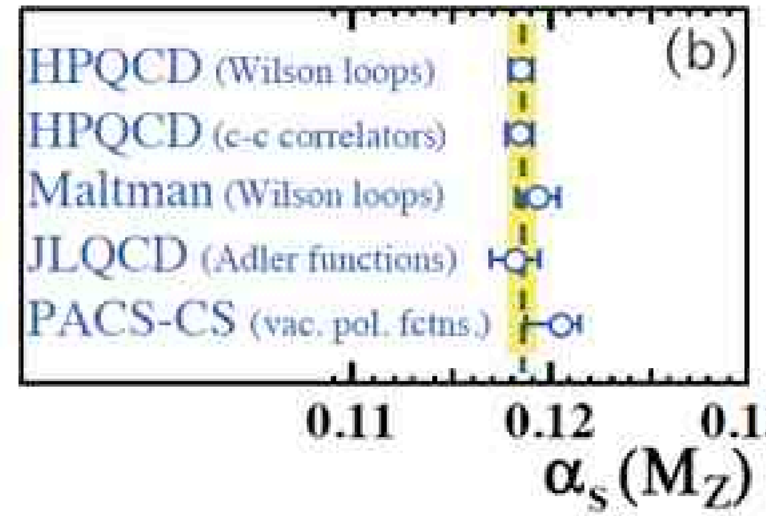
STRONG COUPLING DETERMINATION

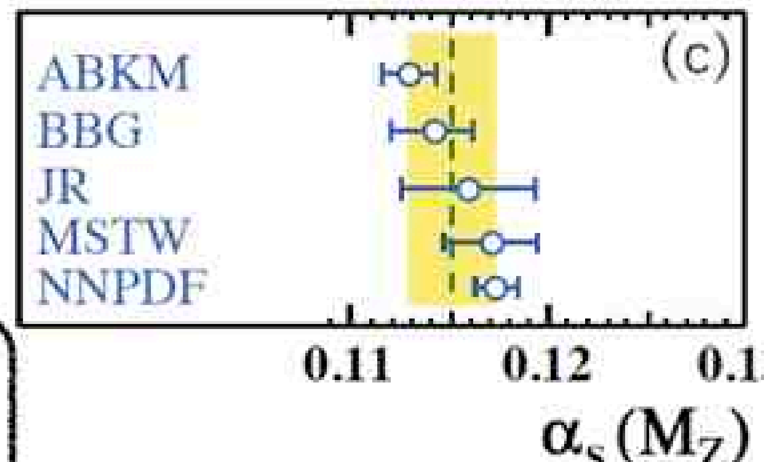
α_s determination: world average

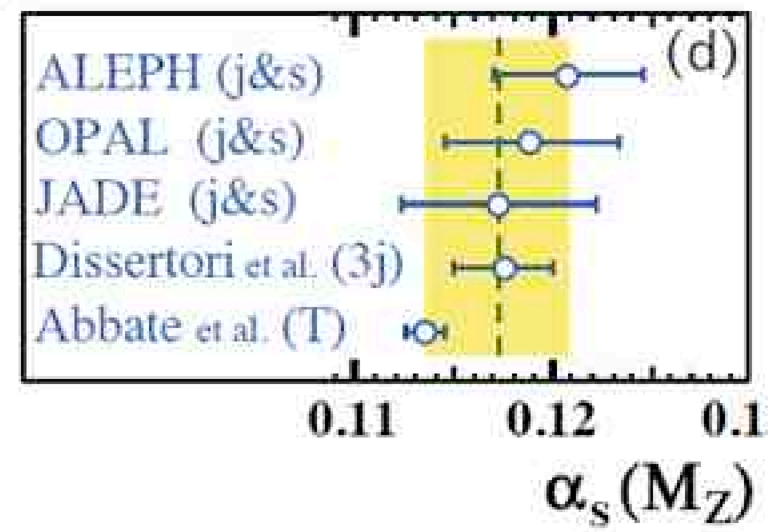
Figures taken from PDG



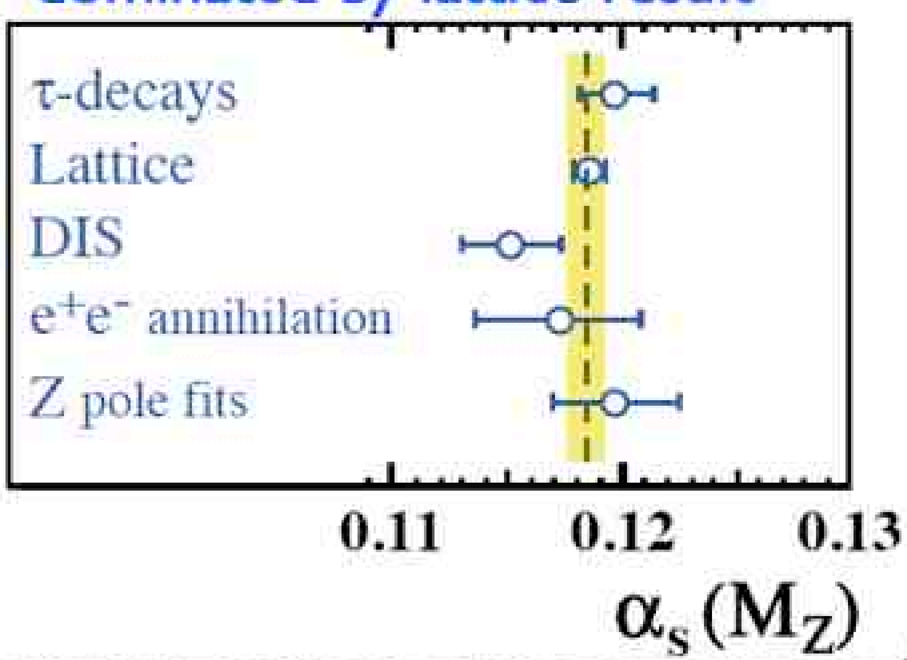








World average completely dominated by lattice result



many details in review *[arXiv: 1110.0016]*

High-precision event shape determinations washed out from average

Determinations are first averaged within a given process

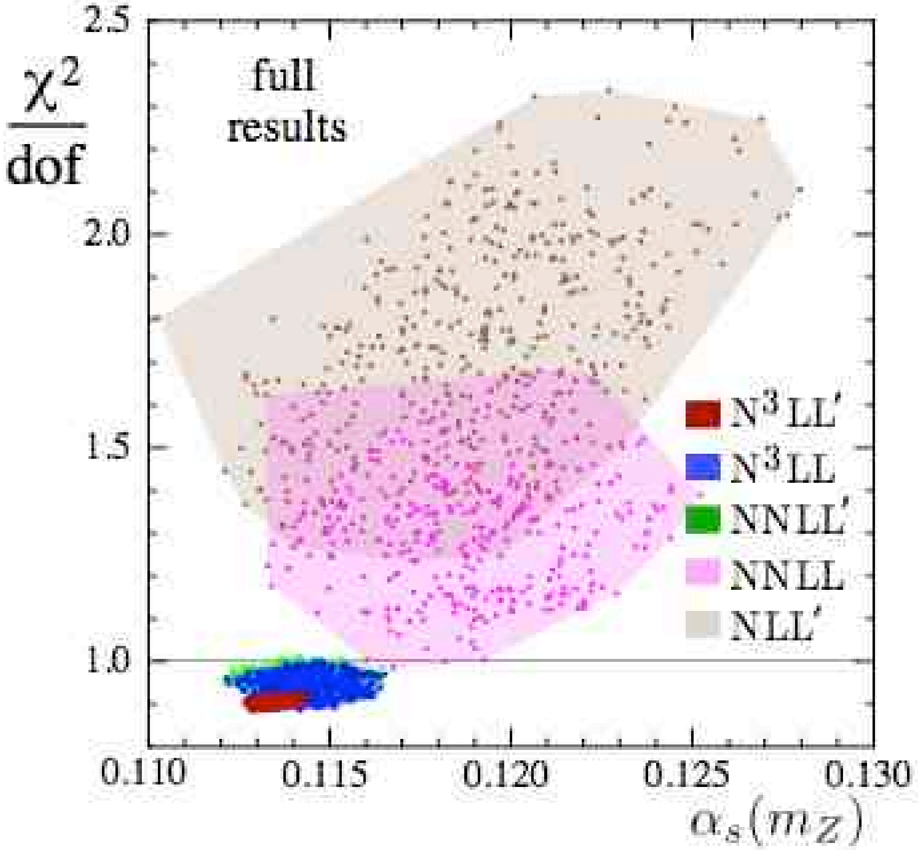
The various averages are later combined together for the final average

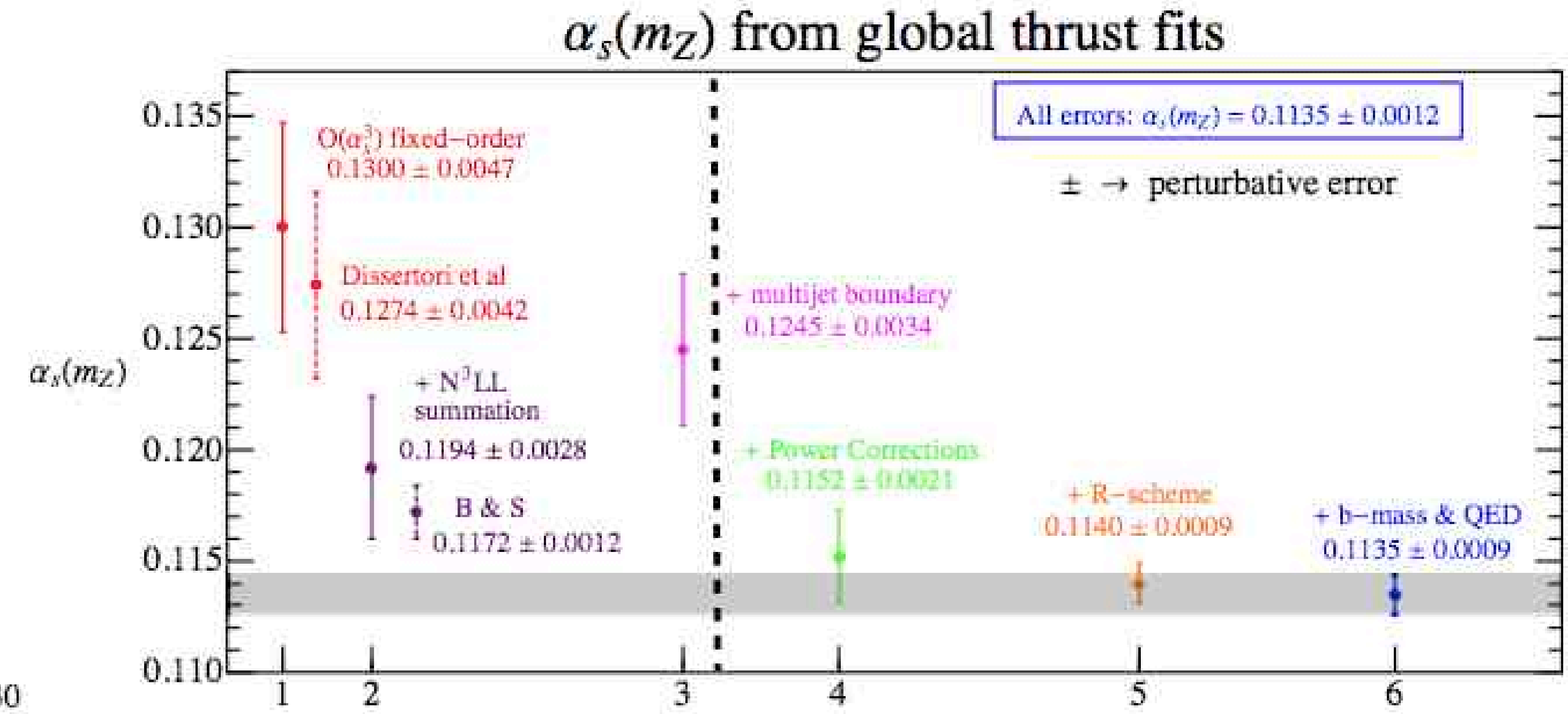
α_s determination: Thrust tail fits

[Abbate, Fickinger, Hoang, VM Stewart 1006.3080]

- N³LL resummation, NNLO matrix elements
- Fits to Q > 34 GeV, global fit
- Thrust analysis only
- Power corrections OPE
- QED and bottom mass effects, axial singlet contribution
- Renormalon subtraction

$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$





α_s determination: Thrust tail fits

[Abbate, Fickinger, Hoang, VM Stewart 1006.3080]

- N^3LL resummation, NNLO matrix elements
- Fits to Q > 34 GeV, global fit
- Thrust analysis only
- Power corrections OPE
- QED and bottom mass effects, axial singlet contribution
- Renormalon subtraction

$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$

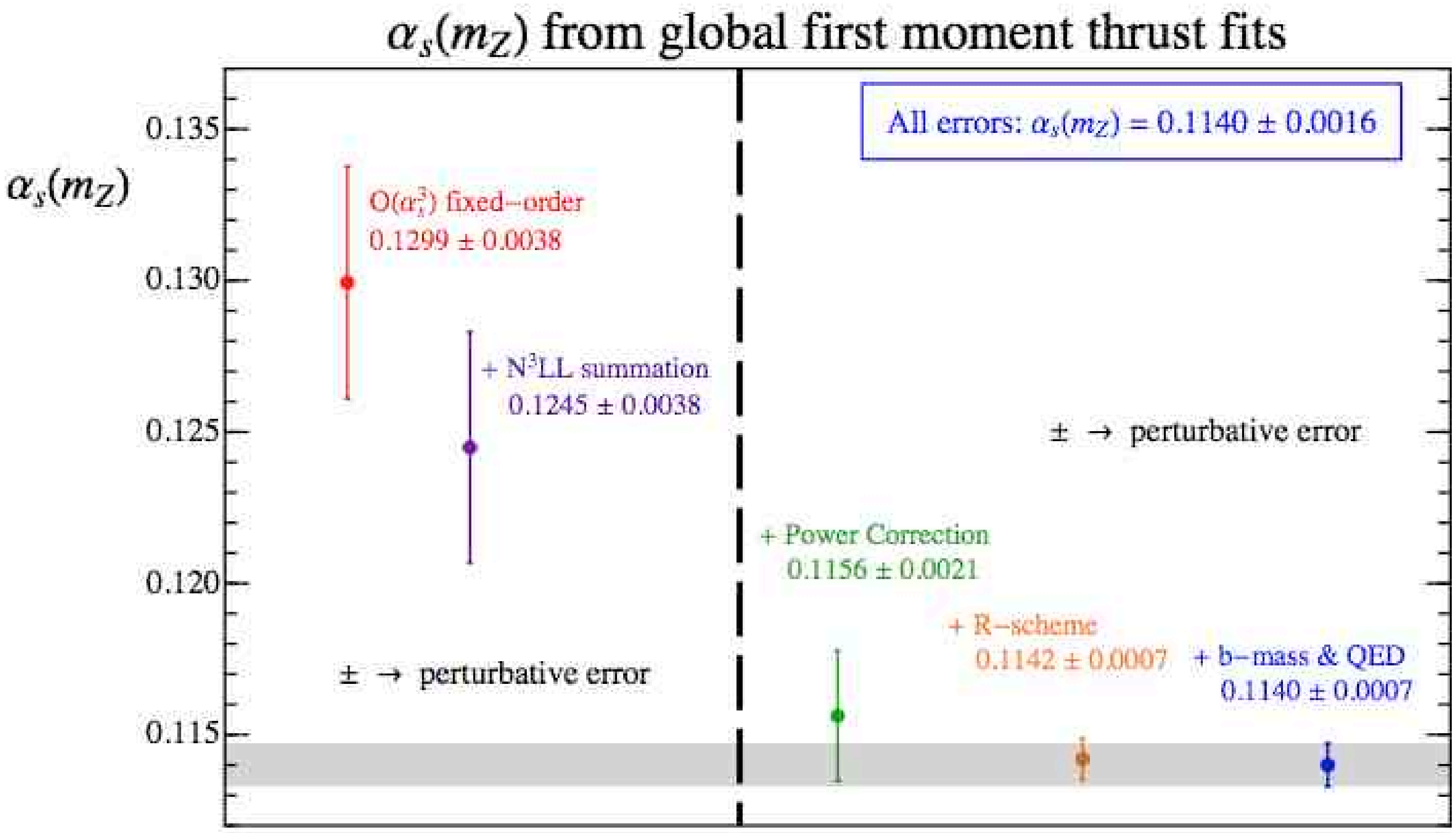
error includes conservative estimates of effects coming from higher order power corrections not included in the fit

