Technicolor on the lattice

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Simulations beyond QCD

- simulations with **light**, **dynamical** fermions w/out compromising the robustness of the formulation
- dynamics different from QCD + we don't know the answer!!
- lattice simulations: robust quantitative tool for NP QFT

SU(N) gauge theories with fermions in higher–dimensional reps

- candidate theory for technicolor: nonperturbative physics at LHC
- framework and implementation
- first numerical results [catterall et al 07/08, ldd et al 08, rummukainen et al 08/09, degrand et al 08, appelquist et al 07/09, deuzeman et al 08/09]

1. Dynamical Electroweak Symmetry Breaking

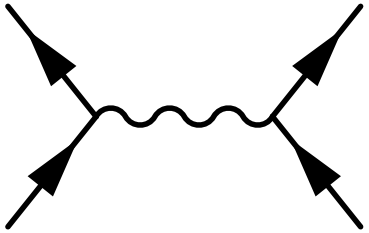
- mechanism for EWSB $\longrightarrow$ hierarchy problem: $\Lambda_{\rm EW} \ll \Lambda_{\rm UV}$
- elegant solution: dynamical EW symmetry breaking [weinberg, susskind]
  $\hookrightarrow$ new strong interaction: $\Lambda_{\rm EW} \sim \Lambda_{\rm UV} e^{-g^2/g^2_{\rm UV}}$
- mechanism already realized in QCD at 100 MeV
- EW sector strongly coupled at TeV scale $\simeq 4\pi v$
- TC Goldstone bosons $\longrightarrow$ longitudinal EW gauge dof
- new resonances $\mathcal{O}(1\ \mathrm{TeV})$

Problems with technicolor

- too much freedom: flavors, colors, representations?
- difficult to compute due to strong dynamics
- precision EW data: do **not** rule out DEWSB in general
- **do** rule out QCD–like behaviour (S parameter)
- **difficult** with large number N of degrees of freedom
  $\hookrightarrow$ large–N_f technicolor, Randall–Sundrum models
- flavor physics from TC: extended TC

Flavor physics

- ETC: mass term for the SM fermions, FCNC



$$\longrightarrow \quad \Delta\mathcal{L} \propto \frac{1}{M^2_{\rm ETC}} \langle \bar{\Psi}\Psi \rangle_{\rm ETC}\ \bar{\psi}\psi,\ \frac{1}{M^2_{\rm ETC}} \bar{\psi}\psi\ \bar{\psi}\psi$$

- running of $\langle \bar{\Psi}\Psi \rangle$: **(near) conformal behaviour**

$$\boxed{\langle \bar{\Psi}\Psi \rangle_{\rm ETC} = \langle \bar{\Psi}\Psi \rangle_{\rm TC}\ \exp\left(\int_{\Lambda_{\rm TC}}^{M_{\rm ETC}} \frac{d\mu}{\mu}\, \gamma(\mu) \right)}$$

Flavor physics

- QCD–like technicolor: $m \sim \Lambda_{\rm TC}^3/M_{\rm ETC}^2$

$$M_t \sim \Lambda_{\rm TC}\left(\frac{\Lambda_{\rm TC}}{m_t}\right)^{1/2} \sim 5\ {\rm TeV}$$

- non QCD–like technicolor: $m \sim \Lambda_{\rm TC}^d/M_{\rm ETC}^{d-1}, \quad d = 3-\gamma$

$$M_t \sim \Lambda_{\rm TC}\left(\frac{\Lambda_{\rm TC}}{m_t}\right)^{1/(d-1)}$$

- technicolor + ladder: $d = 2, \gamma = 1$

- conformal technicolor; smaller values of d [luty 04]

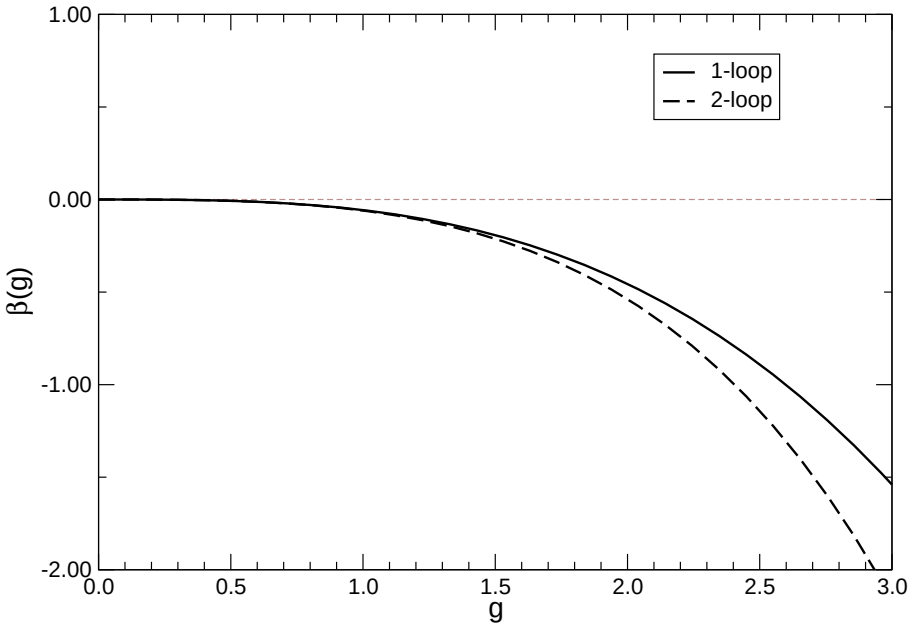
Conformal IR behaviour

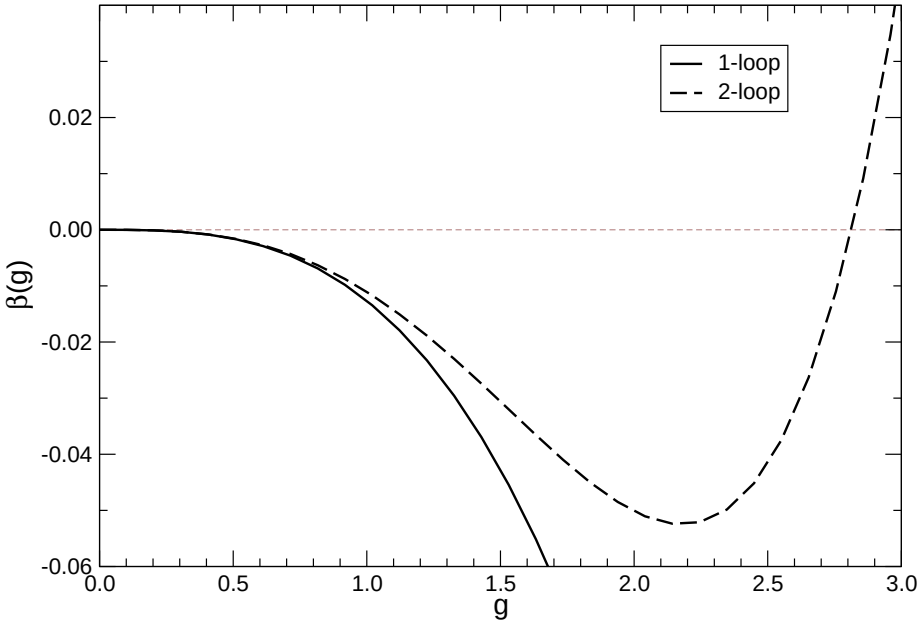
$$\beta(\bar{g}) = \mu \frac{d}{d\mu} \bar{g}(\mu) = -\beta_0 \bar{g}^3 - \beta_1 \bar{g}^5 + O(\bar{g}^7)$$

$$\beta_0 = \frac{1}{(4\pi)^2} \left[\frac{11}{3} C_2(A) - \frac{4}{3} T_R n_f \right]$$

$$\beta_1 = \frac{1}{(4\pi)^4} \left[\frac{34}{3} C_2(A)^2 - \frac{20}{3} C_2(A) T_R n_f - 4 C_2(R) T_R n_f \right]$$

	$(4\pi)^2\beta_0$	$(4\pi)^4\beta_1$
SU(3), $n_f = 3$ fund	9	64
SU(2), $n_f = 2$ adj	2	-40





Walking theories - scheme dependence

- IRFP is scheme-independent

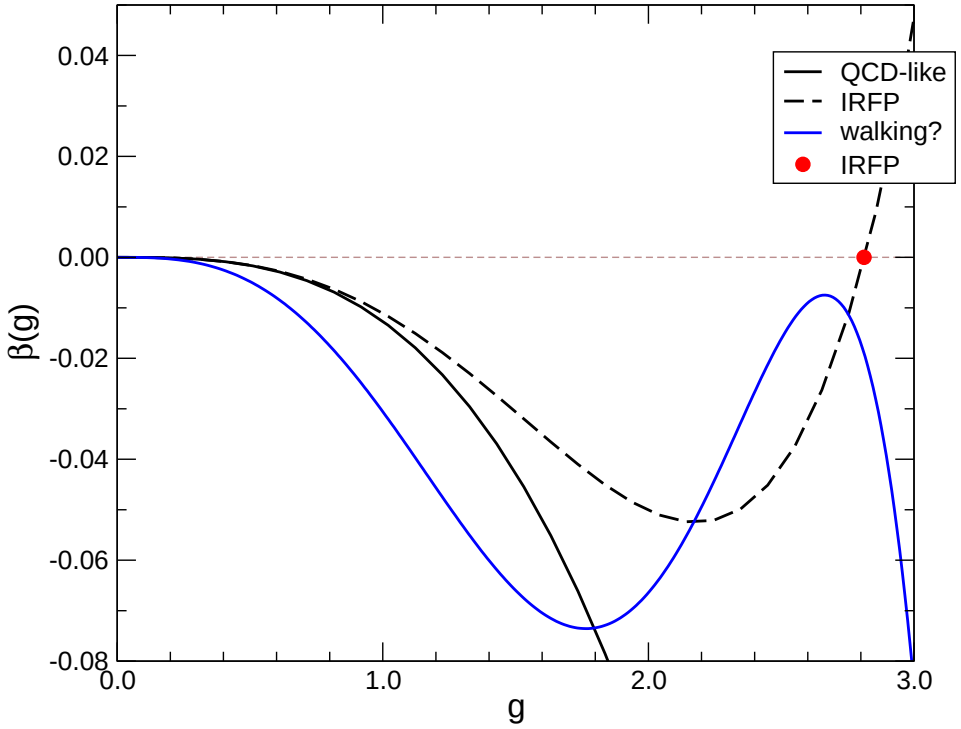
$$\beta(\bar{g}) = \mu \frac{d}{d\mu} \bar{g}(\mu)$$

