Modeling the threshold region in $e^+e^- \to \pi^+\pi^-\gamma(\gamma)$ for Radiative Return at DAPHNE

(based on arXiv: 0910.5335 [hep-ph])

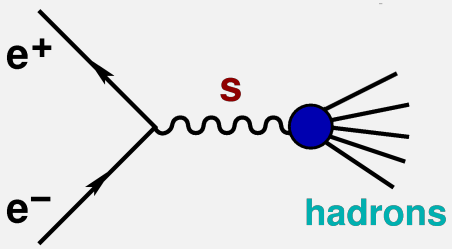
Sergiy IVASHYN [1, 2], H. Czyż[1], A. Korchin[2]

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17 November 2009
IFIC, University of Valencia

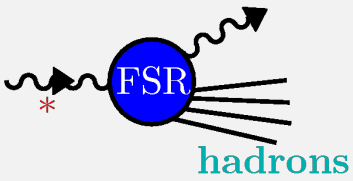
Where, what, how . . .



Context

- $\sigma(e^+e^- \to \text{hadrons})$
- form factors of hadrons

Aim



- off-peak KLOE analysis

Problem

- model dependent ingredients



Tool

- PHOKHARA Monte Carlo



- ✓ Radiative return
- ✓ Experimental situation
- ✓ Formalism
- ✓ FSR tests
- ✓ Conclusions

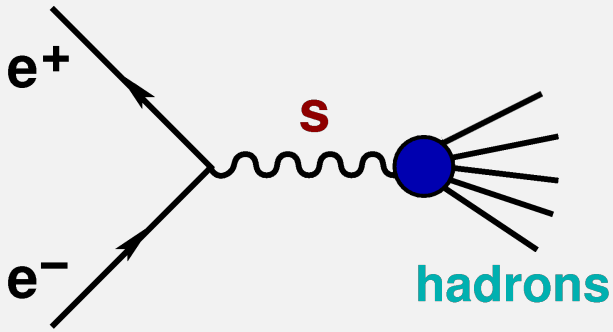


- ✓ Radiative return
- ✓ Experimental situation
- ✓ Formalism
- ✓ FSR tests
- ✓ Conclusions

Motivation: Hadronic cross section

Precise measurement of σ_{had} as function of Q^2 is needed

- ✓ form factors of hadrons



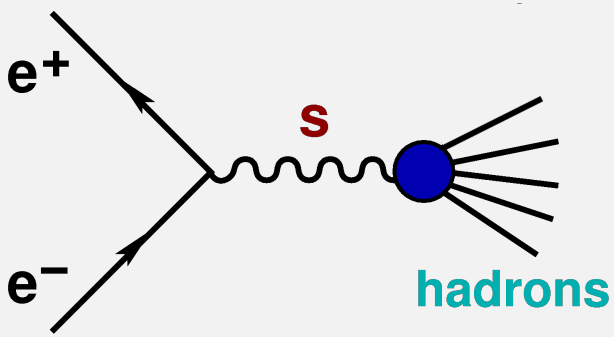
This diagram: $Q^2 = s$

scan $\sigma_{had}(Q^2)$ $\Leftrightarrow$ vary beam energy

but. . . at meson factories the beam energy is fixed

Motivation: Hadronic cross section

Precise measurement of σ_{had} as function of Q^2 is needed



- ✓ form factors of hadrons
- ✓ $R(Q^2) = \sigma_{had}/\sigma_{point}$ over the full range of Q^2

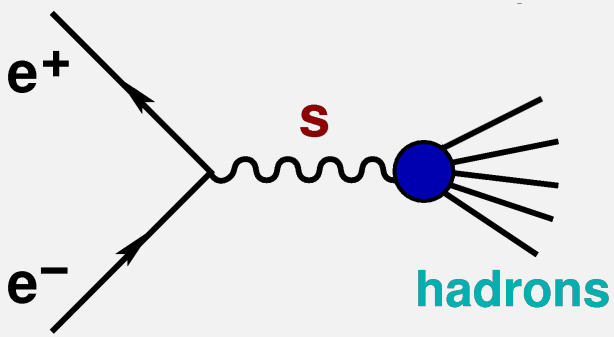
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Motivation: Hadronic cross section

Precise measurement of σ_{had} as function of Q^2 is needed



- ✓ form factors of hadrons
- ✓ $R(Q^2) = \sigma_{had}/\sigma_{point}$ over the full range of Q^2
- ✓ hadronic contribution to $g-2$ of the muon
  $\Rightarrow$ attention to $R(Q^2)$ at low Q^2

[F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1]

This diagram: $Q^2 = s$

scan $\sigma_{had}(Q^2)$ $\Leftrightarrow$ vary beam energy

but. . . at meson factories the beam energy is fixed

Radiative Return method

- How to "scan" the hadronic cross section
  at meson factories, i.e., when the beam energy is fixed

✓ Radiative Return method [Min-Shin Chen and P.M. Zerwas, Phys. Rev. D **11**, 58 (1975)]

[A. B. Arbuzov, E. A. Kuraev, N. P. Merenkov and L. Trentadue, JHEP **9812** (1998) 009]

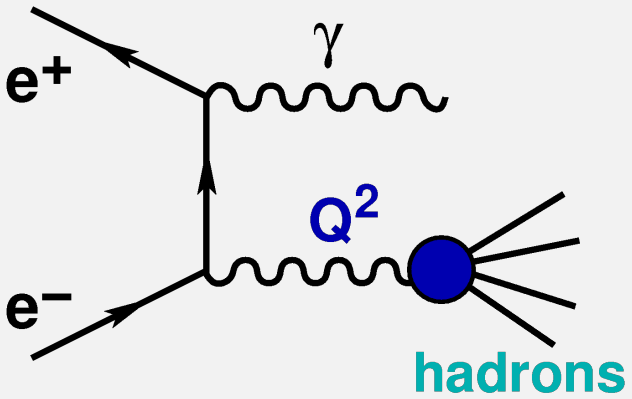
[S. Binner, J. H. Kuhn and K. Melnikov, Phys. Lett. B **459** (1999) 279]

Obstacles:

- radiative corrections
- hadronic interaction modeling

see e.g. [A. Grzelińska, H. Czyż and A. Wapienik, Nucl. Phys. Proc. Suppl. **189** (2009) 216]

Idealistic Radiative Return



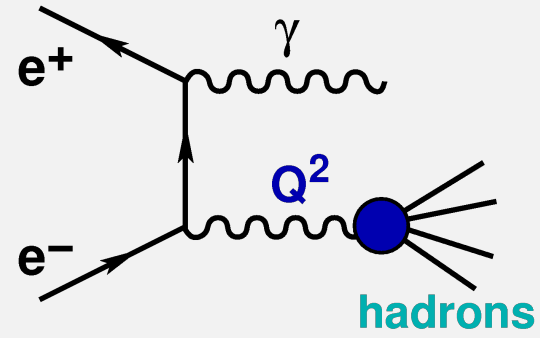
pure Initial State Radiation

$$\mathrm{d}\sigma(Q^2)\big|_{ISR} = (\cdots)\,\mathrm{d}\sigma_{had}(Q^2)$$

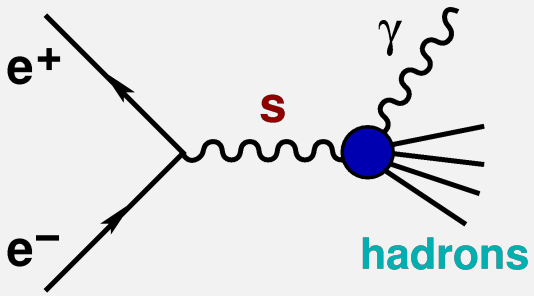
This diagram: $Q^2 \neq s$

scan $\sigma_{had}(Q^2)$ $\Leftrightarrow$ use ISR data + work out "$(\cdots)$" in QED

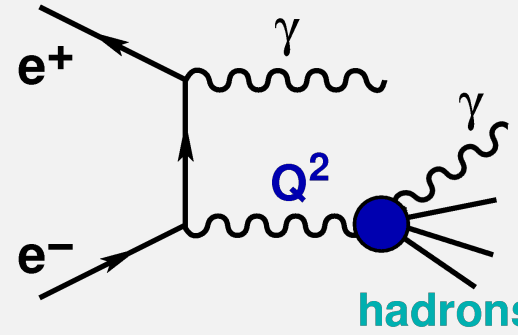
realistic Radiative Return: FSR



ISR QED perturbative calculation



FSR hadronic models



ISR & FSR

NLO QED & modeling

see e.g. [A. Grzelińska, H. Czyż and A. Wapienik, Nucl. Phys. Proc. Suppl. **189** (2009) 216]

PHOKHARA: MC for Radiative Return

[A. Grzelińska, H. Czyż and A. Wapienik, Nucl. Phys. Proc. Suppl. **189** (2009) 216]