α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma}\int d\tau\, \tau^n \frac{d\sigma}{d\tau}$$

Only first moment of thrust
Used N³LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication

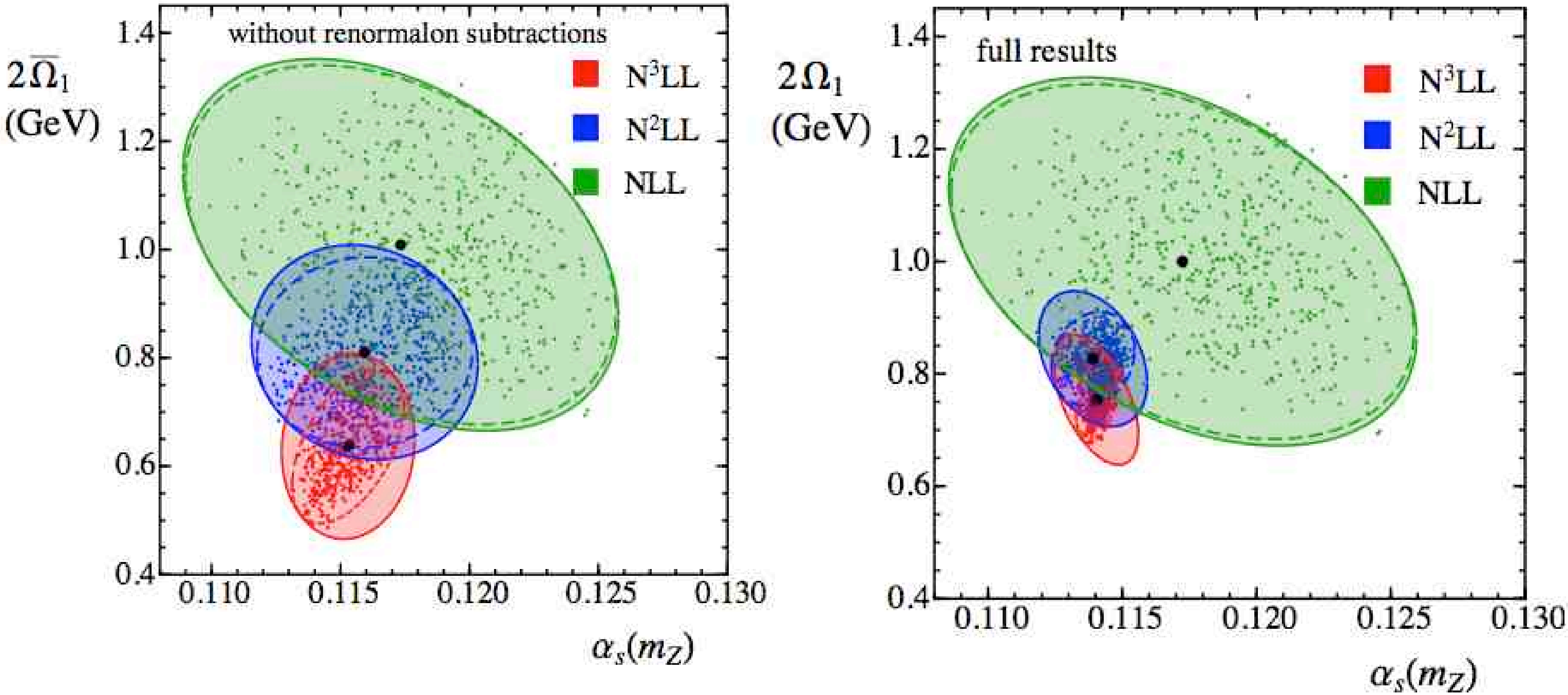


α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma}\int \mathrm{d}\tau\,\tau^n \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

Only first moment of thrust
Used N³LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication
Significant error reduction when renormalon is removed



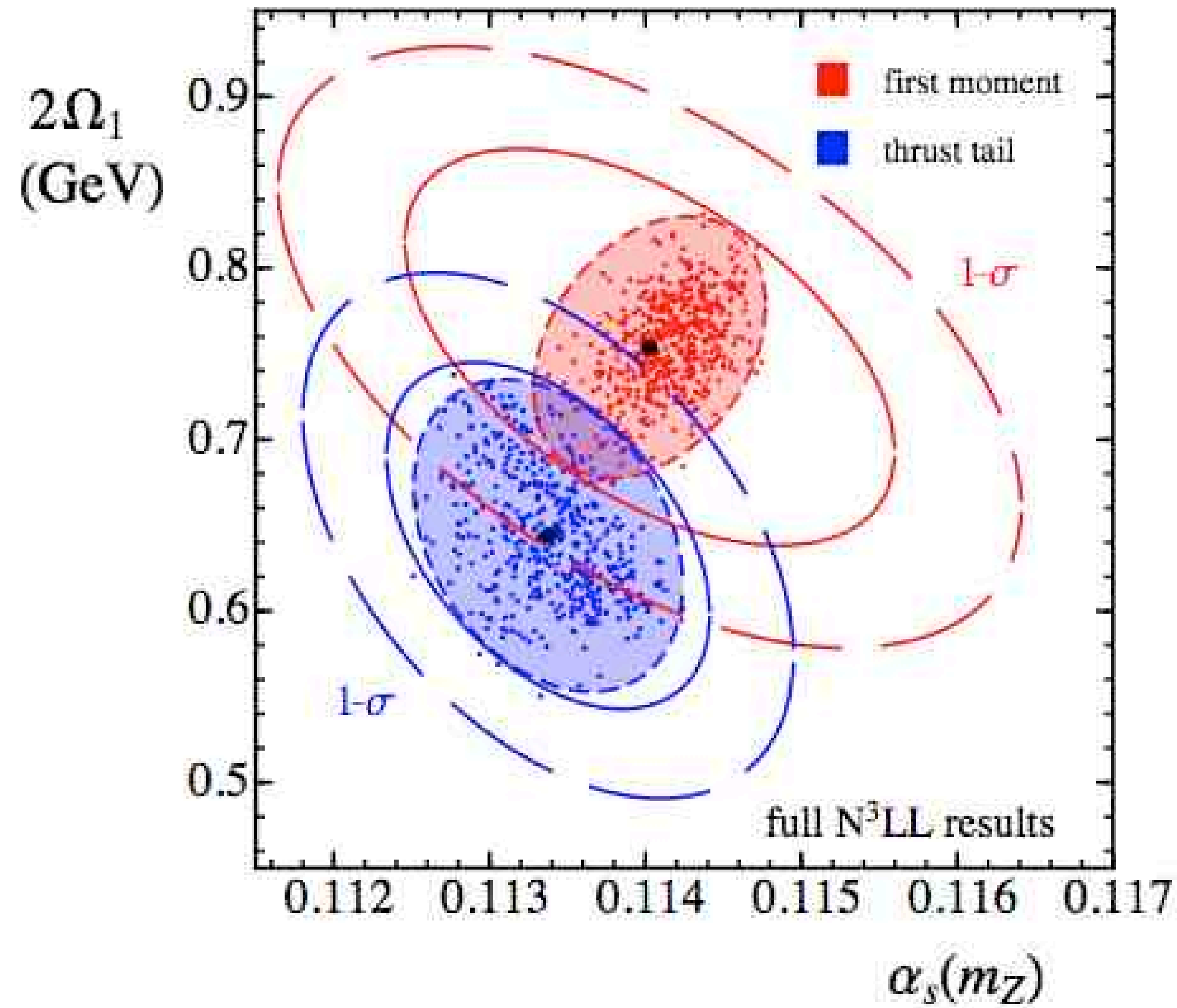
α_s determination: Thrust moment fits

[Abbate, Fickinger, Hoang, VM, Stewart]

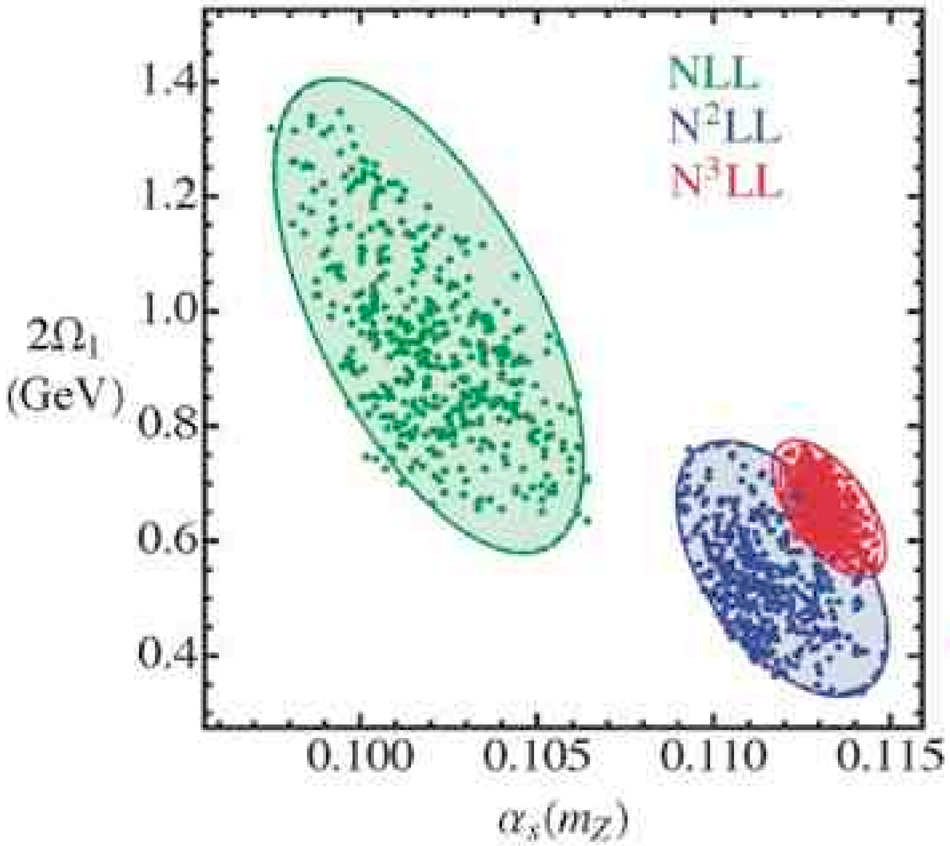
$$M_n = \frac{1}{\sigma}\int \mathrm{d}\tau\, \tau^n \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}$$

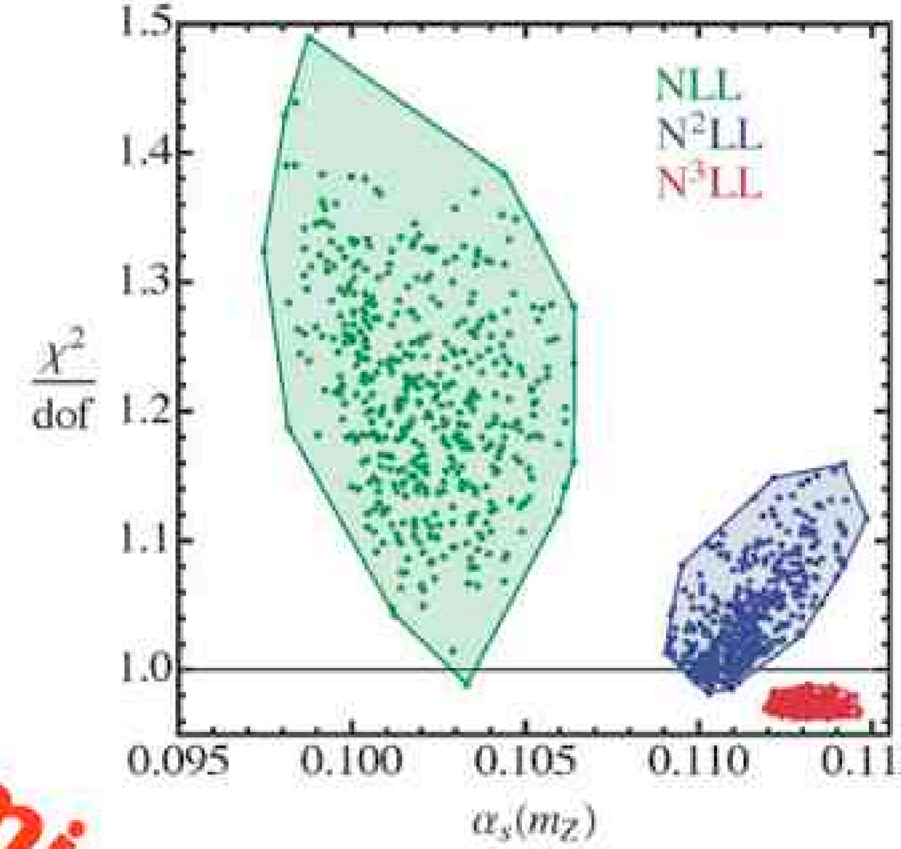
Only first moment of thrust
Used N^{3}LL code, with power corrections and renormalon subtraction
Different levels of theoretical sophistication
Significant error reduction when renormalon is removed

