Conformal neutrinos: a 4D alternative to the See-saw

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Outline
Introduction
Dirac Neutrinos
Majorana Neutrinos
Conclusion

Outline

The outline of this talk is

Outline

- Introduction: neutrino masses from "conformal sequestering"
- Dirac neutrinos
  - Bounds on FCNC: $\mu \longrightarrow e\gamma$
  - Invisible Higgs decay $h \longrightarrow \nu\nu$
- Majorana neutrinos
- Conclusion

Introduction

- It is by now a well established fact that neutrinos have non-zero masses
- The most important theoretical implication is that there exist right-handed neutrinos fields as direct mass term for left-handed neutrinos are forbidden in the SM

The See-saw mechanism

- It is possible to get Dirac mass terms after EWSB: $h_Y \bar{\ell}_L H \nu_R \Longrightarrow m_\nu^D = h_Y v$
- One can also allow for Majorana mass terms for right-handed neutrinos: $m_{\nu_R}^M \nu_R \nu_R$

$$m_\nu \simeq \frac{(m_\nu^D)^2}{m_{\nu_R}^M} .$$

See-saw implies either physics at an inaccessible energy or unnaturally small Yukawa couplings!

- An alternative 4D solution happens if the right-handed neutrino $\mathcal{N}_R$ belongs to a conformal theory with a fixed point at the scale Λ: unparticle
- If the theory is strongly coupled the field $\mathcal{N}_R$ may acquire a large anomalous dimension γ with propagator

Unparticle propagator

$$\Delta(p,\gamma) = -iB_\gamma \bar{\sigma}^\mu p_\mu (-p^2 - i\epsilon)^{-1+\gamma}$$

$$B_\gamma = \frac{8\pi^{3/2}}{(2\pi)^{2+2\gamma}} \frac{\Gamma(1-\gamma)\Gamma(3/2+\gamma)}{\Gamma(2+2\gamma)}$$

- The renormalizable operator with the Standard Model fields

$\Lambda^{-\gamma} \bar{\ell}_L H \mathcal{N}_R$ becomes irrelevant for $\gamma > 0$

- The neutrino Majorana mass

$\Lambda^{1-2\gamma} \mathcal{N}_R \mathcal{N}_R$ also becomes irrelevant if $\gamma > 1/2$

Dirac Neutrinos

- If the conformal theory conserves lepton number the effective Lagrangian at the scale Λ is given by

$$\mathcal{L}(\Lambda) = \Lambda^{-\gamma}\, \bar{\ell}_L H \mathcal{N}_R + h.c.$$

- The fact that the operators are sequestered by the conformal dynamics for scales $\mu < \Lambda$ can be made explicit by redefining $\mathcal{N}_R$ in terms of fields ν_R with canonical dimension as

$$\mathcal{N}_R = B_\gamma^{1/2}\, \mu^\gamma\, \nu_R\,.$$

- For scales $\mu < \Lambda$ one can write the effective Lagrangian

$$\mathcal{L}(\mu) = B_\gamma^{1/2} \left(\frac{\mu}{\Lambda}\right)^\gamma \bar{\ell}_L H \nu_R + h.c.$$

- When $\langle H \rangle = v/\sqrt{2}$ the Dirac mass term triggers the

Conformal breaking

Electroweak breaking is responsible for the breaking of the conformal symmetry

Dirac mass

- Conformal dynamics will still be governing the right-handed neutrino sector until the mass becomes of the order of the renormalization group scale
- This happens when

$$\mu_c = B_\gamma^{1/2} \frac{v}{\sqrt{2}} \left(\frac{\mu_c}{\Lambda}\right)^\gamma \Longrightarrow m_\nu^D = \mu_c = \Lambda \left(\frac{B_\gamma^{1/2} v}{\Lambda\sqrt{2}}\right)^{\frac{1}{1-\gamma}}$$