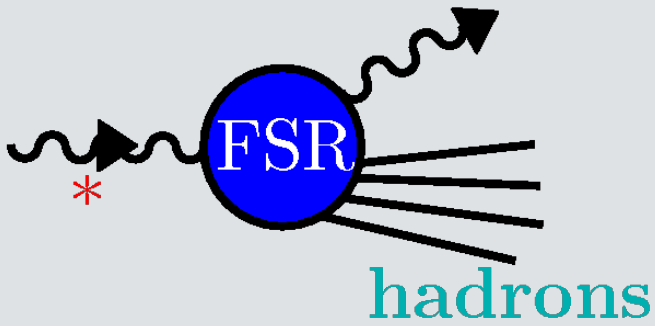
- special tool for Radiative Return
- important radiative corrections

PHOKHARA: MC for Radiative Return

[A. Grzelińska, H. Czyż and A. Wapienik, Nucl. Phys. Proc. Suppl. **189** (2009) 216]

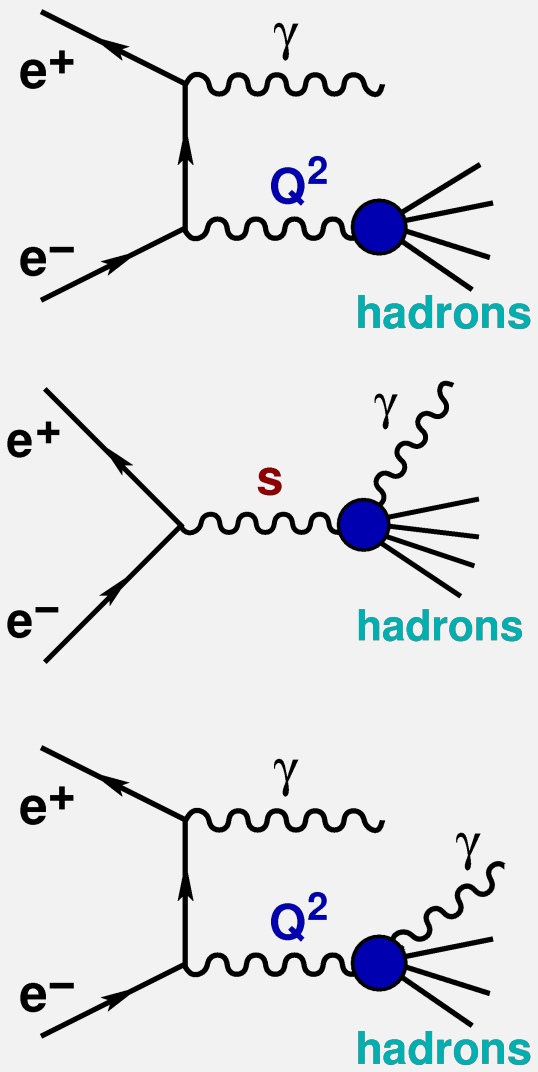
- special tool for Radiative Return
- important radiative corrections
- sQED*VMD already implemented long ago

- models for Final State Radiation tensor can be replaced in the program



✓ we compare several FSR models using PHOKHARA

PHOKHARA for *realistic* Radiative Return



- events with one photon
- events with two photons
- "experimental" cuts, in detail
  - ✓angular cuts on π, γ
  - ✓track-mass cut
  - ✓Ω angle cut
  - ✓missing mass cut



- ✓ Radiative return
- ✓ Experimental situation
- ✓ Formalism
- ✓ FSR tests
- ✓ Conclusions

Radiative Return at B factories

$e^+e^- \to \gamma_{ISR}$+hadrons

- BaBar:
  $e^+e^- \to \ldots$

  $\pi^+\pi^- J/\psi$, 3π, 4π, $2K\pi$, $2K2\pi$, $4K$, $p\bar{p}$, $\Lambda\bar{\Lambda}$

  [recent review: S.Serednyakov, arXiv: 0911.0235 [hep-ex]]

- Belle:
  $e^+e^- \to \ldots$

  $\pi^+\pi^- J/\psi$, $\pi^+\pi^-\psi(2S)$, $\Lambda_c^+\Lambda_c^-$, $\pi^+\pi^-\phi(1020)$

  [recent review: A.Zupanc, arXiv: 0910.3404 [hep-ex]]

Radiative Return at KLOE

$e^+e^- \to \pi^+\pi^-\gamma \Rightarrow e^+e^- \to \pi^+\pi^-$

Previous analyses by KLOE

- Data 2001 [A. Aloisio et al. [KLOE Collaboration], Phys. Lett. B606 (2005) 12]
- Data 2002 [F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 670 (2009) 285]

✓ $F_\pi(Q^2)$ measurement, $0.35 < Q^2 < 0.95$ GeV^2

$\sqrt{s} = m_\phi = 1.02$ GeV $\Rightarrow \phi$ radiative decay was important

Radiative Return at KLOE

$e^+e^- \to \pi^+\pi^-\gamma \Rightarrow e^+e^- \to \pi^+\pi^-$

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"Small-angle" cuts

Suppress FSR by means of kinematic cuts

$\Rightarrow$ decreased statistics

$\Rightarrow$ low Q^2 region is lost

model uncertainties for FSR due to ϕ decay

- extraction of F_π was difficult

Off-peak data analysis

We anticipate "off-peak" Rad. Return data analysis by KLOE

- ✓ $\sqrt{s} = 1$ GeV
- ✓ integrated luminosity 0.2 fb^{-1}
- ✓ two-pion invariant mass Q^2: go as low as possible

Off-peak data analysis

We anticipate "off-peak" Rad. Return data analysis by KLOE

"Large-angle" cuts

- no special FSR-suppression
- preserve statistics
- "contamination" by FSR have to be subtracted

Off-peak data analysis

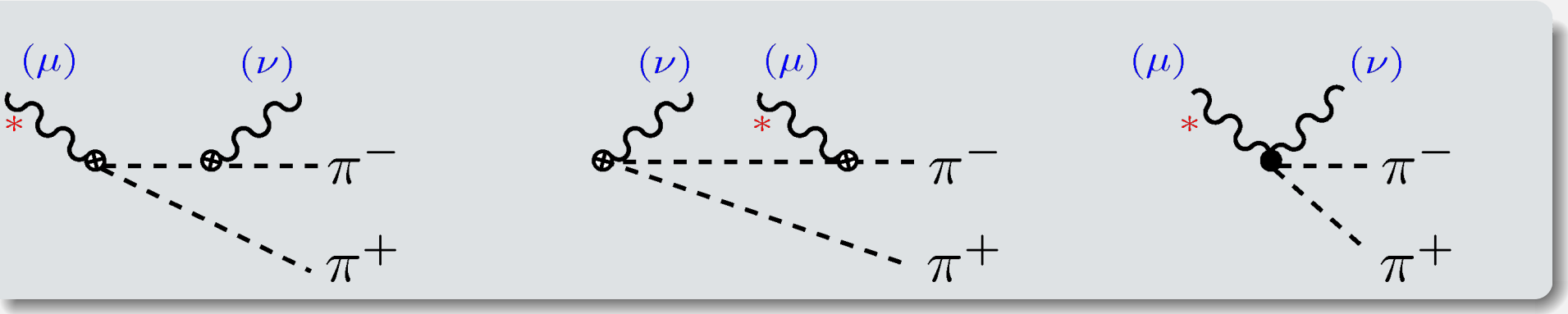
We anticipate "off-peak" Rad. Return data analysis by KLOE

- ✓ FSR is simple, in comparison to $\sqrt{s} = m_\phi$ case
  ($s < m_\phi \quad \Rightarrow \quad \phi$ radiative decay is suppressed)
- comparison of FSR models is important
  ($\Rightarrow$ accuracy of the method)
- ✓ special interest in Q^2 below the ρ resonance
  $\Rightarrow$ ChPT may be useful



- ✓ Radiative return
- ✓ Experimental situation
- ✓ Formalism
- ✓ FSR tests
- ✓ Conclusions

FSR tensor at Born level



scalar QED
(no-structure charged pions)