change of scheme: $\bar{g}' = \Phi(\bar{g}), \quad \Phi'(\bar{g}) > 0$

$$\beta'(\bar{g}') = \Phi'(\bar{g}) \beta(\bar{g})$$

universality of the first two coefficients

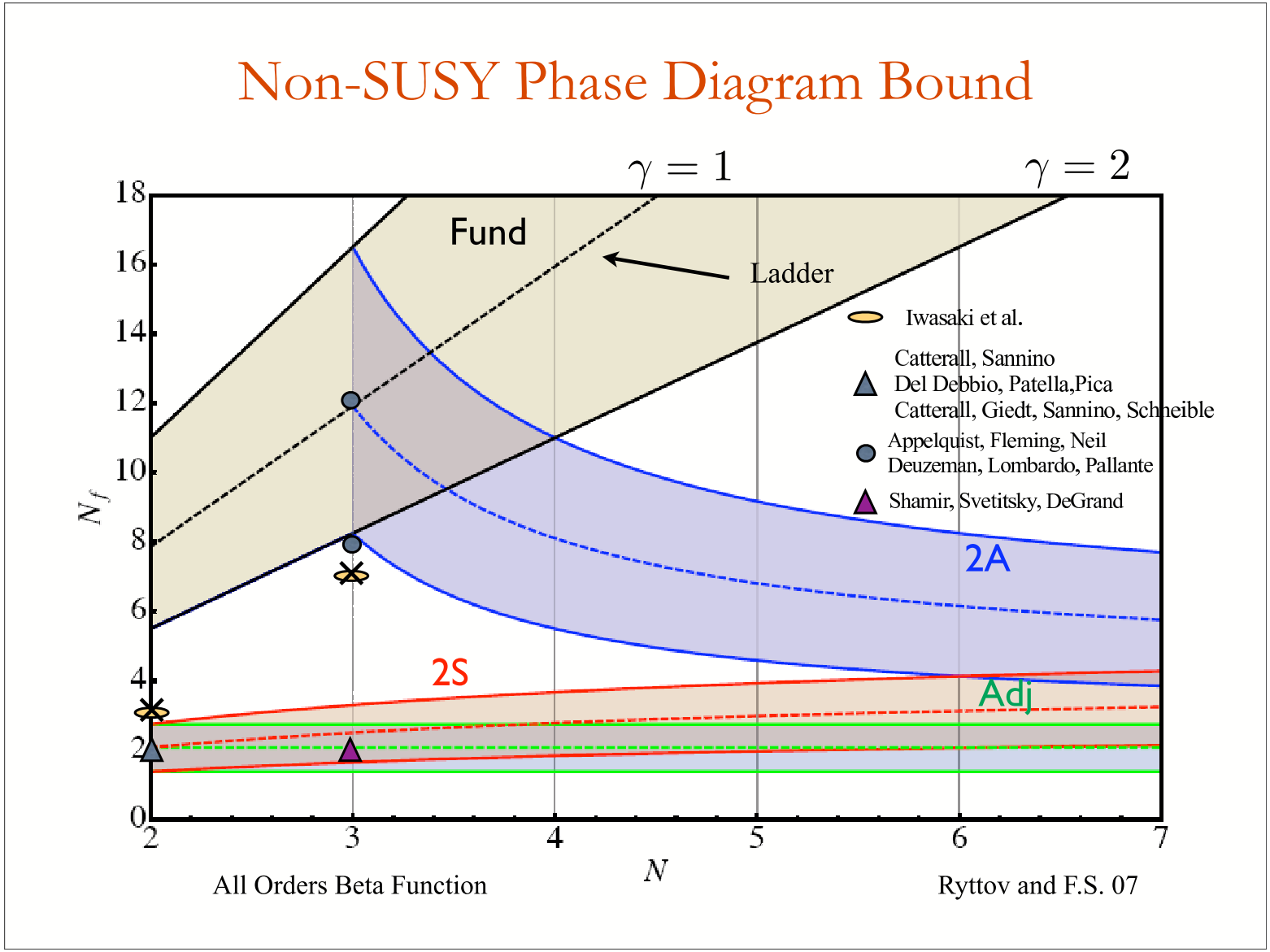
- β-function away from IRFP is not universal!
- change of scheme can produce a “walking” behaviour



DEWSB @ LHC

- collider phenomenology via effective theories
- effective lagrangian for the low–lying states
  LHC observables in terms of the LEC [giudice et al, sannino et al]
- lattice simulations provide first principle calculations
  $\hookrightarrow$ phase structure of the theory, spectrum, LEC
- light dynamical fermions play a key role for non QCD–like behaviour
- **understand the effect of systematic errors in lattice results**

Phase diagram [sannino 08]



Perturbation theory

[ldd, frandsen, panagopoulos, sannino 08]

Perturbative computations can be easily generalized:

- vertices with fermions and gluons involve generators T^a_R
- vertices with 3 and 4 gluons involve generators T^a_F
- at one–loop replace Casimir operators as necessary
- numerical integrals are independent of the color structure

$$am_c(g_0) = g_0^2\Sigma^{(1)} + g_0^4\Sigma^{(2)}$$
$$\Sigma^{(1)} = 2C_2(R)\left[c_1 + c_2\right]$$
$$\Sigma^{(2)} = C_2(R)Nd_1 + 2C_2(R)T_R n_f d_2 + C_2(R)C_2(F)d_3 + C_2(r)^2 d_4$$

$$Z_A = 1 - \frac{g_0^2}{16\pi^2}C_2(R) \times 15.7$$
$$\Lambda_{\overline{\rm MS}}/\Lambda_{\rm lat}\big|_{{\rm SU}(3),n_f=2,2S} = 1469.59$$
$$\Lambda_{\overline{\rm MS}}/\Lambda_{\rm lat}\big|_{{\rm SU}(2),n_f=2,2S} = 6884.36$$

Chiral symmetry

For a generic N 2–index representations are complex:

$$\mathrm{SU}(n_f) \times \mathrm{SU}(n_f) \longrightarrow \mathrm{SU}(n_f)$$

$\Longrightarrow$ usual pattern of chiral symmetry breaking

SU(2) is a special case: $\square\!\square$ is the adjoint representation

chiral symmetry breaking pattern for $n_f = 2$:

$$\mathrm{SU}(4) \longrightarrow \mathrm{SO}(4)$$

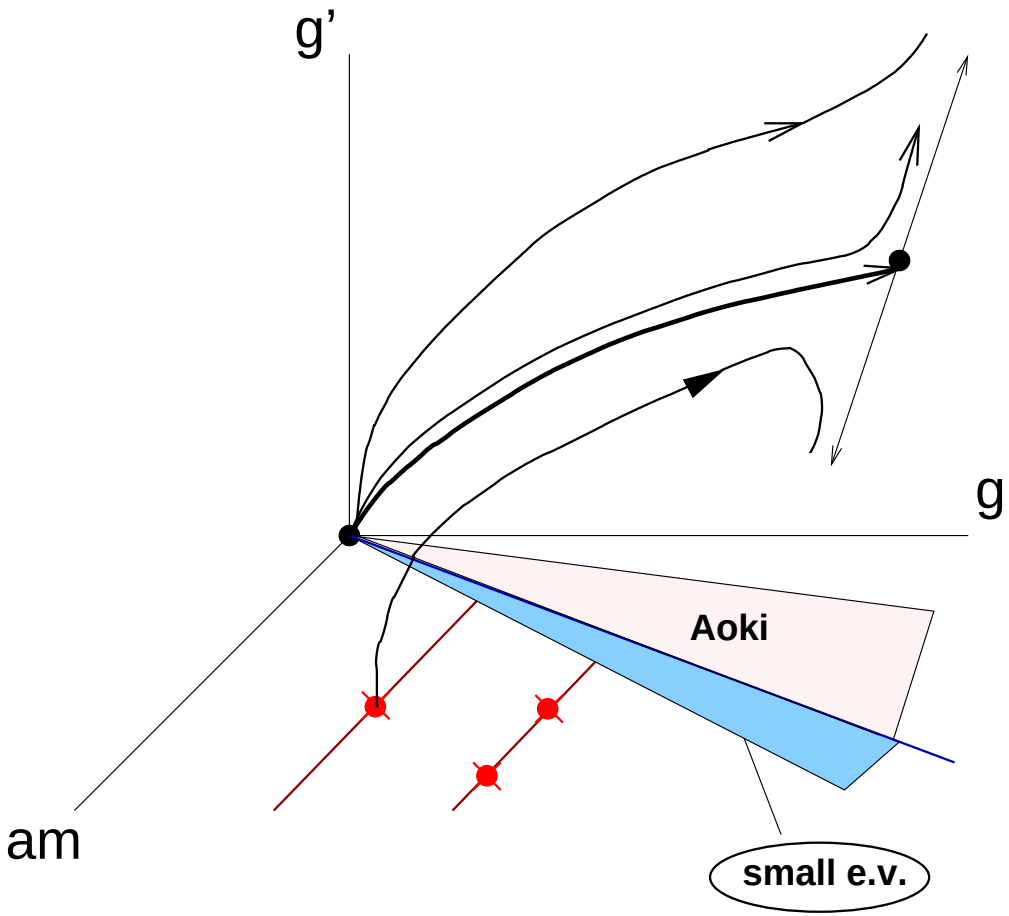
Goldstone bosons are coupled to EW gauge bosons:

$$\boxed{F_{\mathrm{PS}} = v \approx 250\ \mathrm{GeV}}$$

$\hookrightarrow$ can be used to set the scale

Conformal fixed point?

- scale invariance is broken by m AND $1/L$
- HOW to identify a conformal fixed point?
- deviations from QCD spectrum
- fixed physical volume + light masses!
- scaling behaviour of field correlators
- Schrödinger functional/twisted BC allow $m = 0$



2. Higher representations

Wilson–Dirac operator:

$$D = \frac{1}{2}\sum_{\mu}\left[\gamma_\mu\left(\nabla_\mu + \nabla^*_\mu\right) - a\nabla^*_\mu\nabla_\mu\right] + m\,,$$

i.e.

$$\begin{aligned}\sum_y D(x,y)\psi(y) \quad &= \quad -\frac{1}{2a}\Big\{\sum_\mu\Big[(1-\gamma_\mu)\,U^R(x,\mu)\psi(x+\mu) + \\ &\qquad (1+\gamma_\mu)\,U^R(x-\mu,\mu)^\dagger\psi(x-\mu)\Big] - (8+2am)\psi(x)\Big\},\end{aligned}$$

$$\kappa = 1/(8+2am)$$

Link variables:

$$\begin{aligned}U(x,\mu) \quad &= \quad \exp\left[i\,A^a_\mu(x)\,T^a_f\right], \quad N\times N \text{ matrix} \\ U^R(x,\mu) \quad &= \quad \exp\left[i\,A^a_\mu(x,\mu)\,T^a_R\right], \quad d_R\times d_R \text{ matrix}\end{aligned}$$

HMC for higher representations

[Idd, patella, pica 08]

Including the fermionic determinant:

$$\begin{aligned} Z &= \int \mathcal{D}U\mathcal{D}\phi\mathcal{D}\phi^* \exp[-S_G - \left\|Q^{-1}\phi\right\|^2] \\ &= \int \mathcal{D}\Pi\mathcal{D}U\mathcal{D}\phi\mathcal{D}\phi^* \exp[-\frac{1}{2}(\Pi,\Pi) - S_G - \left\|Q^{-1}\phi\right\|^2], \quad Q = \gamma_5 D \end{aligned}$$

Molecular dynamics evolution:

$$\begin{aligned} \frac{d}{dt}\Pi(x,\mu) &= -\frac{\delta\mathcal{H}}{\delta U(x,\mu)} = \sum_k F_k(x,\mu) \\ \frac{d}{dt}U(x,\mu) &= \Pi(x,\mu)U(x,\mu) \end{aligned}$$

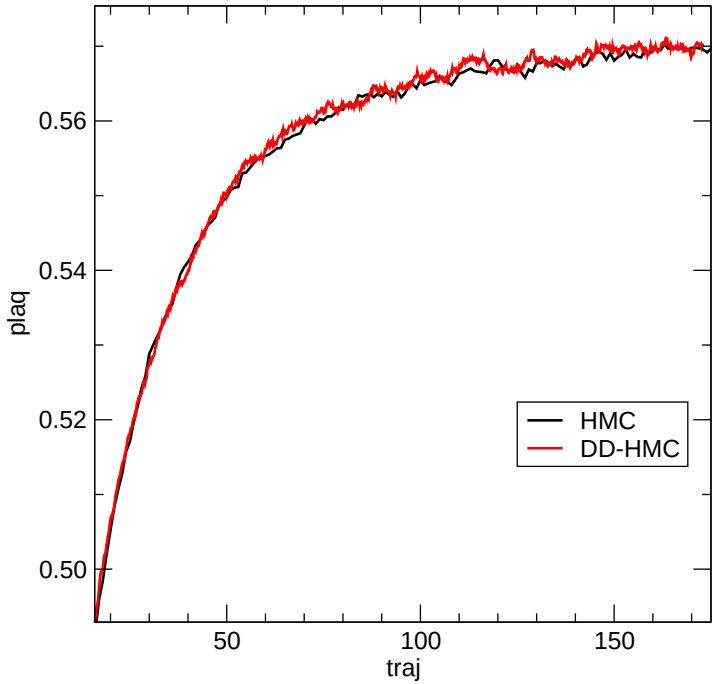
performed numerically through discrete leapfrog integration w. time-step $d\tau$
Total trajectory length $\tau = N d\tau$ – $\text{cost} \propto N\langle N_{\text{iter}}\rangle$

performance of the algorithm for light fermion masses is crucial

Tests of the algorithm (1)

