The high energy limit of QCD: Heavy quark production and the effective action

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Outline

1. The BFKL-equation
2. Heavy quark production
3. The effective action of high energy QCD
4. Vertices in QCD Reggeon-field-theory
5. Conclusions

Perturbative QCD at high energies

- squared center-of-mass energy $s \gg$ all other occuring energy scales
- hard scale Q^2, $\alpha_s(Q^2) \ll 1$, $s \gg Q^2 \gg \Lambda_{\text{QCD}}$
- large logarithm compensate for smallness of the coupling $\alpha_s(Q^2) \ln s \sim 1$
- **L**eading **L**ogarithmic **A**pproximation (LLA): resummation of perturbative terms $(\alpha_s \ln s)^n$

→ **BFKL equation**

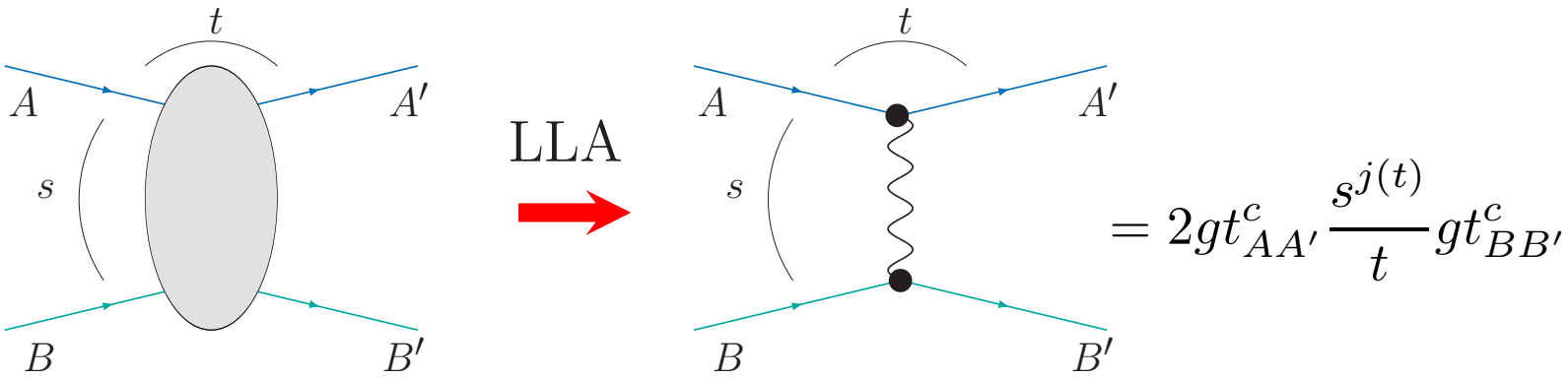
- LLA: [Balitsky, Fadin, Kuraev, Lipatov, 1975-1978]
- NLLA: [Fadin Lipatov, 1998; Ciafaloni, Camici 1998]

description strongly relies on the observation that the gluon reggeizes at high energies

High energy QCD amplitudes: gluon reggeization and BFKL

Reggeization of high energy QCD amplitudes

Scattering of colored objects, resum within LLA all order corrections
[Fadin, Kuraev, Lipatov, 1975 - 1977]



$$= 2gt^c_{AA'} \frac{s^{j(t)}}{t} gt^c_{BB'}$$

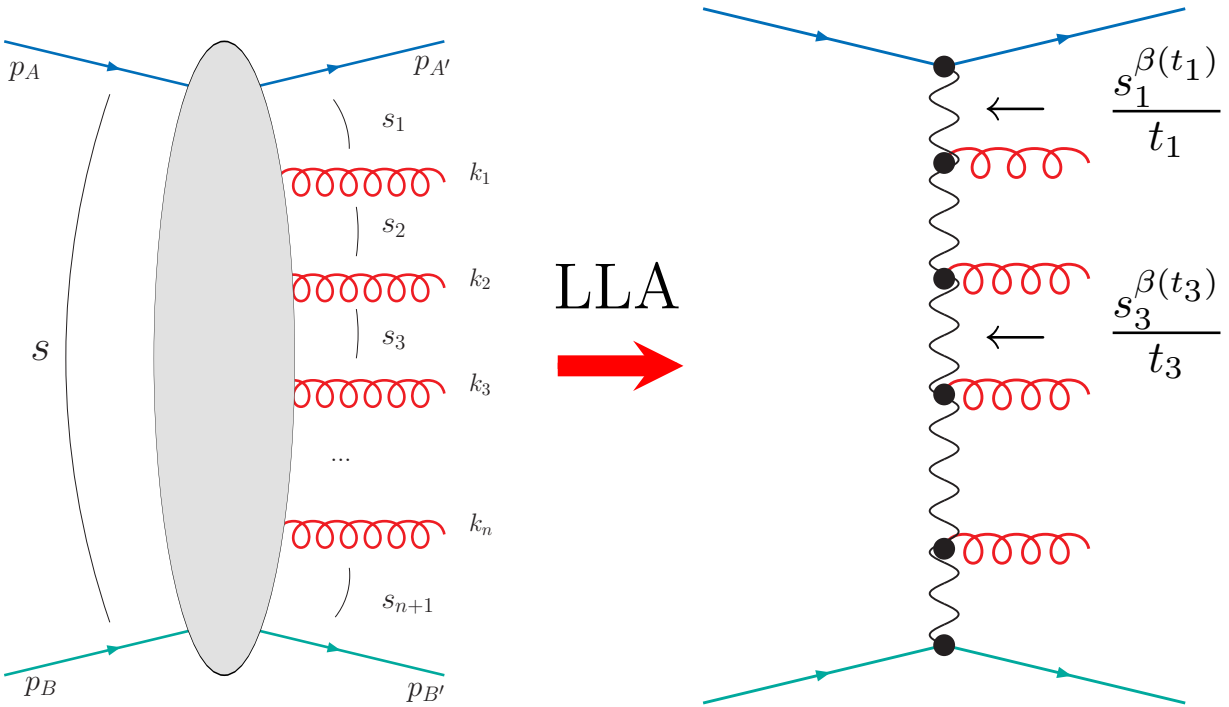
- all interaction gathered in collective excitation of gluon field
- Regge-pole with quantum number of the gluon, trajectory $j(t) = 1 + \beta(t)$, $j(t = 0) = 1$

