Automatic computation of NLO corrections for multiparticle production at the LHC

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Abstract

Techniques based on Unitarity cuts have produced a lot of progresses in the computation of 1-loop amplitudes. However, when working in no more than 4 integer dimensions, only a part of each of these amplitudes can be obtained by means of cuts, thus partially reducing, at least from the conceptual point of view, the complexity of a 1-loop calculation to the easier problem of evaluating tree-level amplitudes. Another part instead, needs a fully dedicated 1-loop computation. I will discuss on the effective Feynman Rules we have developed to allow for the computation of this last part of Standard Model amplitudes, in the framework of the OPP technique. I will also discuss how these methods have been recently merged in the Helac Monte Carlo event generator, allowing for the evaluation at NLO in the strong coupling costant, of elementary scattering processes of crucial interest at LHC. The present limits of the Helac-NLO system will be also pointed out, and the work to be done to further extend this code, even in the Electroweak sector, will be briefly sketched.

Hadron-Hadron cross section

QCD factorization:

$$\sigma_{H_1 H_2 \to X} = \sum_{i,j} \int d\phi_n dx_1 dx_2 f_{i/H_1}(x_i, \mu_F) f_{j/H_2}(x_j, \mu_F)$$
$$|\mathcal{M}_{ij\to X}(x_i p_1, x_j p_2, \alpha_S(\mu_R), \mu_F)|^2$$

valid if hard scattering scale $>>$ hadronization scale $\sim 1GeV$

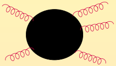
* μ_F factorization scale (IR origin)
* μ_R renormalization scale (UV origin)
* $f_{i,j}$ parton distribution functions
* $\mathcal{M}_{ij}$ matrix element for the hard scattering process (at present LO matrix elements in many MC event generators, even for processes with many external legs, thanks to off-shell recursive relations)
* σ in LO pert. theory has a logarithmic dependence on μ_R and μ_F.....

NLO ?

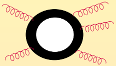
needed for reducing scale dependency uncertainties
(and non always enough...)

NLO radiative corrections

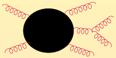
At NLO two different contributions to the amplitude can appear:
real corrections and virtual corrections.



$\mathcal{A}_{\mathrm{LO}}$



$\mathcal{A}_{\mathrm{NLO,\ virtual}}$

.

$\mathcal{A}_{\mathrm{NLO,\ real}}$

* No problem when integrating the LO matrix element on phase-space $\Rightarrow$ 4-dim integration.
* As for NLO, a dimensional regularization procedure has to be introduced $\Rightarrow$ d-dim integration.
* Divergencies of UV origin are re-absorbed through renormalization.
* Both virtual and real NLO corrections include divercencies of IR origin: they cancel each other to give finite physical results.

Subtraction scheme:

$$\int_{n+1} d\sigma^R + \int_n d\sigma^V = \int_{n+1} (d\sigma^R - d\sigma^A)_{\epsilon=0} + \int_n (d\sigma^V - \int_1 d\sigma^A)_{\epsilon=0}$$

$d\sigma^A$ has the same singular behaviour as $d\sigma^R$

Virtual correction computation

* There is an integration $\int d^n q$ of a tensor integrand
* Any 1-loop amplitude can be decomposed in terms of scalar functions (boxes, triangles, bubbles and tadpoles) according to the Master Eq.:

$$\mathcal{A}^{1-loop} = \sum_i d_i D_i + \sum_i c_i C_i + \sum_i b_i B_i + \sum_i a_i A_i + R$$

* 2 ways of calculating it:

- Tensor reduction formalism (Passarino-Veltman, Denner et al., Binoth et al.,...), so far used to compute 1-loop amplitudes for $2 \to 2$, $2 \to 3$ and some $2 \to 4$ processes
- Unitarity inspired methods: powerful for processes involving many external legs

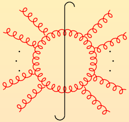
$$\mathcal{A}^{1-loop} = CC + R$$

CC = Cut-Constructible part
R = Rational part (residual of the tensor reduction procedure)

Unitarity methods

$$S = 1 + iT\,, \quad S^\dagger S = 1 \quad \Rightarrow \quad 2\mathrm{Im}(T) = T^\dagger T$$

$$\Rightarrow \mathrm{Im}\, \mathcal{A}^{1-loop} \sim \sum_{cuts} \int dPS_{cut}$$



- Original Unitarity formulation (Bern et al., 1994): only 1 cut
- Generalized Unitarity (Britto et al., 2004): multiple (quadruple) cuts, first introduced to determine the d_i coefficients of boxes in $\mathcal{N} = 4$ Super-Yang-Mills theories.