$\gamma(\Lambda)$ for fixed m_ν^D

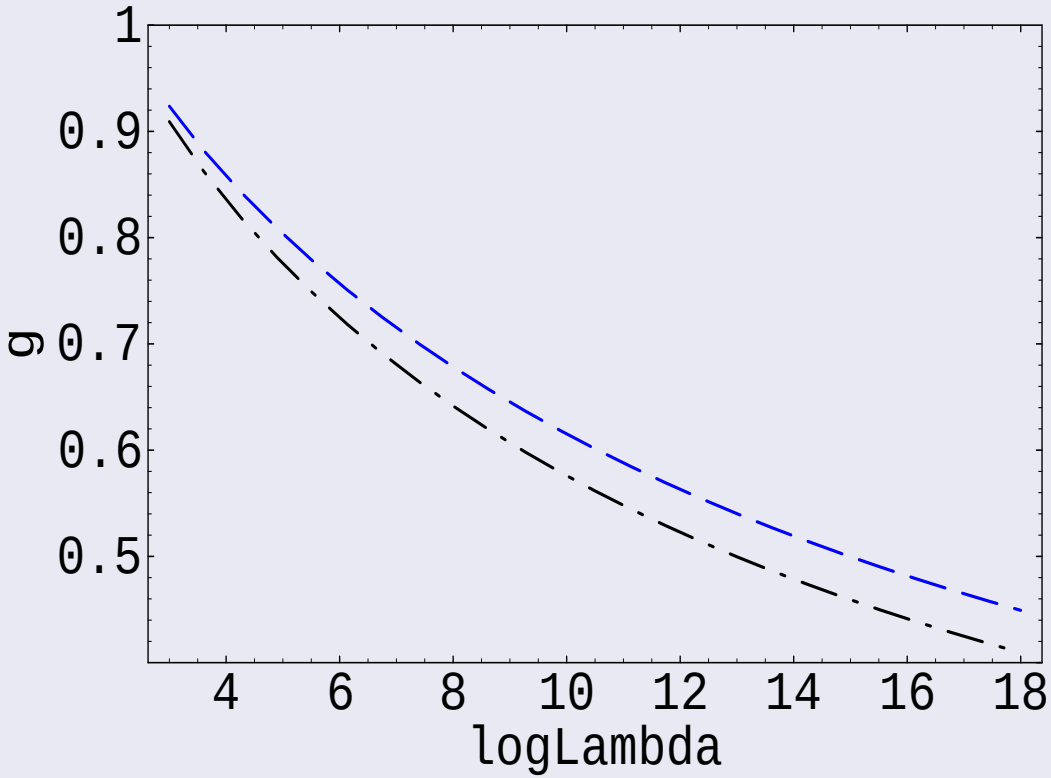


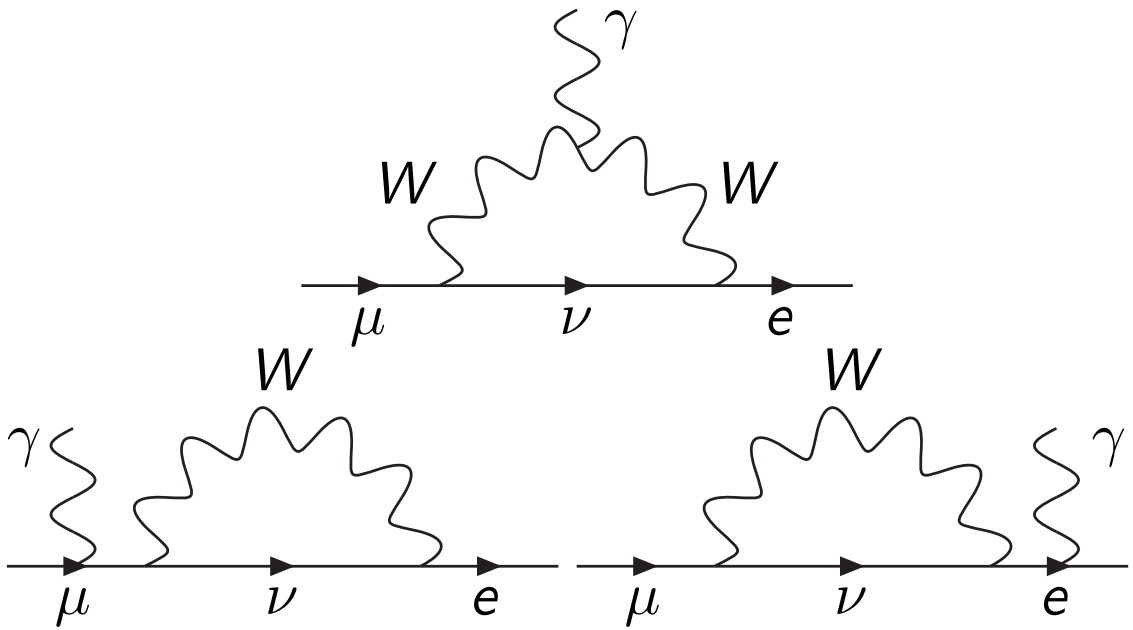
Figure: The anomalous dimension γ of the right-handed neutrino as functions of $\log_{10}(\Lambda/GeV)$. The two lines correspond to $m_\nu^D = 0.01$ eV (dashed blue) and 1 eV (dash-dotted black)