→ **the reggeized gluon**

n-gluon production amplitude within the LLA

LLA: production takes place in the Multi-Regge-Kinematics (MRK)

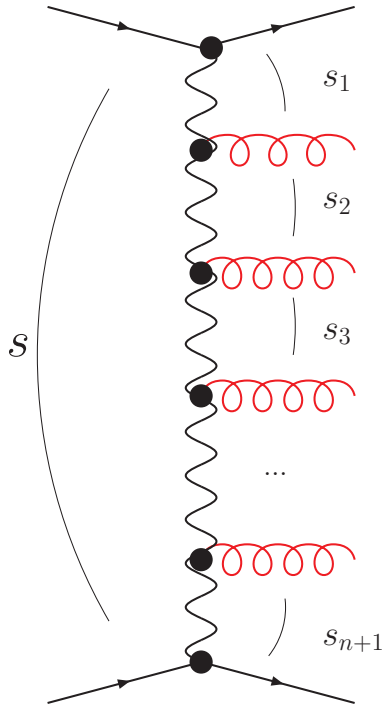
$$s \gg s_r \gg -t_r \qquad t_r, \simeq \boldsymbol{k}_i^2 \simeq m^2$$



$$s = (p_A + p_B)^2,\ s_r = (k_r + k_{r-1})^2,\ t_r = -\boldsymbol{q}_r^2,\ k_r = q_{r+1} - q_r$$

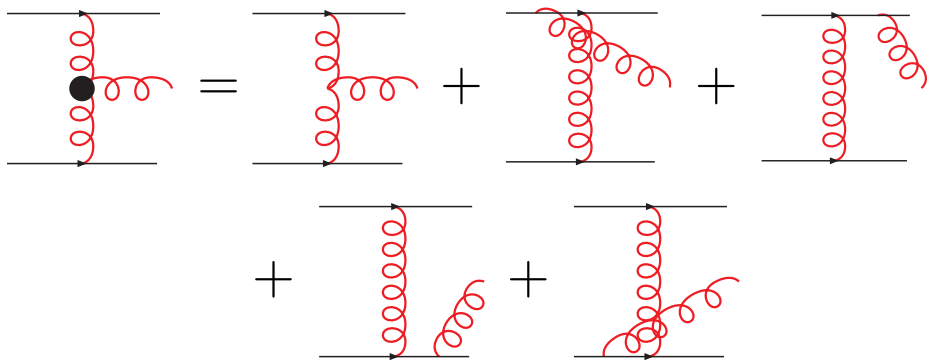
- Interaction mediated by the reggeized gluon ➡ effective degree of freedom in the t-channel of high energy QCD amplitudes

n-gluon production amplitude within the LLA



- reggeized gluons interact with QCD particles by gauge invariant vertices:

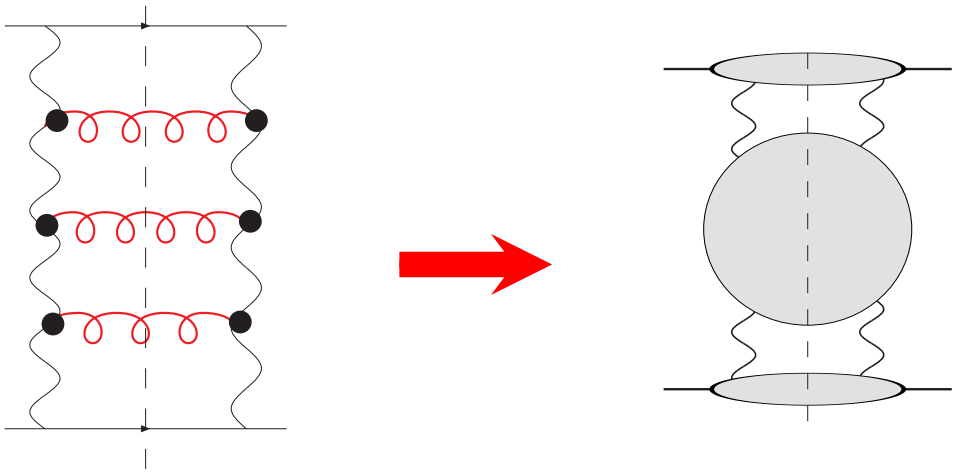
Yang-Mills supplemented by 'non-local' terms



➡ gauge invariant reggeized gluon

Essential to achieve 'Regge'/high energy -factorization of amplitudes

The total cross-section



$$\sigma(s) = \int d^2\mathbf{k}_a \int d^2\mathbf{k}_b \phi_A(\mathbf{k}_a)\, f\left(\frac{s}{s_0}, \mathbf{k}_a, \mathbf{k}_b\right)\, \phi_B(\mathbf{k}_b)$$

- impact factors $\phi_A(\mathbf{k}_a)$, $\phi_B(\mathbf{k}_b)$: coupling to external particles
- BFKL-Green's function $f\left(\frac{s}{s_0}, \mathbf{k}_a, \mathbf{k}_b\right)$: carries complete dependence on s, process independent
- s_0 energy scale, typically of the size of the transverse scale of the process $\longrightarrow$ see below

The BFKL-equation: state of two reggeized gluon

BFKL-Green's function usually expressed in terms of its Mellin transform

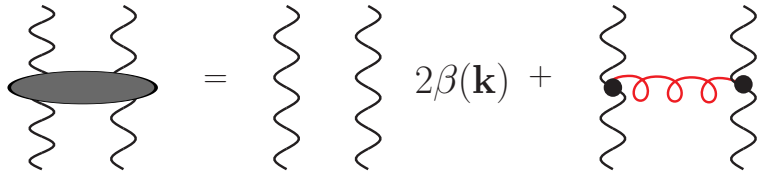
$$f\left(\frac{s}{s_0}, \mathbf{k}_a, \mathbf{k}_b\right) = \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} f_\omega(\mathbf{k}_a, \mathbf{k}_b)$$

summed up by the BFKL-equation (= Bethe-Salpeter integral equation)

$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) + \int d^2\mathbf{k} K_{\text{BFKL}}(\mathbf{k}_a, \mathbf{k}) f_\omega(\mathbf{k}, \mathbf{k}_b)$$

