Top quark mass reconstruction using the Globalχ^2 *algorithm*

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Introduction

Goals

- Top quark mass reconstruction in the semi-leptonic channel.
  - Production: $\sigma_{tt\ (LHC)}$ = (833±100) pb
  - Top decay (99.9%) through $t \longrightarrow W^+b$, where $t\bar{t} \longrightarrow W^+bW^-\bar{b}$
  - Final states depending on the W decay channel:
    - Fully-Hadronic (4/9): 6 jets (2x $W \longrightarrow jj$)
    - Fully-Leptonic (1/9): 2 leptons, 2 neutrinos and 2 jets (2x $W \longrightarrow l\nu$)
    - Semi-Leptonic (4/9): 1 lepton, 1 neutrino and 4 jets ($W \longrightarrow l\nu$ + $W \longrightarrow jj$)
  - Golden channel (lepton = e, μ): 2.5 10^6 events/year.
  - Reproduce the results from the CSC T9 note.
  - Introduce the GlobalChi2 method.

Data Sample

- Based on the official TopView ntuples (generated with v12.14.0.3)
- Simulation: ATLAS-CSC-01-02-00
  - New magnetic field description, misaligned geometry with material distortion.
- Reconstruction: ATLAS-CSC-01-00-00
  - New magnetic field description, perfect geometry with calibration constants.
- 146 ntuples with 3750 events/ntuple: 547.5 kevents
- Cross section:
  - Fully-Hadronic channel: $\sigma_{tt\ (LHC)} \simeq$ 367 pb
  - Fully-Leptonic channel: $\sigma_{tt\ (LHC)} \simeq$ 90 pb
  - Semi-Leptonic channel (l = e,μ): $\sigma_{tt\ (LHC)} \simeq$ 250 pb

Kinematic Fit

- Several analysis perform with this technique.
- Explicit reconstruction of the event topology (assuming a particle decay model).
- Typical chi2 function:

$$\chi^2 = \sum_{4jets+lepton} \left(\frac{E_i^m - E_i^{fit}}{\sigma_{E_i}}\right)^2 + \left(\frac{M_{jj} - M_W^{PDG}}{\Gamma_W^{PDG}}\right)^2 + \left(\frac{M_{l\nu} - M_W^{PDG}}{\Gamma_W^{PDG}}\right)^2 + \left(\frac{M_{jjb_H} - M_{top_H}^{fit}}{\sigma_{top_H}}\right)^2 + \left(\frac{M_{l\nu b_L} - M_{top_L}^{fit}}{\sigma_{top_L}}\right)^2$$

- Parameters: E_i^{fit}, M_{top}
- All the parameters fitted altogether with kinematic constraints, for example $M_{jj} \rightarrow M_W(PDG)$.
  - When all the parameters (leptonics and hadronics) are fitted together, the fit is called "*global*".
- The data calibration is done *in-situ*: $E_i^{fit} = \alpha_i E_i^{reco}$.
  - This allows to study the Jet Energy Scale (JES), denoted as α_i, for light quarks and b quarks separately.
- Using MINUIT: a blackbox tool for finding (numerically) the minimum value of a multi-parameter function, i.e. a chi2 function.

Globalχ^2

- Analytic fit instead of numerical.
- Idea: apply all the alignment knowledge as they are similarities. In both cases a nested minimization can be introduce:
  - Alignment case:
    1. Chi2 minimization wrt he track parameters (local variables)
       - Track parameters depend linearly on the alignment parameters (not viceversa).
    2. Then, chi2 minimization wrt the alignment parameters (global variables).
  - Top quak mass case:
    1. Chi2 minimization wrt the W parameters.
       1. W parameters depend linearly with the top parameters. We assume that the W params depend on the top quark params.
    2. Then, chi2 minimization wrt the top parameters.

GlobalChi2

In the GlobalChi2, we consider a basic chi2 term and the other terms are (kinematic) constraints. This is just notation:

$$\chi^2 = \sum_{4jets+lepton} \left(\frac{E_i^m - E_i^{fit}}{\sigma_{E_i}} \right)^2 + \text{constraints} \xrightarrow{\textit{Matrix notation}} \chi^2 = r^T V^{-1} r + \text{constraints}$$

Our residuals, denoted by **r**, are:

$$\mathbf{r} = \begin{pmatrix} E_{j1}^{reco} - E_{j1}^{fit} \\ E_{j2}^{reco} - E_{j2}^{fit} \\ E_{b_{had}}^{reco} - E_{b_{had}}^{fit} \\ E_{b_{lep}}^{reco} - E_{b_{lep}}^{fit} \\ E_{l}^{reco} - E_{l}^{fit} \end{pmatrix} = \begin{pmatrix} E_{j1}^{reco} - \alpha_{j1} E_{j1}^{reco} \\ E_{j2}^{reco} - \alpha_{j2} E_{j2}^{reco} \\ E_{b_{had}}^{reco} - \alpha_{b_{had}} E_{b_{had}}^{reco} \\ E_{b_{lep}}^{reco} - \alpha_{b_{lep}} E_{b_{lep}}^{reco} \\ E_{l}^{reco} - \alpha_{l} E_{l}^{reco} \end{pmatrix}$$