Three neutrino flavors: normal hierarchy

- The first possibility is having all the three right-handed neutrinos with identical anomalous dimension
- The ratio of the two heavier masses is close to unity, and the small splitting might be accounted for by SM corrections to the Yukawa couplings
- The same holds true if the lightest neutrino mass is of the same order
- A much lighter state can also be naturally achieved if
  - The light state has a different anomalous dimension γ_1
  - The lightest state has a smaller Yukawa coupling
- The solution depends on the theory (conformal theory vs. weakly coupled theory) below μ_c
- The last (obvious) possibility is that different neutrinos ν_i belong to different conformal theories at the scale Λ_i and develop different anomalous dimensions γ_i

FCNC: $\mu \to e\gamma$

- There are strong experimental bounds on decay channels such as $\mu \to e\gamma$ and $\mu \to 3e$
- One needs to evaluate the diagrams



- The leading contribution comes from two Dirac mass insertions and one right-handed neutrino (unparticle) propagator

$\mu \longrightarrow e\gamma$

- Normalizing to the main channel $\mu \longrightarrow e\nu\nu$

γ-dependence: unparticle effect

$$B(\mu \longrightarrow e\gamma) = \frac{3}{32}\frac{\alpha}{\pi}\left|\frac{\pi\gamma}{\sin(\pi\gamma)}\sum_i U_{ei}U^*_{\mu i}\left(\frac{m_i}{M_W}\right)^{2-2\gamma}\right|^2$$

- The result for the SM with massive Dirac neutrinos (the limit $\gamma \longrightarrow 0$) is many orders of magnitudes below the experimental bound $B(\mu \longrightarrow e\gamma) < 1.2 \times 10^{-11}$
- Due to its exponential dependence on the anomalous dimension $B(\mu \longrightarrow e\gamma)$ can reach this bound if γ becomes close enough to one
- E.g. the existing bound implies that $\gamma \lesssim 0.86$ for the case of a regular hierarchy with $m_1 \ll m_2, m_3$
- Future experiments will improve the sensitivity to $\sim 10^{-14}$ which $\Longrightarrow \gamma \lesssim 0.81$