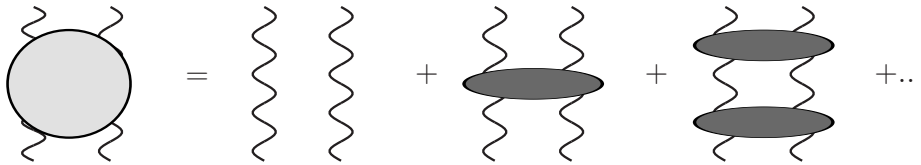
with the BFKL-Kernel given by

$$K^{(\text{LO})}_{\text{BFKL}}(\mathbf{k}_a, \mathbf{k}) =$$



this yields

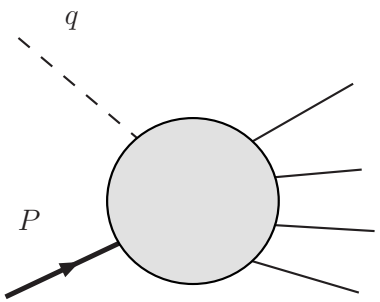
$$f_\omega(\mathbf{k}_a, \mathbf{k}_b) =$$



BFKL predicts powerlike rise of total cross-section

$$\sigma_{\text{tot}} \sim f(s) \sim s^{\lambda}, \qquad \lambda = \frac{\alpha_s N_c}{\pi} 4 \ln 2$$

- λ: hard or LO BFKL Pomeron intercept, $\lambda \simeq 0.5$ for $\alpha_s \simeq 0.2$
- if infinitly continued, such a rise is known to violate unitarity
  ➡ unitarity corrections, see later
- when theoretically discovered, experimental observation impossible
  $s \sim$ hard scale



* 92-07: Deep Inelastic Scattering (DIS) at HERA

* proton structure function $F_2(x, Q^2)$, $x \sim Q^2/s$

* observe strong rise of F_2 for small x ($\equiv$ large s)

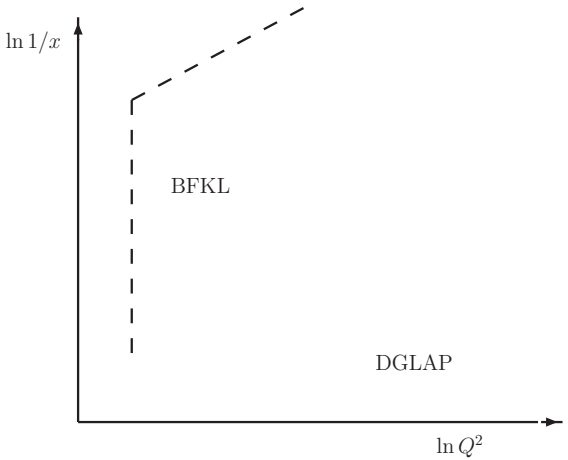
➡ believed to be driven by BFKL-evolution, also described by conventional DGLAP-evolution

DGLAP and BFKL

- conventional description of hadronic scattering processes, in presence of hard scale Q^2: collinear factorization

$$\sigma_{\gamma^* P} = \sum_i \int_x^1 \frac{dz}{z} \hat{\sigma}_{\gamma^* p_i} \left(\frac{x}{z}, \frac{Q^2}{\mu_F^2} \right) f_i(z, \mu_F^2)$$

- μ_F^2 dependence of parton distribution functions f_i described by DGLAP-evolution = evolution in Q^2



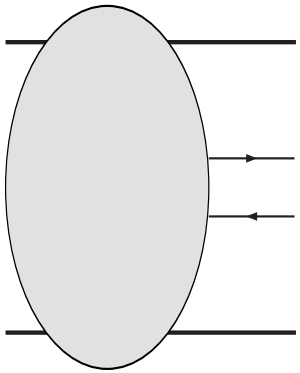
- BFKL evolution in $\ln 1/x$
- defines at small x kT factorization + unintegrated gluon densitity
- probability to find gluon with proton momentum fraction x and transverse momentum **k**
- complementary approach to DGLAP

$$\sigma_{\gamma^* P} = \int_x^1 \frac{dz}{z} \int d^2\mathbf{k}\, \hat{\sigma}_{\gamma^* p_i} \left(\frac{x}{z}, Q^2, \mu, \mathbf{k} \right) g\left(z, \mathbf{k}, \mu\right)$$

Heavy quark production at the LHC

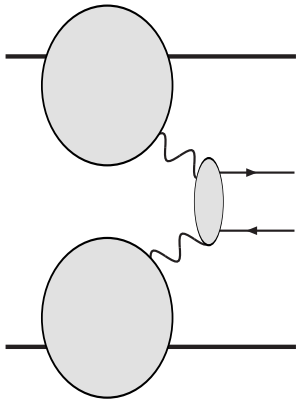
BFKL at hadron collider: heavy quark production

- goal: test this formalism (and BFKL in general) at the LHC [Chachamis, MH, Sabio Vera, Salas]
- high center of mass energy $\longrightarrow$ natural enviroment for BFKL
- suitable process: heavy quark production at central rapidities



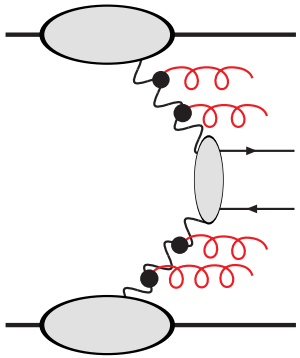
- heavy quark mass allows for perturbative treatment
- top quark mass large - $x \sim M^2/s$ not very small
- top at Tevatron: major corrections from the threshold region
- bottom: proton probed at relatively small $x \sim M^2/s$, BFKL logarithms become sizeable

Exclusive heavy quark production at the LHC



previous studies:

- inclusive study (total heavy flavor cross-section)
- restricted to LO BFKL effects
- good description of Tevatron data, large scale dependence (20 % - 50 %)



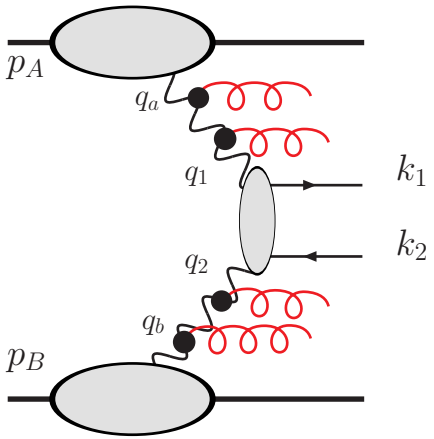
- LHC: high luminosity $\rightarrow$ huge amount of heavy quarks produced

