Automation of NLO calculations

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   - Obtaining the numerator in an automatic way

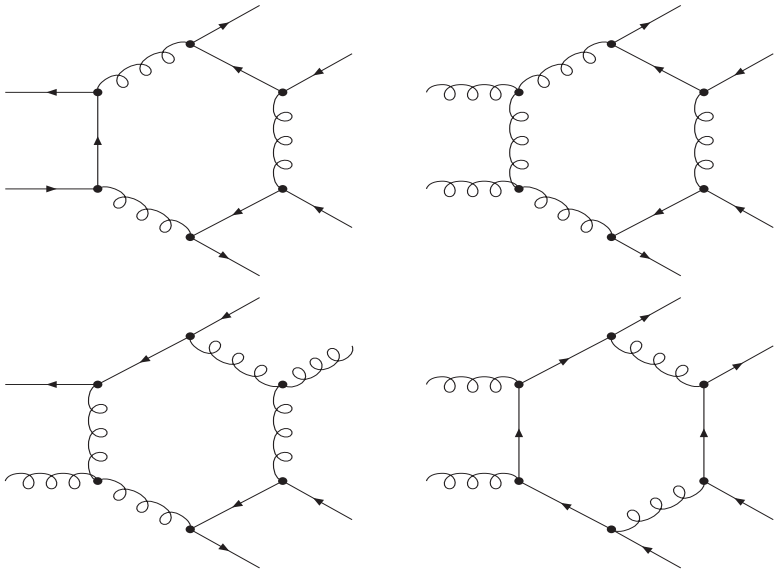
The processes of interest for LHC

NLO Process	relevant to
$VV +$ jet	$t\bar{t}H$
$H + 2$ jets	H production by VBF
$t\bar{t}b\bar{b}$	$t\bar{t}H$
$b\bar{b}b\bar{b}$	$t\bar{t}H$
$VV + 2$ jets	VBF $\rightarrow H \rightarrow VV$
$VV + 3$ jets	various new physics
VVV	SUSY trilepton

Huge number of diagrams

Take for example the process $pp \to t\bar{t}b\bar{b}$ at NLO

- Channel $q\bar{q} \to t\bar{t}b\bar{b}$ has ~ 250 graphs
- Channel $gg \to t\bar{t}b\bar{b}$ has ~ 1000 graphs!



The structure of a NLO amplitude

In 4 dimensions NLO amplitudes are usually divergent so in general we work in n dimensions

$$A^{NLO} = \int d^n\bar{q} \frac{\bar{N}(\bar{q})}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}}$$

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

$\bar{N}(\bar{q})$ is made of products of the external momenta, the loop momentum, fermion wavefunctions and polarization vectors and is a tensor in general.
All the singularities appear as poles in $\varepsilon = n - 4$:

$$\frac{1}{\varepsilon^2}, \ \frac{1}{\varepsilon}, \ \varepsilon^0$$

The structure of a NLO amplitude

Every amplitude can be written in terms of four scalar basis integrals, the Box, the Triangle, the Bubble and the Tadpole (massive case only), plus a remainder:

$$A^{NLO} = \sum_i d_i Box_i + \sum_i c_i Tri_i + \sum_i b_i Bub_i + \sum_i a_i Tad_i + R$$

- all the singularities are contained in the basis integrals
- the coeffs d, c, b, a depend on the kinematics
- the rational term R is finite and is left over from the reduction from n to 4 dimensions

Reducing the amplitude to the base

There are several techniques for the reduction:

- Passarino-Veltman tensor reduction
- Unitarity-based methods (Britto, Feng, Mastrolia,..)
- OPP reduction (Ossola,Papadopoulos,Pittau)

Splitting in $n = 4 + \epsilon$ dimensions

The start of the *OPP* method is to separate the 4-dim from the n-dim numerator. We do this as follows:

$$\begin{aligned} \bar{q} &= q + \tilde{q}\,, \\ \bar{D} &= D + \tilde{q}^2\,, \\ \bar{\gamma}_{\bar{\mu}} &= \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}}\,, \\ \bar{g}^{\bar{\mu}\bar{\nu}} &= g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}} \end{aligned}$$