Good agreement with tail fits

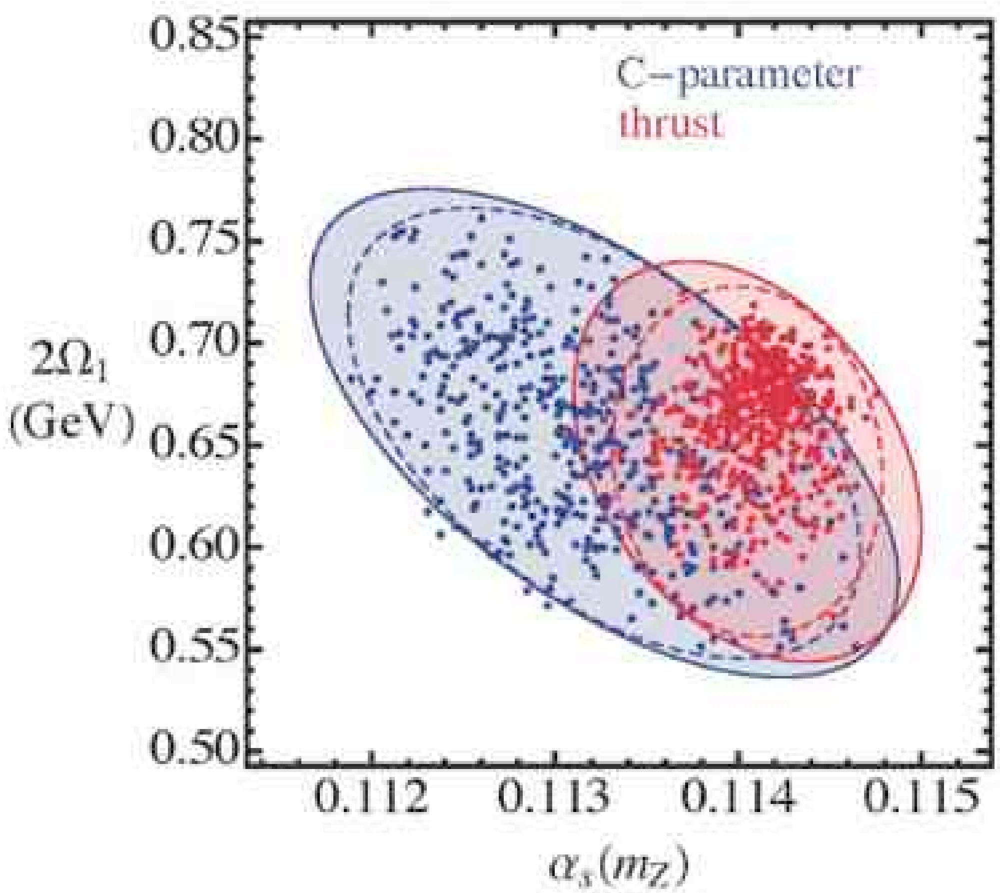


α_s determination: C-parameter tail fits





Preliminary results

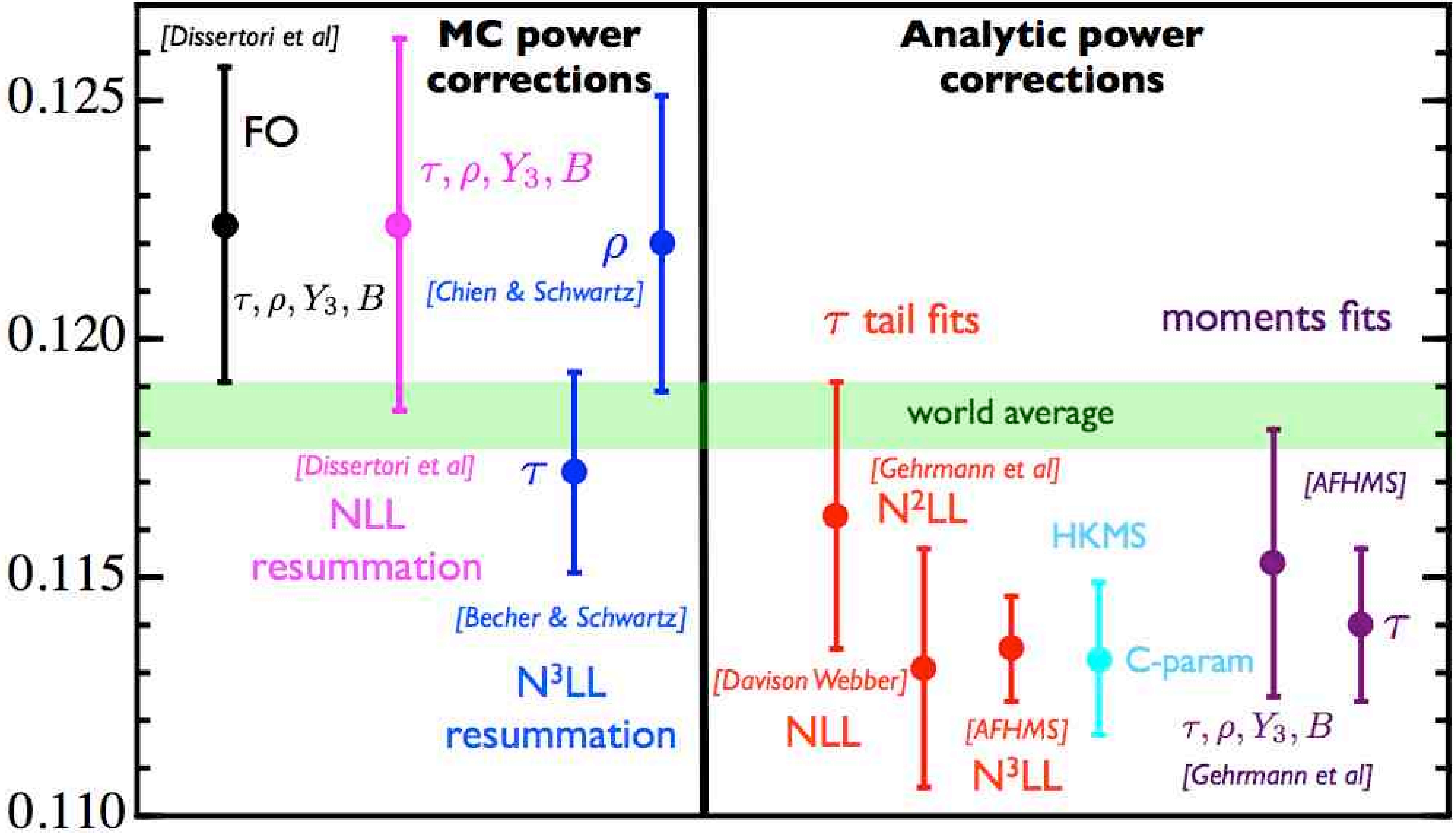


$$\alpha_s(m_Z) = 0.1133 \pm 0.0016$$

α_s determination: compendium

Only consider analysis with 3-loop input

$\alpha_s(m_Z)$ determination from event shape fits



UNIVERSALITY OF POWER CORRECTIONS

Studies of Universality

- Dispersive approach *[Dokshitzer & Webber 1995]*
  - Predicts universality for a bunch of event shapes, including recoil sensitive ones
  - They are based on a model using the one-gluon approximation. Modification of (effective coupling) below a cutoff scale
  - Milan factor takes into account two-gluon effects *[Dokshitzer, Webber, Salam]*
- SCET-CSS approach *[Lee & Sterman 2006]*
  - Predicts universality for non-recoil-sensitive event shapes.
  - They are model-independent, formulated in terms of QCD matrix elements.
  - Do not rely on one-gluon approximation.

$$\Omega_1^e = c_e \, \Omega_1^\rho$$

Massless predictions for universality

Thrust	$\tau = 1 - \max_{\vec{n}} \frac{\sum_i \lvert\vec{p}_i \cdot \vec{n}\rvert}{\sum \lvert\vec{p}_i\rvert}$	$c_\tau = 2$
Two-Jetiness	$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i \lvert\vec{p}_i \cdot \vec{n}\rvert}{Q}$	$c_{\tau_2} = 2$
C-parameter	$C = \frac{3}{2} \frac{\sum_{i,j} \lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert \sin^2(\theta_{ij})}{(\sum_i \lvert\vec{p}_i\rvert)^2}$	$c_C = 3\pi$
Angularities	$\tau_{(a)} = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a (1 - \lvert\cos\theta_i\rvert)^{1-a}$	$c_{\tau_{(a)}} = \frac{2}{1-a}$
Jet Masses	$\rho_\pm = \frac{1}{Q^2} \Big(\sum_{i\in\pm} p_i \Big)^2$	$c_\rho = 1$

Studies of Universality

- Dispersive approach *[Dokshitzer & Webber 1995]*
  - Predicts universality for a bunch of event shapes, including recoil sensitive ones
  - They are based on a model using the one-gluon approximation. Modification of (effective coupling) below a cutoff scale
  - Milan factor takes into account two-gluon effects *[Dokshitzer, Webber, Salam]*
- SCET-CSS approach *[Lee & Sterman 2006]*
  - Predicts universality for non-recoil-sensitive event shapes.
  - They are model-independent, formulated in terms of QCD matrix elements.
  - Do not rely on one-gluon approximation.

Both approaches assume particles are massless!!

$$\Omega_1^e = c_e\, \Omega_1^\rho$$

Kinematics of Event Shapes

We will concentrate on event shapes that are **not recoil sensitive** and can be written in the dijet limit as

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp} f_e(r_i, y_i)$$

$y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right)$	rapidity	All event shapes can be expressed in terms of these two variables
$r \equiv \frac{p^{\perp}}{m^{\perp}}$	transverse velocity	

$$m^{\perp} = \sqrt{p_T^2 + m^2}$$ transverse mass

$$\eta = \ln\left(\frac{\sqrt{r^2+\sinh^2 y}+\sinh y}{r}\right)$$ pseudo-rapidity

$$v = \frac{\sqrt{r^2+\sinh^2 y}}{\cosh y}$$ velocity

massless limit

$$v = r = 1$$

$$y = \eta$$

$$m^{\perp} = p^{\perp}$$

Massless Universality in SCET-CSS

In the massless limit one has

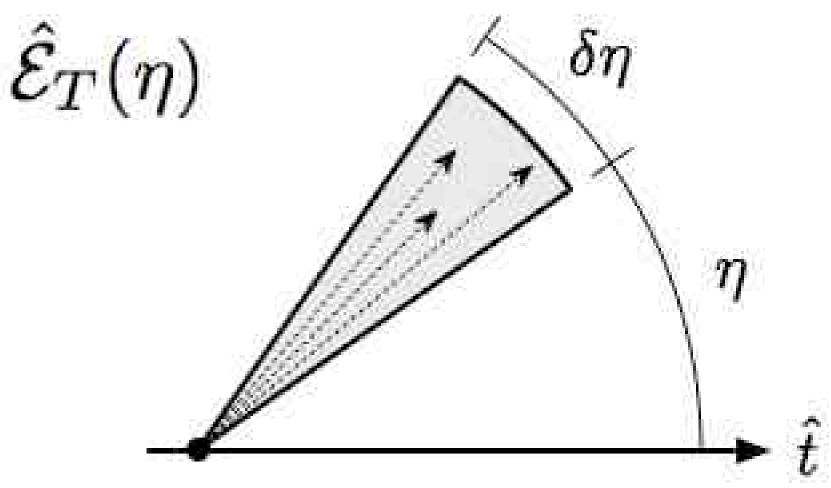
$$e(N) = \frac{1}{Q} \sum_{i \in N} p_i^{\perp} \, f_e(1, y_i)$$

Massless Universality in SCET-CSS

In the massless limit one has $$e(N) = \frac{1}{Q}\sum_{i\in N} p_i^\perp\, f_e(1, y_i)$$

Transverse energy-flow operator $$\mathcal{E}_T(y)|\,N\,\rangle = \sum_{i\in N} p_i^\perp \delta(y - y_i)|\,N\,\rangle$$

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F.V.Tkachov Ore Sterman]



[Bauer, Fleming, Lee, Sterman]

Measures all momenta flowing in a given rapidity

$$\mathcal{E}_T(y) = \frac{1}{\cosh^3 y}\int_0^{2\pi} \mathrm{d}\phi \lim_{R\to\infty}\int_0^\infty \mathrm{d}t\, \hat{n}_i T_{0i}(t, R\hat{n})$$

[unfortunately there is no physical limit in which this is the correct operator to use for power correction...]

Massless Universality in SCET-CSS

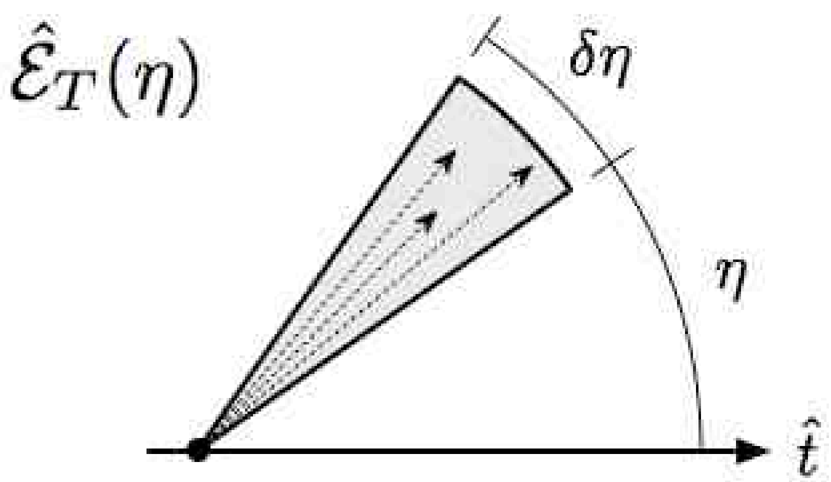
In the massless limit one has

$$e(N) = \frac{1}{Q} \sum_{i \in N} p_i^\perp \, f_e(1, y_i)$$

Transverse energy-flow operator

$$\mathcal{E}_T(y) | N \rangle = \sum_{i \in N} p_i^\perp \delta(y - y_i) | N \rangle$$

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F.V.Tkachov Ore Sterman]



[Bauer, Fleming, Lee, Sterman]

Measures all momenta flowing in a given rapidity

$$Q\,\hat{e} = \int \mathrm{d}y \, f_e(1, y) \mathcal{E}_T(y) \longrightarrow \hat{e} \, | N \rangle = e(N) \, | N \rangle$$

Event shape operator

$$\mathcal{E}_T(y) = \frac{1}{\cosh^3 y} \int_0^{2\pi} \mathrm{d}\phi \, \lim_{R \to \infty} \int_0^\infty \mathrm{d}t \, \hat{n}_i T_{0i}(t, R\hat{n})$$

[unfortunately there is no physical limit in which this is the correct operator to use for power correction...]