$\Rightarrow$ allows for study of exclusive differential cross-sections

$\Rightarrow$ stronger test than previous inclusive studies

- use NLO BFKL to increase of accuracy and reduce large scale dependence
- give formulation that allows to keep additional gluon radiation exclusive

The differential cross-section in Regge factorization



$s_1 = (p_A + q_2)^2 = ys,\ s_2 = (p_B + q_1)^2 = xs$

$\alpha_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{\eta_i},\ \beta_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{-\eta_i}$, i=1,2

$\eta_i,\ \mathbf{k}_i,\ i = 1, 2$: rapidity/transverse momentum of produced heavy quarks

M : heavy quark mass

$$\frac{d^6\sigma}{d\eta_1 d\eta_2 d\mathbf{k}_1 d\mathbf{k}_2} = \frac{1}{16\pi^2} \int dx \int dy \int \frac{d^2\mathbf{q}_1}{(2\pi)^3} \int \frac{d^2\mathbf{q}_2}{(2\pi)^3}$$

$$\left(\int d^2\mathbf{q}_a \frac{\Phi_A(\mathbf{q}_a)}{\mathbf{q}_a^2} f(\frac{s_1}{s_{0,2}}, \mathbf{q}_a, \mathbf{q}_1) \right) |\Gamma_{\mathrm{RR}\to\mathrm{Q\bar{Q}}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{k_1}, \mathbf{k}_2, \alpha_1/x)|^2$$

$$\left(\int d^2\mathbf{q}_b \frac{\Phi_B(\mathbf{q}_b)}{\mathbf{q}_b^2} f(\frac{s_2}{s_{0,2}}, \mathbf{q}_2, \mathbf{q}_b) \right) (2\pi)^4 \delta^{(2)}(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{k}_1 - \mathbf{k}_2)$$

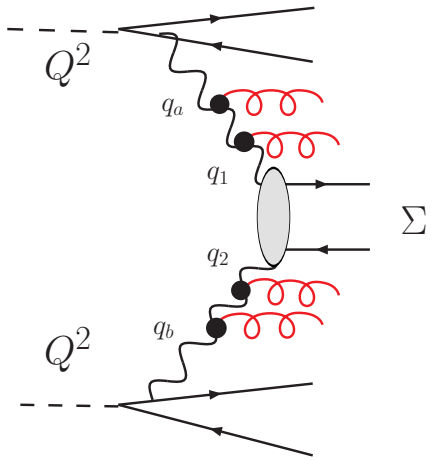
$$\delta(x - \alpha_1 - \alpha_2)\delta(y - \beta_1 - \beta_2)$$

The transverse energy scale $s_{0,1}$

BFKL Green's function: $f\left(\frac{s_1}{s_{0,1}}, \mathbf{k}_1, \mathbf{q}_1\right) = \int \frac{d\omega}{2\pi i} \left(\frac{s_1}{s_{0,1}}\right)^\omega f_\omega(\mathbf{k}_1, \mathbf{q}_1)$

f_ω as solution of BFKL equation

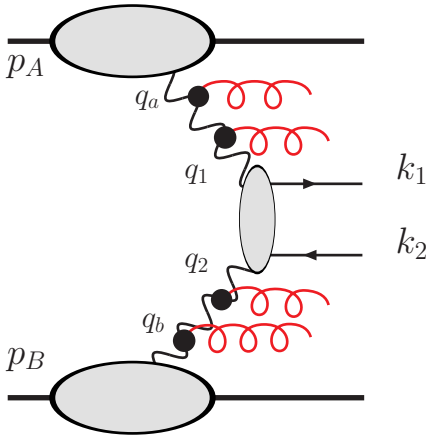
- choice of energy scale $s_{0,1}$ arbitrary for LO BFKL (source of large scale dependence), constrained at NLO



- theory: Multi-Regge-Kinematics: all (transverse) scales are of the same order of magnitude
- ideal process: $\gamma^*\gamma^*$-scattering
- symmetric choice $s_{0,1} = \sqrt{Q^2\Sigma}$, $\Sigma = (k_1 + k_2)^2 + (\mathbf{k}_1 + \mathbf{k}_2)^2 = xys$ transverse scale of heavy quark system

- yields $\left(\frac{s_1}{s_{0,1}}\right)^\omega = e^{\omega(y_A - y_{Q\bar{Q}})}$
- $y_A, y_{Q\bar{Q}}$: rapidity of the proton/heavy quark system

Unintegrated gluon density at LO and NLO



hadronic collision: transverse scales asymmetric $\Sigma \gg \Lambda_{\text{QCD}}$

here: natural choice: $s_{0,1} = \Sigma, \quad \left(\frac{s_1}{s_{0,1}}\right)^{\omega} = x^{-\omega}$

➡ unintegrated gluon density = probability to resolve off-shell gluon with momentum fraction x and transverse momentum $\mathbf{k}$

LO: choice of $s_{0,1}$ not constrained

$$g^{\text{LO}}(x,\mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi} \frac{\Phi_P(\mathbf{q})}{\mathbf{q}^2} f(x,\mathbf{q},\mathbf{k})$$

NLO: obtain modification of both proton impact factor and NLO Green's function

$$g^{\text{NLO}}(x,\mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi} \frac{\tilde{\Phi}_P(\mathbf{q})}{\mathbf{q}^2} \tilde{f}(x,\mathbf{q},\mathbf{k})$$

NLO itself independent of choice, but exponentiation of kernel in the Green's function leads to residual dependence

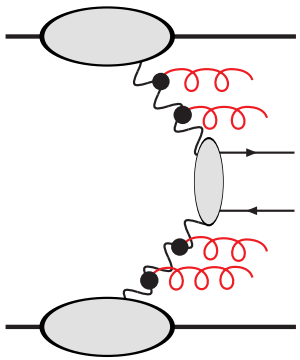
Collinear correction terms due to change of energy scales