$\Leftrightarrow$

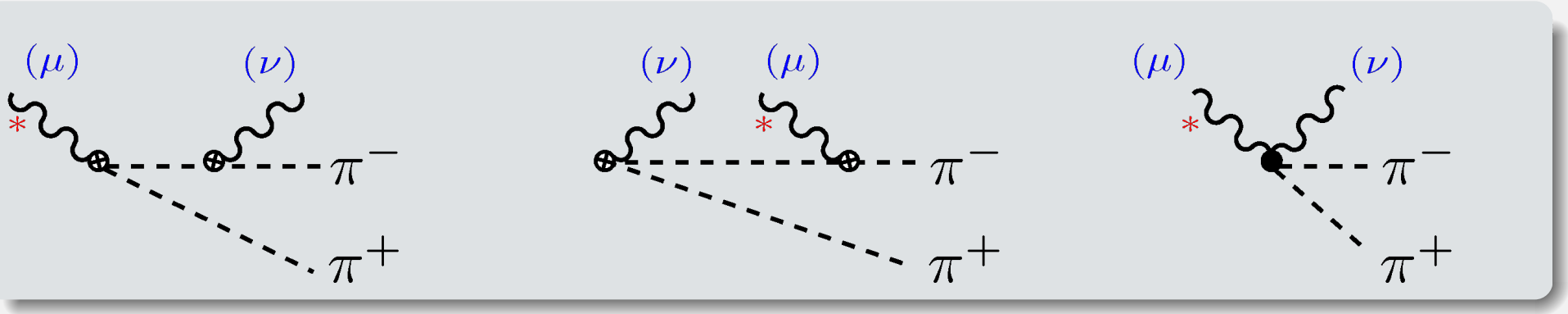
lowest-order
Chiral Perturbation Theory

[Unkmeir, Scherer, L'vov, Drechsel, PR D61, 034002 (1999)]

[Czyż, Grzelińska, Kühn, Rodrigo, EPJ C33, 333 (2004)]

[. . . and others]

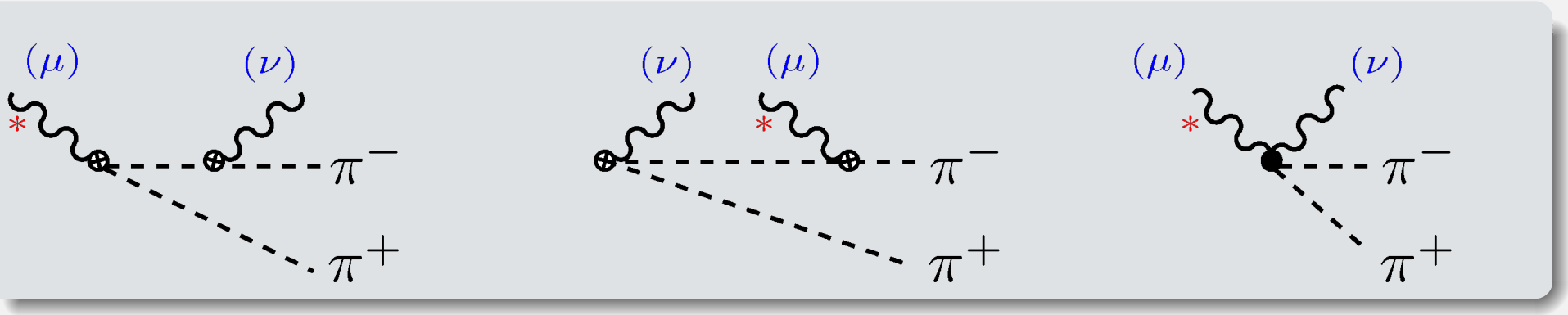
FSR tensor at Born level



"Born" tensor

$$M_B^{\mu\nu} = -ie^2 \left(\frac{(k + q_1 - q_2)^\mu q_1^\nu}{q_1 \cdot k} + \frac{(k + q_2 - q_1)^\mu q_2^\nu}{q_2 \cdot k} - 2\, g^{\mu\nu} \right)$$

FSR tensor at Born level



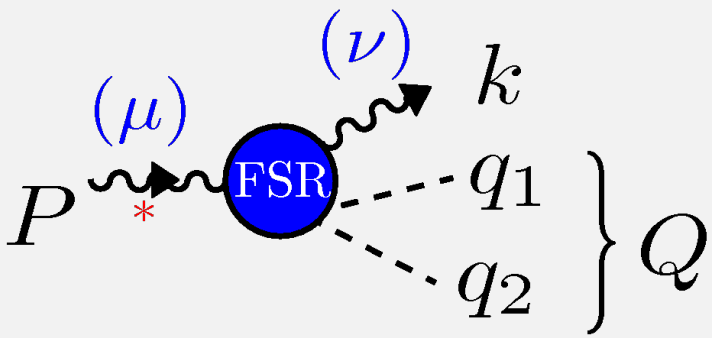
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"Generalized Born" tensor ($\equiv$ sQED*VMD)

$$M_B^{\mu\nu} = -ie^2 F_\pi(P^2) \left(\frac{(k+q_1-q_2)^\mu q_1^\nu}{q_1 \cdot k} + \frac{(k+q_2-q_1)^\mu q_2^\nu}{q_2 \cdot k} - 2\, g^{\mu\nu} \right)$$

Final State Radiation: generic structure



k — real photon momentum

q_1, q_2 — pion momenta

Q^2 — two-pion invariant mass

P^2 — photon virtuality

$r \equiv q_1 - q_2$

FSR: 3 gauge invariant and independent tensors

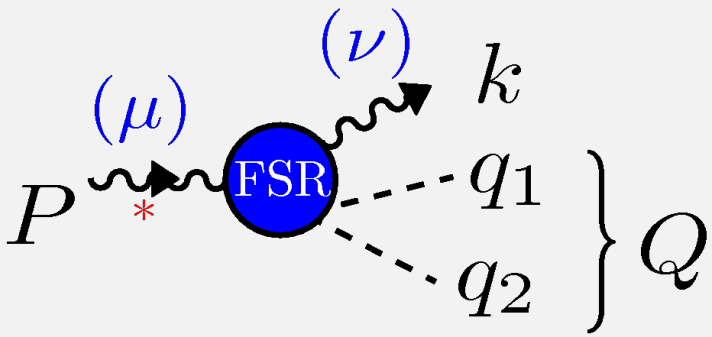
[Dubinsky, Korchin, Merenkov, Pancheri, Shekhovtsova, EPJ C40, 41 (2005)]

[Drechsel, Knöchlein, Metz, Scherer, PR C55, 424 (1997)]

$$M^{\mu\nu}(Q,k,r) = -ie^2(\tau_1^{\mu\nu} f_1 + \tau_2^{\mu\nu} f_2 + \tau_3^{\mu\nu} f_3),$$

$$\tau_1^{\mu\nu} = k^\mu Q^\nu - g^{\mu\nu}\ k\cdot Q,$$
$$\tau_2^{\mu\nu} = k\cdot r\ (r^\mu Q^\nu - g^{\mu\nu}\ Q\cdot l) + r^\nu\ (k^\mu\ Q\cdot r - r^\mu\ k\cdot Q),$$
$$\tau_3^{\mu\nu} = Q^2\ (g^{\mu\nu}\ k\cdot r - k^\mu r^\nu) + Q^\mu\ (r^\nu\ k\cdot Q - Q^\nu\ k\cdot r)$$

Final State Radiation: generic structure



k — real photon momentum

q_1, q_2 — pion momenta

Q^2 — two-pion invariant mass

P^2 — photon virtuality

$r \equiv q_1 - q_2$

FSR: 3 gauge invariant and independent tensors

[Dubinsky, Korchin, Merenkov, Pancheri, Shekhovtsova, EPJ C40, 41 (2005)]

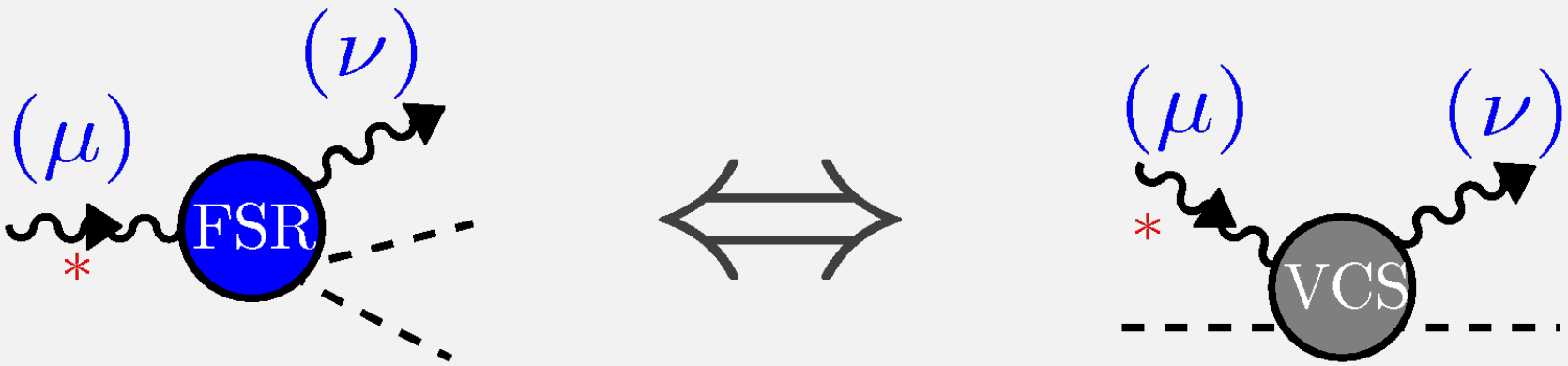
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$$M^{\mu\nu}(Q,k,r) = -ie^2(\tau_1^{\mu\nu} f_1 + \tau_2^{\mu\nu} f_2 + \tau_3^{\mu\nu} f_3),$$