The OPP method

The OPP method (Ossola et al., 2006) is a systematical way to determine ALL coefficients d_i, c_i, b_i and a_i appearing in the CC part of 1-loop amplitudes

- OPP works at the integrand level
- algebraic technique
- universal: it can be applied to QCD, EW and even BSM virtual amplitudes

It is based on a universal decomposition of the numerator $N(q)$ appearing in any m-point amplitude in terms of denominators:

$$\mathcal{A}^{1-loop} = \int d^n q \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}} \,, \text{ with } \bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

(dimensional regularization is understood $n = 4 + \epsilon$)

Universal OPP decomposition of $N(q)$ in 4 dim

$$\begin{aligned}
N(q) \;=\; & \sum_{i_0<i_1<i_2<i_3}^{m-1} \Big(d(i_0,i_1,i_2,i_3) + \tilde{d}(q,i_0,i_1,i_2,i_3)\Big) \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \left(c(i_0,i_1,i_2) + \tilde{c}(q,i_0,i_1,i_2)\right) \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \Big(b(i_0,i_1) + \tilde{b}(q,i_0,i_1)\Big) \prod_{i\neq i_0,i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} \left(a(i_0) + \tilde{a}(q,i_0)\right) \prod_{i\neq i_0}^{m-1} D_i \\
+ & P(\tilde{q}) \prod_{i}^{m-1} D_i
\end{aligned}$$

Determination of d_i, c_i, b_i and a_i coefficients - part I

- d_{ijkl} coefficients are determined by quadruple cuts $\equiv$ to putting on mass-shell 4 propagator denominators:

$$D_i = D_j = D_k = D_l = 0 \Rightarrow$$

$$N(q^\pm) = \left(d(i,j,k,l) + \tilde{d}(q^\pm, i,j,k,l) \right) \prod_{r \neq i,j,k,l} D_r(q^\pm)$$

  Two q solutions (due to the quadratic nature of the propagators): $q^\pm$, allowing to determine d and $\tilde{d}$.

- Then, the c_{ijk} coefficients are determined by putting on mass-shell 3 propagator denominators:

$$D_i = D_j = D_k = 0$$

Determination of d_i, c_i, b_i and a_i coefficients - part II

- Then, the b_{ij} coefficients are determined by putting on mass-shell 2 propagator denominators:

$$D_i = D_j = 0$$

- Finally, the a_i coefficients are determined by putting on mass-shell 1 propagator denominator:

$$D_i = 0$$

At the end of this iterative procedure, and taking into account that libraries of scalar integrals already exist [e.g. QCDloop (Zanderighi et al.), OneLoop (van Hameren), etc..], the CC part of the amplitude is completely known.

....and what about the Rational Part ?

* R is the result of the tensor reduction process, leading to scalar integrals
* R comes from the dimensional regularization procedure one need to introduce to perform the reduction (divergencies may appear in the intermediate steps, even when the initial tensor integrals are finite)
* In particular, in the framework of OPP, R can be decomposed in

$$R = R_1 + R_2$$

(Ossola et al., 2008)

- R_1 terms arise from the mismatch between the 4-dim numerators (appearing in the universal OPP decomposition shown before) and the d-dim denominators in the integrand of 1-loop amplitude
- R_2 terms arise from the residual ϵ-dim dependent part of numerators.