[Bartels, Sabio Vera, Schwennsen, 06] can aborb change in scale s_0 at NLO in a redefinition of impact factors, BFKL-Kernel and production vertex:

$$\Phi_P^{\text{NLO}}(\mathbf{q}) \to \tilde{\Phi}_P^{\text{NLO}}(\mathbf{q}), \qquad K_{\text{BFKL}}^{\text{NLO}}(\mathbf{l}_a, \mathbf{l}_b) \to \tilde{K}_{\text{BFKL}}^{\text{NLO}}(\mathbf{l}_a, \mathbf{l}_b),$$
$$|\Gamma_{\text{RR}\to\text{Q}\bar{\text{Q}}}^{\text{NLO}}|^2 \to |\tilde{\Gamma}_{\text{RR}\to\text{Q}\bar{\text{Q}}}^{\text{NLO}}|^2$$

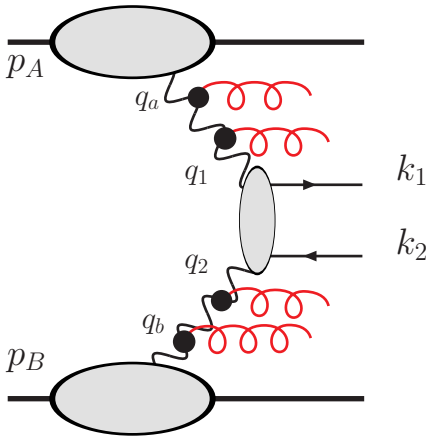
For instance the BFKL-Kernel (which yields the modified Green's function $\tilde{f}(x_1, \mathbf{q}_a, \mathbf{q}_1)$)

$$\tilde{K}_{\text{BFKL}}^{\text{NLO}}(\mathbf{l}_a, \mathbf{l}_b) = K_{\text{BFKL}}^{\text{NLO}}(\tilde{\mathbf{l}}_a, \mathbf{l}_b) - \frac{1}{2}\int d\mathbf{l}^2 K_{\text{BFKL}}^{\text{LO}}(\mathbf{l}_a, \mathbf{l}) K_{\text{BFKL}}^{\text{LO}}(\mathbf{l}, \mathbf{l}_b) \ln \frac{\mathbf{l}^2}{\mathbf{l}_b^2}.$$



- include evolution in the transverse scale in the BFKL-evolution
- onset of collinear/DGLAP resummation

The differential cross-section with unintegrated gluon densities



$$\alpha_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{\eta_i},\ \beta_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{-\eta_i},\ \text{i=1,2}$$

$\eta_i,\ \mathbf{k}_i,\ i = 1,2$: rapidity/transverse momentum of produced heavy quarks

M : heavy quark mass

$$\frac{d^6\sigma}{d\eta_1 d\eta_2 d\mathbf{k}_1 d\mathbf{k}_2} = \frac{1}{4}\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} g^{\text{NLO}}(\alpha_1+\alpha_2, \mathbf{q}_1)$$
$$\times |\tilde{\Gamma}_{\text{RR}\to\text{Q}\bar{\text{Q}}}(\mathbf{q}_1, \mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2, \alpha_1/(\alpha_1+\alpha_2))|^2$$
$$\times g^{\text{NLO}}(\beta_1+\beta_2, \mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_1)$$

The NLO unintegrated gluon density

$$g^{\text{NLO}}(x,\mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi}\frac{\tilde{\Phi}_P(\mathbf{q})}{\mathbf{q}^2}\tilde{f}(x,\mathbf{q},\mathbf{k})$$

- proton impact factors is non-perturbative; needs to be modeled and fitted to i.e. HERA DIS data
- LO Green's function: known analytically
- NLO Green's function: solution not known, can represent it in terms of anomalous dimension and conformal spin, requires numerical treatment

here: solve BFKL-equation iteratively in x, k_T-space:

BFKL kernel given as sum of virtual (trajectory) and real part:

$$\tilde{K}_{\text{BFKL}}(\mathbf{q}_a,\mathbf{q}) = 2\beta(\mathbf{q}_a)\delta^{(2+2\epsilon)}(\mathbf{q}_a-\mathbf{q}) + K^{\text{real}}(\mathbf{q}_a,\mathbf{q})$$

idea: treat real (gluon emission) and virtual (gluon trajectory, no gluon emission) parts on different grounds [Andersen, Sabio Vera, 03-04]

Iterative solution of the NLO BFKL-equation

problem: real and virtual parts IR divergent, only sum is finite

➡ introduce phase space slicing parameter λ, to obtain finite real and virtual parts

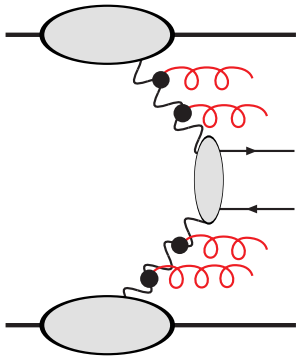
$$K_\lambda(\mathbf{q}_a,\mathbf{q}) = \beta_\lambda(\mathbf{q})\delta^{(2)}(\mathbf{q}_a-\mathbf{q}) + K_\lambda^{\text{real}}(\mathbf{q}_a,\mathbf{q})$$

iterative solution of BFKL Green's function in (x,k_T) space given by

$$f(x,\mathbf{q},\mathbf{k}) = x^{-\beta_\lambda(\mathbf{q})}\left\{\delta^{(2)}(\mathbf{q}-\mathbf{k}) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{l}_i\left[K_\lambda^{\text{real}}(\mathbf{q}+\sum_j^{i-1}\mathbf{l}_j,\mathbf{q}+\sum_j^{i}\mathbf{l}_j)\right.\right.$$
$$\left.\left.\times\int_{x_{i-1}}^{1}\frac{dx_i}{x_i}x_i^{-\beta_\lambda(\mathbf{q}+\sum_{j=1}^{i}\mathbf{l}_j)+\beta_\lambda(\mathbf{q}+\sum_{j=1}^{i-1}+\mathbf{l}_j)}\right]\delta^{(2)}(\mathbf{q}+\sum_{j=1}^{n}\mathbf{l}_j-\mathbf{k}),\right\}$$

- close connection to underlying physical processes
- next task: numerical implementation + fit proton impact factors to HERA data

NLO matrix elements

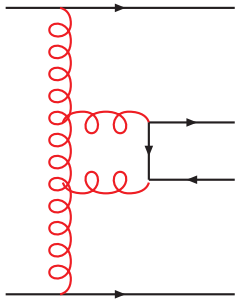


- NLO correction to heavy quark production vertex Γ^{NLO} currently unknown
- small x: main contribution from Green's function, restriction to LO production vertex good approximation
- prepare numerical study with Γ^{LO} alone