Massless Universality in SCET-CSS

In the massless limit one has

$$e(N) = \frac{1}{Q} \sum_{i\in N} p_i^\perp \, f_e(1, y_i)$$

Transverse energy-flow operator

$$\mathcal{E}_T(y) | \, N \, \rangle = \sum_{i\in N} p_i^\perp \delta(y - y_i) | \, N \, \rangle$$

Event shape operator

$$\hat{e} = \frac{1}{Q} \int \mathrm{d}y \, \mathcal{E}_T(y) f_e(1, y)$$

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F.V.Tkachov Ore Sterman]

$$\Omega_1^e = \langle \, 0 \, | \, \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q \hat{e} Y_n \overline{Y}_{\bar{n}} \, | \, 0 \, \rangle$$

$$\Omega_1^e = \int \mathrm{d}y \; f_e(1, y) \langle \, 0 \, | \, \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \mathcal{E}_T(y) Y_n \overline{Y}_{\bar{n}} \, | \, 0 \, \rangle$$

Massless Universality in SCET-CSS

In the massless limit one has

$$e(N) = \frac{1}{Q}\sum_{i\in N} p_i^{\perp}\, f_e(1, y_i)$$

Transverse energy-flow operator

$$\mathcal{E}_T(y)|\,N\,\rangle = \sum_{i\in N} p_i^{\perp}\delta(y - y_i)|\,N\,\rangle$$

Event shape operator

$$\hat{e} = \frac{1}{Q}\int \mathrm{d}y\, \mathcal{E}_T(y) f_e(1, y)$$

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F.V.Tkachov Ore Sterman]

$$\Omega_1^e = \langle\, 0\,|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} Q\hat{e} Y_n \overline{Y}_{\bar{n}}\,|\,0\,\rangle$$

$$\Omega_1^e = \int \mathrm{d}y\, f_e(1, y)\langle\, 0\,|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger}\mathcal{E}_T(y) Y_n \overline{Y}_{\bar{n}}\,|\,0\,\rangle = c_e \times \Omega^E$$

Boost invariance requires this term is y-independent

Universal power correction

Calculable coefficient, depends on the event shape

$$\Omega_1^E = \langle\, 0\,|\,\overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger}\mathcal{E}_T(0) Y_n \overline{Y}_{\bar{n}}\,|\,0\,\rangle$$

Operator definition of power correction

MASS EFFECTS ON POWER CORRECTIONS

Mass Effects in SCET

[VM, I.W. Stewart, J.Thaler]
arXiv: 1209.3781

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp}\, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

Mass Effects in SCET

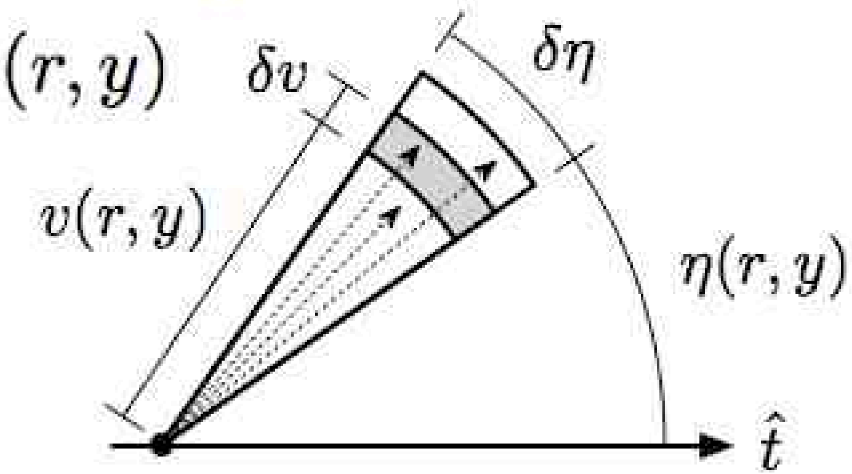
[VM, I.W. Stewart, J.Thaler]
arXiv: 1209.3781

$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^{\perp} f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

Transverse velocity operator

$\hat{\mathcal{E}}_T(r,y)$



$$v = v(r,y)$$
$$\eta = \eta(r,y)$$

$$\mathcal{E}_T(r,y)|\,N\,\rangle = \sum_{i \in N} m_i^{\perp} \delta(r - r_i)\delta(y - y_i)|\,N\,\rangle$$

measures momenta of particles with given transverse velocity flowing at a given rapidity

$$\hat{e} = \frac{1}{Q} \int \mathrm{d}y\, \mathrm{d}r\, \mathcal{E}_T(r,y) f_e(r,y)$$

two integrals

$$\mathcal{E}_T(v,\eta) = -\frac{v(1 - v^2 \tanh^2 y)^{\frac{3}{2}}}{\cosh\eta} \lim_{R\to\infty} R^3 \int_0^{2\pi} \mathrm{d}\phi\, \hat{n}_i\, T_{0i}(R, v\, R\, \hat{n})$$

Mass Effects in SCET

[VM, I.W. Stewart, J. Thaler]
arXiv: 1209.3781

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp}\, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

$$\Omega_1^e = \int \mathrm{d}r\,\mathrm{d}y\, f_e(r,y)\langle 0\,|\,\overline{Y}_{\bar n}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,y) Y_n \overline{Y}_{\bar n}\,|\,0\,\rangle$$

Mass Effects in SCET

[VM, I.W. Stewart, J.Thaler]
arXiv: 1209.3781

$$e(N) = \frac{1}{Q}\sum_{i\in N} m_i^{\perp}\, f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

$$\Omega_1^e = \int \mathrm{d}r\, \mathrm{d}y\, f_e(r,y) \langle 0 |\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,y) Y_n \overline{Y}_{\bar{n}} \,| 0 \rangle = c_e \int \mathrm{d}r\, g_e(r)\Omega_1(r)$$

Boost invariance requires this term is y-independent

Operator definition of power correction $\Omega_1(r) = \langle 0 |\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} \,| 0 \rangle$

Same as for massless computation $c_e = \int_{-\infty}^{\infty} \mathrm{d}y\, f_e(1,y)$