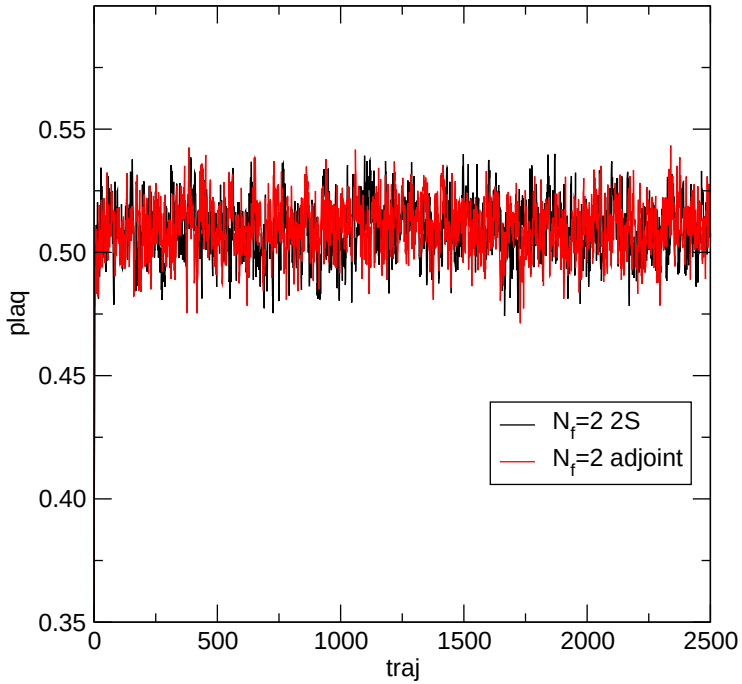
SU(3) fund repr

$16^3 \times 16, \beta = 5.6, \kappa = 0.15750$

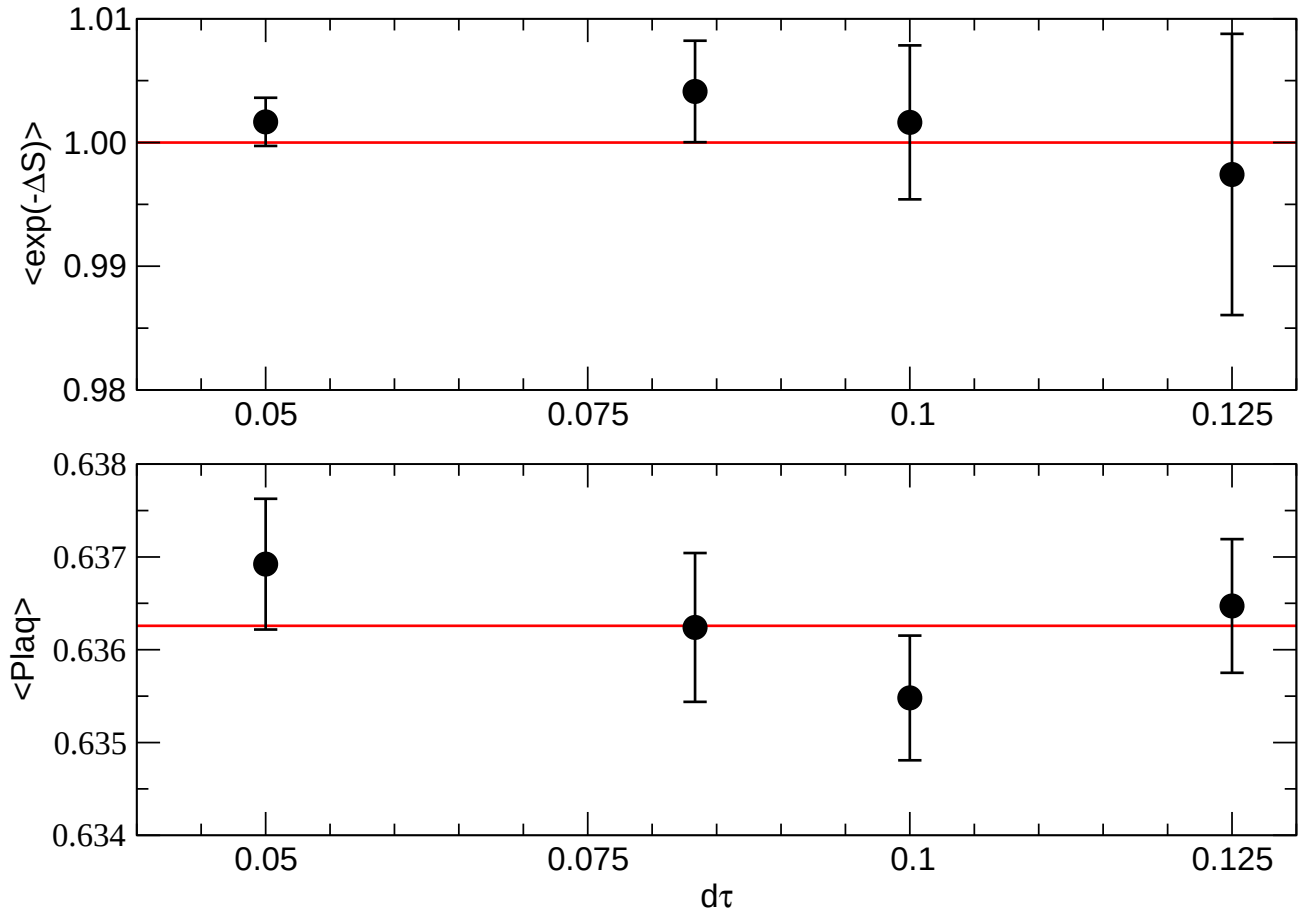


SU(2) 2S/adjoint repr

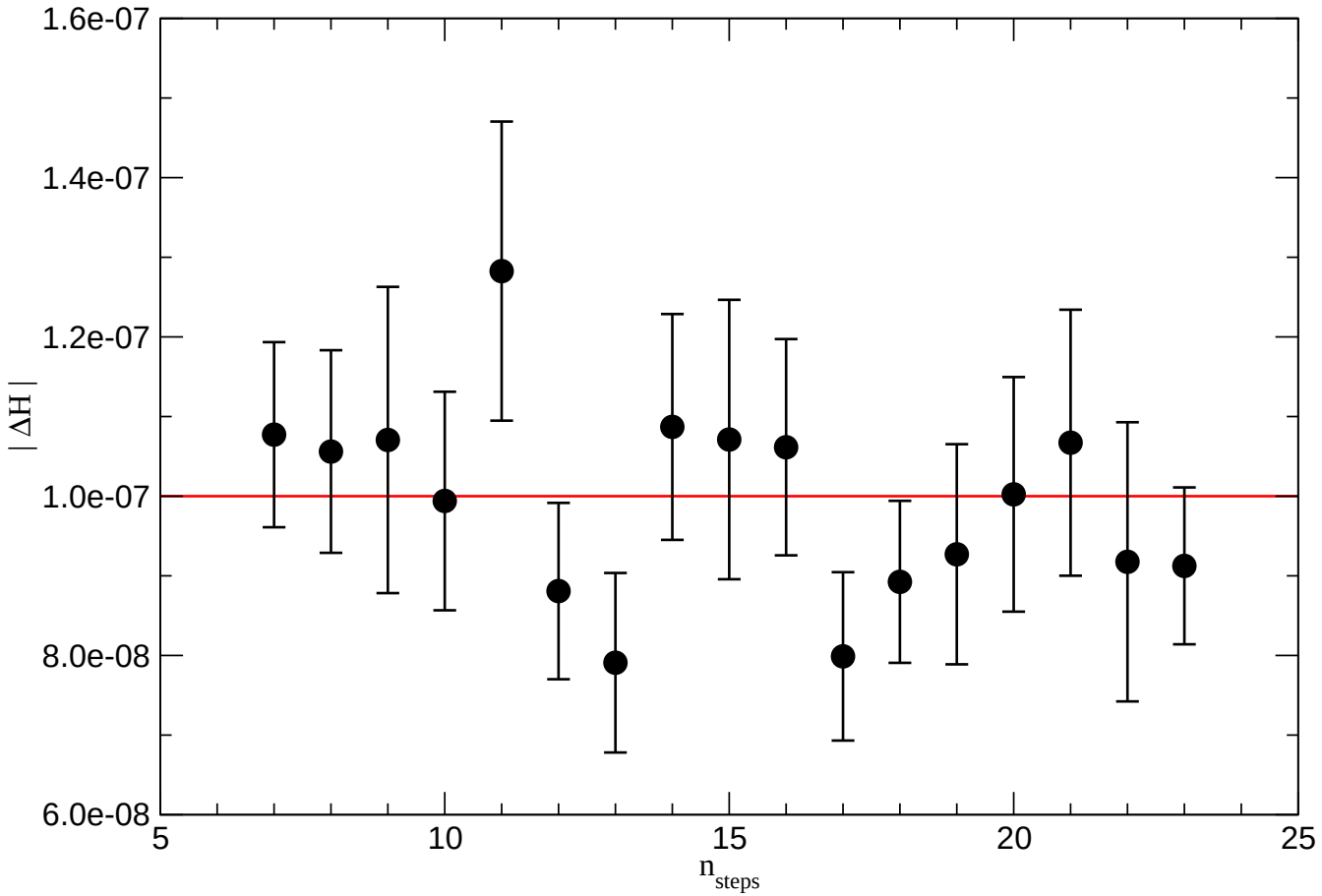
$4^3 \times 8, \beta = 2.0, \kappa = 0.12500$



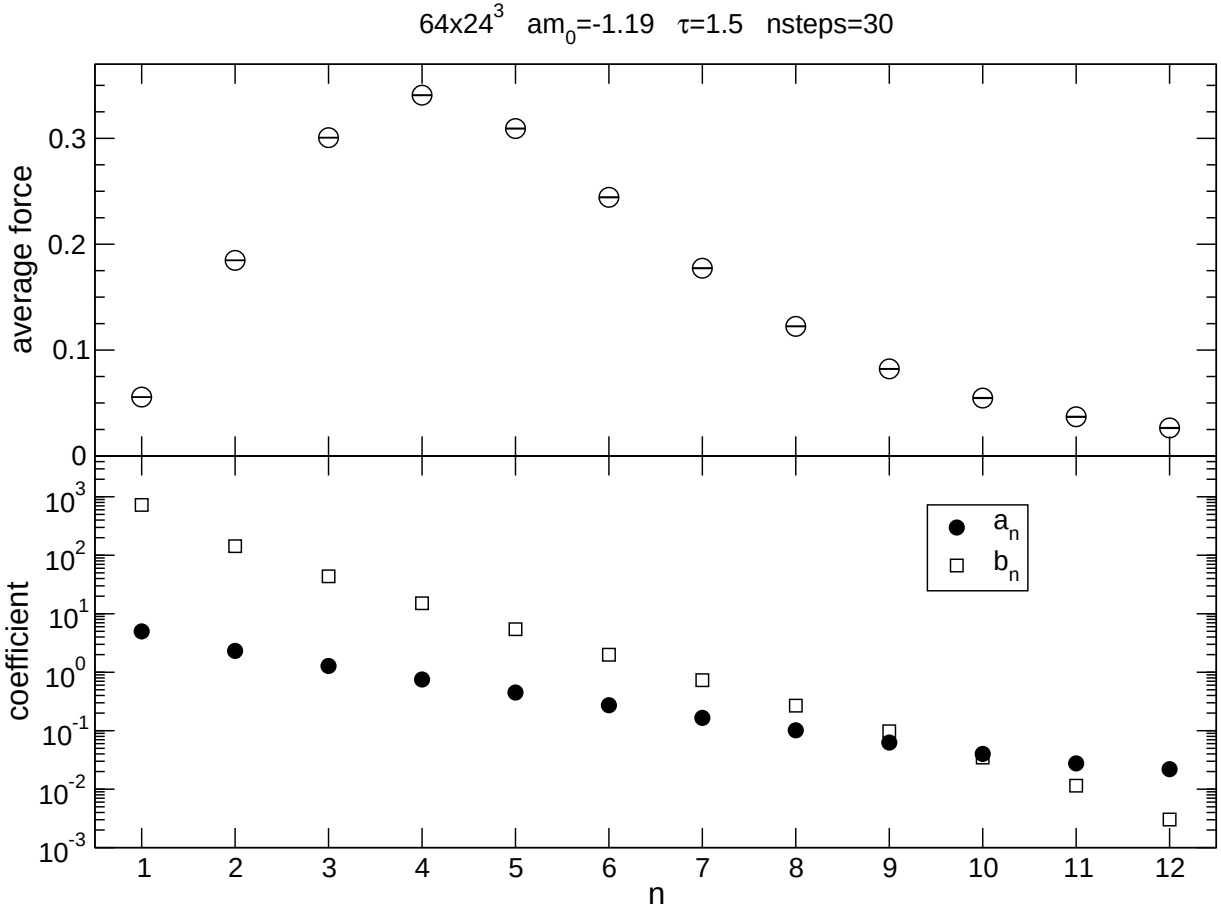
Tests of the algorithm (2)