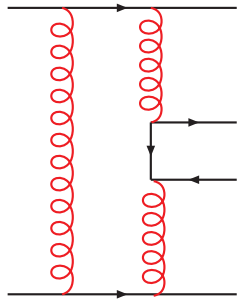
- Final goal: determine Γ^{NLO}
- problem: scattering of *off-shell reggeized gluons* ➡ gauge invariance requires to include "non-local " diagrams

Sufficent: replace protons by quarks:

'local': low order reggeized gluon corresponds to single gluon



'non-local': reggeized gluon contains 2 gluon state

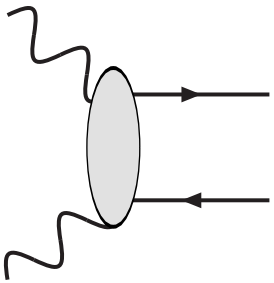


Can try to extract NLO matrix element from $2 \to 4$ process at 1-loop
➡ systematic approach to calculate matrix elements: **effective action**

The gauge invariant effective action of high energy QCD

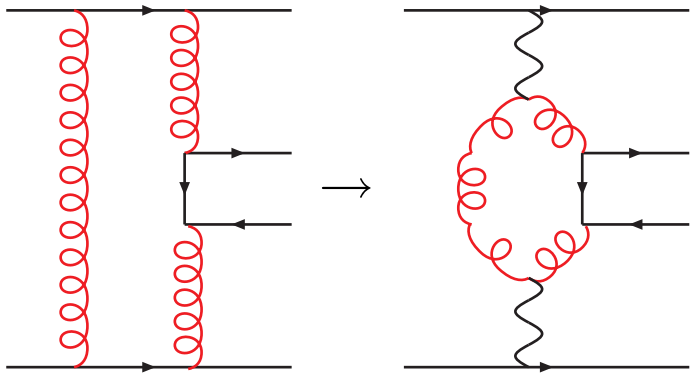
Systematic treatment: effective action

basic idea of th effective action: introduce new 'induced' vertices that allow to evaluate matrix elements *local in rapidity*



calculation of $g^*g^* \to Q\bar{Q}$ factorizes from scattering hadron, quark, gluon ...

achieve this by redistribution of diagrams in the $\mathbf{8_A}$-sector



➡supplement QCD Lagrangian by induced term to achieve redistribution

The gauge invariant effective action

[Lipatov, 1995]

$$S_{\text{eff}} = \int d^4x \left[\mathcal{L}_{\text{QCD}}(v_\mu, \psi) + \mathcal{L}_{\text{ind}}(v_\mu, A_+, A_-)\right]$$

- describes interaction of reggeized gluons $(A^a_\pm)$ with quark- (ψ) and gluon-fields (v_μ)
- $A^a_\pm$ invariant under local gauge transformations, but transforms globally in the adjoint representation of $SU(N_c)$
- new induced part is given by

$$\mathcal{L}_{\text{ind}}(v_\pm, A_\pm) = \text{tr}\left[(A_-(v) - A_-)\,\partial_\sigma^2 A_+\right] + \text{tr}\left[(A_+(v) - A_+)\,\partial_\sigma^2 A_-\right]$$

where $A_\pm(v) = -\frac{1}{g}\partial_\pm U(v_\pm) = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \ldots.$

- yields the induced vertices and reggeized gluon propagator

Gauge invariance and locality in rapidity

effective action factorizes amplitude into gauge invariant pieces, local in rapdidity

* interaction between particles and reggeized gluons restriced to small rapidity interval $\Delta Y < \eta$
* non-local interaction $\Delta Y > \eta$ mediated by gauge invariant reggeized gluon field

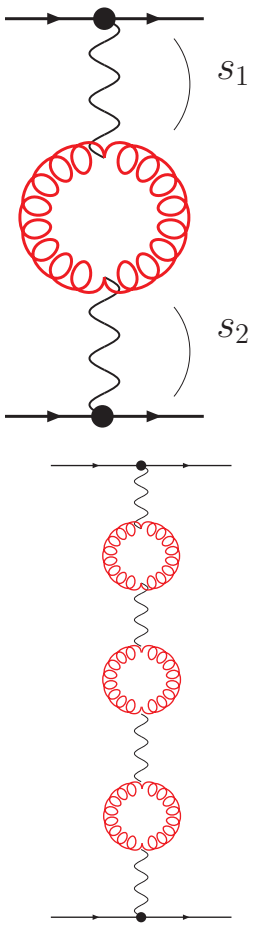
Effective action ready to calculate? NO!

- need additional rules and a supplementary term

Why?

- derived from study of production amplitudes in 'Quasi-Multi-Regge-Kinematics'
- so far no formal derivation

Regularization for longitudinal integrals



- exchange of reggeized gluon only meaningful if s_1, s_2 are large
- simple cut-off regularization not sufficent ➡ improved regularization method:

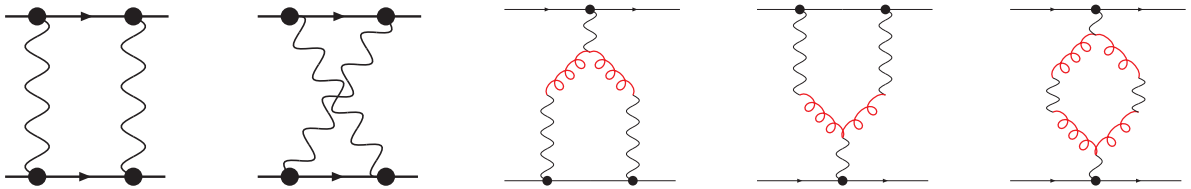
$$\lim_{\nu\to 0}\int_{-i\infty}^{i\infty}\frac{d\omega}{4\pi i}\frac{1}{\omega+\nu}\left[\left(\frac{-s_1-i\epsilon}{\Lambda}\right)^{\omega}+\left(\frac{s_1-i\epsilon}{\Lambda}\right)^{\omega}\right]=\theta\left(\left|\frac{s_1}{\Lambda}\right|-1\right)$$

  - allows to take into account imaginary part of the reggeized gluon
  - Reobtain full reggeized gluon within the LLA with correct real and imaginary part