$\log_{10} B(\mu \longrightarrow e\gamma)$ for fixed m_ν^D

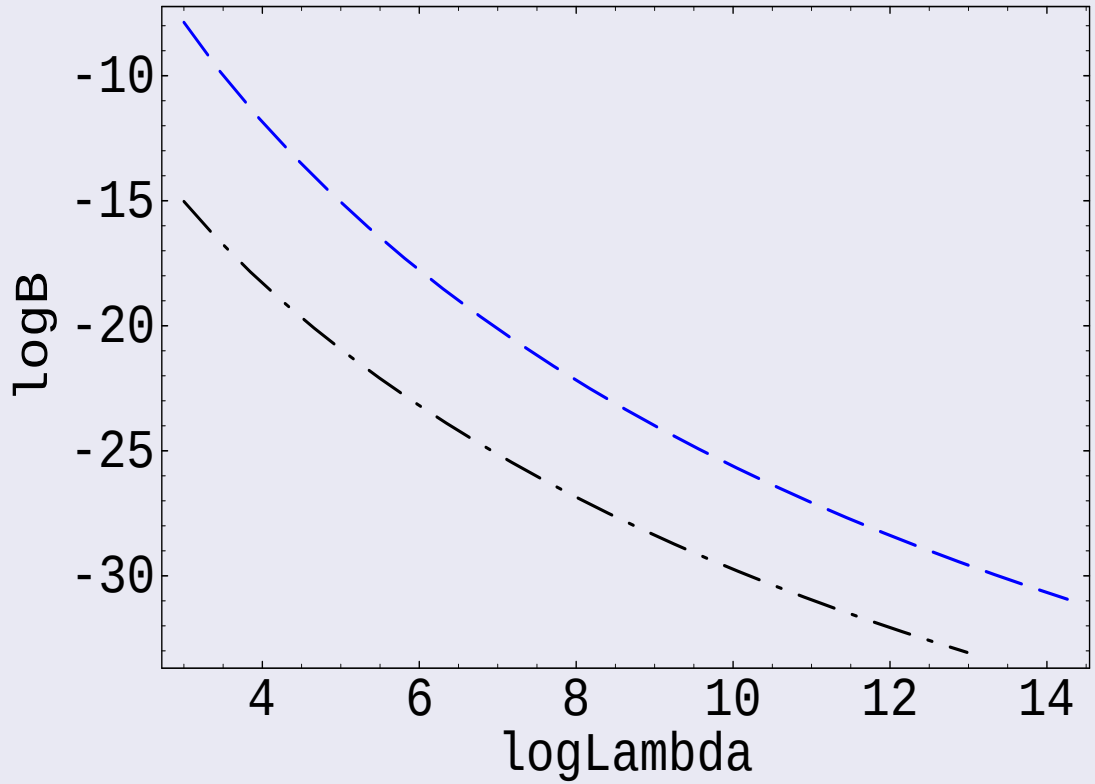


Figure: The branching ratio as a function of $\log_{10}(\Lambda/GeV)$. The two lines correspond to $m_3 = 0.05$ eV (dashed blue) and 1 eV (dash-dotted black)