R_1 terms

$N(q)$ has been decomposed in terms of 4-dim denominators, whereas in the integrand expression of 1-loop amplitude d-dim denominators appear:

$$N(q) = \sum_{i_0<i_1<i_2<i_3} \left(d(i_0,i_1,i_2,i_3) + \tilde{d}(q,i_0,i_1,i_2,i_3) \right) \prod_{i \neq i_0,i_1,i_2,i_3} D_i + ...$$

$$\bar{A}(\bar{q}) = \frac{N(q) + \tilde{N}(q,\tilde{q}^2,\epsilon)}{\bar{D}_0 \bar{D}_1 .... \bar{D}_{m-1}}$$

To face the mismatch, one writes these d-dim denominators in terms of 4-dim ones:

$$\frac{1}{\bar{D}_i} = \frac{D_i}{\bar{D}_i D_i} = \frac{\left(\bar{D}_i - \tilde{q}^2\right)}{\bar{D}_i} \frac{1}{D_i} = \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right) \frac{1}{D_i} \equiv \frac{\bar{Z}_i}{D_i}$$

It follows:

$$\frac{N(q)}{\bar{D}_0 \bar{D}_1 .... \bar{D}_{m-1}} =$$

$$= \sum_{i_0<i_1<i_2<i_3} \left(d(i_0,i_1,i_2,i_3) + \tilde{d}(q,i_0,i_1,i_2,i_3) \right) \prod_{i \neq i_0,i_1,i_2,i_3} \bar{Z}_i + ...$$

R_1 terms arise together with the CC part

$$\bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

The first part of this expression (1) originates the CC part of the amplitude, whereas the second part ($\frac{\tilde{q}^2}{\bar{D}_i}$) originates R_1:

$$R_1 \equiv \int d^d q \frac{f(q, \tilde{q}^2)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \ldots\ldots \bar{D}_{m-1}}\,.$$

R_1 can be computed at the same time of the CC part.

These methods have been implemented in a Fortran90 code, called CutTools (Ossola et al., 2007), able to numerically compute both CC and R_1 for any given 1-loop amplitude. CutTools is available on the web at: `http://www.ugr.es/~pittau/CutTools`

R_2 terms

$$\bar{A}(\bar{q}) = \frac{N(q) + \tilde{N}(q, \tilde{q}^2, \epsilon)}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}}$$

$$\mathrm{R}_2 \equiv lim_{\epsilon \to 0} \int d^d \bar{q} \frac{\tilde{N}(q, \tilde{q}^2, \epsilon)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \dots \dots \bar{D}_{m-1}} \, .$$

R_2 can not be determined just in terms of 4-dim cuts, in the framework of the OPP we perform a full 1-loop computation.

Computational strategy

* Due to their UV nature (no IR contribution), as proven in Binoth et al., 2006, it is enough to build, for each theory of interactions, all possible R_2 contributions from irreducible diagrams with up to 4 external legs:

$\Rightarrow$ R_2 effective vertices and Feynman Rules,

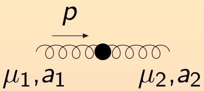
specific for the theory at hand (QED, QCD, EWSM, etc..)!

* These effective Feynman Rules can then be used to build the R_2 contribution to any given helicity amplitude, by considering all tree-level diagrams joining the external legs including one and only one R_2 effective vertex.

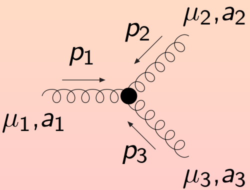
QCD R_2 effective vertices (Draggiotis et al., 2009)

- QCD corrections to pure QCD processes:
  gg - $q\bar{q}$ - ggg - $gq\bar{q}$ - $gggg$ effective vertices
- QCD corrections to mixed QCD/EW processes:
  $vq\bar{q}$ - $sq\bar{q}$ - vgg - sgg - $vvgg$ - $ssgg$ - $vggg$ effective vertices

* pure QCD examples:



$$= \frac{ig^2 N_{col}}{48\pi^2}\,\delta_{a_1 a_2}\left[\frac{p^2}{2} g_{\mu_1\mu_2} + \lambda_{HV}\left(g_{\mu_1\mu_2}p^2 - p_{\mu_1}p_{\mu_2}\right) + \frac{N_f}{N_{col}}\left(p^2 - 6\,m_q^2\right) g_{\mu_1\mu_2}\right]$$