Contraction with a 4-dim vector selects only the 4-dim part:

$$\bar{q} \cdot v = q \cdot v \quad \text{and} \quad \bar{\not{v}} = \not{v}$$

Splitting in $n = 4 + \epsilon$ dimensions

We have two pieces for the numerators

$$\underbrace{\bar{N}(\bar{q})}_{n-dim} = \underbrace{N(q)}_{4-dim} + \underbrace{\tilde{N}(\tilde{q}^2, q, \epsilon)}_{\epsilon-dim}$$

and we split the amplitude in two pieces as well:

$$A^{NLO} = \int d^n\bar{q} \frac{N(q)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} + \int d^n\bar{q} \frac{\tilde{N}(\tilde{q}^2, q, \epsilon)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}}$$

The first integral will give us the coeffs in the scalar basis and part of the rational part, R_1. The second integral will give us the remainder of the rational part.

The 4 dimensional part

We start by writing the numerator in terms of the 4 dimensional denominators:

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \bar{d}(i_0 i_1 i_2 i_3) \prod_{k\neq i_0\neq i_1\neq i_2\neq i_3}^{m-1} D_k \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \bar{c}(i_0 i_1 i_2) \prod_{k\neq i_0\neq i_1\neq i_2}^{m-1} D_k \\
&+ \sum_{i_0<i_1}^{m-1} \bar{b}(i_0 i_1) \prod_{k\neq i_0\neq i_1}^{m-1} D_k + \sum_{i_0}^{m-1} \bar{a}(i_0) \prod_{k\neq i_0}^{m-1} D_k
\end{aligned}
$$

The 4 dimensional part

Now, we can obtain the coeffs numerically, by putting $4, 3, 2$ or 1 propagator on-shell (equivalent to unitary cuts) and solving the (complex) systems of equations:

Box coeff $D_i = D_j = D_k = D_l = 0 \rightarrow d_{ijkl}$

Triangle coeff $D_i = D_j = D_k = 0 \rightarrow c_{ijk}$

Bubble coeff $D_i = D_j = 0 \rightarrow b_{ij}$

Tadpole coeff $D_i = 0 \rightarrow a_i$

BE CAREFUL THOUGH!

There is a mismatch from $\bar{D} \rightarrow D$. This gives the R_1 part of R

The 4 dimensional part: Example

The integral to be computed is:

$$\int d^n\bar{q}\,\frac{\bar{N}(\bar{q})}{\bar{D}_0\bar{D}_1\bar{D}_2\bar{D}_3\bar{D}_4}$$

with:

$$\begin{aligned}\bar{D}_0 &= q^2-m^2\ ,\ \bar{D}_1=(q+p_1)^2-m^2\ ,\ \bar{D}_2=(q+p_1+p_2)^2-m^2\\ \bar{D}_3 &= (q+p_1+p_2+p_3)^2-m^2\ ,\ \bar{D}_4=(q+p_1+p_2+p_3+p_4)^2-m^2\end{aligned}$$

We expand the numerator

$$\begin{aligned}N(q) &= \bar{d}_{0123}D_4+\bar{d}_{0124}D_3+\cdots+\bar{d}_{1234}D_0\\ &+ \bar{c}_{012}D_3D_4+\bar{c}_{013}D_2D_4+\cdots+\bar{c}_{234}D_0D_1\\ &+ \bar{b}_{01}D_2D_3D_4+\bar{b}_{02}D_1D_3D_4+\cdots+\bar{b}_{34}D_0D_1D_2\\ &+ \bar{a}_0D_1D_2D_3D_4+\cdots+\bar{a}_4D_0D_1D_2D_3\end{aligned}$$

The 4 dimensional part: Example

To determine the box coefficients we put

$$D_0 = D_1 = D_2 = D_3 = 0 \rightarrow q^{\pm}$$

and solve a system of equations

$$\begin{aligned} N(q^+) &= \left(d_{0123} + \tilde{d}_{0123}(q^+)\right) D_4(q^+) \\ N(q^-) &= \left(d_{0123} + \tilde{d}_{0123}(q^-)\right) D_4(q^-) \end{aligned}$$