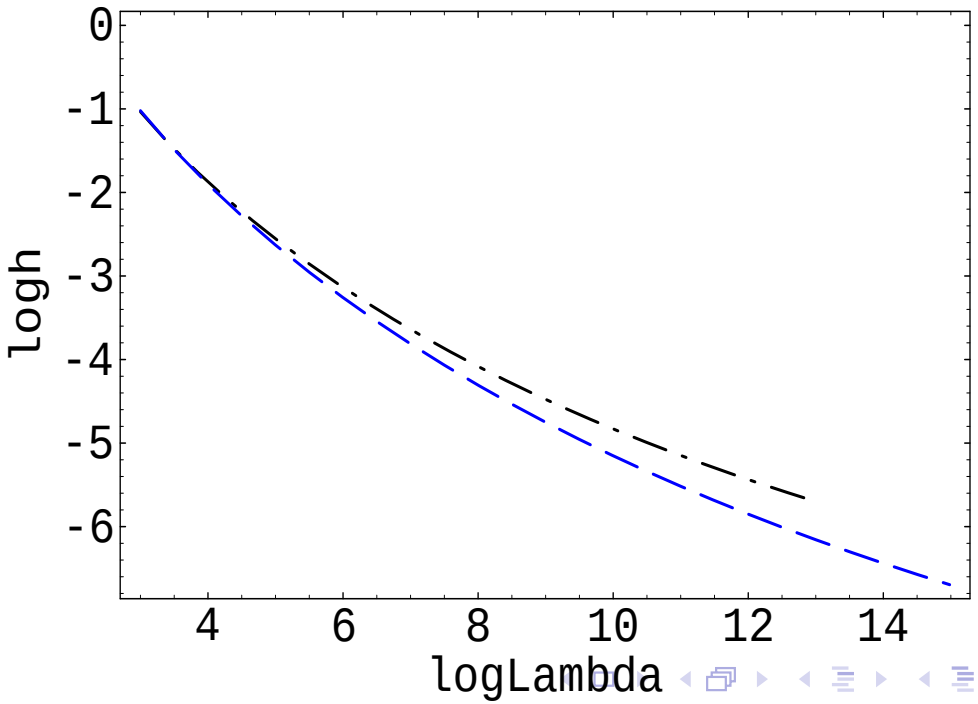
$h \longrightarrow \nu\bar{\nu}$-DECAY

- A very interesting effect is that at a given scale μ the neutrino Yukawa coupling is given by

Yukawa coupling

$$h_\nu(\mu) = B_\gamma^{1/2} \left(\frac{\mu}{\Lambda}\right)^\gamma h_\nu(\Lambda)$$

Yukawa coupling $h_\nu(v)$ as a function of $\log_{10} \Lambda/GeV$ for $m_\nu^D = 0.01$ eV (dashed blue) and 1 eV (dash-dotted black)



$\Gamma(h \to \nu\bar{\nu})$

- One can easily compute the width $\Gamma(h \to \nu\bar{\nu})$ as the imaginary part of the one-loop correction to the Higgs inverse propagator with an internal loop with an unparticle right-handed neutrino propagator

$$\Gamma(h \to \nu\bar{\nu}) = h_\nu^2(m_H)\,\frac{m_H}{16\pi}\,\frac{2}{\Gamma(1-\gamma)\Gamma(3+\gamma)}$$

- In the particle limit $\gamma \to 0$ one recovers the Standard Model expression

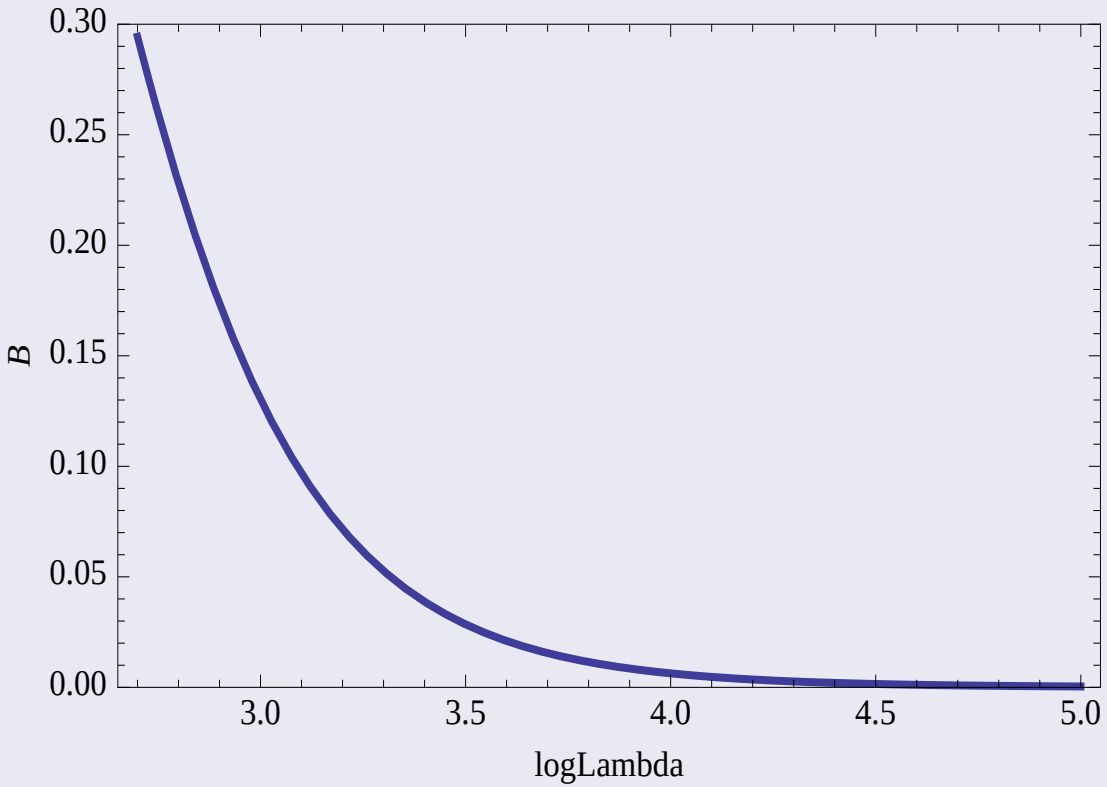
$$\Gamma_{SM}(h \to \nu\bar{\nu}) = h_{\nu\,SM}^2\,\frac{m_H}{16\pi}$$