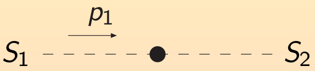


$$= -\frac{g^3 N_{col}}{48\pi^2}\left(\frac{7}{4} + \lambda_{HV} + 2\frac{N_f}{N_{col}}\right) f^{a_1 a_2 a_3}\, V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$$

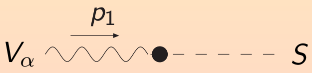
EW R_2 effective vertices (Garzelli et al., 2010)

- EW corrections to QCD processes:
  $q\bar{q}$ - $gq\bar{q}$ effective vertices
- EW corrections to EW processes:
  ss - vv - sv - $f\bar{f}$ - sss - vss - svv - vvv - $sf\bar{f}$ - $vf\bar{f}$ - $ssss$ - $ssvv$ - $vvvv$ effective vertices

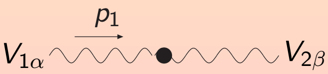
  * 2-point non vanishing vertices:



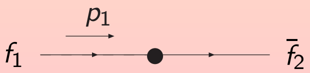
$= \mathrm{Vert}(S_1, S_2)$



$= \mathrm{Vert}(V, S)$



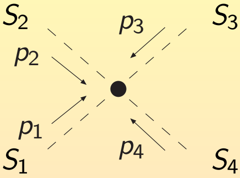
$= \mathrm{Vert}(V_1, V_2)$



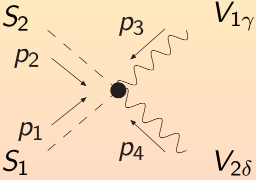
$= \mathrm{Vert}(f_1, f_2)$

EW R_2 effective vertices (Garzelli et al., 2010)

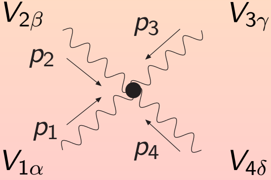
* 4-point non-vanishing vertices:



$= \mathrm{Vert}(S_1, S_2, S_3, S_4)$



$= \mathrm{Vert}(S_1, S_2, V_1, V_2)$



$= \mathrm{Vert}(V_1, V_2, V_3, V_4)$

Example of diagrams contributing to the R_2 EW *ssss*, *ssvv*, *vvvv* effective vertices

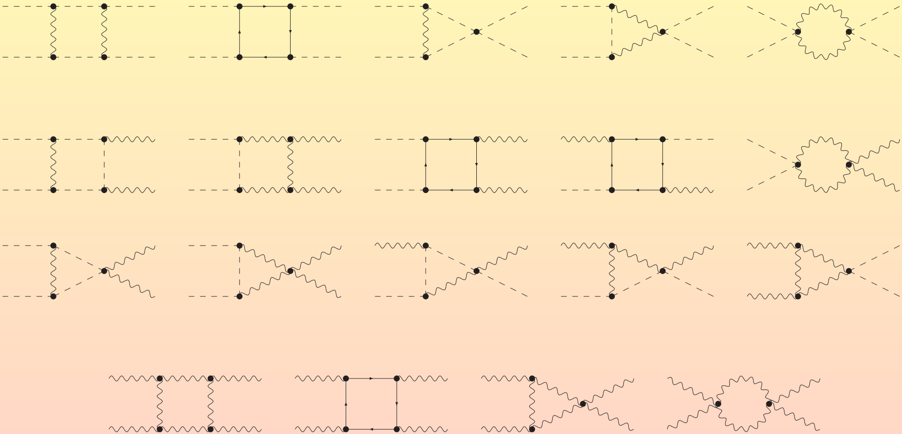


Figure: Non null contributions to the ssss, ssvv and vvvv R_2 effective vertices in the generalized R_ξ gauges, with generic finite ξ, ξ_Z, ξ_A. The corresponding analytical formulas associated to selected topologies of each generic diagram are available within the R2SM package.

The R2SM package (Garzelli & Malamos, 2010)

- The code written in FORM to calculate all R_2 effective vertices in the EW sector of SM is available on the web at `http://www.ugr.es/~garzelli/R2SM`
- The analytical expressions associated to generic diagrams (i.e. diagrams filled by means of *generic scalars*, *vectors* and *fermions*) are automatically dressed with *specific* particles (i.e. H, Z, etc.) by do-loop procedures (no need to draw specific Feynman diagrams).