- $f_{1,2,3} \equiv f_{1,2,3}(Q^2, k\cdot Q, k\cdot r)$ — Lorentz invariant functions
- model-independent decomposition
- explicit form of $f_{1,2,3}$ is model-dependent

$\Rightarrow$ source of theoretical uncertainty

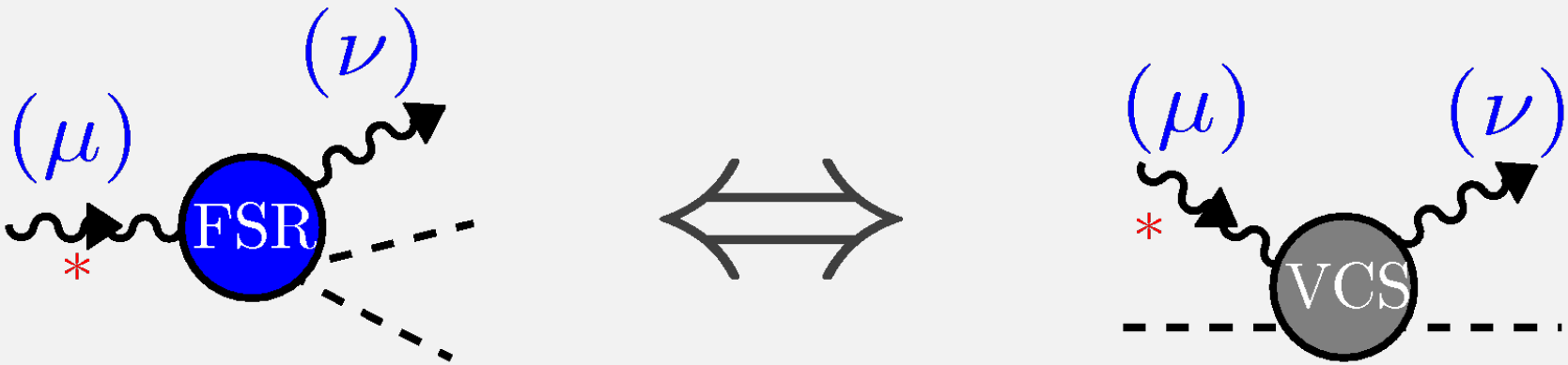
Virtual Compton Scattering



Huge human work has been devoted so far to the
Virtual Compton Scattering off pion

- direct sensitivity to “non-Born” part of amplitude
- a number of models have been applied

Virtual Compton Scattering



We use the ChPT prediction for Virtual Compton Scattering

- SU(2) [Unkmeir, Scherer, L'vov, Drechsel, PR D61, 034002 (1999)]
- SU(3) [Fuchs, Pasquini, Unkmeir, Scherer, Czech. J. Phys. **52**, B135 (2002)]

"Non-Born" correction in ChPT at $\mathcal{O}(p^4)$

Chiral Perturbation Theory at 1 loop level

$$M^{\mu\nu}_{NB}(Q,k,r) = -ie^2(\tau_1^{\mu\nu} f_1^{NB} + \tau_2^{\mu\nu} \cancel{f_2^{NB}} + \tau_3^{\mu\nu} \cancel{f_3^{NB}}),$$

- $\tau_1^{\mu\nu} = k^\mu Q^\nu - g^{\mu\nu}\ k \cdot Q,$
- only f_1^{NB} does not vanish in this model

"Non-Born" correction in ChPT at $\mathcal{O}(p^4)$

$$M^{\mu\nu}_{NB}(Q,k,r) = -ie^2\,(k^\mu Q^\nu - g^{\mu\nu}\; k\cdot Q)\; f_1^{NB}$$

SU(2): pion loop

[Unkmeir, Scherer, L'vov, Drechsel, PR D61, 034002 (1999)]

$$f_1^{NB} = \frac{-1}{16\,\pi^2\,F^2}\left(\frac{2}{3}(\bar{l}_6 - \bar{l}_5) + \frac{P^2 - 2\;P\cdot k}{P\cdot k}\,G_\pi(P^2,\;P\cdot k)\right)$$

SU(3): pion and kaon loop

[Fuchs, Pasquini, Unkmeir, Scherer, Czech. J. Phys. **52**, B135 (2002)]

$$f_1^{NB} = \frac{-1}{16\,\pi^2\,F_0^2}\,\Big(128\pi^2(L_9^r + L_{10}^r) + \frac{P^2 - 2\;P\cdot k}{P\cdot k}\,G_\pi(P^2,\;P\cdot k) + \frac{1}{2}\,G_K(P^2,\;P\cdot k)\Big)$$

KLOE model

Phenomenological parametrization/fit of $\pi^0\pi^0\gamma$ KLOE data

- worked out by KLOE and N.N. Achasov for scalar meson studies
- transformed to $\gamma^* \to \pi^+\pi^-\gamma$ via isospin symmetry
- implemented in PHOKHARA 6.1 and FASTERD by O. Shekhovtsova

[PHOKHARA 6.1, URL: http://ific.uv.es/řodrigo/phokhara/]

[Shekhovtsova, Venanzoni, Pancheri, arXiv:0901.4440 [hep-ph] (2009)]

KLOE model

Phenomenological parametrization/fit of $\pi^0\pi^0\gamma$ KLOE data
Includes:

- ϕ radiative decay (N.N. Achasov)
- vector + double-vector contributions



Other FSR studies

Resonance Chiral Theory - based approaches

- with axial-vector meson a_1 contribution

  [Dubinsky, Korchin, Merenkov, Pancheri, Shekhovtsova, EPJ C40, 41 (2005)]

  numerical FSR studies [Pancheri, Shekhovtsova, Venanzoni, Phys. Lett. B642, 342 (2006)]

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  numerical FSR studies [Pancheri, Shekhovtsova, Venanzoni, Phys. Lett. B642, 342 (2006)]

- $\pi^0\pi^0\gamma$ FSR with scalar mesons $\sigma(600)$, $f_0(980)$, vector & double-vector contributions

  [Eidelman, Ivashyn, Korchin, Shekhovtsova, in preparation]

Other FSR studies

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  [Eidelman, Ivashyn, Korchin, Shekhovtsova, in preparation]

Other related studies

- Czyż et al. [Czyz, Grzelińska, Kühn, PL B611, 116 (2005)]
- Pancheri et al. [Pancheri, Shekhovtsova, Venanzoni, J.Exp.Theor.Phys. 106, 470 (2008)]
- Gallegos et al. [Gallegos, Lucio, Moreno, Napsuciale, arXiv:0908.2192]
- Oset and Roca [Roca, Oset arXiv:0911.0994]



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Cross section

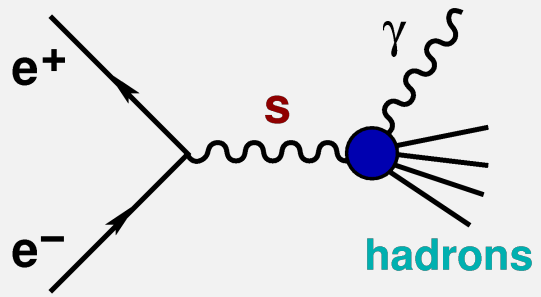
Pion form factor

sQED*VMD ("generalized Born")

$$M_B^{\mu\nu} = -\mathrm{i}e^2 F_\pi(P^2)\left(\frac{(k+q_1-q_2)^\mu q_1^\nu}{q_1\cdot k} + \frac{(k+q_2-q_1)^\mu q_2^\nu}{q_2\cdot k} - 2\,g^{\mu\nu}\right)$$