$$g_e(r) = \frac{1}{c_e}\int \mathrm{d}y\, f_e(r,y)$$

encodes all mass effects

each $g_e(r)$ defines a universality class of events with same power correction

Event shapes considered

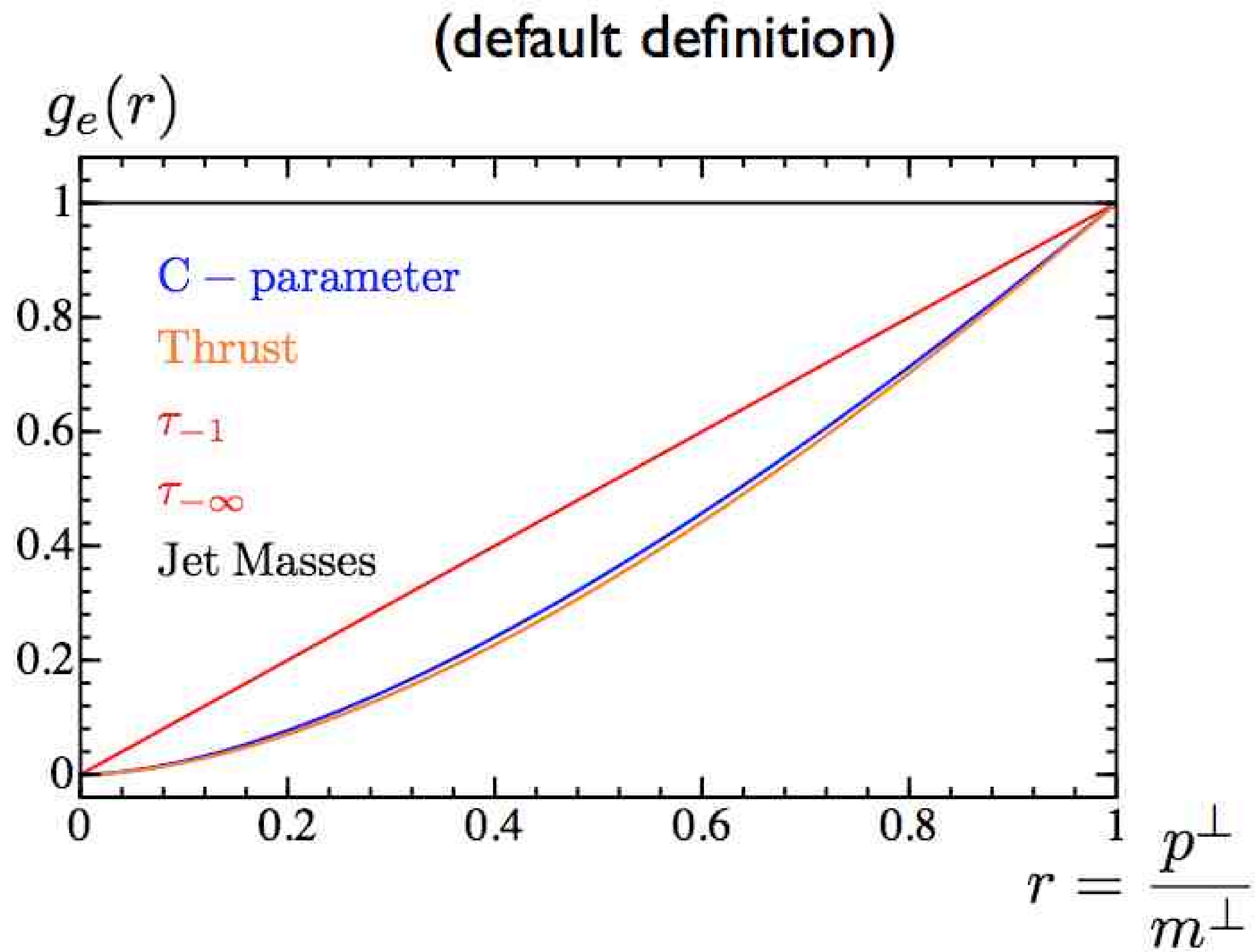
Thrust

Jet Masses

C-parameter

Angularities

2-Jettiness



Same **color** means same **power correction**

Event shapes considered

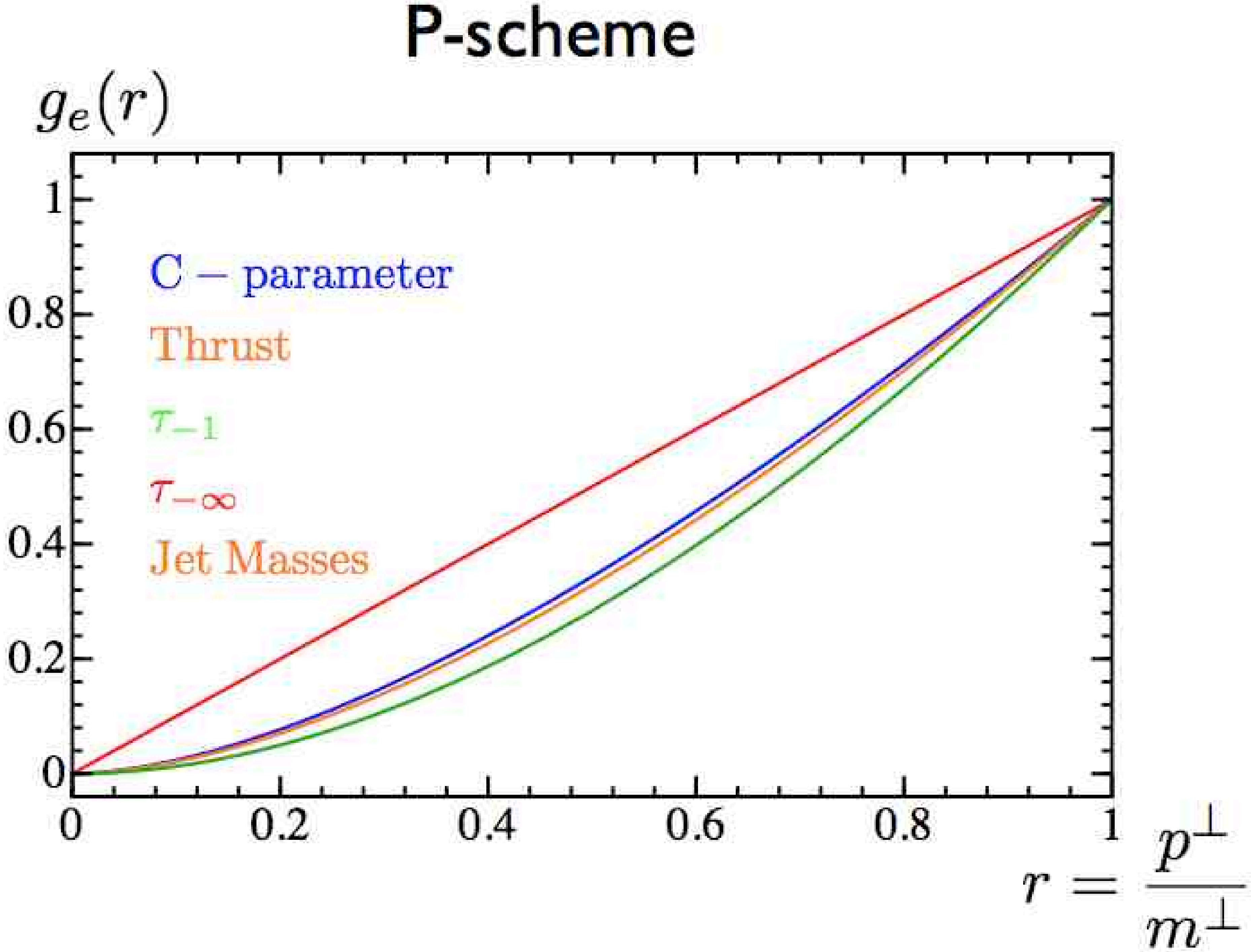
Thrust

Jet Masses

C-parameter

Angularities

2-Jettiness



→ Scheme changes event shape definition

Event shapes considered

Thrust

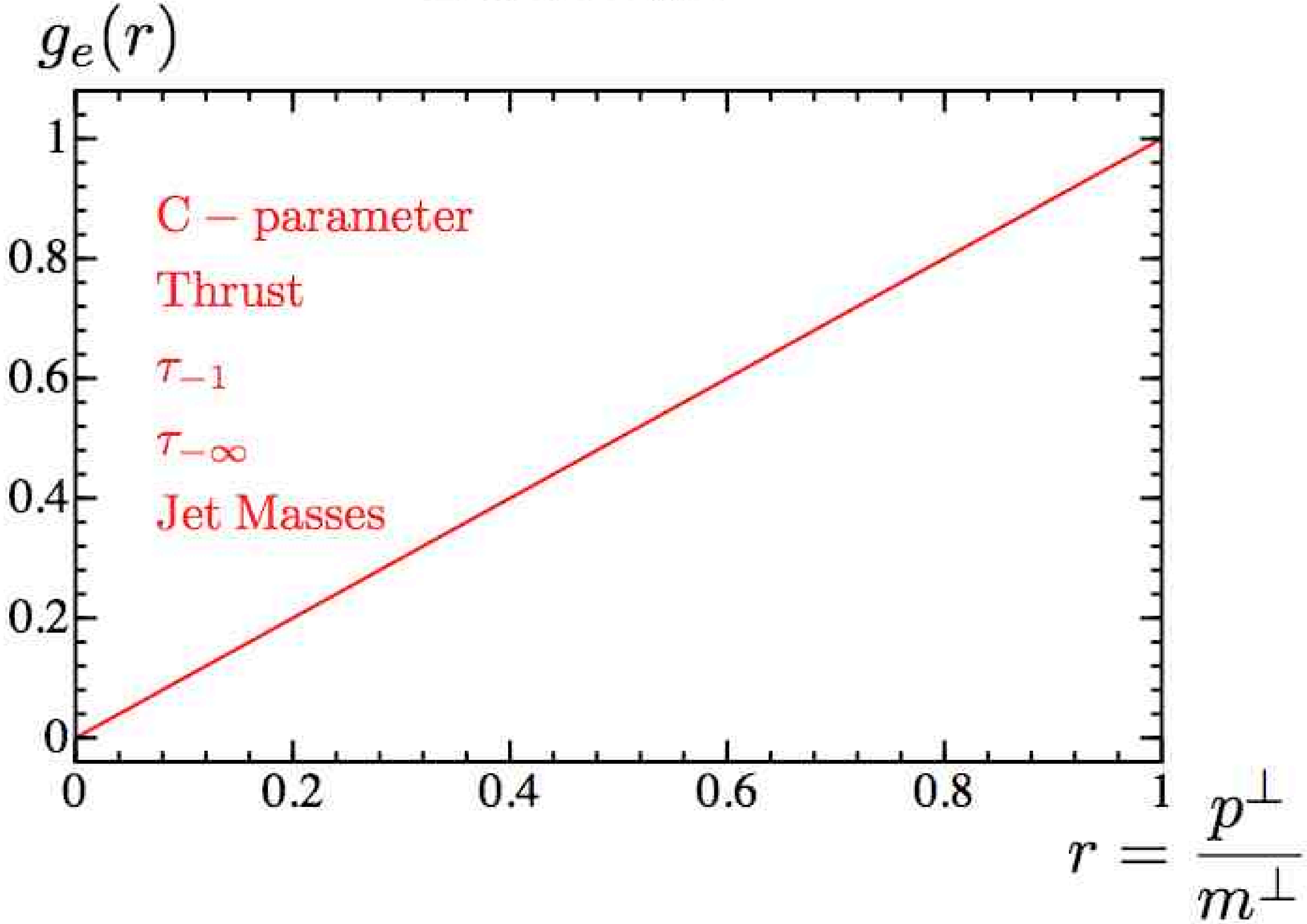
Jet Masses

C-parameter

Angularities

2-Jettiness

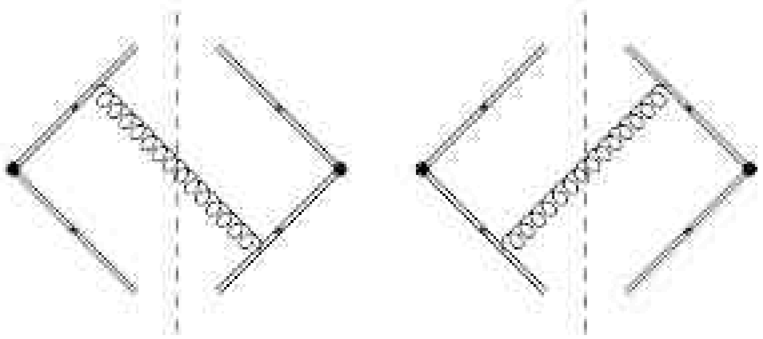
E-scheme



→ Scheme changes event shape definition

ANOMALOUS DIMENSION

Anomalous dimension computation



$$\Omega_1(r) = \langle\, 0\, |\overline{Y}^\dagger_{\bar{n}} Y^\dagger_n \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} |\, 0\, \rangle$$

One needs to compute diagrams
that probe the operator

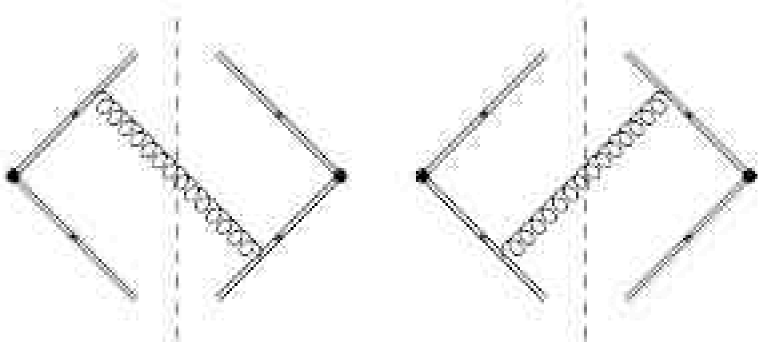
The measured gluon is off-shell
This probes value of $r \neq 0$

$$A^{\mu A}(x) \to A^{\mu A}(x) + J^{\mu A}(x)$$

massless
quantum field

off-shell
background
gauge field

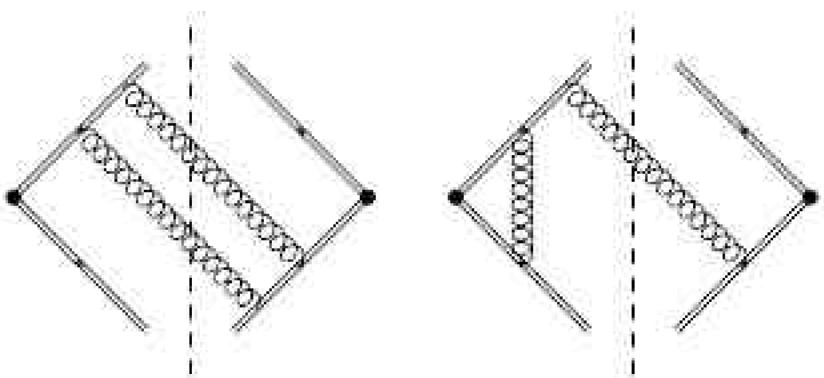
Anomalous dimension computation



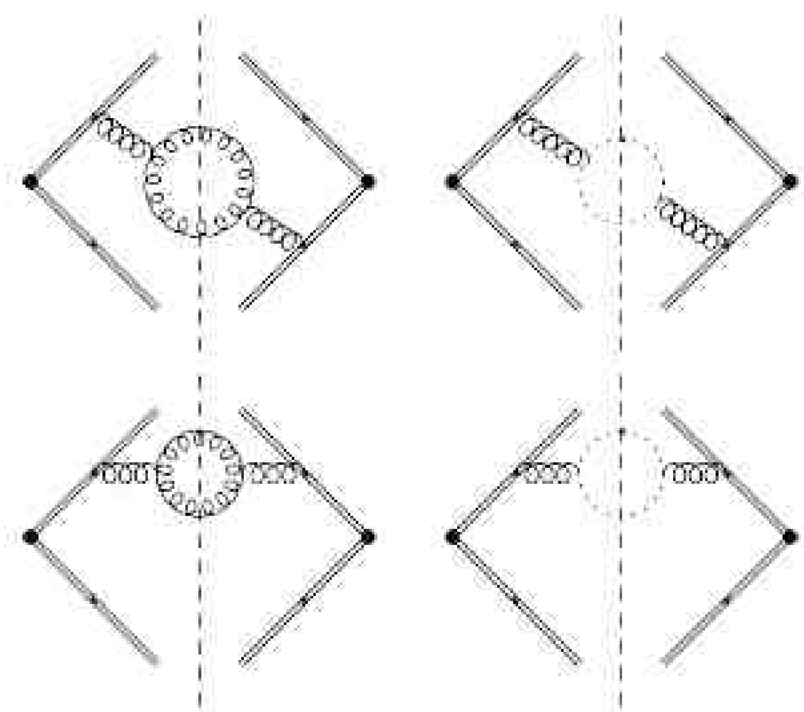
$$\Omega_1(r) = \langle\, 0\, |\overline{Y}^\dagger_{\bar{n}} Y^\dagger_n \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} |\, 0\, \rangle$$

One needs to compute diagrams that probe the operator

The measured gluon is off-shell
This probes value of $r \neq 0$

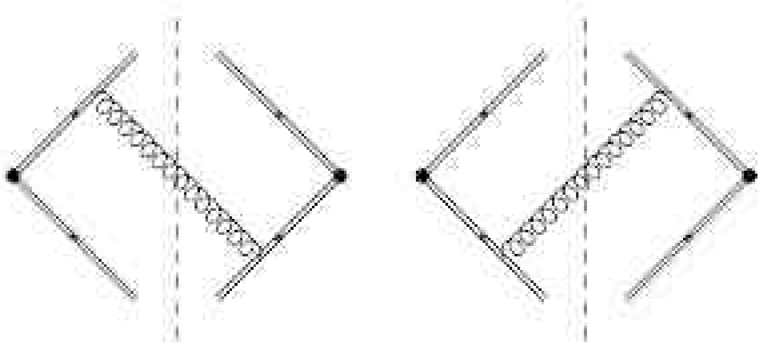


Abelian contribution exactly vanish when adding real and virtual radiation



Self-energy diagrams are IR and UV finite

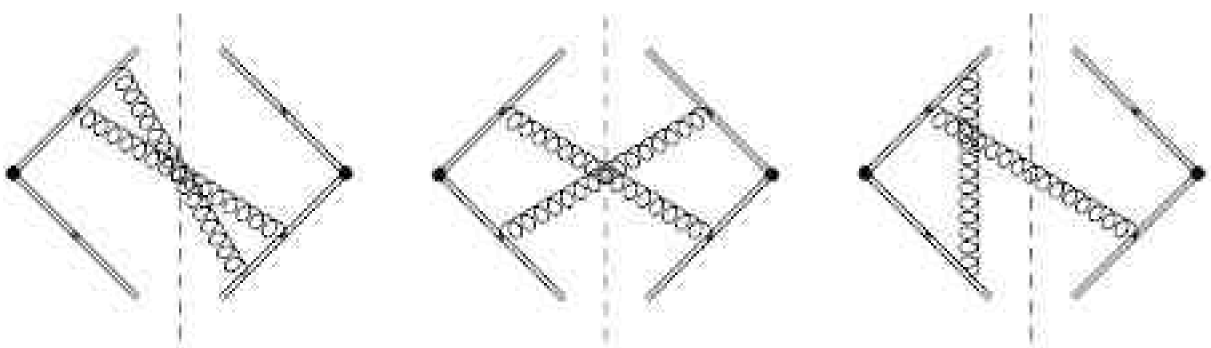
Anomalous dimension computation

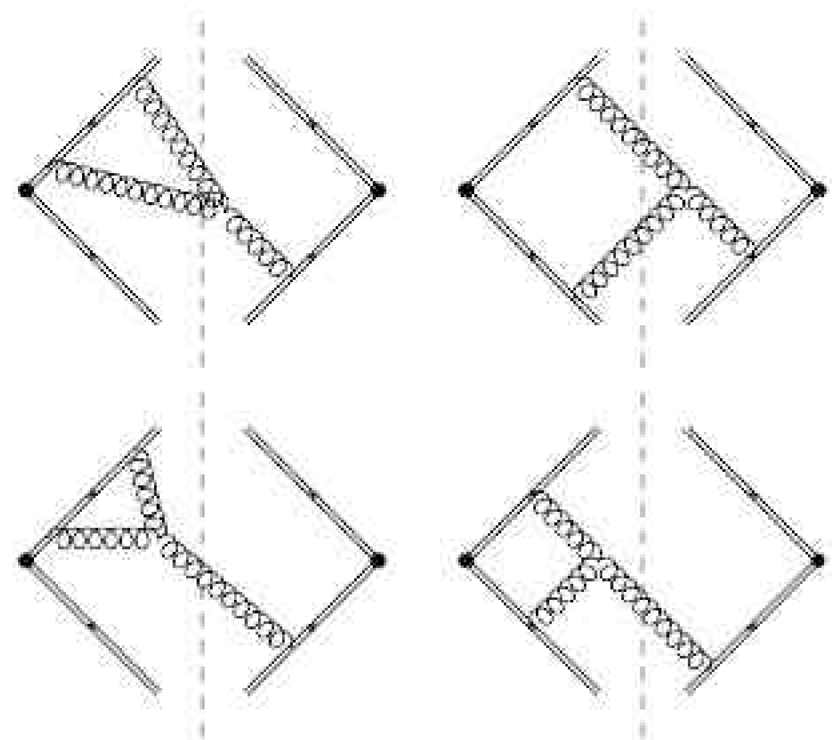


$$\Omega_1(r) = \langle 0 | \overline{Y}^\dagger_{\bar{n}} Y^\dagger_n \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

One needs to compute diagrams
that probe the operator

The measured gluon is off-shell
This probes value of $r \neq 0$





Only purely non-abelian diagrams contribute

We obtain an IR-finite anomalous dimension

Results and consequences

$$\gamma^{\Omega_1} = -\frac{\alpha_s C_A}{\pi} \log(1-r^2)$$

r-dependent anomalous dimension
no mixing between various r values

RGE solution at NLL

Shape Function F contains perturbative evolution

$$\Omega_1(r,\mu) = \Omega_1(r,\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{2C_A}{\beta_0}\log(1-r^2)}$$

$$\sim \Omega_1(r,\mu_0)\left[1 - \frac{\alpha_s(\mu_0)C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\log(1-r^2)\right]$$

Expanding it out

There is not a resummation formula for Ω_1^e

$$\Omega_1^e(\mu) = \int \mathrm{dr}\, \mathrm{g_e(r)}\, \Omega_1(\mathrm{r},\mu_0)\left(\frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(\mu_0)}\right)^{\frac{2\mathrm{C_A}}{\beta_0}\log(1-\mathrm{r}^2)}$$

Unknown function !