The covariance matrix, denoted by V^{-1}, is:

$$V^{-1} = \begin{pmatrix} 1/\sigma_{E_{j1}}^2 & 0 & 0 & 0 & 0 \\ 0 & 1/\sigma_{E_{j2}}^2 & 0 & 0 & 0 \\ 0 & 0 & 1/\sigma_{E_{b_{had}}}^2 & 0 & 0 \\ 0 & 0 & 0 & 1/\sigma_{E_{b_{lep}}}^2 & 0 \\ 0 & 0 & 0 & 0 & 1/\sigma_{E_l}^2 \end{pmatrix}$$

The residuals depend on the W parameters and on the top parameters: $\mathbf{r} = \mathbf{r}(\boldsymbol{\pi}_{W_{Had}}, \boldsymbol{\pi}_{top})$

The GlobalChi2 is based on a chi2 function minimization wrt the top parameters, so the minimum condition has to be apply:

$$\frac{d\chi^2}{d\boldsymbol{\pi}_{top}} = 0 \longrightarrow \left(\frac{d\mathbf{r}}{d\boldsymbol{\pi}_{top}} \right)^T V^{-1} r \; + \text{constraints} \; = 0$$

Therefore, we need to know the differential: $d\mathbf{r} = \dfrac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{W_{Had}}} d\boldsymbol{\pi}_{W_{Had}} + \dfrac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}} d\boldsymbol{\pi}_{top}$

GlobalChi2

And the derivatives:

$$\frac{d\mathbf{r}}{d\boldsymbol{\pi}_{top}} = \frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{W_{Had}}}\frac{d\boldsymbol{\pi}_{W_{Had}}}{d\boldsymbol{\pi}_{top}} + \frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{top}}$$

$$\frac{d\mathbf{r}}{d\boldsymbol{\pi}_{W_{Had}}} = \frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{W_{Had}}} \quad \textit{where} \quad \frac{d\boldsymbol{\pi}_{top}}{d\boldsymbol{\pi}_{W_{Had}}} = 0$$

Assumption:

- The W parameters depend on the top parameters (Ws come from the top decays).

- The top parameters do not depend on the W parameters as the top mother particle and it is fixed (at simulation level the only particle with a fixed mass is the top).

Then:

$$\left(\frac{d\mathbf{r}}{d\boldsymbol{\pi}_{top}}\right)^T V^{-1}r + \text{constraints} = \left(\frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{W_{Had}}}\frac{d\boldsymbol{\pi}_{W_{Had}}}{d\boldsymbol{\pi}_{top}} + \frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{top}}\right)^T V^{-1}r + \text{constraints} = 0$$

The point now is to calculate $\frac{d\boldsymbol{\pi}_{top}}{d\boldsymbol{\pi}_{W_{Had}}}$. To do it, we need to minimize the chi2 function wrt the W parameters:

$$\frac{\partial\chi^2}{\partial\boldsymbol{\pi}_{W_{Had}}} = 0 \longrightarrow \left(\frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{W_{Had}}}\right)^T V^{-1}r + \text{constraints} = 0$$

where $E \equiv \left.\frac{\partial\mathbf{r}}{\partial\boldsymbol{\pi}_{W_{Had}}}\right|_{\boldsymbol{\pi}_{WHad_0}}$

After some matrix manipulation, we obtain:

$$\boldsymbol{\pi}_{W_{Had}} = \boldsymbol{\pi}_{W_{Had_0}} + \delta\boldsymbol{\pi}_{W_{Had}} \longrightarrow \frac{d\boldsymbol{\pi}_{W_{Had}}}{d\boldsymbol{\pi}_{top}} = -(E^TV^{-1}E + M_{\text{W constraints}})^{-1}E^TV^{-1}\frac{\partial r}{\partial\boldsymbol{\pi}_{top}}$$

GlobalChi2 basis

Then we have to introduce $\frac{d\boldsymbol{\pi}_{top}}{d\boldsymbol{\pi}_{W_{Had}}}$ in $\left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{W_{Had}}}\frac{d\boldsymbol{\pi}_{W_{Had}}}{d\boldsymbol{\pi}_{top}} + \frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}}\right)^T V^{-1} r \; + \text{constraints} \; = 0$

Now, after some matrix manipulation....

$$\left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}}\right)^T (I - X_W)\mathbf{r} + \text{Term}_{\text{top constr}} + \left[\left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}}\right)^T (I - X_W)\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}} + \text{Term}_{\text{top constr}}\right]\delta\boldsymbol{\pi}_{top} = 0$$

where: $X \quad \equiv (E \;\; V^{-1}E + M_{[illegible]})^{-1}E \;\; V^{-1}$

Finally:

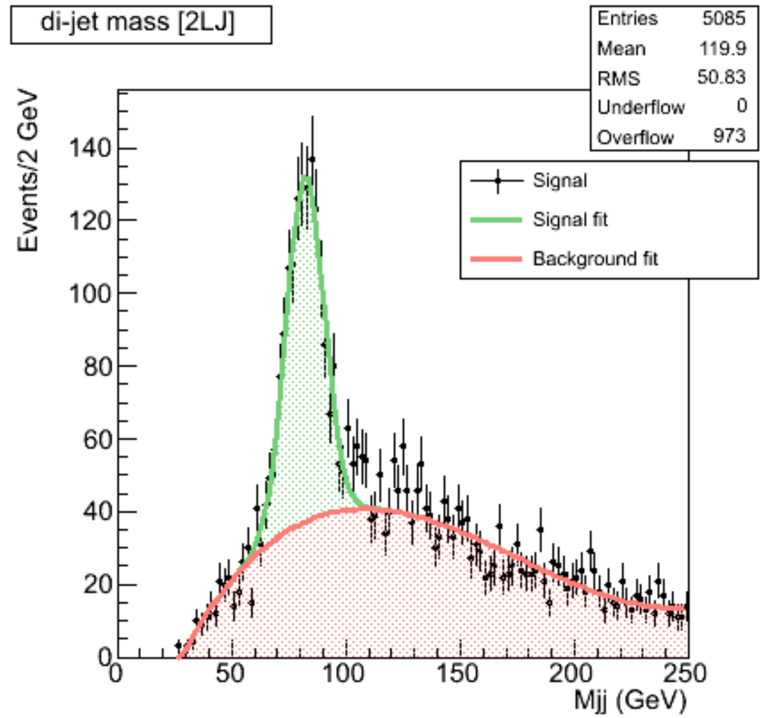
$$\delta\boldsymbol{\pi}_{top} = \underbrace{\left[\left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}}\right)^T (I - X_W)\mathbf{r} + \text{Term}_{\text{top constr}}\right]^{-1}}_{\mathcal{M}} \cdot \underbrace{\left[\left(\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}}\right)^T (I - X_W)\frac{\partial \mathbf{r}}{\partial \boldsymbol{\pi}_{top}} + \text{Term}_{\text{top constr}}\right]}_{\nu}$$

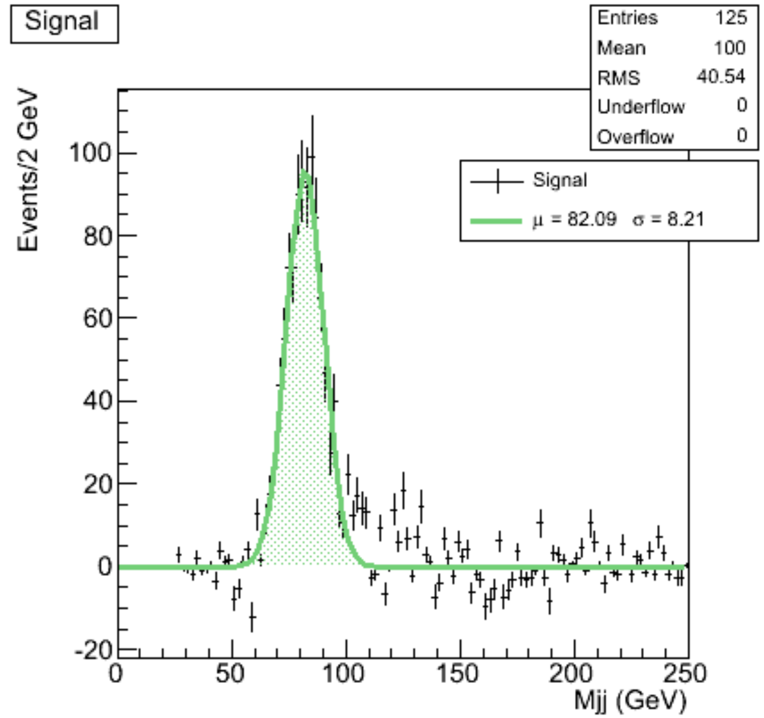
So, the final top parameters are:

$$\boxed{\boldsymbol{\pi}_{top} = \boldsymbol{\pi}_{top_0} + \delta\boldsymbol{\pi}_{top} = \boldsymbol{\pi}_{top_0} - \mathcal{M}^{-1}\nu}$$

Preliminary results

The invariant mass of the two light jets is used to select right events.





See the monitor plots....

Outline

Near Future

- Tune the algorithm
- Run over background data samples.
- Try the method with different data samples where the generated top quark mass is not 175 GeV:
  - trig1_misal1_csc11.005573.AcerMCttbar160.recon.AOD.v12000601
  - trig1_misal1_csc11.005574.AcerMCttbar165.recon.AOD.v12000601
  - trig1_misal1_csc11.005575.AcerMCttbar170.recon.AOD.v12000601
  - trig1_misal1_csc11.005577.AcerMCttbar180.recon.AOD.v12000601
  - trig1_misal1_csc11.005578.AcerMCttbar185.recon.AOD.v12000601
  - trig1_misal1_csc11.005579.AcerMCttbar190.recon.AOD.v12000601

In the next talk.....