- The main difference between both expressions is the "conformal running" of the neutrino Yukawa coupling

$B(h \to \nu\bar{\nu})$ Vs. $\log_{10}(\Lambda/GeV)$

The branching ratio with respect to $h \to b\bar{b}$ for $m_H = 130$ GeV and three (almost) degenerate neutrinos with mass $m_\nu^D \simeq 0.1$ eV

$$B(h \to \nu\bar{\nu}) = \frac{\sum_i \Gamma(h \to \nu_i\bar{\nu}_i)}{\sum_i \Gamma(h \to \nu_i\bar{\nu}_i) + \Gamma(h \to b\bar{b})}$$



Majorana neutrinos

- Let us finally comment on the possibility that the theory above the scale Λ violates lepton number
- For values of $\gamma > 1/2$ the right handed

Majorana mass operator

$$\mathcal{L}^M = \frac{1}{2}\Lambda^{1-2\gamma}\mathcal{N}_R\mathcal{N}_R + h.c.\,,$$

is an irrelevant perturbation and does not lead to a breakdown of the conformal symmetry

- It can be immediately verified that at the scale m_ν^D the right-handed neutrino

Majorana mass

$$m_{\nu_R}^M(\mu) = B_\gamma\,(\mu/\Lambda)^{2\gamma}\Lambda$$

is parametrically suppressed with respect to the Dirac mass at the scale of conformal breaking

Dirac mass

- Conformal dynamics will still be governing the right-handed neutrino sector until the Dirac mass becomes of the order of the renormalization group scale
- This happens when

$$\mu_c = B_\gamma^{1/2} \frac{v}{\sqrt{2}} \left(\frac{\mu_c}{\Lambda}\right)^\gamma \Longrightarrow m_\nu^D = \mu_c = \Lambda \left(\frac{B_\gamma^{1/2} v}{\Lambda\sqrt{2}}\right)^{\frac{1}{1-\gamma}}$$

Majorana mass

- The right-handed Majorana mass operator remains irrelevant after electroweak breaking

$$m_{\nu_R}^M = m_\nu^D \left(\frac{m_\nu^D}{\Lambda}\right)^{2\gamma-1}$$

- For $\gamma > 1/2$ ($m_{\nu_R}^M \ll m_\nu^D$) the scale at which conformal breaking is induced by electroweak breaking is self-consistently fixed to m_ν^D

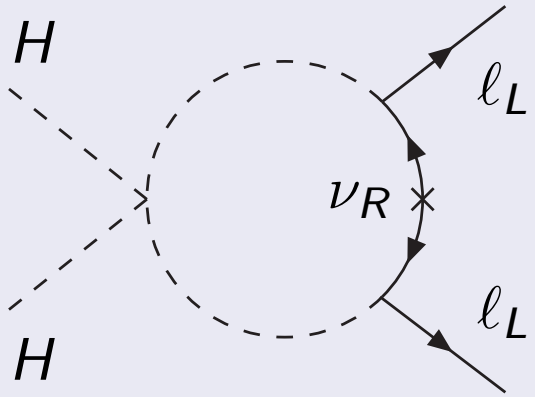
- The fact that lepton number is violated also allows for further higher dimension operators, the most important one being

$$\mathcal{L}^{\not L} = c\,\Lambda^{-1}(H\ell_L)^2$$

- If this operator is generated from integrating out heavy right-handed neutrinos at Λ the constant c will be of $\mathcal{O}(1)$. This is of course the standard see-saw mechanism
- On the other hand if the right-handed neutrinos are conformal (as we are assuming) we cannot integrate them out
- Nevertheless due to the presence of the irrelevant lepton-number violating operator $\mathcal{L}^M$ loop corrections will still generate $\mathcal{L}^{\not L}$