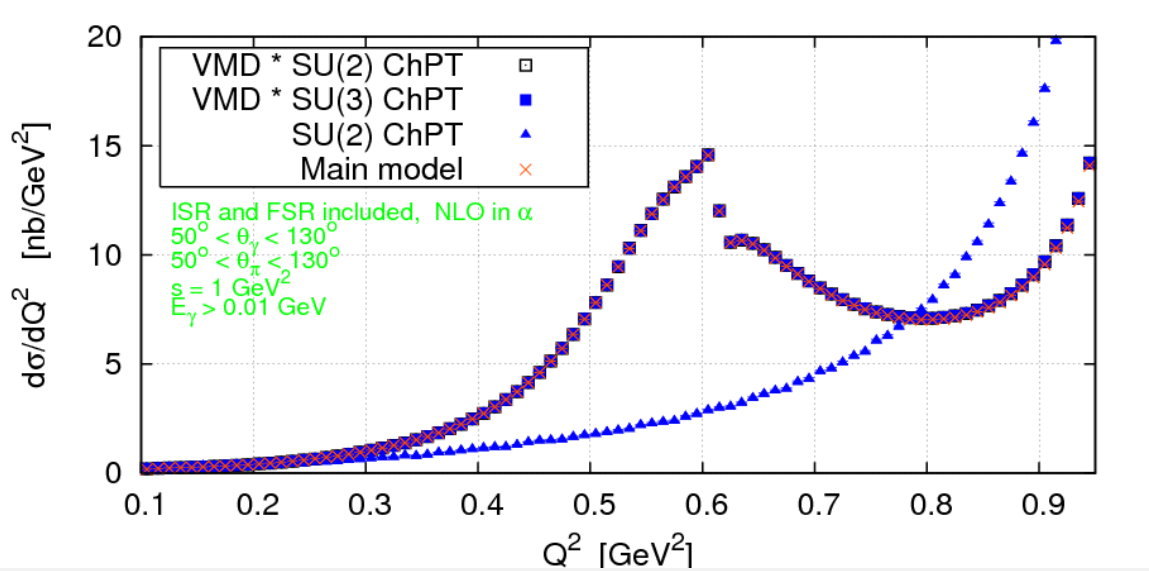
Pion form factor $F_\pi(P^2)$ in the FSR tensor is very important
(see explicit numerical example further)

- at LO in α_{QED}, $P^2 = s$, e.g. : $F_\pi(1\mathrm{GeV}^2)$



The value of the form factor is taken at P^2
(even when the two-pion invariant mass is small $Q^2 \ll P^2$)

Numerical results: Cross section



$$e^+e^- \to \pi^+\pi^-\gamma(\gamma)$$

$$\sqrt{s} = 1 \text{ GeV}$$

$$50^o < \theta_{\gamma,\pi} < 130^o$$

Pion form factor is crucial

- ChPT $\mathcal{O}(p^4)$ FF $\Rightarrow$ wrong estimate for cross section
- KLOE model uses Gounaris-Sakurai parametrization for F_π

[Bruch, Khodjamirian, Kühn, EPJ C39, 41 (2005)]

“Non-Born” correction

$$M^{\mu\nu}_{NB}(Q,k,r) = -ie^2(\tau_1^{\mu\nu} f_1^{NB} + \tau_2^{\mu\nu} f_2^{NB} + \tau_3^{\mu\nu} f_3^{NB}),$$

- explicit form of $f^{NB}_{1,2,3}$ is model-dependent

We now work with

- SU(2) and SU(3) Chiral Perturbation Theory

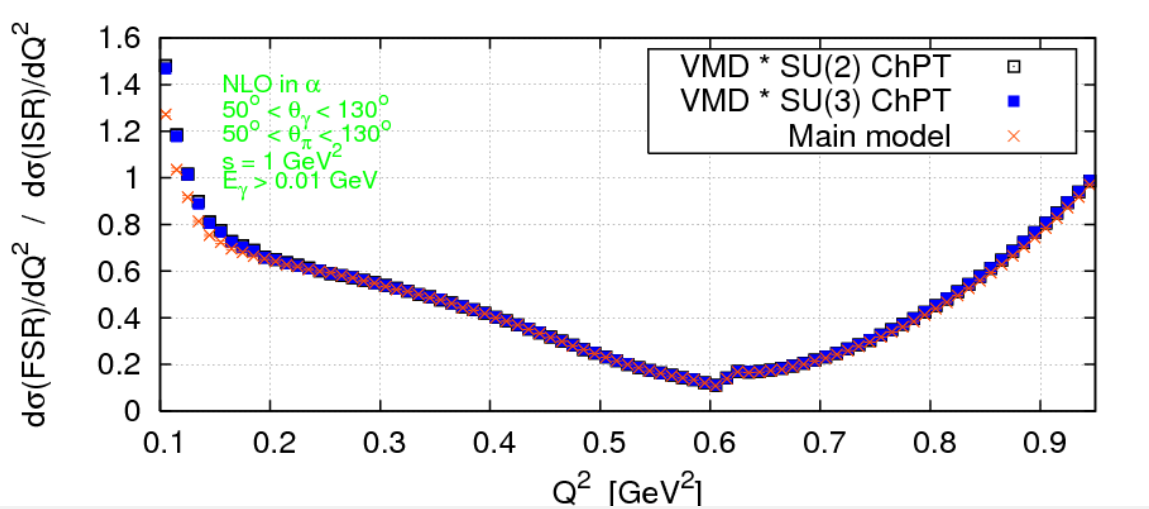
[Unkmeir, Scherer, L’vov, Drechsel, PR D61, 034002 (1999)]

[Fuchs, Pasquini, Unkmeir, Scherer, Czech. J. Phys. **52**, B135 (2002)]

- KLOE model as implemented in PHOKHARA 6.1 and FASTERD by Olga Shekhovtsova [Shekhovtsova, Venanzoni, Pancheri, arXiv:0901.4440 [hep-ph] (2009)]

These models give “predictions” for FSR:
parameters are fixed independently

Numerical results: Cross section. FSR / ISR



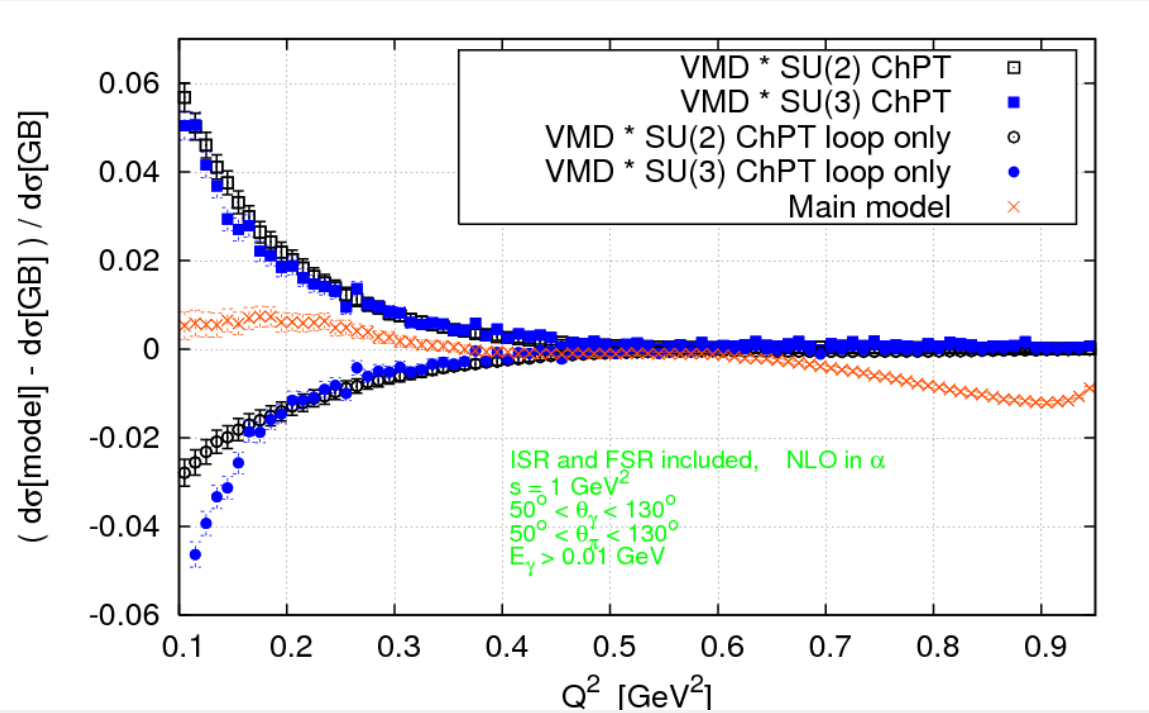
$$e^+e^- \to \pi^+\pi^-\gamma(\gamma)$$

$$\sqrt{s} = 1 \text{ GeV}$$

$$50^o < \theta_{\gamma,\pi} < 130^o$$

Final State Radiation is important
as generally expected for “large-angle” analysis

Role of “non-Born” correction



$$e^+e^- \to \pi^+\pi^-\gamma(\gamma)$$

$$\sqrt{s} = 1 \text{ GeV}$$

$$50^o < \theta_{\gamma,\pi} < 130^o$$

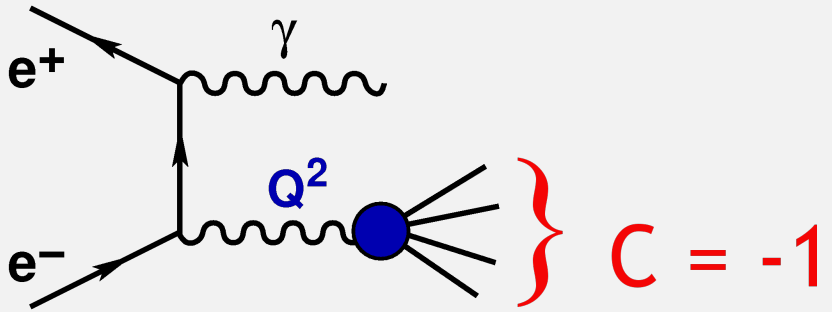
- model dependence is small
- at KLOE statistics the ChPT-corrections are not visible
- effect is enhanced at low Q^2