Using expanded out result

$$\Omega_1^e(\mu) = \Omega_1^e(\mu_0) - \frac{\alpha_s(\mu_0)C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\Omega_{\log}^e(\mu_0)$$

$$\Omega_{\log}^e(\mu_0) = \int \mathrm{dr}\, \log(1-\mathrm{r}^2)\, \mathrm{g_e(r)}\, \Omega_1(\mathrm{r},\mu_0)$$

New nonperturbative parameter

Conclusions from Power Corrections

- Hadron mass effects are O(1), cannot be treated as a correction
- Universality broken by hadron masses.

consequences

$$\Omega_1^C \simeq \frac{3\pi}{2}\Omega_1^\tau$$

$$\Omega_1^{\mathrm{HJM}} \neq 2\,\Omega_1^\tau$$

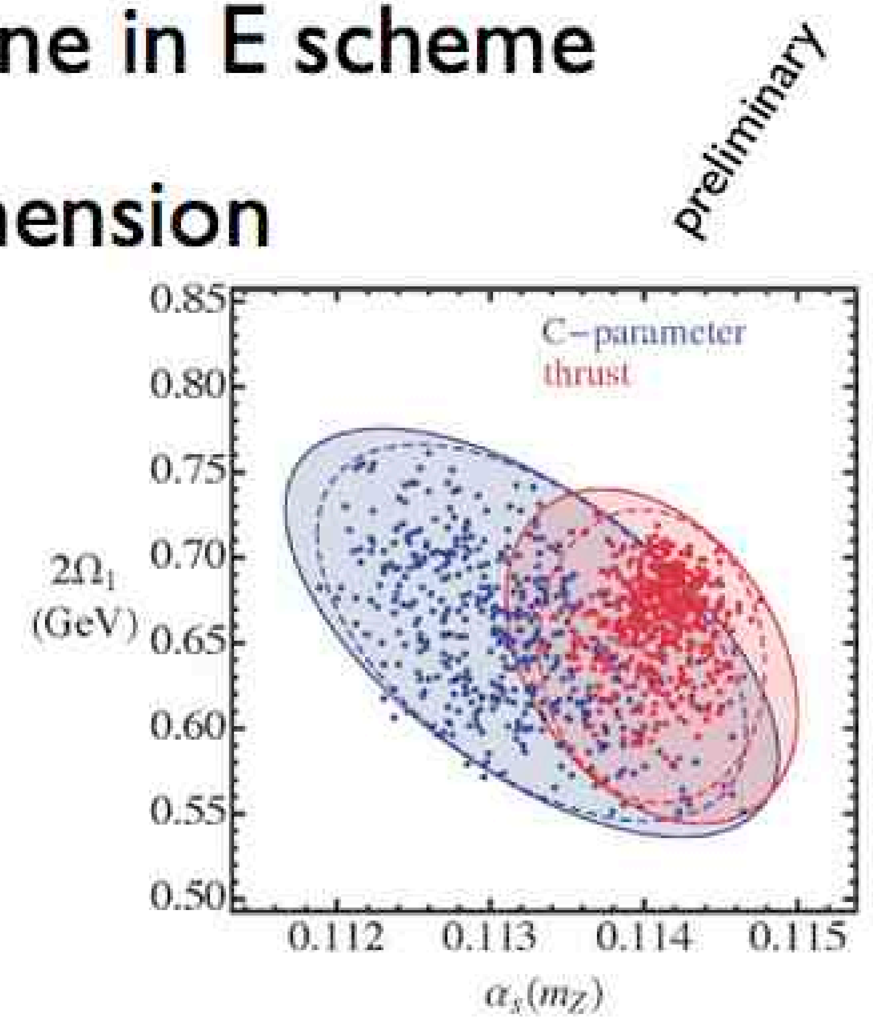
- Universality restored if measurements done in E scheme
- Power corrections carries anomalous dimension
  1. Perturbative remnant in power correction
  2. New power-correction parameters appear

Conclusions from Power Corrections

- Hadron mass effects are O(1), cannot be treated as a correction
- Universality broken by hadron masses.

consequences
$$\Omega_1^C \simeq \frac{3\pi}{2}\Omega_1^\tau$$
$$\Omega_1^{\rm HJM} \neq 2\,\Omega_1^\tau$$

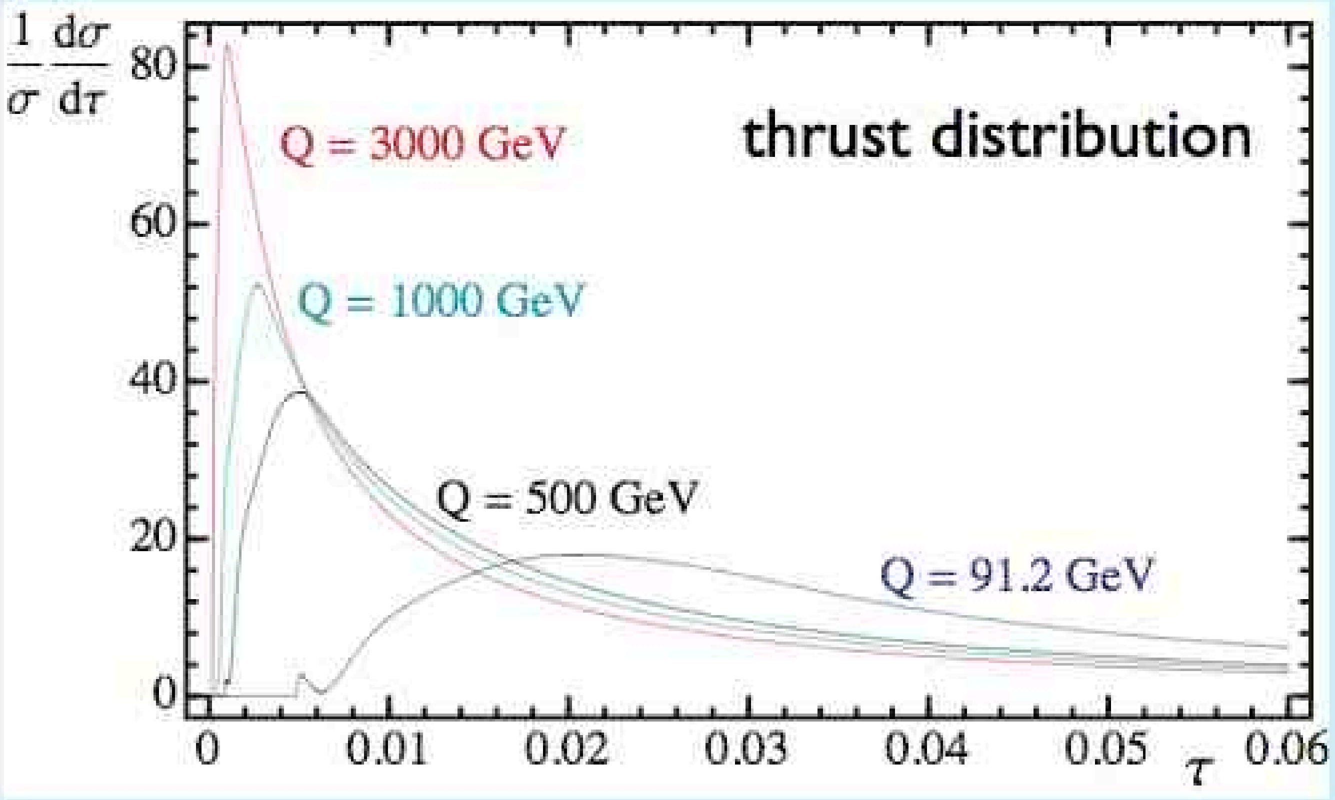
- Universality restored if measurements done in E scheme
- Power corrections carries anomalous dimension
  1. Perturbative remnant in power correction
  2. New power-correction parameters appear



Nice agreement between thrust and C-parameter!