Tests of the algorithm (3)

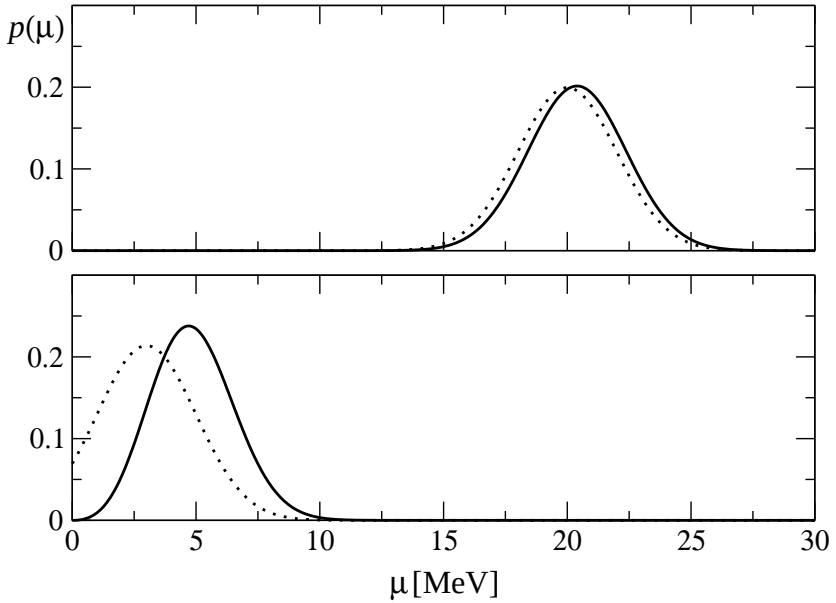


Force profile



Stability of the HMC

[ldd, giusti, luscher, petronzio, tantalo 05]



- integration instabilities / reversibility
- ergodicity problem: HMC stuck in a sector with $\eta \neq 0$
- sampling of observables: $p(\mu)/\mu^2$

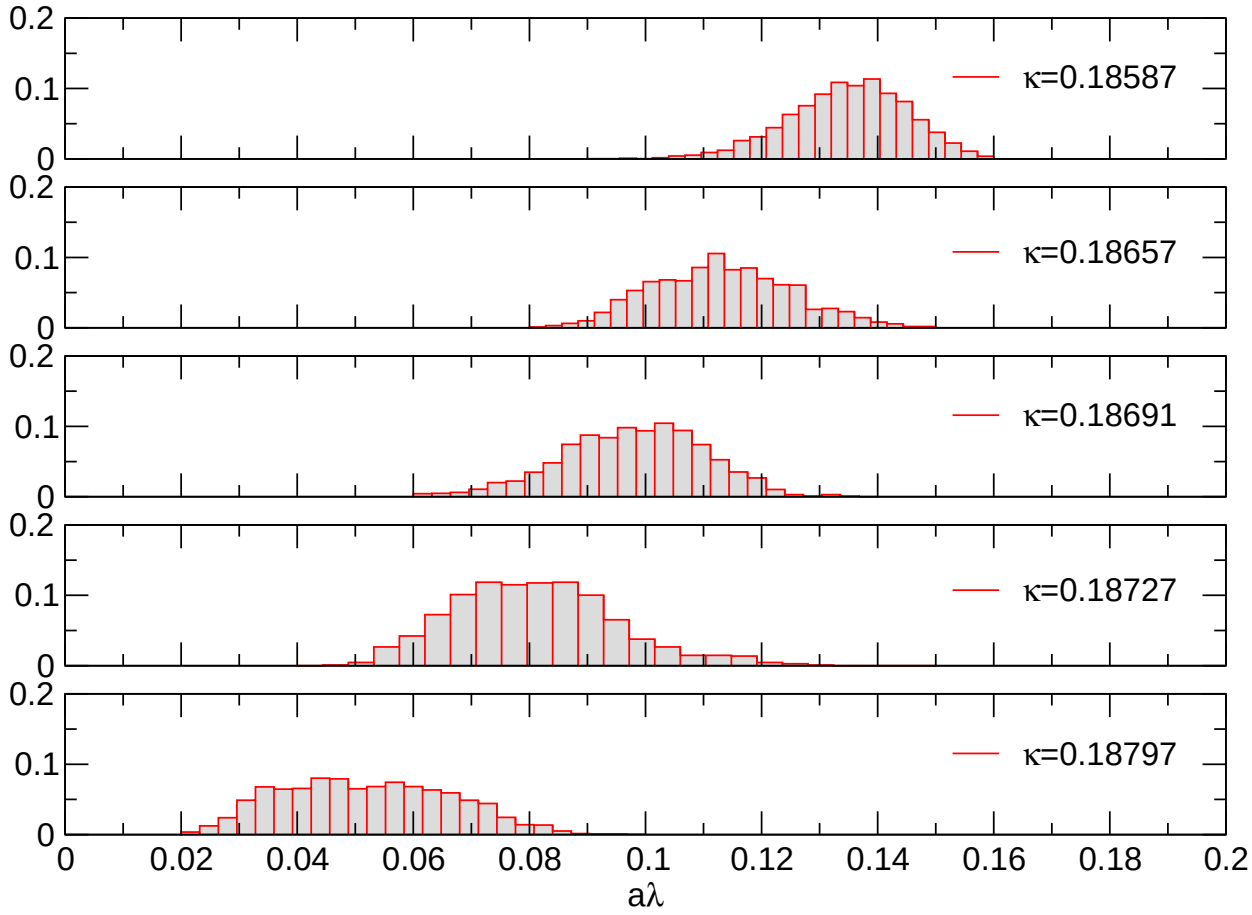
difficult regime for simulations:

$$m \to 0$$
$$a, V \ \text{ fixed}$$

$\hookrightarrow$ check that runs are in a safe regime!!

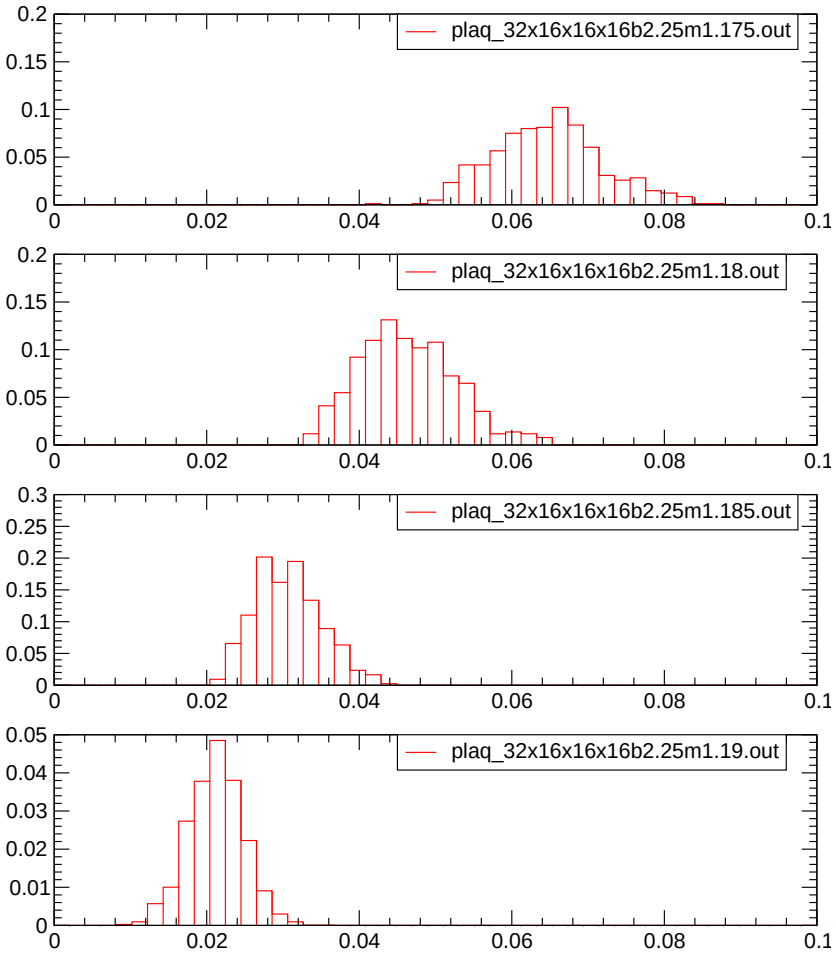
Numerical results for SU(2) adj

[ldd, patella, pica 08]