Asymmetry

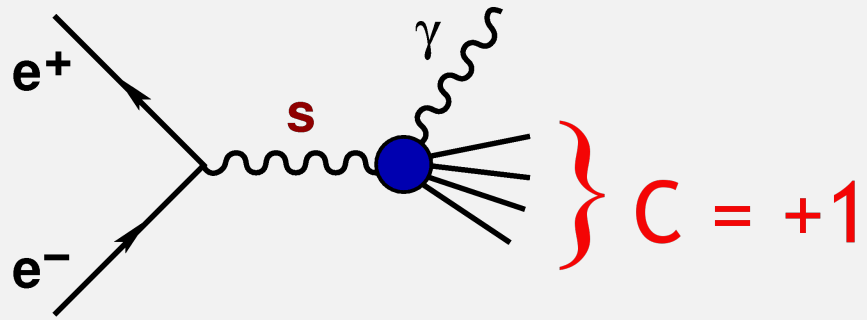
Origin of the asymmetries

Interference of C-odd and C-even amplitudes

ISR



FSR



- ✓ charge asymmetry

$$A_{Charge}(\theta) = \frac{N(\pi^+) - N(\pi^-)}{N(\pi^+) + N(\pi^-)}(\theta)$$

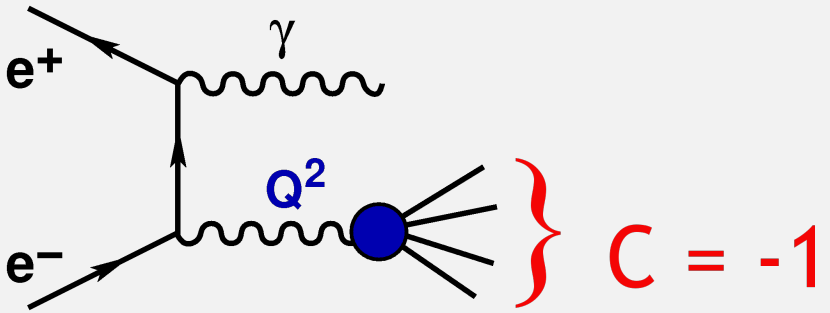
- ✓ forward-backward asymmetry

$$A_{FB}(Q^2) = \frac{N(\theta_{\pi^+} > 90^o) - N(\theta_{\pi^+} < 90^o)}{N(\theta_{\pi^+} > 90^o) + N(\theta_{\pi^+} < 90^o)}(Q^2)$$

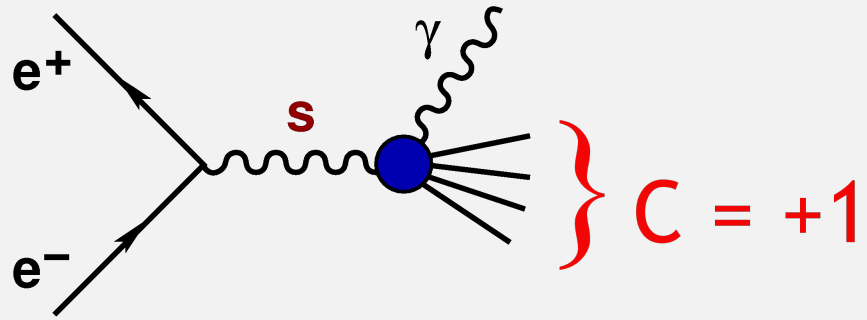
Origin of the asymmetries

Interference of C-odd and C-even amplitudes

ISR



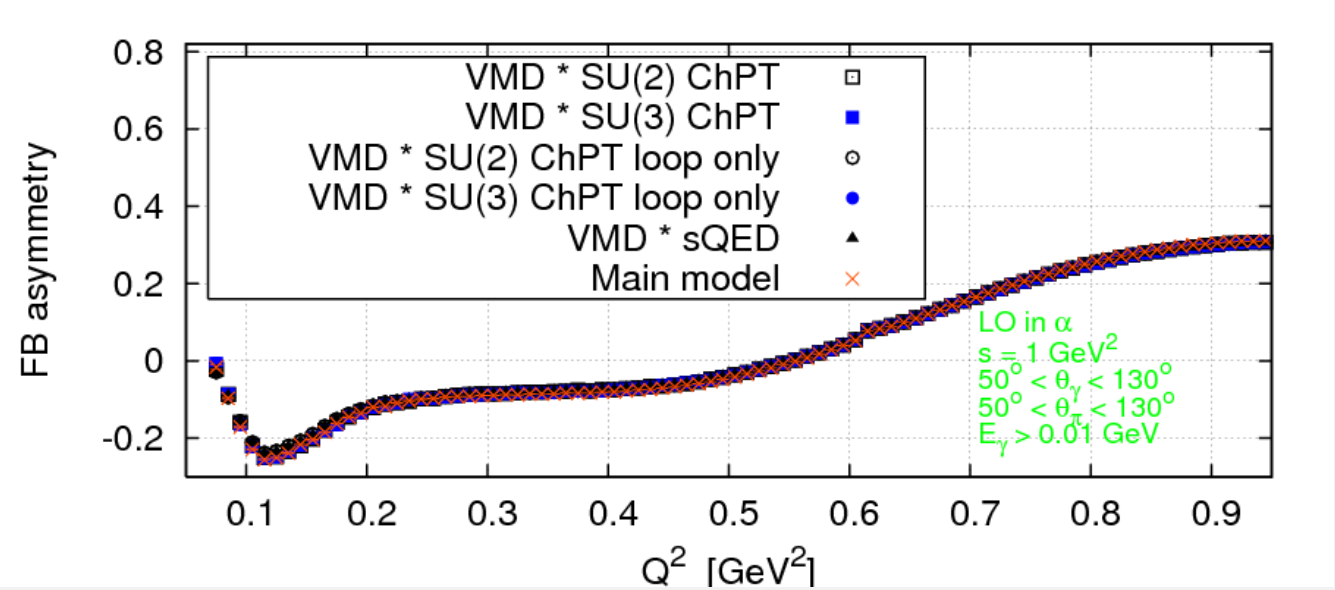
FSR



model $\Rightarrow$ relative phase $\Rightarrow$ interference $\Rightarrow$ asymmetry

- ✓ asymmetry is a good test for models
- ✓ experimental data on asymmetry and cross section are independent

Numerical results: Asymmetry



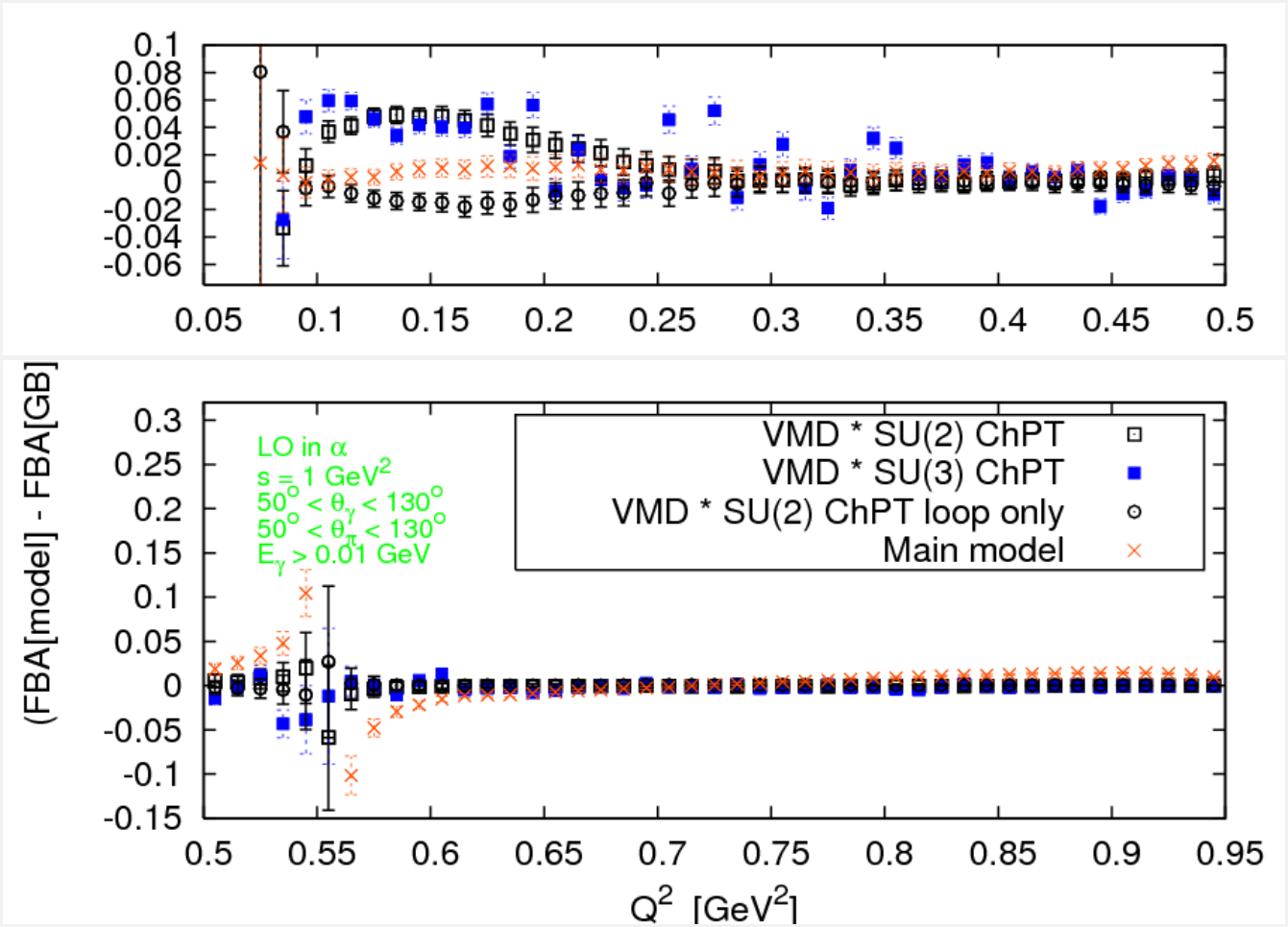
$e^+e^- \to \pi^+\pi^-\gamma$
$\sqrt{s} = 1$ GeV
$50^o < \theta_{\gamma,\pi} < 130^o$