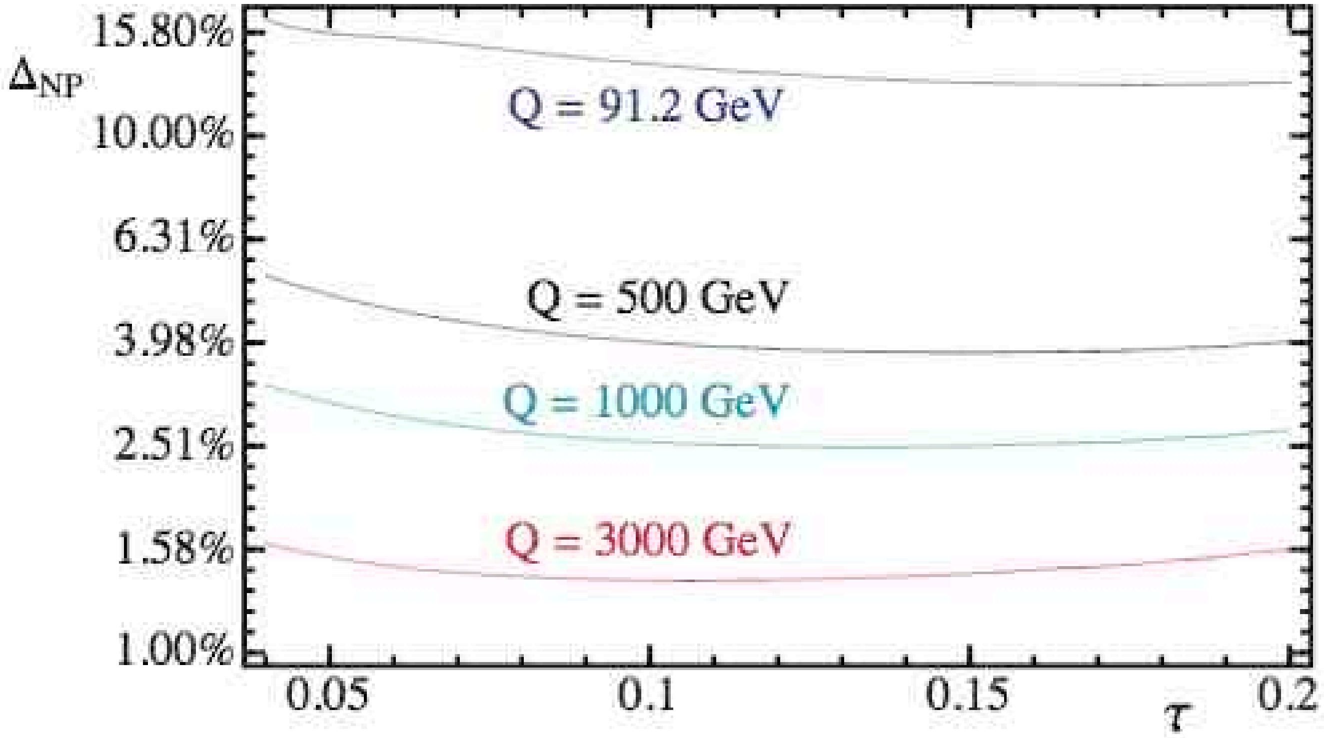
APPLICATIONS FOR A LINEAR COLLIDER

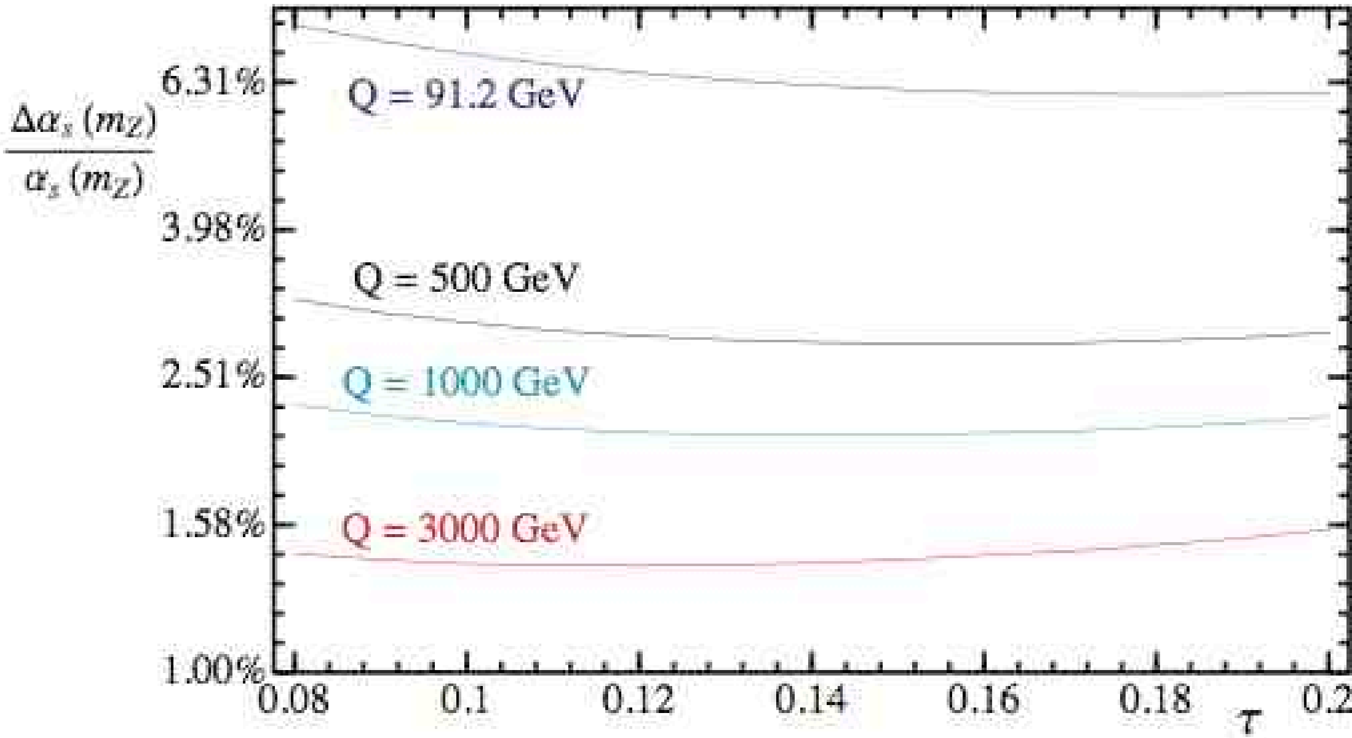
Size of non-perturbative effects



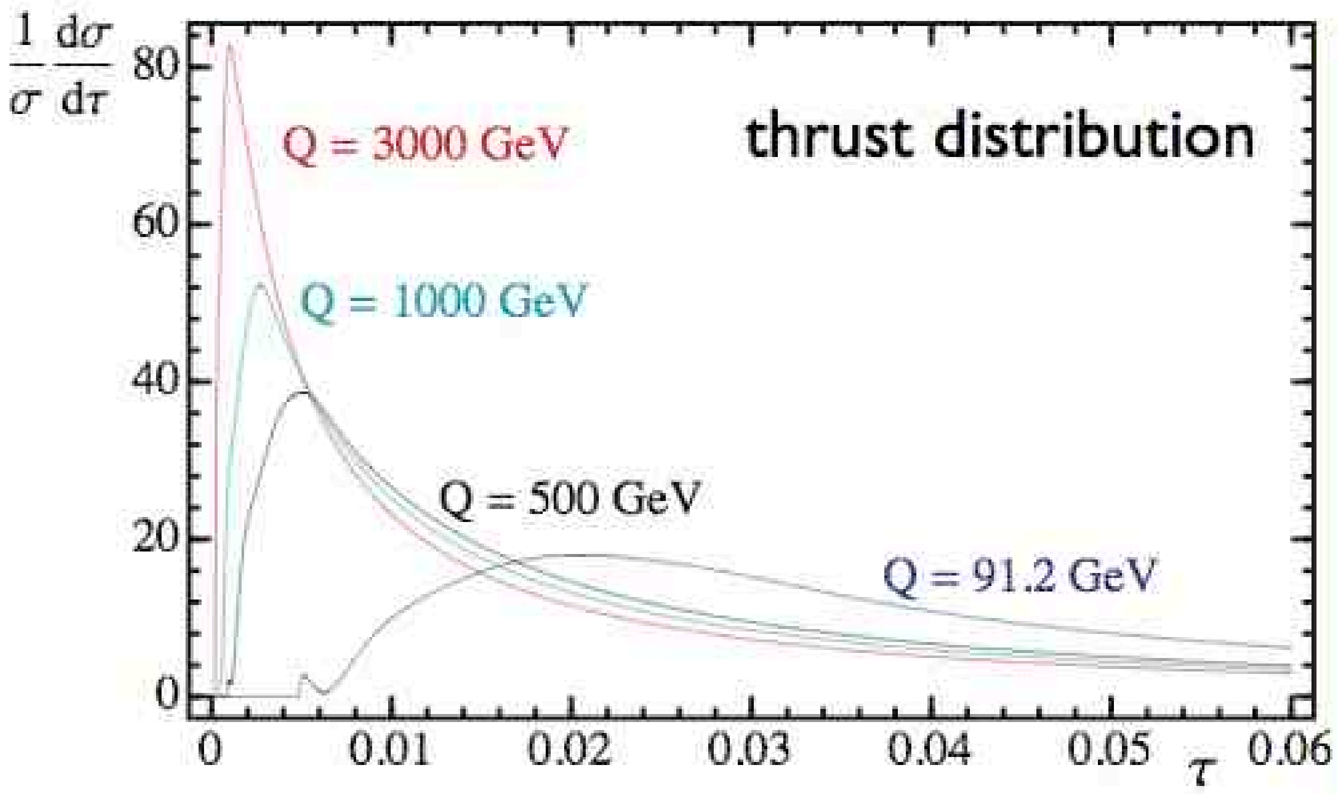
For increasing Q

- Peak moves toward smaller τ
- Events tend to accumulate at very small τ region
- The tail regions becomes longer but less populated



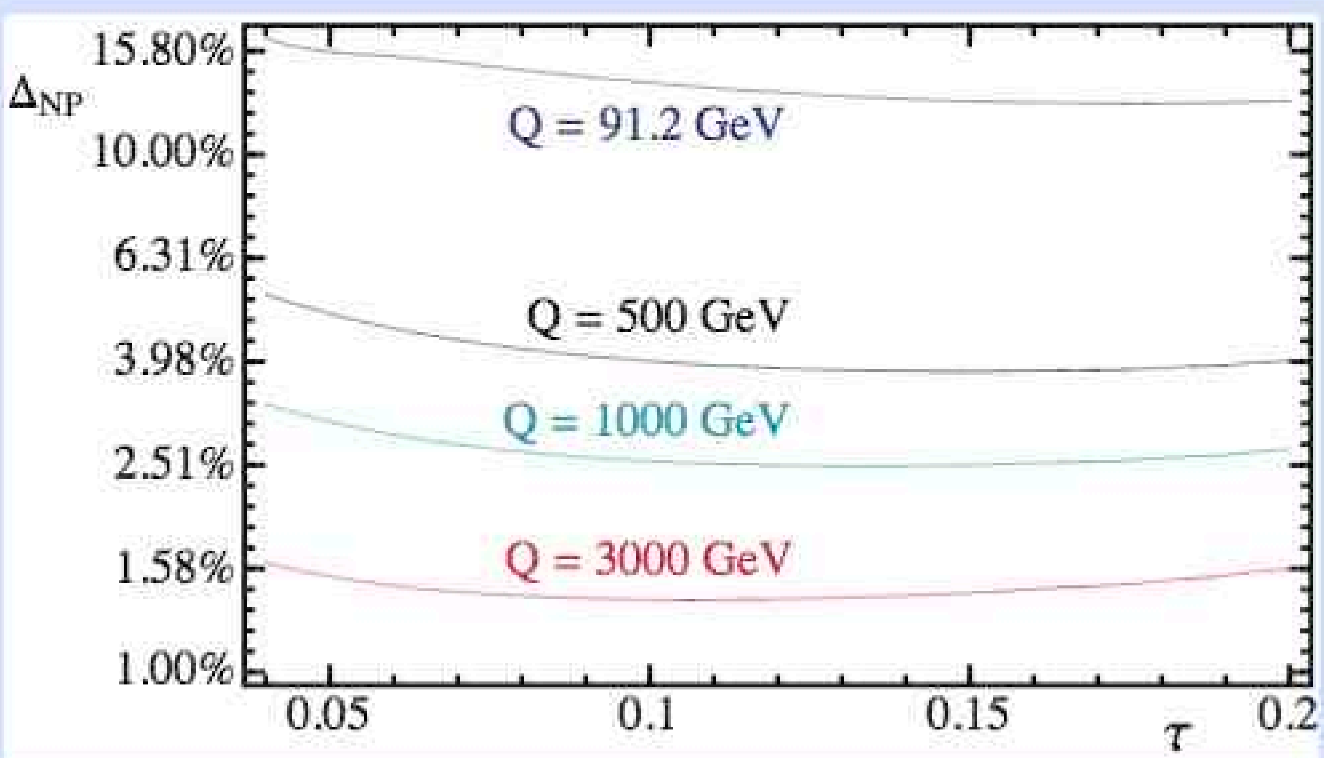


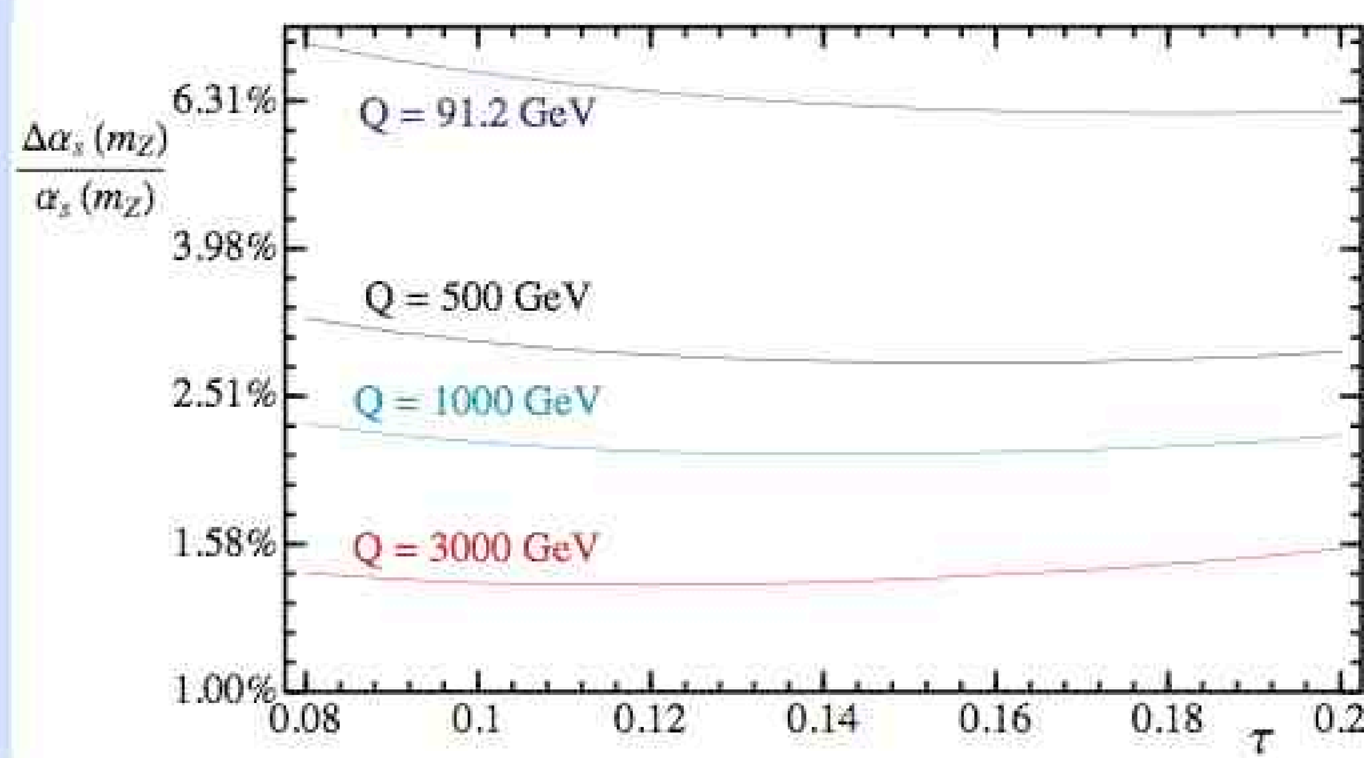
Size of non-perturbative effects



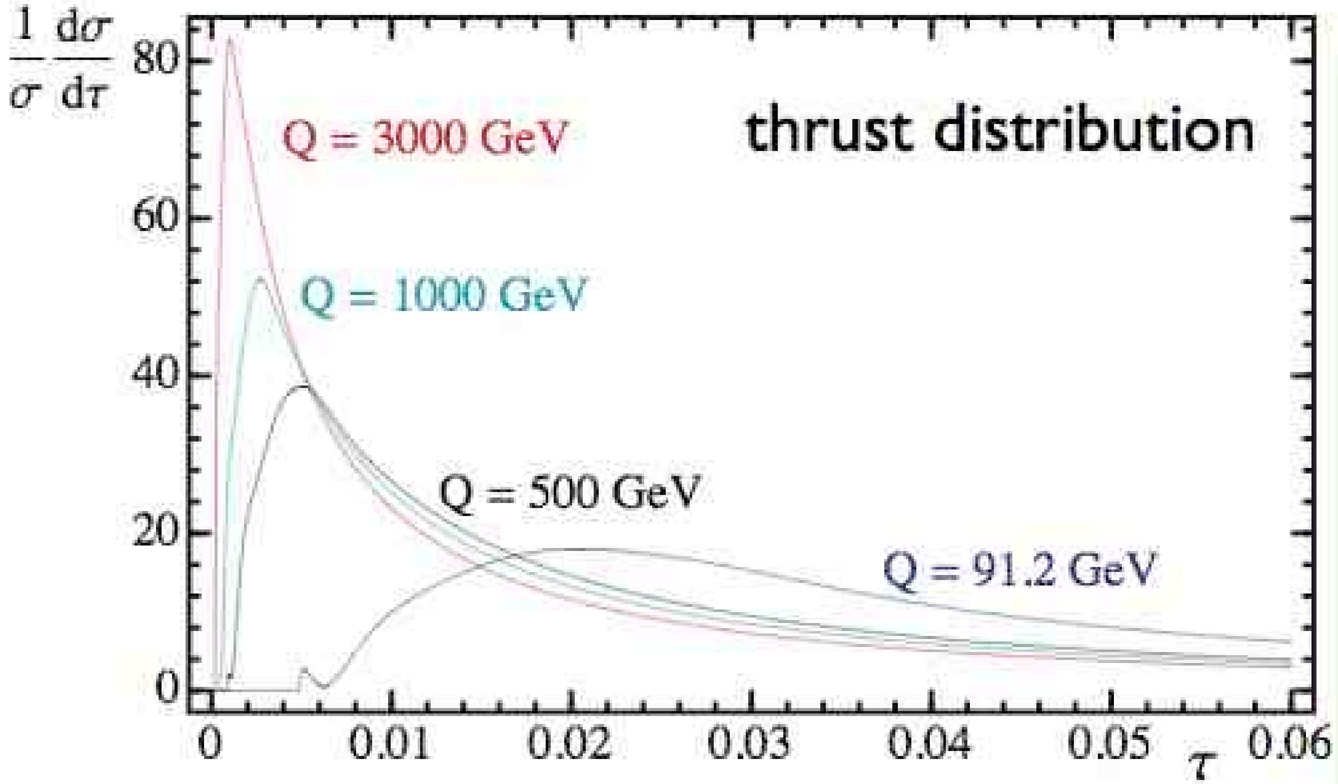
For increasing Q

- Size of nonperturbative effects decreases with Q
- They scale as 1/Q.
- At very high Q, nonperturbative effects become smaller than expt. errors **may be neglected**