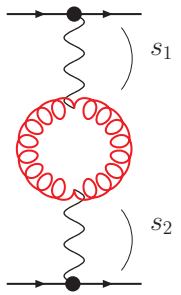
$$\mathcal{M}^{\text{tree}}(s,t)\int\frac{d\omega}{4\pi i}\frac{1}{\omega-\beta(t)}\left[\left(\frac{-s-i\epsilon}{s_R}\right)^{\omega}+\left(\frac{s-i\epsilon}{s_R}\right)^{\omega}\right]$$

Corrective term for n reggeized gluons exchange

To reobtain the BFKL-equation from the effective action, consider t-channel state of 2 reggeized gluons:



- Diagrams contain contribution counted already in the trajectory



➡ remove overcounted terms by a supplementary term to the effective Lagrangian + yields convergence of integrals

$$\tilde{S}_{\text{eff}} = \int d^4x [\mathcal{L}_{\text{QCD}}(v_\mu, \psi) + \mathcal{L}_{\text{ind}}(v_\pm, A_\pm) - 2\mathcal{L}_{\text{ind}}(A_\pm, A_\pm)]$$

- not in conflict with the original derivation of the effective action
- allows to reobtain (LLA) BFKL-equation from effective action

Unitarization of the BFKL-Pomeron

Goals of effective action two-fold:

I Determination of $N^x LO$-corrections to the BFKL-equation and corresponding interactions with external particles

II systematic way to unitarize the BFKL-Pomeron ($=$ two reggeized gluon state in color singlet)

Why (II)? powerlike rise predicted by BFKL-Pomeron for total cross-section would finally violate unitarity, if infinitly continued

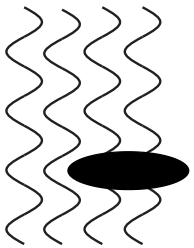
➡ expect occurence of unitarity corrections for certain value of s

Effective action II: Transition vertices in QCD Reggeon-field-theory

Vertices in QCD Reggeon field theory

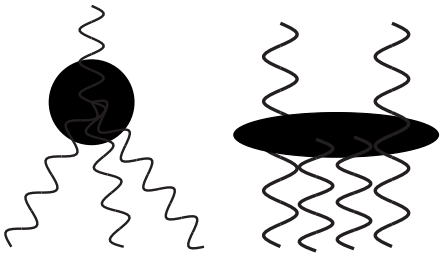
- most general approach: reformulate QCD as a $2+1$ dimensional Reggeon Field Theory (RFT) with both s-channel and t-channel unitarity build in
- know (from 'old' Regge theory), that a RFT of QCD exists [Gribov '68, ...], however only some of its elements known so far

- BKP: states of n reggeized gluons [Bartels; Kwiecinski, Praszalowicz, '80]



  * ($\triangleq$ QM of n particles)
  * integrable in large N_c limit [Lipatov, '94; Faddeev, Korchemsky, '95]

- Number changing elements
  * ($\triangleq$ QFT)



  * Reggeon field theory

➡ Use the effective action to derive these vertices

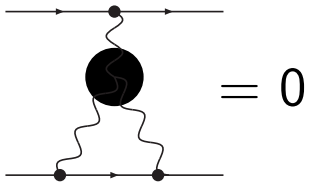
RFT, transition vertices and signature conservation

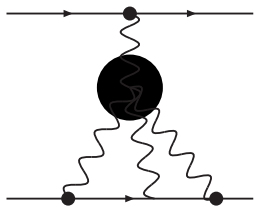
Goals:

- derive elements of RFT
- understand effective action and show that it is capable to reproduce non-trivial results

Find:

- suitable symmetric decomposition of color factors
- 'Old' RFT had additional conserved quantum number: signature
  - * signature conservation fulfilled within the effective action, $U_{1\to 2}$ and $U_{2\to 3}$ vanish inside the elastic amplitude
  - * non-zero transition $U_{1\to 3}$, $U_{2\to 4}$ inside the elastic amplitude





The $1 \to 3$ transition in transverse momentum space

obtain as a result

$$U_{1\to 3}^{a;b_1b_2b_3}(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \mathrm{tr}\left(T^aT^{b_1}T^{b_2}T^{b_3}\right)U(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \mathrm{tr}\left(T^aT^{b_2}T^{b_1}T^{b_3}\right)U(\boldsymbol{q};\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + \mathrm{tr}\left(T^aT^{b_1}T^{b_3}T^{b_2}\right)U(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_3,\boldsymbol{k}_2)$$

with

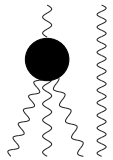
$$U(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = g^4\frac{\boldsymbol{q}^2}{6}\int\frac{d^2\boldsymbol{l}}{(2\pi)^3}\left[\frac{\boldsymbol{q}^2}{\boldsymbol{l}^2(\boldsymbol{q}-\boldsymbol{l})^2} - \frac{(\boldsymbol{q}-\boldsymbol{k}_3)^2}{\boldsymbol{l}^2(\boldsymbol{q}-\boldsymbol{l}-\boldsymbol{k}_3)^2} + \frac{\boldsymbol{k}_1^2}{\boldsymbol{l}^2(\boldsymbol{l}-\boldsymbol{k}_1)^2}\right]$$

- symmetric under simultaneous exchange of transverse momenta and color
- $T^b_{cc'} = if^{cbc'}$ $SU(N_c)$ generator in the adjoint representation

The $2 \to 4$ transition vertex of the effective action

$$U_{2\to 4}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = U_{\text{connect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) + U_{\text{disconnect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$

$$U_{\text{disconnect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) =$$



$$+ \text{perm}$$

$$U_{\text{connect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \sum_{j_1, \ldots, j_4} \left(T^{b_{j_1}} T^{b_{j_2}} T^{b_{j_3}} T^{b_{j_4}}\right)_{a_1 a_2} U_C(\boldsymbol{l}_{i_1}, \boldsymbol{l}_{i_2}; \boldsymbol{k}_{j_1}, \boldsymbol{k}_{j_2}, \boldsymbol{k}_{j_3}, \boldsymbol{k}_{j_4})$$