✓ sQED*VMD

Forward-backward asymmetry

$$A_{FB}(Q^2) = \frac{N(\theta_{\pi^+} > 90^o) - N(\theta_{\pi^+} < 90^o)}{N(\theta_{\pi^+} > 90^o) + N(\theta_{\pi^+} < 90^o)}(Q^2)$$

Asymmetry: ChPT-corrections to sQED*VMD



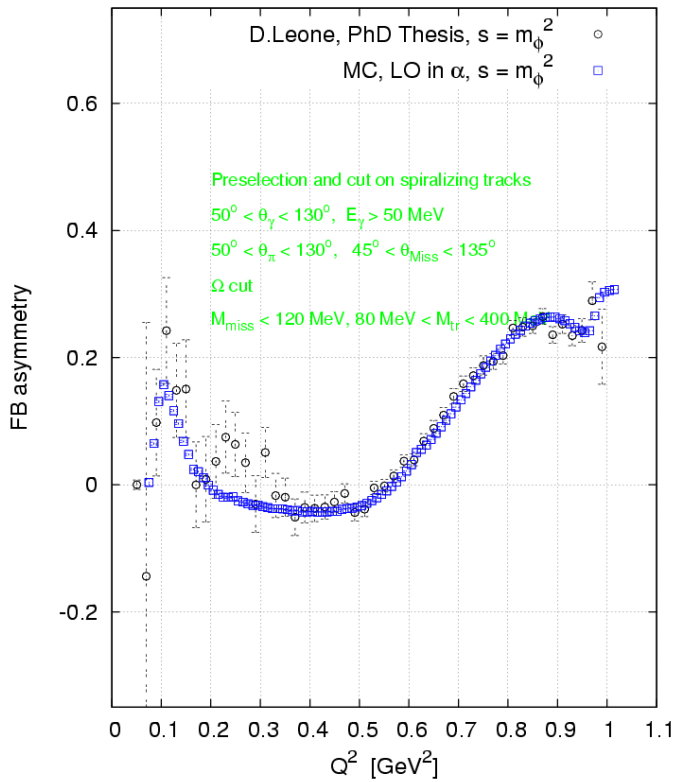
$e^+e^- \to \pi^+\pi^-\gamma$

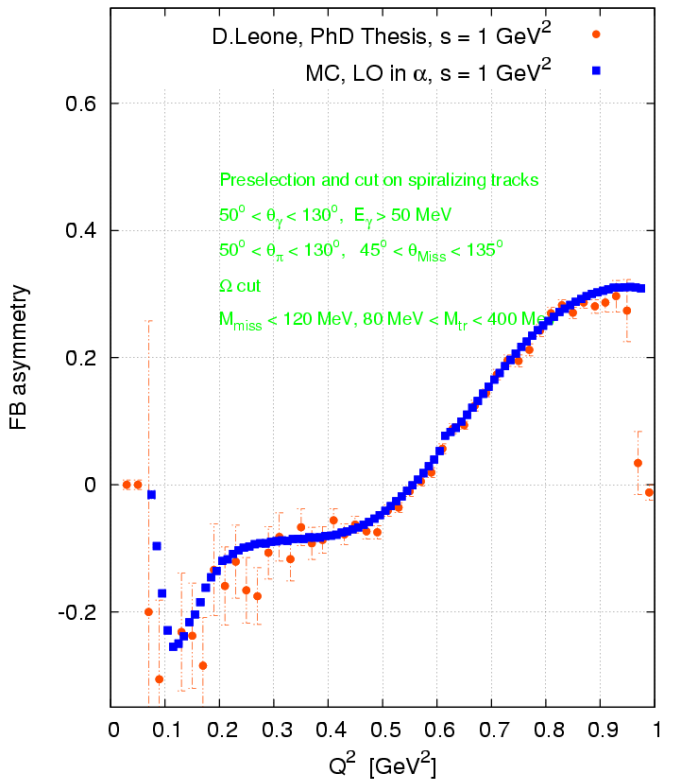
$\sqrt{s} = 1$ GeV

$50^o < \theta_{\gamma,\pi} < 130^o$

- ✓ high statistics is needed to see the difference
- small sensitivity to corrections

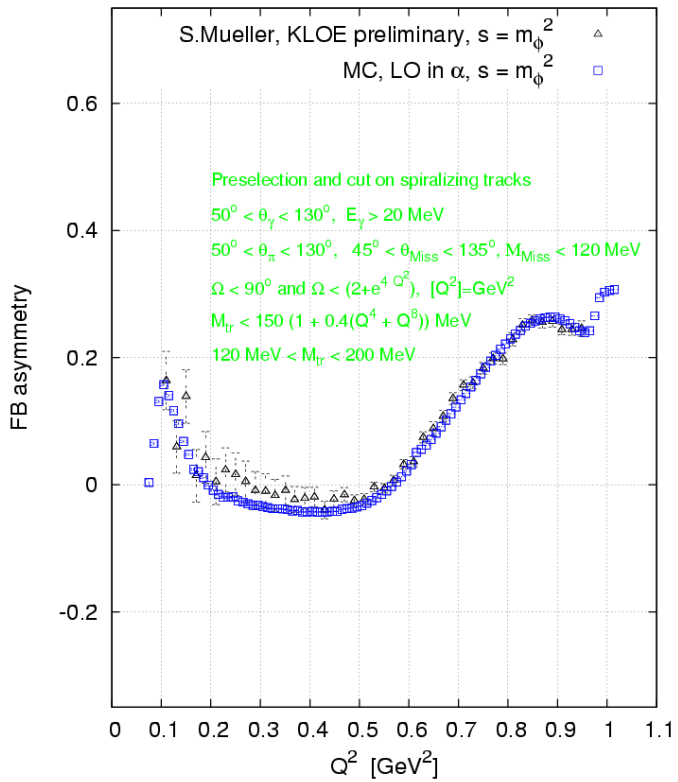
Asymmetry Vs. Preliminary data (D.Leone)