Choice of a gauge and R_2 behaviour under gauge transformations

- We consider the following gauges:
  - Generalized R_ξ gauge with 3 different finite ξ's: ξ_A, ξ_Z and ξ.
    $\Rightarrow$ Standard R_ξ gauge follows as the particular case
    $\xi_A = \xi_Z = \xi$
    $\Rightarrow$ 't Hooft Feynman gauge follows as the particular case
    $\xi_A = \xi_Z = \xi = 1$
  - Unitary gauge (it follows from the limit ξ_Z, $\xi \to \infty$, $\xi_A \to 1$)
- R_2 by itself is not invariant under gauge transformations
- The $R_1 + R_2$ contribution to any physical quantity is gauge invariant (since 1-loop amplitudes are gauge invariant, and the same is true for their CC part).
  - example 1: We verified that the S-matrix element for the $H \to \gamma\gamma$ decay at 1-loop turns out to be ξ independent, and to be the same in all gauges.
  - example 2: We verified that the contributions to vv self-energies proportional to $g_{\mu\nu}$ calculated on mass-shell ($p^2 = m^2$) are independent of gauge parameters and the same in all gauges.

Check of R_2 Feynman Rules: Ward identities

- We explicitly wrote down many 2, 3 and 4-point Ward identities.
- We calculated analytically R_1 effective vertices up to 4 external legs.
- We verified that the Ward identities are fullfilled by $R_1 + R_2$.

This is a non-trivial check! In fact,

- tensor reduction leads to R_1 analytical expressions involving a huge amount of terms with different combinations/powers of Gram determinants
- many of these Ward identities involve more than one effective vertex at the same time (e.g. *vss*, *sss* and *ss* vertices all together)

(Towards) Helac 1-loop....

- Helac 1-loop is an extension of the LO event generator Helac (Kanaki & Papadopoulos, 2000, Cafarella et al., 2007), written in Fortran.
- CutTools has been merged to compute the $CC + R_1$ part of 1-loop amplitudes (van Hameren et al., 2009)
  - Berends-Giele off-shell recursion relations have been used to build cut (sub-)amplitudes involving $2 \to n+2$ particles.
  - Loops are generated by gluing together 2 external particles.
- The R_2 QCD Feynman Rules have been implemented in Fortran.
  - Recursion relations are used to build the R_2 contribution to each $2 \to n$ 1-loop amplitude. It is enough to recursively calculate tree-level (sub-)amplitudes involving standard vertices and one and only one "special" R_2 effective vertex ("special" means special couplings and, sometimes, special Lorentz structure)

⇒ At present, Helac 1-loop can be used to automatically compute 1-loop QCD corrections to SM processes.

σ^{LO+V} computation in Helac-1-loop

It is based on an unweighting and reweighting procedure (Bevilacqua et al., 2009):

$$\sigma_{ij}^{LO+V} = \int d\phi_n dx_1 dx_2 f_i(x_i,\mu_F) f_j(x_j,\mu_F)(|\mathcal{M}|^2 + \mathcal{M}\mathcal{L}_v^* + \mathcal{M}^*\mathcal{L}_v)$$
$$= \int ....|\mathcal{M}|^2 \left(1 + \frac{\mathcal{M}\mathcal{L}_v^* + \mathcal{M}^*\mathcal{L}_v}{|\mathcal{M}|^2}\right)$$

- Many LO events are generated by sampling the phase-space.
- A sample of unweighted events is selected, i.e. a sample $g(x_1, x_2, \phi_m)$ of events distributed like $\frac{d\sigma_{ij}^{LO}}{\sigma_{ij}^{LO} dx_1 dx_2 d\phi_m}$ (n.b. $\int dx_1 dx_2 d\phi_m g(x_1, x_2, \phi_m) = 1$)
- L_v is calculated only in the corresponding phase-space points (x_1, x_2, ϕ_m)
- $\sigma_{ij}^{LO+V} \simeq 1/N_{unweighted} \sum_{i\in S} \left(1 + \frac{\mathcal{M}\mathcal{L}_v^* + \mathcal{M}^*\mathcal{L}_v}{|\mathcal{M}|^2}\right) \sigma_{ij}^{LO}$, as follows from MC integration properties: $\int dX g(X) O(X) \simeq \frac{1}{N_S} \sum_{i\in S} O(X_i)$

σ^{real} computation

In the Helac framework, real NLO corrections and IR divergence subtraction are computed by means of the Catani-Seymour approach, extended to states with arbitrary polarizations, as implemented in the Helac-Dipole fortran code (Czakon et al., 2009), available on the web.