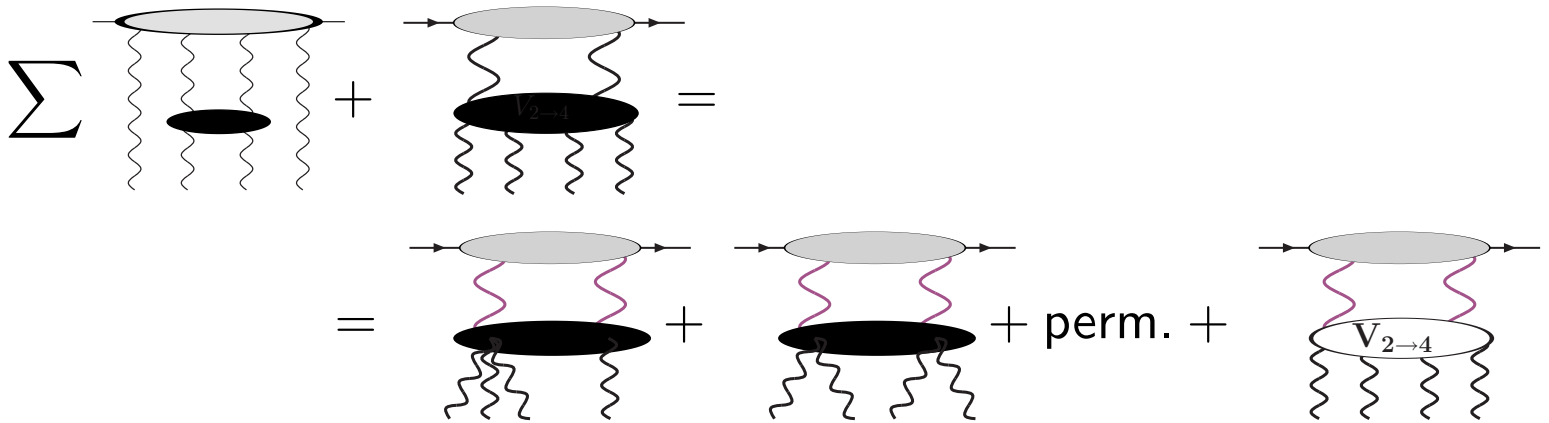
$$U_C(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = g^4 \boldsymbol{l}_1^2 \boldsymbol{l}_2^2 \Bigg[\frac{1}{24} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)^2}{\boldsymbol{l}_1^2 \boldsymbol{l}_2^2} - \frac{1}{12} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)^2}{\boldsymbol{l}_1^2 (\boldsymbol{l}_2 - \boldsymbol{k}_4)^2}$$
$$- \frac{1}{12} \frac{(\boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)^2}{\boldsymbol{l}_2^2 (\boldsymbol{l}_2 - \boldsymbol{k}_1)^2} + \frac{1}{8} \frac{(\boldsymbol{k}_2 + \boldsymbol{k}_3)^2}{(\boldsymbol{l}_1 - \boldsymbol{k}_1)^2 (\boldsymbol{l}_2 - \boldsymbol{k}_4)^2} + \frac{1}{24} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2}{\boldsymbol{l}_1^2 (\boldsymbol{l}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)^2}$$
$$- \frac{1}{24} \frac{\boldsymbol{k}_2^2}{(\boldsymbol{l}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)^2 (\boldsymbol{l}_1 - \boldsymbol{k}_1)^2} + \frac{1}{24} \frac{(\boldsymbol{k}_3 + \boldsymbol{k}_4)^2}{\boldsymbol{l}_2^2 (\boldsymbol{l}_1 - \boldsymbol{k}_1 - \boldsymbol{k}_2)^2} - \frac{1}{24} \frac{\boldsymbol{k}_3^2}{(\boldsymbol{l}_1 - \boldsymbol{k}_1 - \boldsymbol{k}_2)^2 (\boldsymbol{l}_2 - \boldsymbol{k}_4)^2} \Bigg]$$

The state of 4 reggeized gluons

There exists an earlier result on a $2 \to 4$ transition [Bartels, Wüsthoff, 1995]

- derived from triple discontinuity of six-point-amplitude, for overall color singlet
- also contained in BK-equation [Balitsky '96, Kovchegov '99]

Find for overall color singlet find following decomposition



- Additional reggeization in the symmetric sector
- $V_{2\to4}$ is the transition vertex of [Bartels, Wüsthoff, 1995]: infrared finite, 'triple Pomeron vertex'

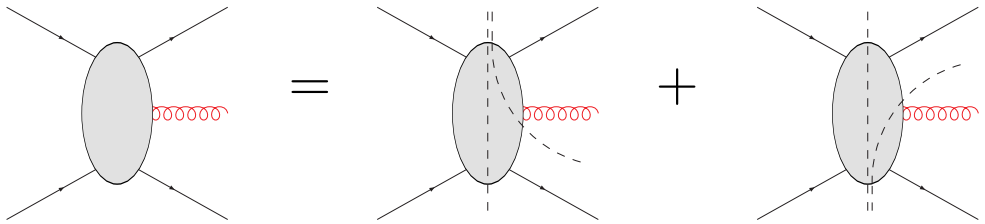
Summary and Conclusions

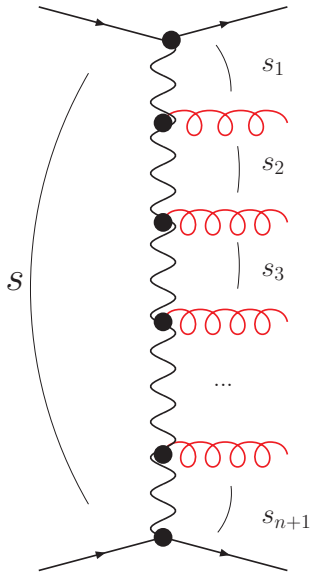
- heavy flavor production :
  - set theoretical framework for heavy flavor production within NLO k_T factorization
  - increase accuracy and reduce scale dependence by NLO unintegrated gluon densities
  - iterative solution of BFKL-Green's function that allows for numerical solution
  - outlook: perform numerical implementation and obtain unintegrated gluon densities from fit to HERA data
- effective action:
  - requires supplementary rules + new term
  - allow rederivation of LLA reggeized gluon and BFKL-equation
  - derrived transition vertices of QCD Reggeon-field-theory and showed agreement with 'triple Pomeron vertex'
  - outlook: make action ready for NLO calculation

Phase structure of the reggeized gluon

Production amplitudes: Constraint on discontinuities/imaginary parts due to the Steinmann relations: *no simultaneous discontinuities in overlapping channels*

Example:
$2 \to 3$ production amplitude:





- LLA: both production vertex and reggeized gluon are real
- beyond: check by 1-loop correction to production vertex, that overall phase structure is correct - positive result