data from 16×8^3 lattice, $\beta = 2.0$

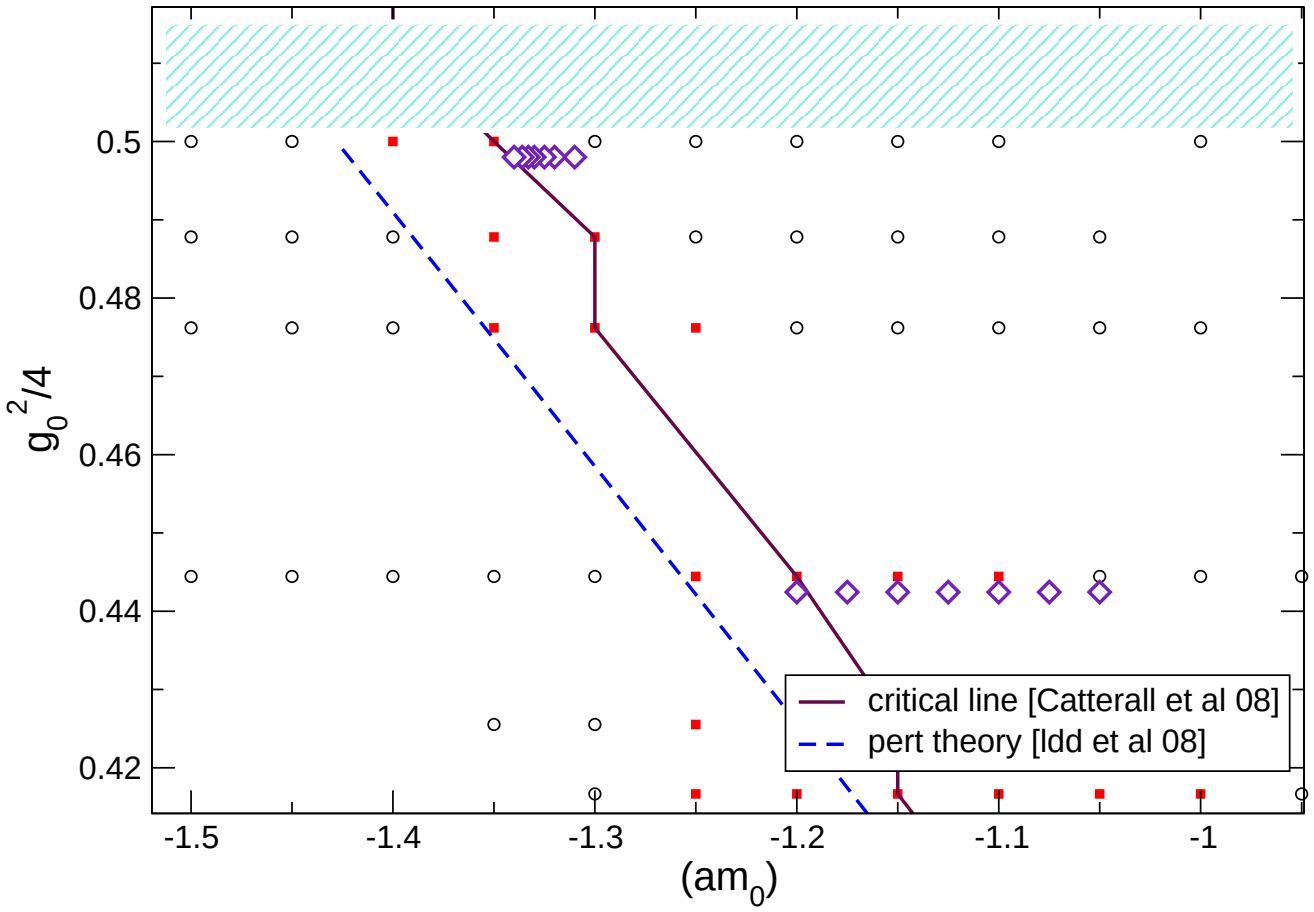
Numerical results for SU(2) adj



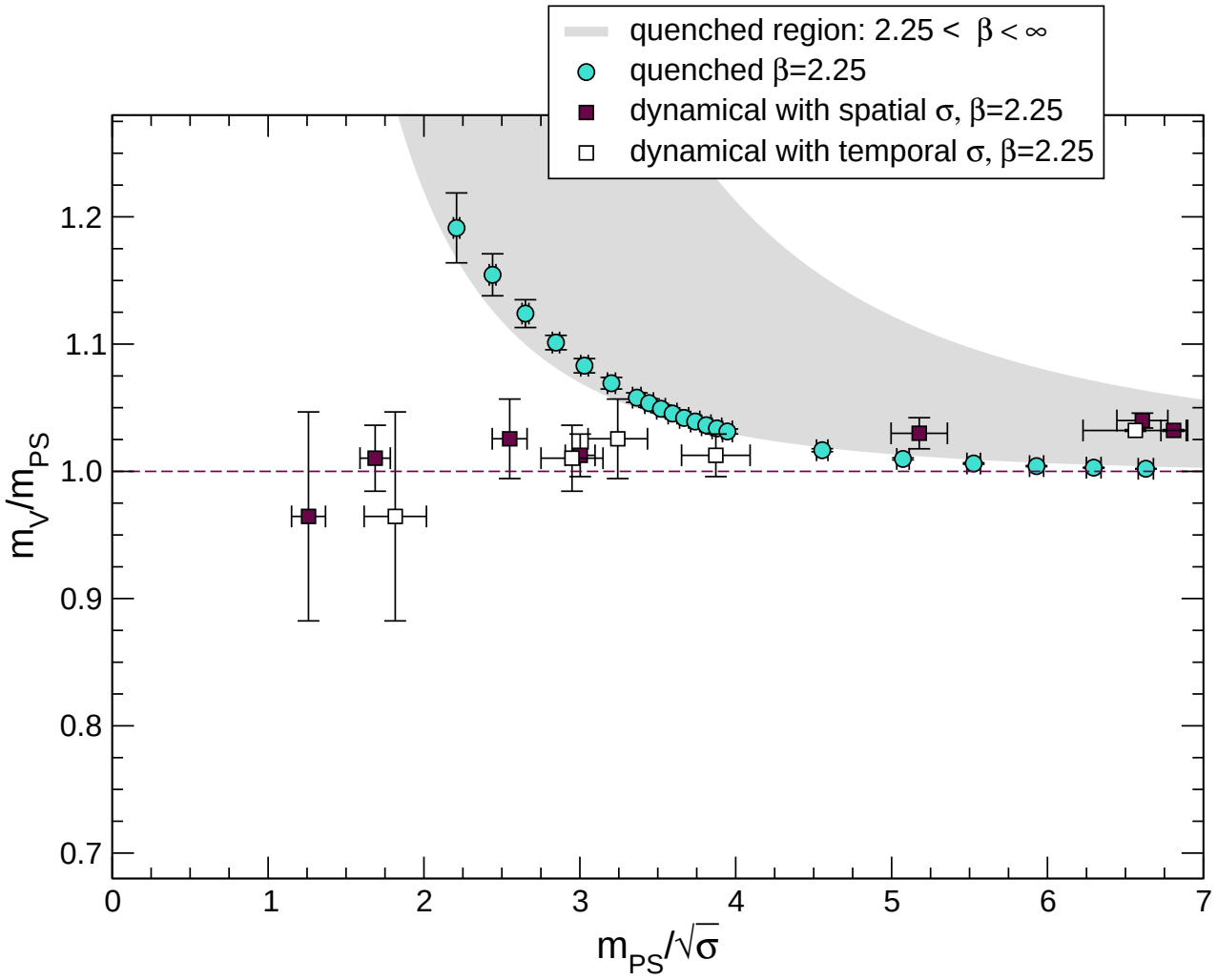
data from 32×16^3 lattice, $\beta = 2.25$

3. Exploring the chiral regime

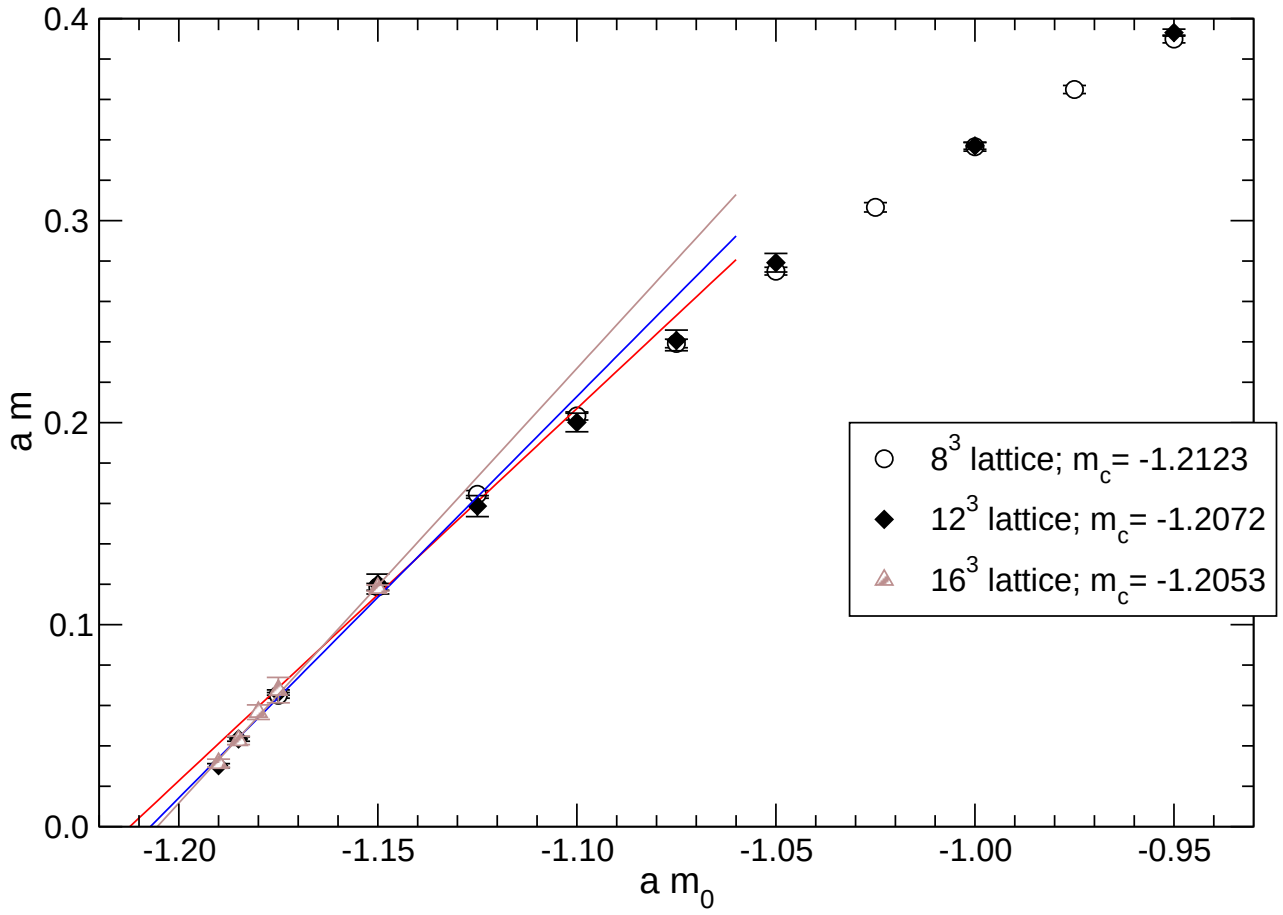
[edinburgh-swansea 09]



The large–mass limit

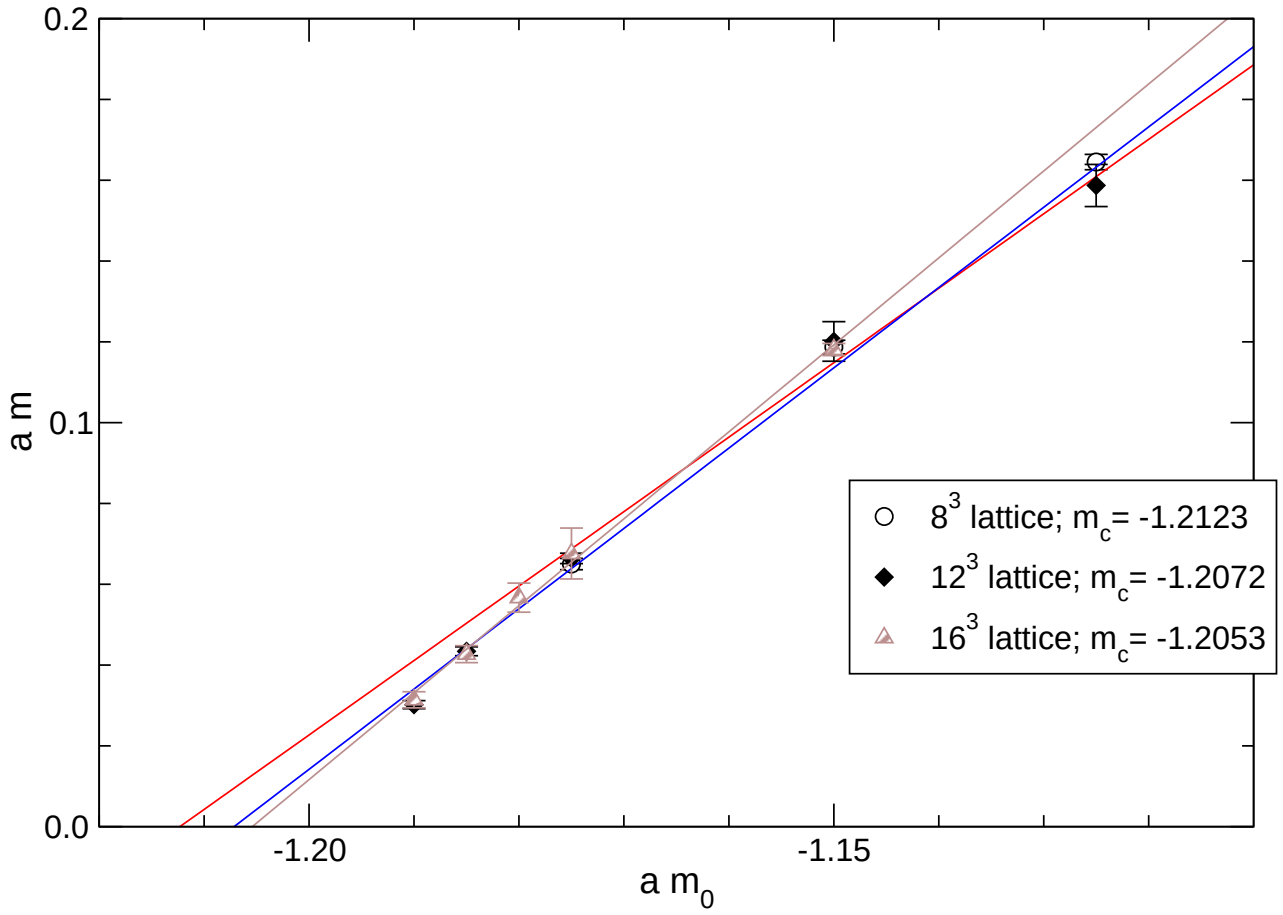


Chiral limit (1)