- For instance the diagram

Diagram contributing to $\mathcal{L}^{\not L}$



gives

$$c(\gamma) = \frac{\lambda\,(B_\gamma)^2}{16\pi^2(2\gamma-1)}\left[1-\left(\frac{v}{\Lambda}\right)^{4\gamma-2}\right]$$

where λ is the Higgs quartic coupling and one Majorana-mass insertion was used

- Once electroweak symmetry is broken a left-handed neutrino mass is generated $m_{\nu_L}^M = c\,\frac{v^2}{\Lambda}$

- For $\mu < v$ this mass will run much less compared to the strongly running Dirac and right-handed Majorana masses and hence the dominant neutrino mass will be $m_{\nu_L}^M$
- The neutrino mass matrix at the scale of conformal breaking

Mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} m_{\nu_L}^M & m_\nu^D \\ m_\nu^D & m_{\nu_R}^M \end{pmatrix}$$

- Due to the hierarchy $m_{\nu_L}^M \gg m_\nu^D \gg m_{\nu_R}^M$ we now have an *inverted* see-saw mechanism, with an extremly light and almost completely sterile right-handed neutrino

Light mass eigenvalue and mixing

$$m_{\nu_R}^{\prime M} = m_{\nu_L}^M \alpha^2 , \qquad \alpha = \left(\frac{B_\gamma^{1/2}}{\sqrt{2}c} \right)^{\frac{\gamma}{1-\gamma}} \left(\frac{m_{\nu_L}^M}{v} \right)^{\frac{2\gamma-1}{1-\gamma}} .$$