At present, this is the most time consuming part of NLO computations.....

- Too many dipole subtraction terms may contribute to $d\sigma^A$ in case of multiparticle amplitudes.
- The integration of $d\sigma^A$ can be cumbersome, due to its complicated analytical expression.
- A parameter α_{max} has been introduced in the attempt of cutting the number of contributing dipole subtraction terms.
- The final results for σ^{real} should be independent of α_{max} (check).

σ^{NLO}: examples of $2 \to 4$ processes at LHC

$$\sigma^{NLO} = \int_n (\sigma^{LO+V} + d\sigma^A) + \int_{n+1} (d\sigma^{Real} - d\sigma^A)$$

Check of the correctness of the calculation: IR divergencies ($1/\epsilon$ and $1/\epsilon^2$ poles) cancel in the sum, according to KLN theorem (Kinoshita et al. 1963)!

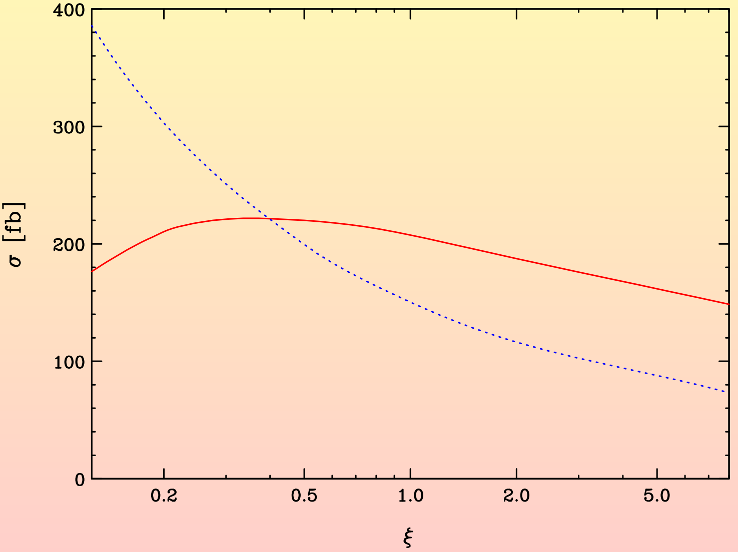
Example at LHC: $pp \to t\bar{t}H \to t\bar{t}b\bar{b}$ signal vs. $pp \to t\bar{t}b\bar{b}$ background (G. Bevilacqua et al., 2010, contrib. to the LHC Higgs σ Working Group)

Assumptions and cuts: CTEQ6 pdf sets used consistently, $\mu_R = \mu_F = \mu_0$, $m_{top} = 172.6$ GeV, $m_{otherquarks} = 0$, $m_H = 130$ GeV, recombination of b and g with $|\eta| < 5$ into jets via k_T-algorithm, rapidity of 2 recombined b jets $|y_b| < 2.5$ and $p_{T,b} > 20 GeV$.