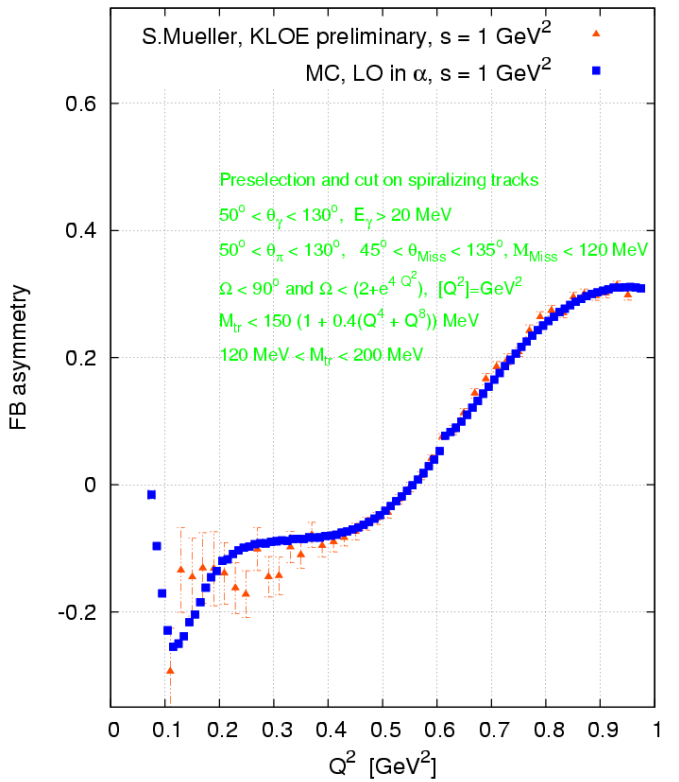




[Data: D.Leone, PhD Thesis, IEKP-KA/2004-20]

Asymmetry Vs. Preliminary data (S.Müller)





[Data: S.Müller, Presentation at “Phi to Psi 2009”, Beijing]



- ✓ Radiative return
- ✓ Experimental situation
- ✓ Formalism
- ✓ FSR tests
- ✓ Conclusions

Summary



High-statistics
Monte Carlo studies of
model-dependent effects in
$e^+e^- \to \pi^+\pi^-\gamma(\gamma)$

$> 3 \times 10^9$ MC evenst used

- ✓ FB asymmetry
- ✓ Cross section

- scalar QED * VMD
- "KLOE model"
- $SU(2)$ and $SU(3)$ Chiral Perturbation Theory

✓ Virtual Compton Scattering amplitudes can be used

A few conclusions

Cross section

- ✓ In general FSR is as much important as ISR
- ✓ Studied NLO contributions are not small

FB asymmetry

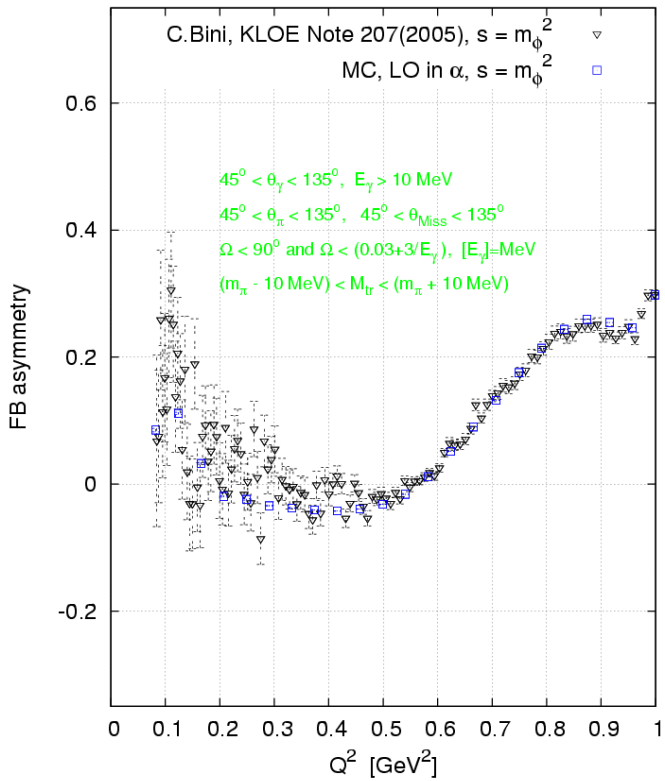
- ✓ small sensitivity to corrections at $\sqrt{s} = 1$ GeV

- ✓ Steps forward to estimate the theoretical ambiguity
- experimental study is in order

Spare parts



Asymmetry Vs. On-peak data (C.Bini)



[F.Ambrosino et al., Phys.Lett. B634 (2006) 148]

Numerical input

SU(2)

$F = 92.4\text{MeV}$

[Bijnens, Talavera, NP B489, 387 (1997)]

[Bijnens, arXiv:0904.3713 [hep-ph] (2009)]

$(\bar{l}_6 - \bar{l}_5) = 3.0 \pm 0.3$

SU(3)

[Bijnens, arXiv:0904.3713 [hep-ph] (2009)]

$F_0 = 87.7\text{MeV}$

[Unterdorfer, Pichl, EPJ C55, 273 (2008)]

$(L_9^r + L_{10}^r) = (1.32 \pm 0.14) \times 10^{-3}$ at $\mu = M_\rho$

ChPT loop functions

[Belucci, Gasser, Sainio, NP B423, 80 (1994); B431, 413(1994)]

[Unkmeir, Scherer, L'vov, Drechsel, PR D61, 034002 (1999)]

[. . . and others]

$$
\begin{aligned}
G_{\pi,K}(P^2,\ P\cdot k) &= 1 + \frac{m^2_{\pi,K}}{P\cdot k}\left(J^{(-1)}(a) - J^{(-1)}(b)\right) \\
&\quad - \frac{P^2}{2\ P\cdot k}\left(J^{(0)}(a) - J^{(0)}(b)\right), \\
a &\equiv P^2/m^2_{\pi,K}, \qquad b \equiv (P^2 - 2\ P\cdot k)/m^2_{\pi,K}. \\
J^{(0)}(x) &= -2 - \sigma \ln\frac{\sigma-1}{\sigma+1}, \\
J^{(-1)}(x) &= \frac{1}{2}\ln^2\frac{\sigma-1}{\sigma+1}, \\
\sigma(x) &\equiv \sqrt{1-4/x}.
\end{aligned}
$$

Implemented “Non-Born” corrections to FSR

SU(2): C. Unkmeir, S. Scherer, A. I. L’vov and D. Drechsel, Phys. Rev. D **61** (2000) 034002 [arXiv:hep-ph/9904442].

SU(3): T. Fuchs, B. Pasquini, C. Unkmeir and S. Scherer, Czech. J. Phys. **52** (2002) B135 [arXiv:hep-ph/0010218].

KLOE: model for KLOE, as implemented in PHOKHARA 6.1 and FASTERD

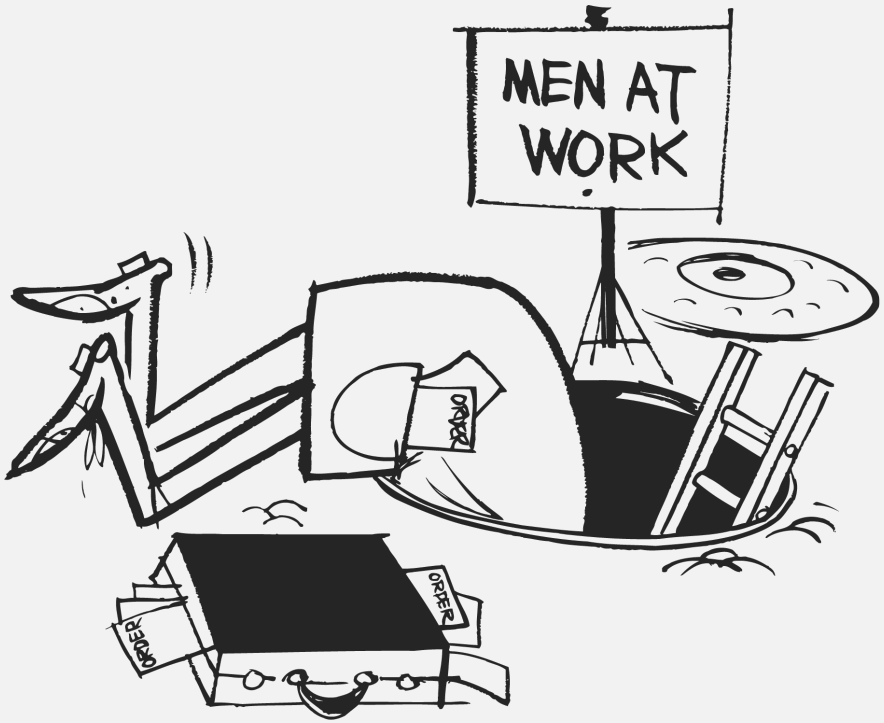
[PHOKHARA 6.1, URL: http://ific.uv.es/řodrigo/phokhara/]

[Shekhovtsova, Venanzoni, Pancheri, arXiv:0901.4440 [hep-ph] (2009)]

Meson factories

- DAPhNE (KLOE)
  $\sqrt{s} = m_\phi \approx 1.02$ GeV
- SLAC (BaBar)
  $\sqrt{s} = m_{\Upsilon(4S)} \approx 10.5$ GeV
- KEK (BELLE)
  $\sqrt{s} = m_{\Upsilon(4S)} \approx 10.5$ GeV

Work is in progress



Ongoing/planned

- axial-vector meson a_1 contribution to FSR
- cross-check with on-peak case
- cuts on track-mass, Ω angle, missing mass