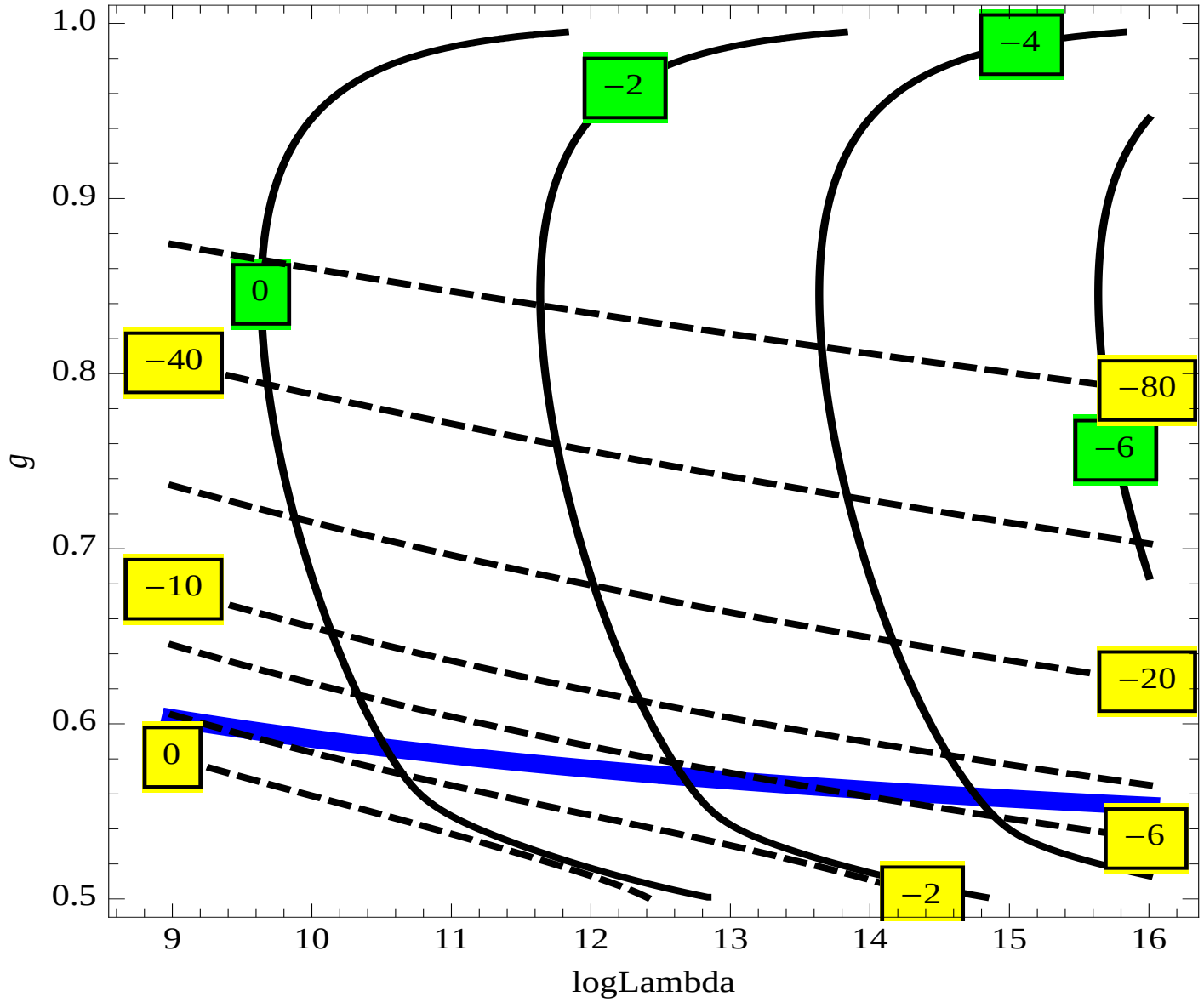


Figure: Heaviest –mainly ν_L– (solid) and the lightest –mainly ν_R– (dashed) mass eigenstates. Labels indicate m as $\log_{10}(m/\mathrm{eV})$. The region below the thick (blue) line is $\alpha > 0.1$.

- For $\gamma \simeq 1/2$ the mixing becomes of $\mathcal{O}(1)$ and the two eigenvalues are similar. Such a scenario would lead to new sterile neutrinos participating in the oscillations
- Although mixing schemes with sterile neutrinos have been proposed to explain the LSND anomaly, global fits including recent data (seem to) produce poor results
  We therefore impose a rough cutoff of $\alpha < 0.1$
- It is thus interesting to notice that neutrino oscillation experiments are able to probe our model even for large values of Λ, as required in the lepton number violating case
- Needless to say in this case Majorana neutrino masses are sensitive to neutrino-less double beta-decay experiments
- Finally, when lepton number is violated in the conformal sector, since the only γ-dependence for left-handed masses comes from the prefactor $c(\gamma)$, it is natural to expect near-degenerate neutrino masses

Conclusion

- A conformally invariant right-handed neutrino sector represents a natural way to obtain sub-eV neutrino masses if the anomalous dimensions lie in the interval $1/2 < \gamma < 1$
- Electroweak symmetry breaking triggers conformal breaking at the neutrino mass scale
- Our model predicts lepton flavor violating reactions such as $\mu \to e\gamma$ at a much larger rate than in the SM and even opens up the possibility to experimentally determine the anomalous dimensions in forthcoming experiments

- For rather low scales $\Lambda \sim 10$ TeV the neutrino Yukawa couplings can be comparable with those for charm and tau at the weak scale and induce sizable (invisible) Higgs decay into the $\nu\bar{\nu}$ channel
- If the conformal theory violates lepton number there are sterile (Majorana) neutrinos with mixing angles which depend on the anomalous dimensions γ. The mixing angles can be large for $\gamma \simeq 1/2$

Open questions

- What is the 4D conformal theory? A sort of (non-perturbative) Banks-Zaks model? SQCD in the conformal window?
- What is (if any) the 5D theory? A 5D soft-wall model with right-handed propagating neutrinos?
- Other phenomenological applications: $\mu \to 3e$, $\mu \to e$ conversion, LHC/NLC Higgs phenomenology,...