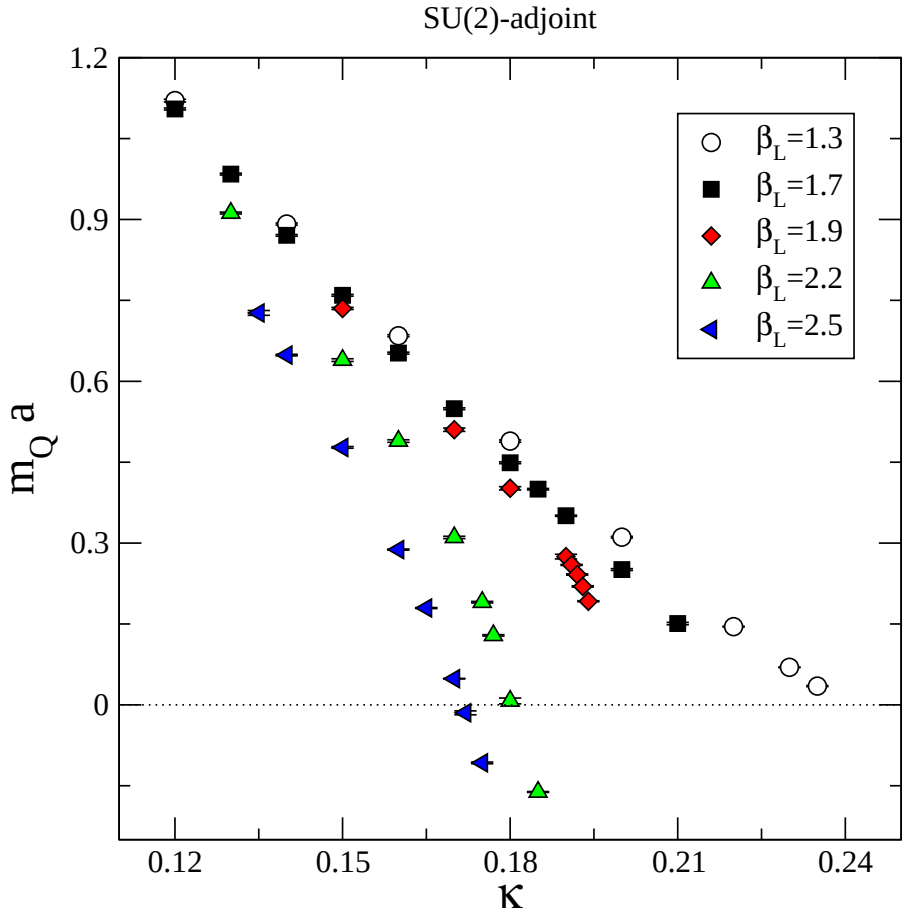


data from $\beta = 2.25$ lattices

Chiral limit (2)

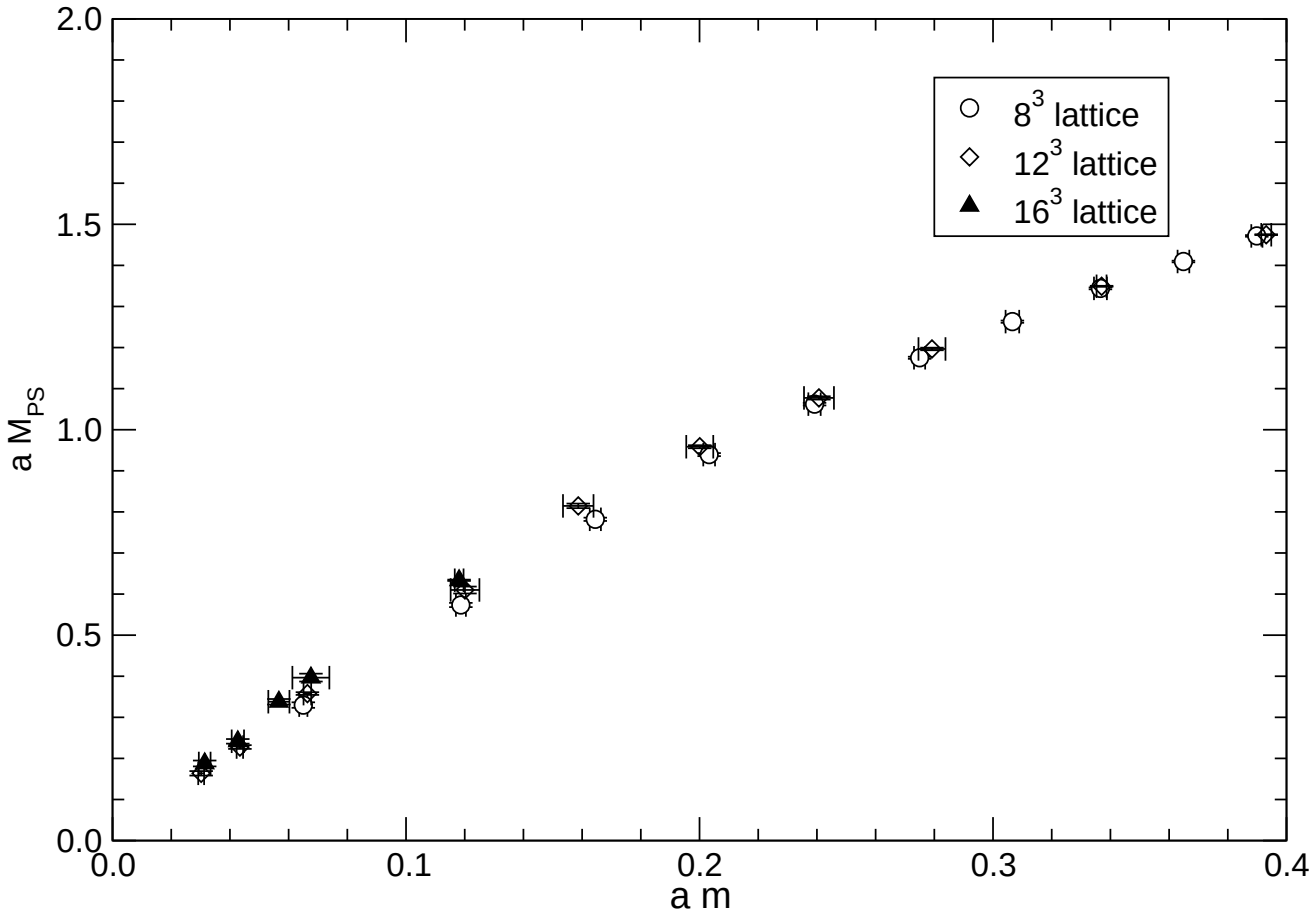


Comparison with other studies

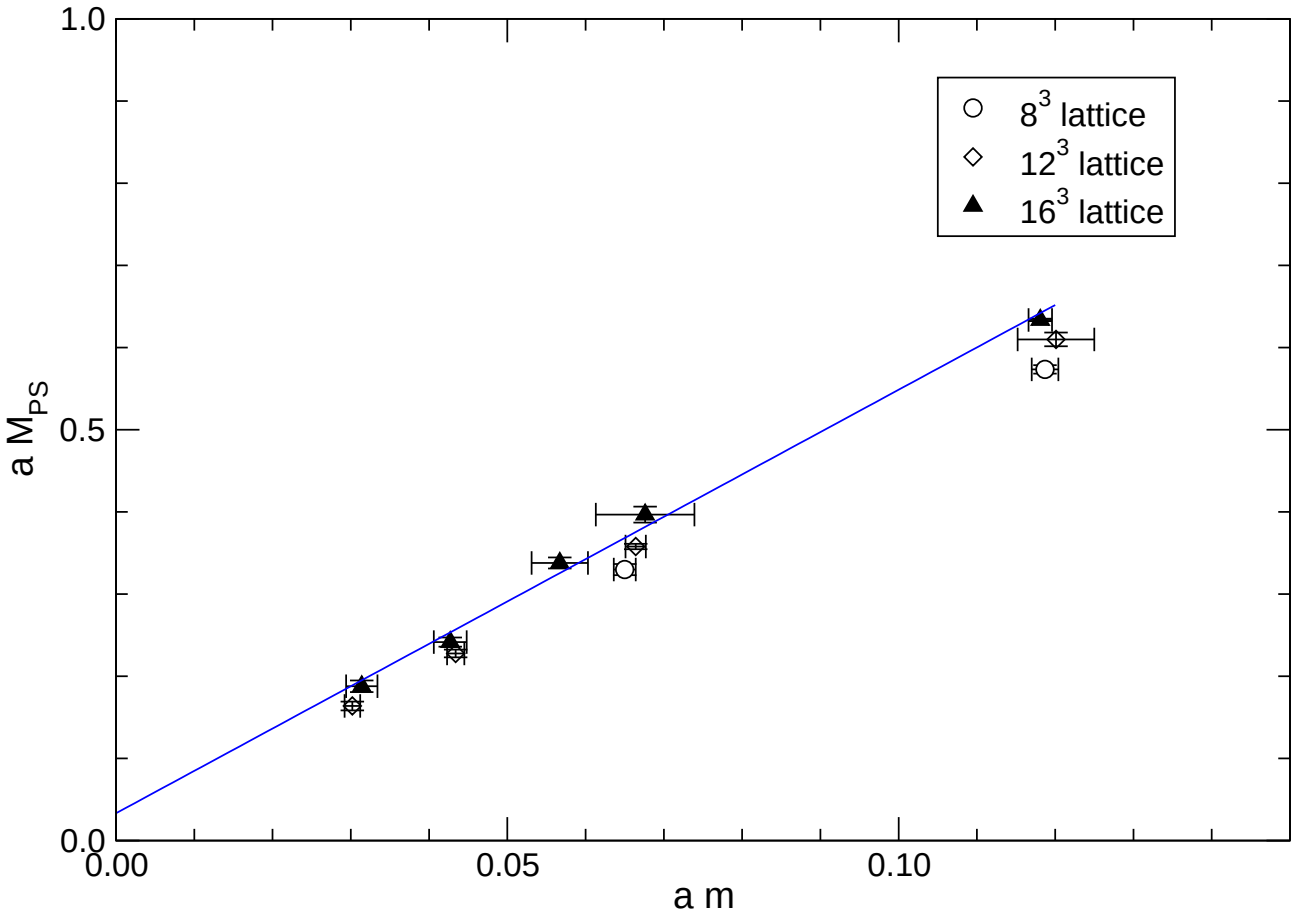


[rummukainen et al 08]

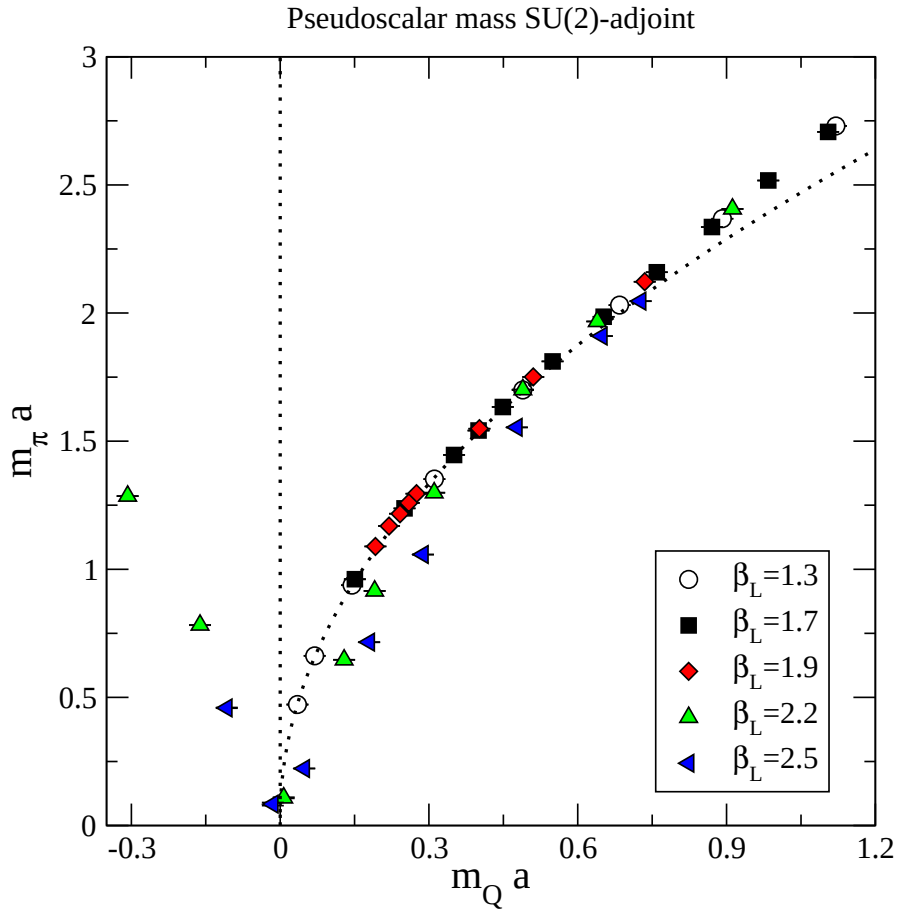
Pseudoscalar mass (1)