For the Triangle coeffs, similar

$$N'(q) \equiv N(q) - \sum \bar{d} D = \bar{c} DD + \cdots + \bar{b} DDD + \cdots + \bar{a} DDDD$$

with a system to solve:

$$D_0 = D_1 = D_2 = 0 \rightarrow q_i$$

PD, G. Ossolla, C. Papadopoulos, R. Pittau

The 4 dimensional part: Example

For the bubble coeff:

$$N''(q) \equiv N(q) - \sum \bar{d} D - \sum \bar{c} DD = \bar{b} DDD + \cdots + \bar{a} DDDD$$

$$D_0 = D_1 = 0 \rightarrow q_i$$

For the R_1 part of the rational terms we re-establish the dimensionality

$$m_i^2 \rightarrow m_i^2 - \tilde{q}^2$$

and we have to determine 3 more coeffs

$$\begin{aligned} R_1 &= -\frac{i}{96\pi^2} d^{(2m-4)} - \frac{i}{32\pi^2} \sum_{i_0<i_1<i_2}^{m-1} c^{(2)}(i_0 i_1 i_2) \\ &- \frac{i}{32\pi^2} \sum_{i_0<i_1}^{m-1} b^{(2)}(i_0 i_1) \left(m_{i_0}^2 + m_{i_1}^2 - \frac{(p_{i_0} - p_{i_1})^2}{3} \right) \end{aligned}$$

The ϵ dimensional part

The $n - 4$ part of the amplitude is given by:

$$R_2 \equiv \frac{1}{(2\pi)^4} \int d^n \bar{q} \frac{\tilde{N}(\tilde{q}^2, q, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

The whole of the rational part is now $R = R_1 + R_2$.

- A practical way of obtaining the R_2 is to compute, once and for all, tree-level like Feynman Rules
- Only up to four legs are needed, due to the UV nature of the amplitude
- Numerically, there is no additional computing cost because this is a LO calculation

PD, M. Garzelli, C. Papadopoulos, R. Pittau

The ϵ dimensional part

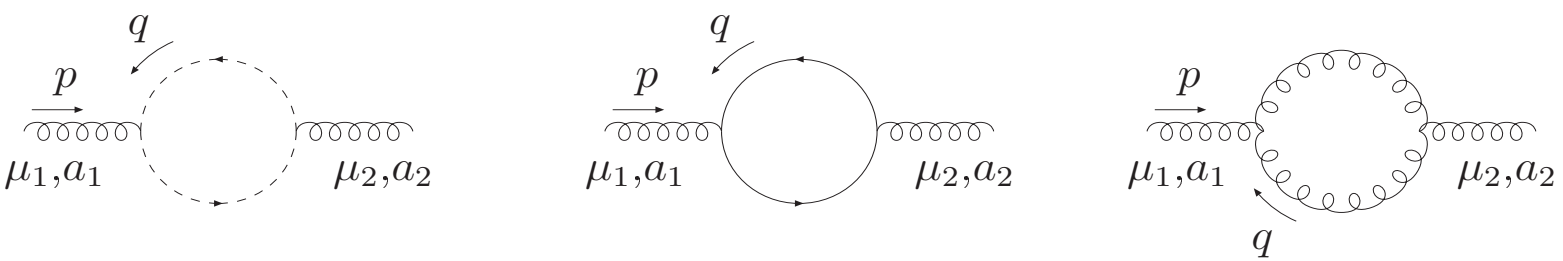
As an example consider the gluon self-energy:

$$\bar{N}(\bar{q}) = -\frac{g^2}{(2\pi)^4} N_q \frac{\delta_{a_1 a_2}}{2} \mathrm{Tr}[\gamma^{\mu_1}(\bar{\not{q}} + m_q)\gamma^{\mu_2}(\bar{\not{q}} + \not{p} + m_q)]$$