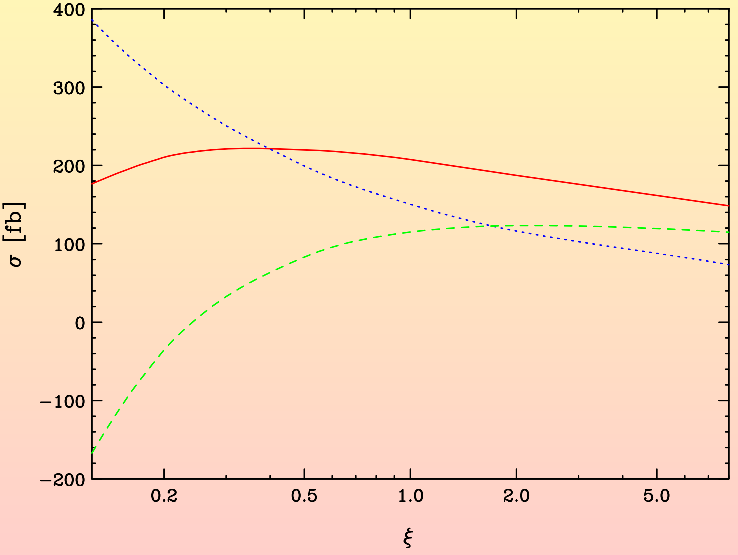
$$K_{signal}(\mu_0 = m_t + m_H/2) = \frac{\sigma^{NLO}_{signal}}{\sigma^{LO}_{signal}} = \frac{207.268 \pm 0.150 fb}{150.375 \pm 0.077 fb} = 1.38$$

$$K_{background}(\mu_0 = m_t) = \frac{\sigma^{NLO}_{background}}{\sigma^{LO}_{background}} = \frac{2642 \pm 3 fb}{1489.2 \pm 0.9 fb} = 1.77$$

Signal σ^{NLO} vs σ^{LO} as a function of $\xi = \mu/\mu_0$: scale dependence attenuation....



....Experimental cuts can strongly affect the result: effect of an additional jet-veto $p_{T,cut} = 50\,GeV$



Open issues in Helac 1-loop and further extensions

- Modularity: we should go in this direction to allow for flexibility in using different combinations of LO event generators (HELAC, SHERPA, ALPGEN, MadGraph....) and 1-loop calculators (CutTools, BlackHat, Rocket, Samurai....), and to better compare results from different generators (see Les Houches 2009 effort, Binoth et al.)
- Many legs: in line of principle possible, but attention on the memory required to store skeleton info.
- EW corrections to SM processes:
  - CutTools is ready to compute the CC part of 1-loop amplitudes.
  - Helac-1-loop is not yet ready to compute the R_2 EW contributions.
  - We have to implement in the generator all effective R_2 Feynman rules we have calculated analytically so far.
  - EW ghosts have to be included, as well.
  - Gauge issues: R_ξ vs. Unitary.
- These developments will allow for phenomenology in the EW sector at LHC and e^+e^- colliders.
- Effects of using different pdf sets have also to be explored.
- Interface with shower MCs (POWHEG, MC@NLO, new approaches....) and matching at NLO.

Summary and Conclusions

- New techniques have been developed for the calculation of 1-loop amplitudes based on Unitarity cuts.
- The OPP method implemented in the CutTools code is one of these techniques, allowing for the calculation of the CC part and of a part of R.
- A residual part of R exist, called R_2, that needs a dedicated 1-loop computation, at least when working in no more than 4 integer dim.
- We calculated analytically all R_2 effective vertices in QCD and in the SM of EW interactions.
- We studied their dependence on gauge, expressing a strong favour for the calculation of R_2 in the R_ξ one (in the unitary gauge the UV behaviour of the theory appears worser....).
- CutTools and the QCD R_2 vertices have been implemented in Helac, originating Helac-1-loop.
- This, together with a method and a code to compute real radiation emissions, allows for the computation of NLO QCD corrections to processes like $pp \to t\bar{t}b\bar{b}$ and $pp \to t\bar{t}H \to t\bar{t}b\bar{b}$, important for Higgs and New Physics search at hadron colliders

Collaborators & Competitors

* In this seminar results have been presented obtained thanks to collaboration with G. Bevilacqua, M. Czakon, P. Draggiotis, A. van Hameren, I. Malamos, C.G. Papadopoulos, R. Pittau and M. Worek.

* Other groups working on Unitarity and Generalized Unitarity inspired methods:

- BlackHat + SHERPA collaboration (Z. Bern et al.)
- Rocket + MCFM collaboration (G. Zanderighi et al.)
- Samurai + GOLEM collaboration (P. Mastrolia et al.)

A serious comparison between different methods has still to be performed!