Pseudoscalar mass (2)

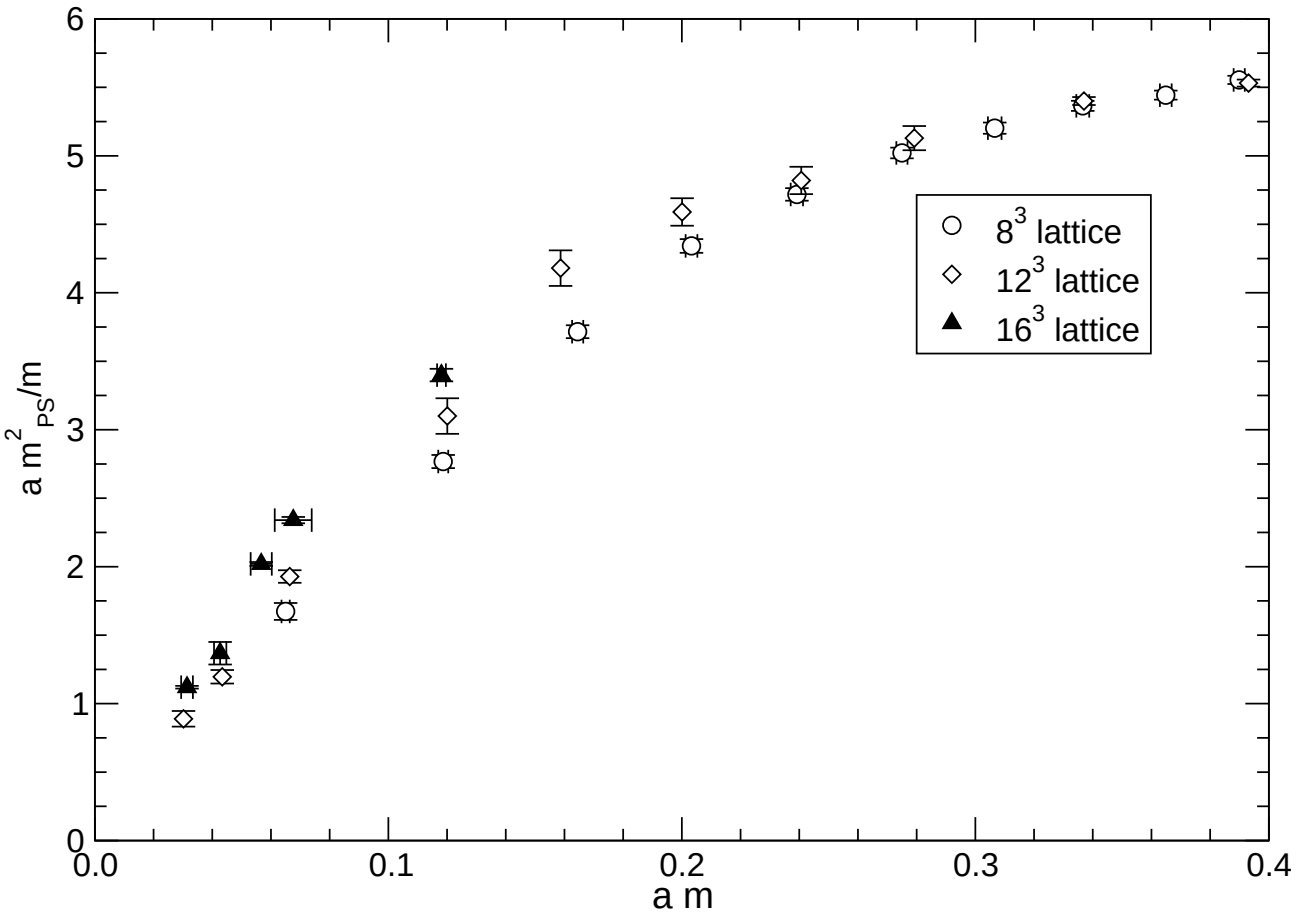


Comparison with other studies

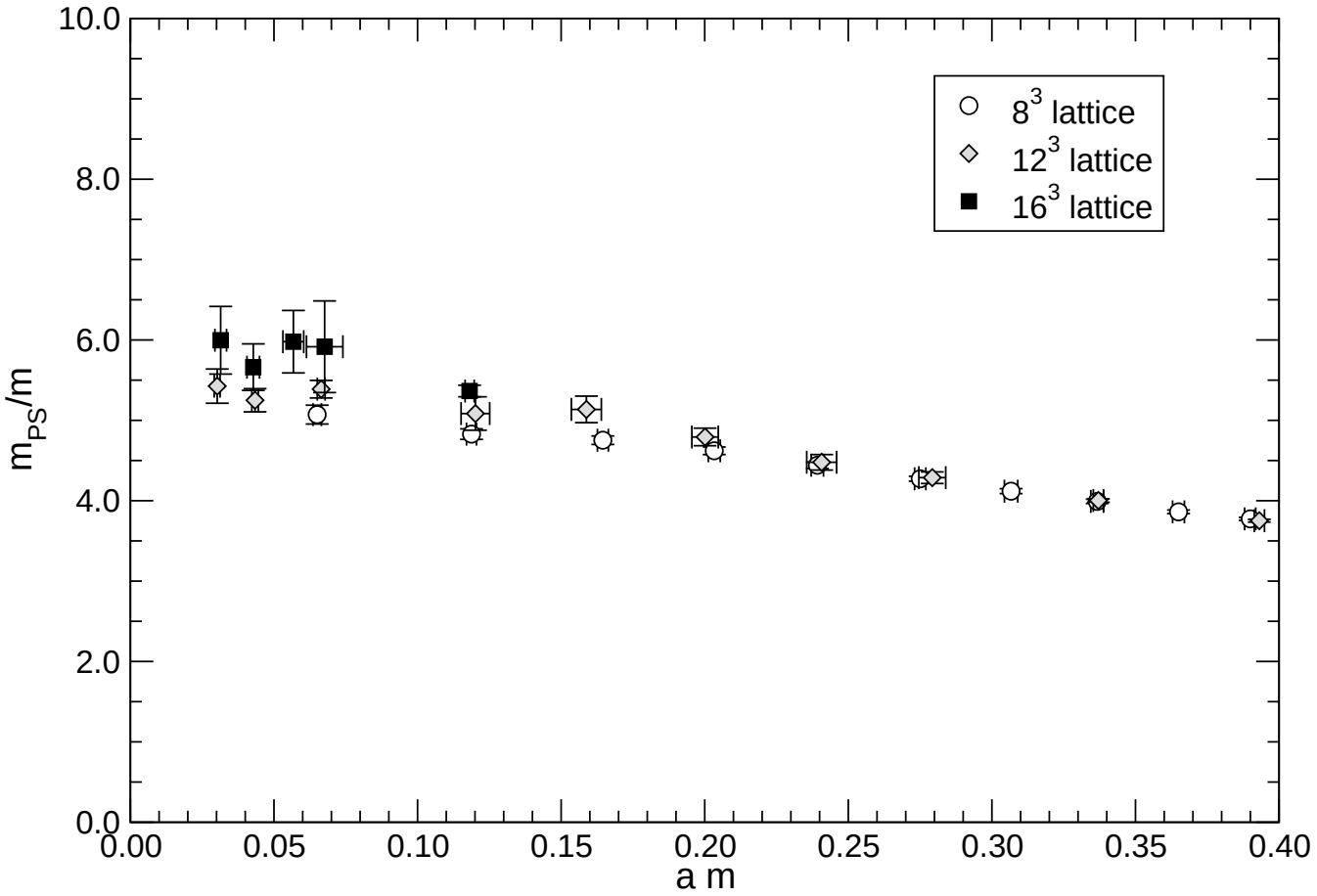


[rummukainen et al 08]

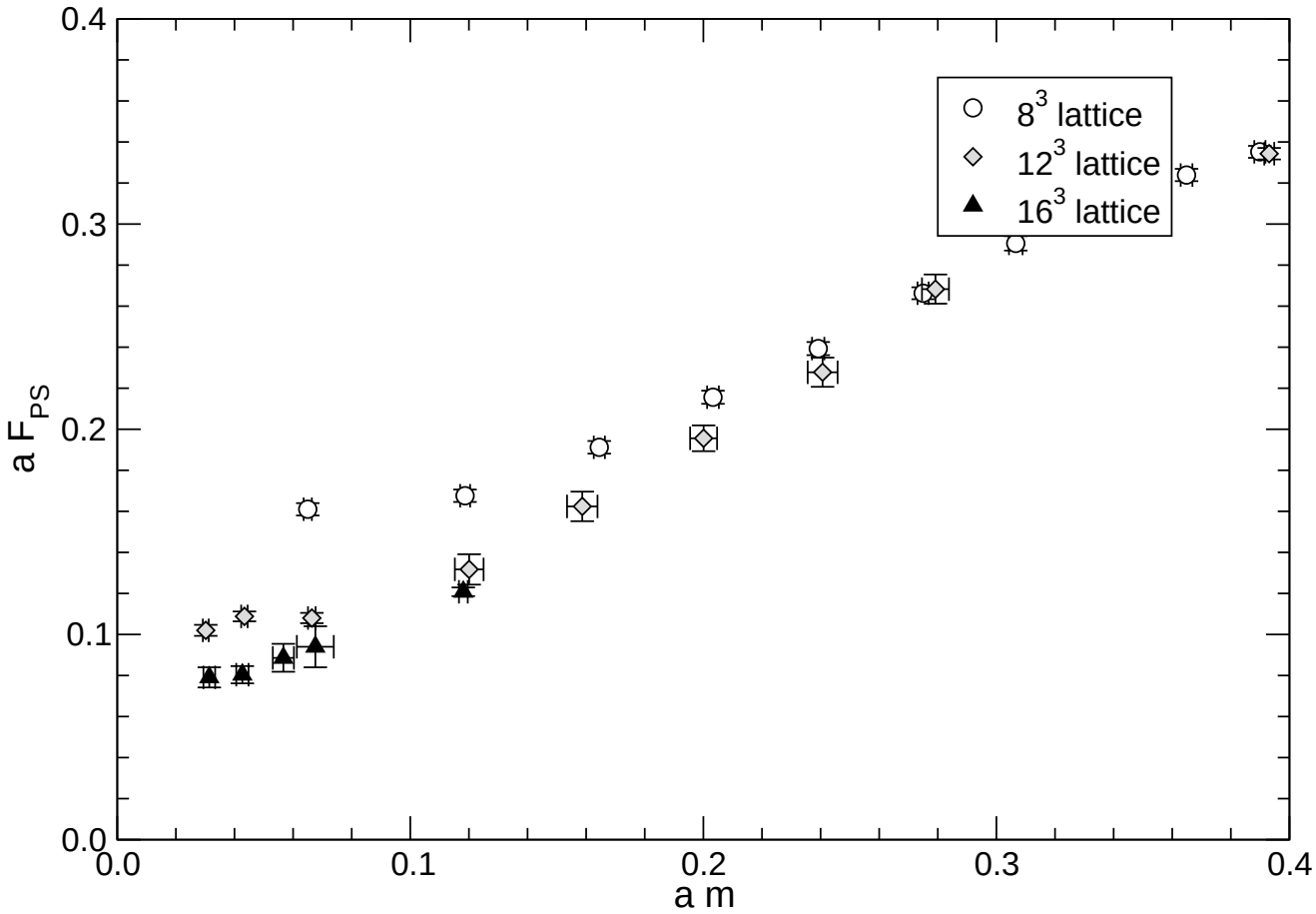
Pseudoscalar mass (3)