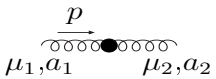
$$\longrightarrow \underbrace{\tilde{N}(\tilde{q}^2) = \frac{g^2}{8\pi^4} N_q \, \delta_{a_1 a_2} \, g_{\mu_1 \mu_2} \tilde{q}^2}_{\epsilon-\mathrm{dim}}$$

Using the integral:

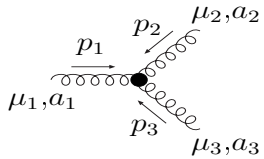
$$\int d^n \bar{q} \frac{\tilde{q}^2}{D_i D_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{p_{ij}^2}{3}\right] + O(\epsilon)$$



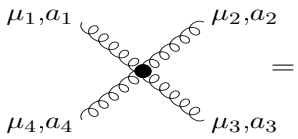
The ϵ dimensional part



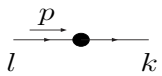
$$= \frac{ig^2 N_{col}}{48\pi^2}\,\delta_{a_1a_2}\left[\frac{p^2}{2}g_{\mu_1\mu_2} + \lambda_{HV}\left(g_{\mu_1\mu_2}p^2 - p_{\mu_1}p_{\mu_2}\right) + \frac{N_q}{N_{col}}\left(p^2 - 6\,m_q^2\right)g_{\mu_1\mu_2}\right]$$



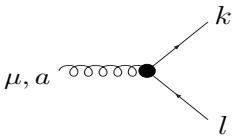
$$= -\frac{g^3 N_{col}}{48\pi^2}\left(\frac{7}{4} + \lambda_{HV} + 2\frac{N_q}{N_{col}}\right) f^{a_1a_2a_3}\,V_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)$$



$$= -\frac{ig^4 N_{col}}{96\pi^2}\sum_{P(234)}\left\{\left[\frac{\delta_{a_1a_2}\delta_{a_3a_4} + \delta_{a_1a_3}\delta_{a_4a_2} + \delta_{a_1a_4}\delta_{a_2a_3}}{N_{col}} + 4\,Tr(t^{a_1}t^{a_3}t^{a_2}t^{a_4} + t^{a_1}t^{a_4}t^{a_2}t^{a_3})\,(3+\lambda_{HV}) - Tr(\{t^{a_1}t^{a_2}\}\{t^{a_3}t^{a_4}\})\,(5+2\lambda_{HV})\right]g_{\mu_1\mu_2}g_{\mu_3\mu_4} + 12\frac{N_q}{N_{col}}Tr(t^{a_1}t^{a_2}t^{a_3}t^{a_4})\left(\frac{5}{3}g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_2\mu_3}g_{\mu_1\mu_4}\right)\right\}$$



$$= \frac{ig^2}{16\pi^2}\frac{N_{col}^2 - 1}{2N_{col}}\delta_{kl}(-\not{p} + 2m_q)\,\lambda_{HV}$$



$$= \frac{ig^3}{16\pi^2}\frac{N_{col}^2 - 1}{2N_{col}}\,t^a_{kl}\gamma_\mu\,(1+\lambda_{HV})$$

Obtaining the numerator in an automatic way

- Automation of calculating cross sections has been going on since 1989.
- For LO: HELAC (PD, Papadopoulos, Kleiss, Worek), ALPGEN (Pittau,Mangano,..), SHERPA(Gleisberg,Krauss,..) to name but a few
- For NLO: HELAC@NLO (Papadopoulos,Pittau,Worek,..) , MCFM (Ellis), BLACKHAT(Kossower,Bern,Dixon,..), ROCKET(Giele, Zanderighi)

Obtaining the numerator in an automatic way

HELAC@NLO

Is based on a recursion solution of the Schwinger-Dyson equations. The basic steps are:

- Cut one line, compute the tree level amplitude and reconnect the line, in all posssible ways, thus obtaining the numerator
- Pass the numerator through OPP to obtain the coeffs of the reduction and the rational part
- Add renormalization
- Compute the real corrections to cancel IR singularities