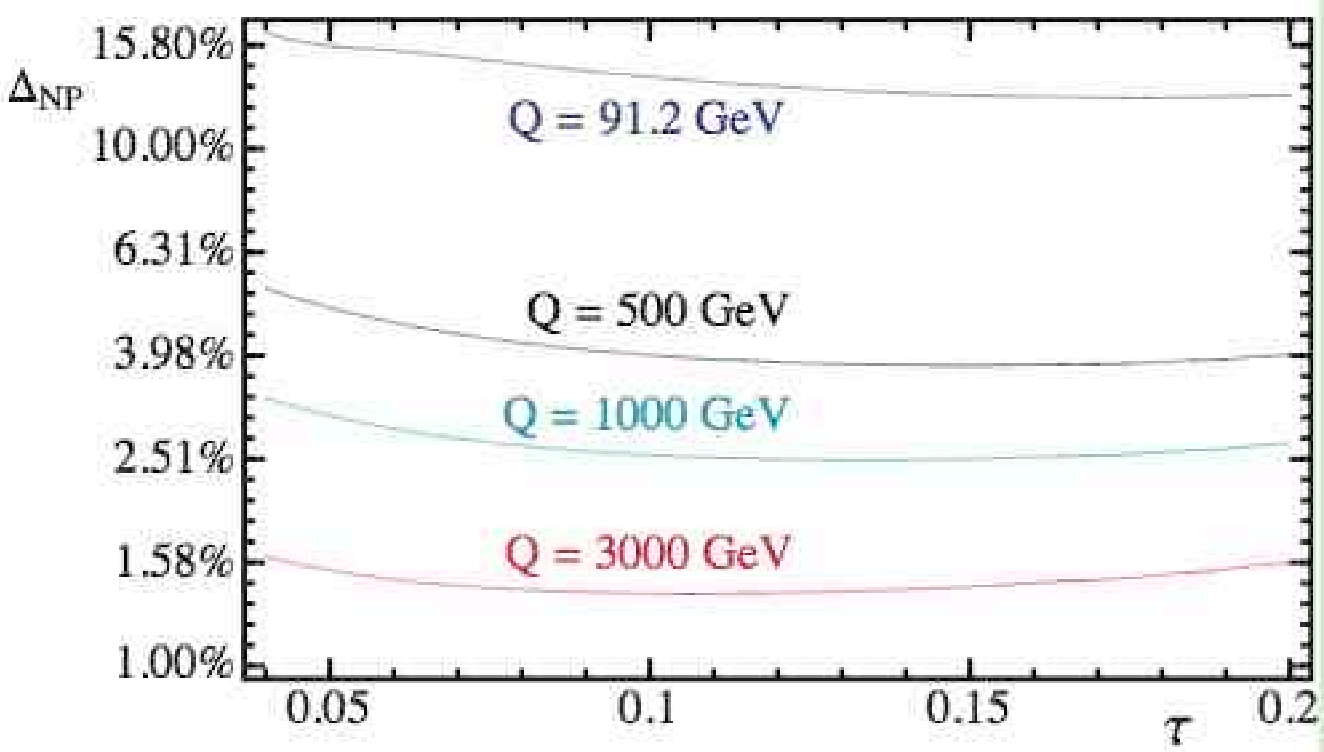
Size of non-perturbative effects

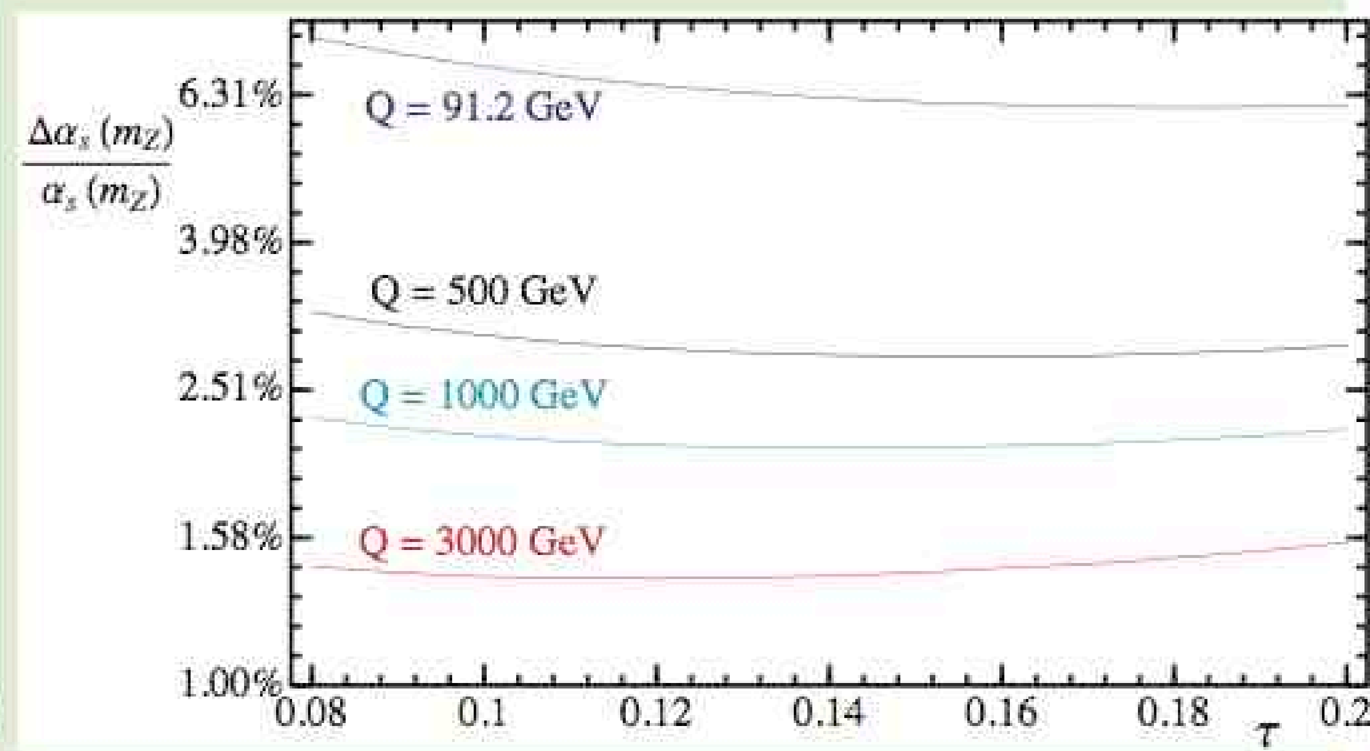


For increasing Q

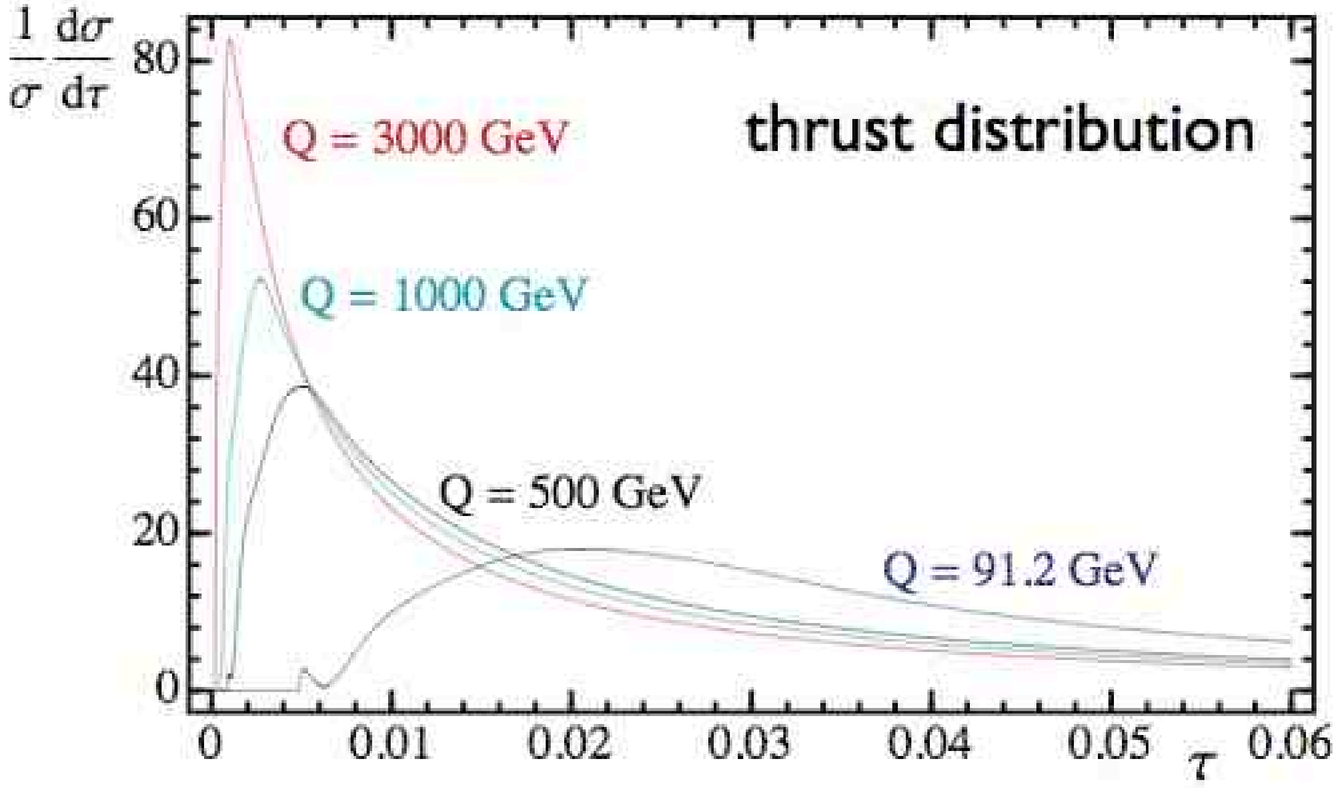
- Impact of nonperturbative effects when determining α_s is less important at higher energies.
- There is also a "running-downwards" effect, which increases uncertainties.

Mimic power corr. with change in α_s

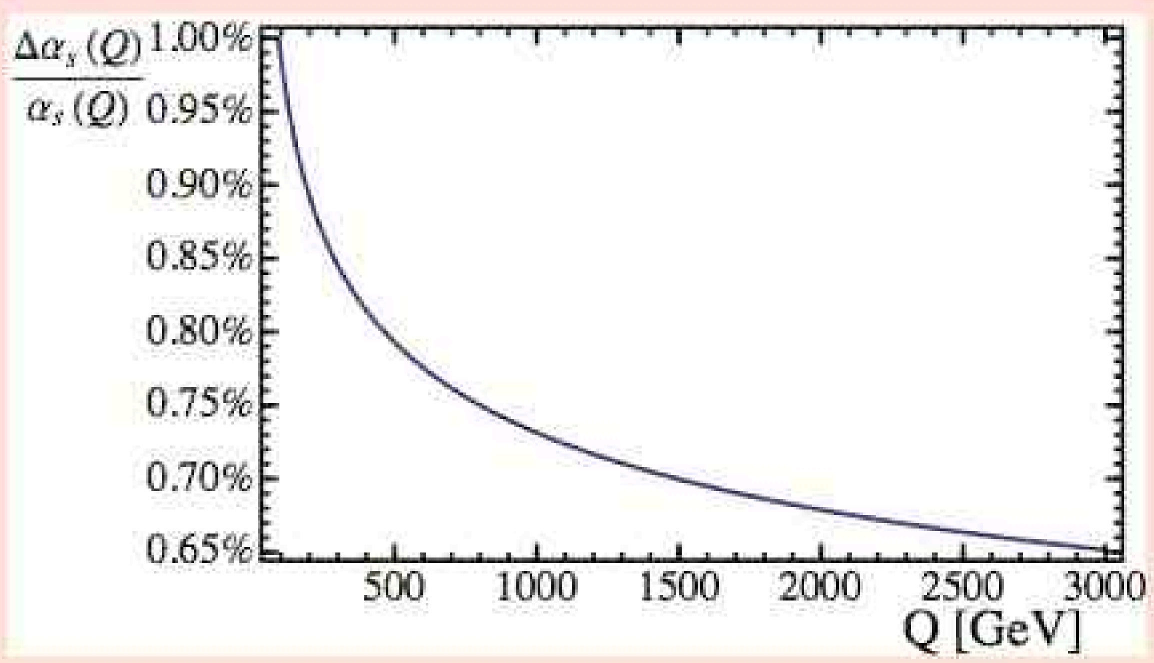




Size of non-perturbative effects

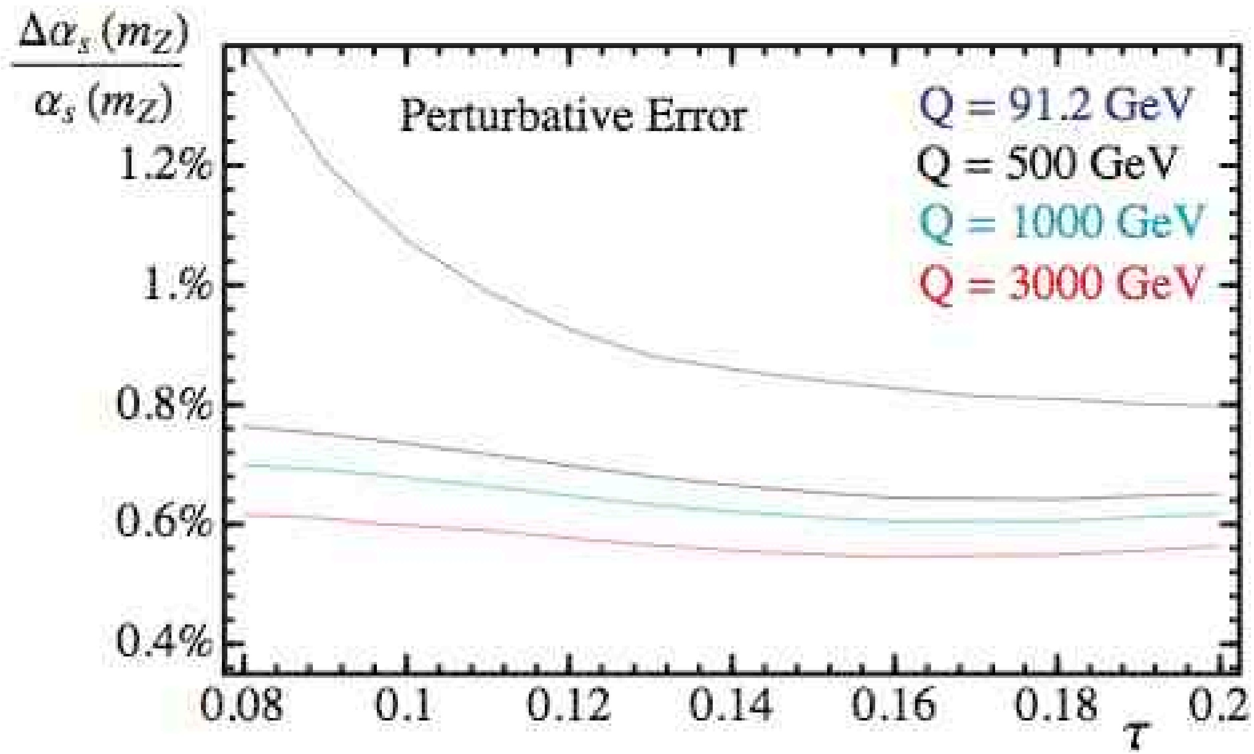


running downwards effect



Size of Perturbative errors

- Perturbative errors significantly reduced at high energies
- Reduction overcomes the "running down" effect.
- Could compensate for the growth of experimental uncertainties



At very high energies one can make fits using a single Q distribution

Fits for α_s at very high energies can probe for new particles through vac. pol. & shed light on lattice vs. thrust results

CONCLUSIONS

CONCLUSIONS

- Event shapes are a powerful tool to understand non-perturbative corrections and to determine the strong coupling constant. Very high precision cross sections!
- Power corrections and hadron mass effects now understood from a QCD operator formalism. **Universality breaking**.
- We find that power corrections carry a non-trivial anomalous dimensions.
- Event Shapes from an ILC would fuel a new era of precision QCD and New Physics analyses