Pseudoscalar mass (4)

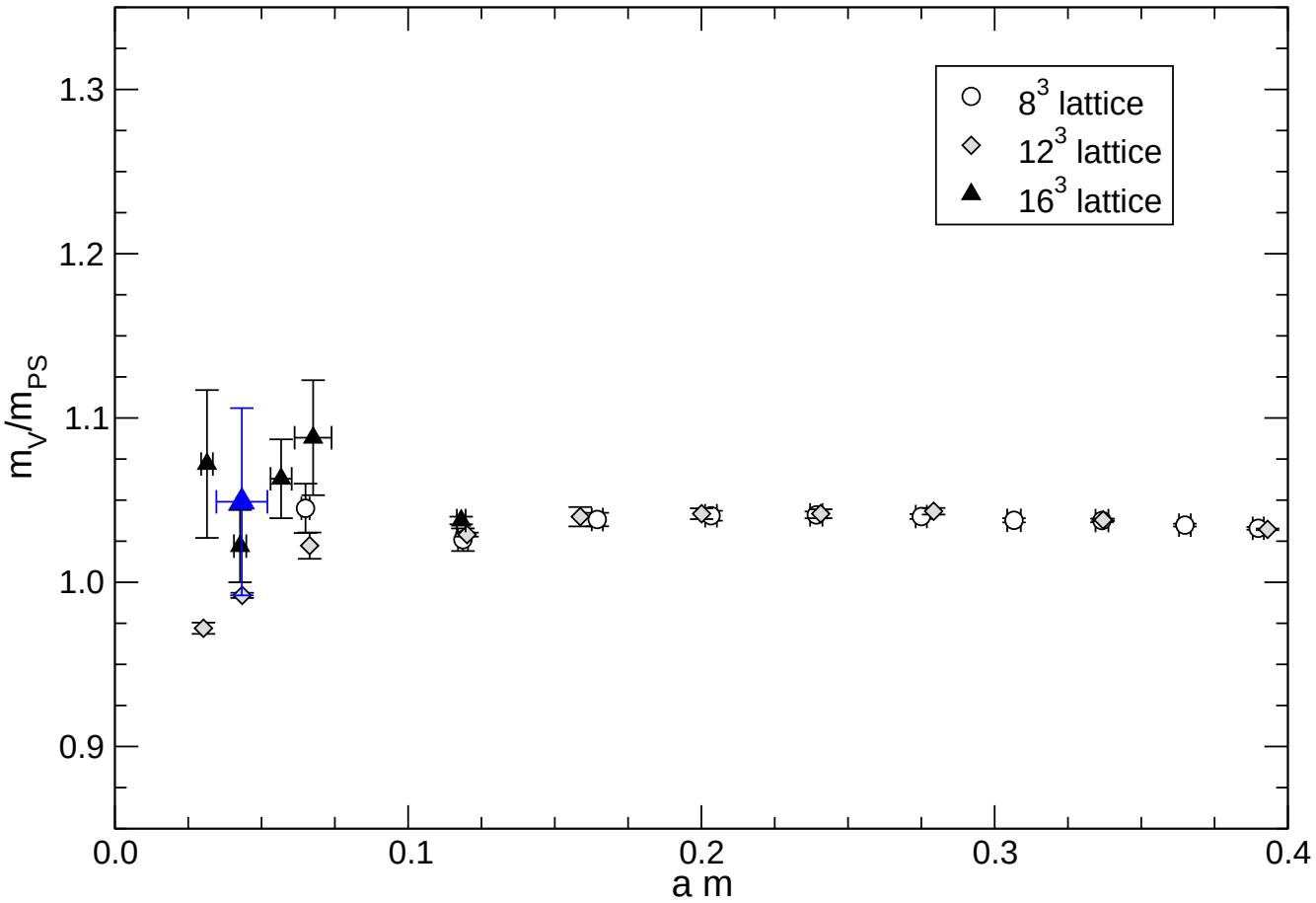


Decay constant

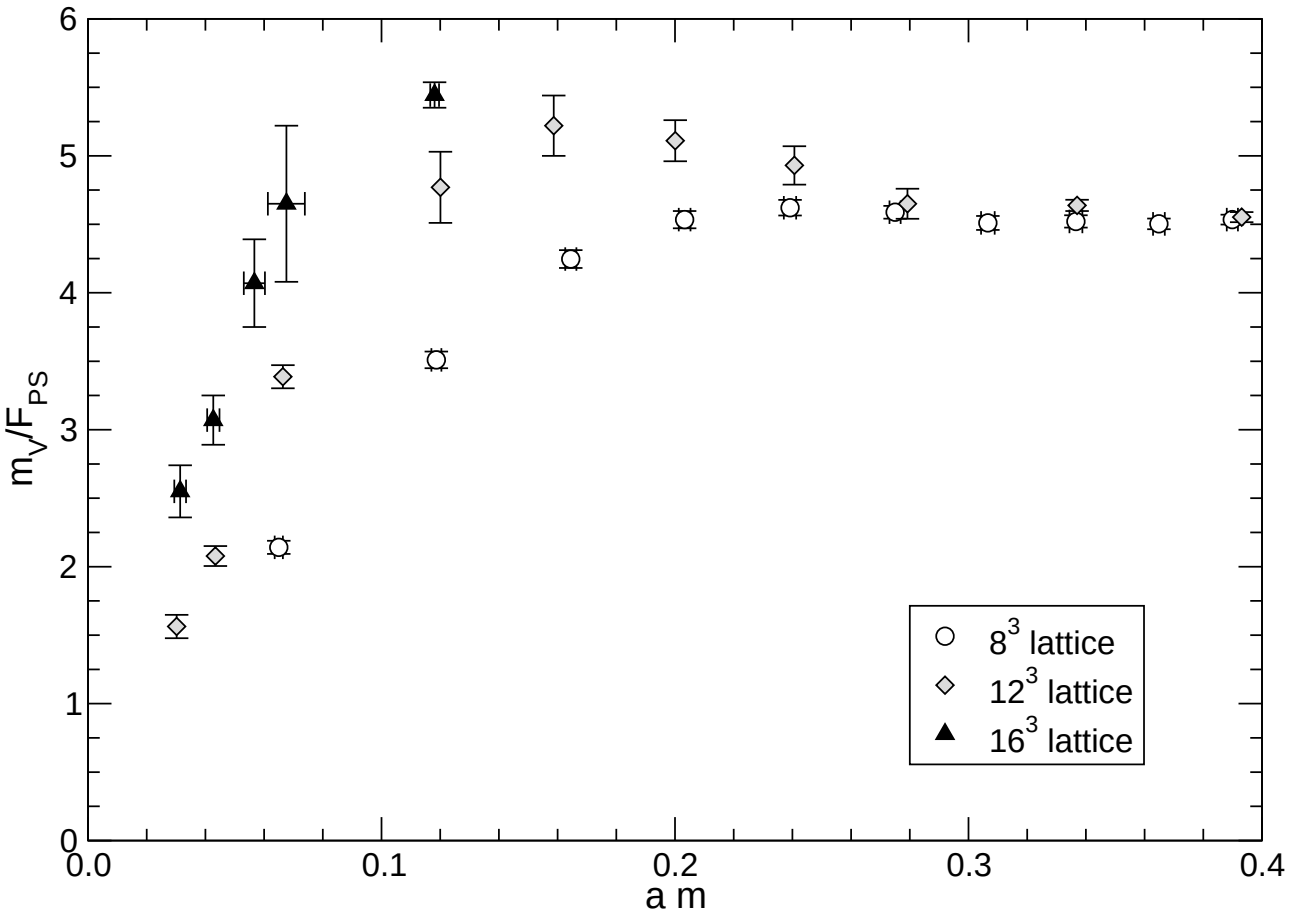


$$Z_A = 0.602 \Longrightarrow aF_{\rm PS} = 0.0612$$

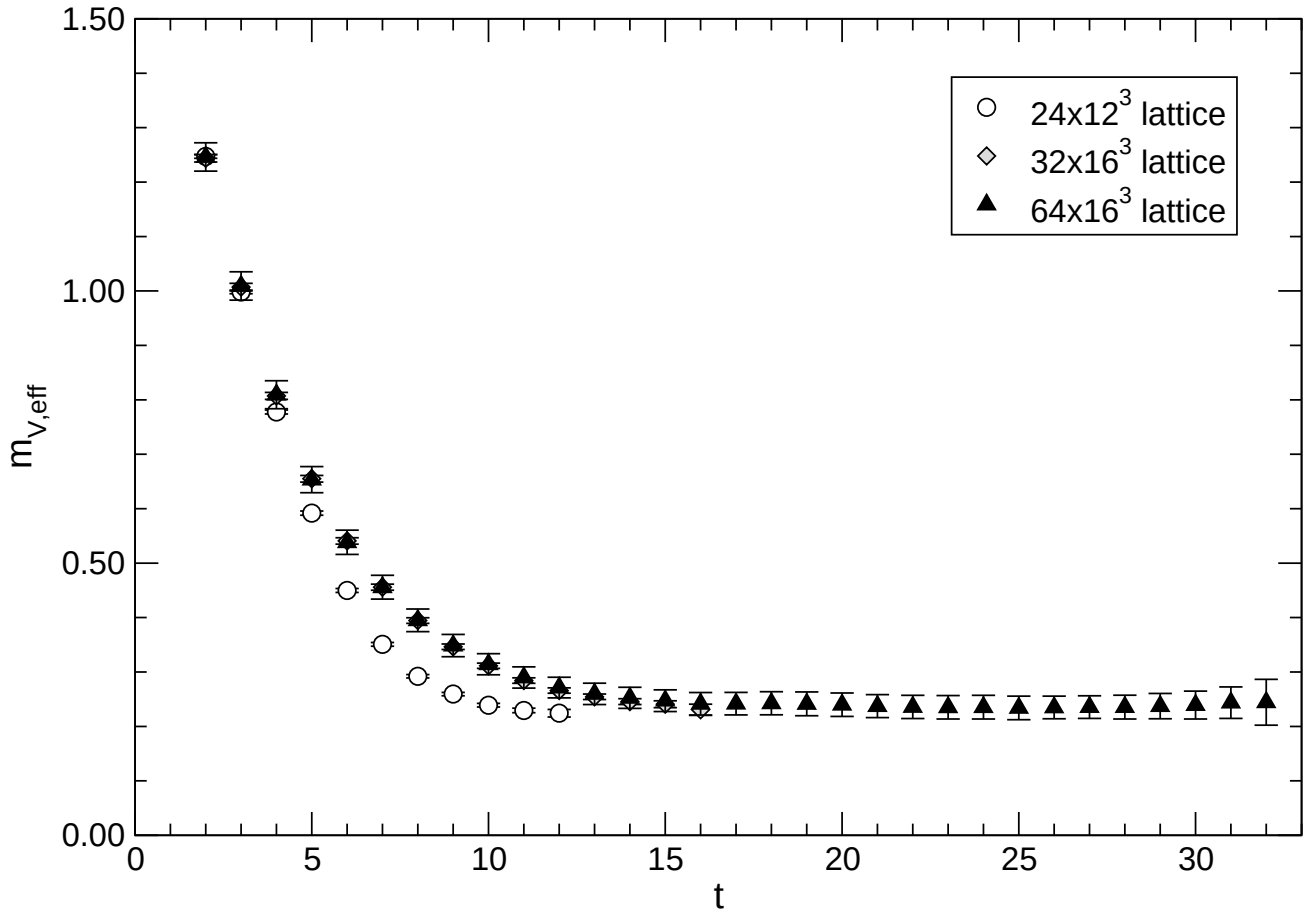
Vector Mass (1)



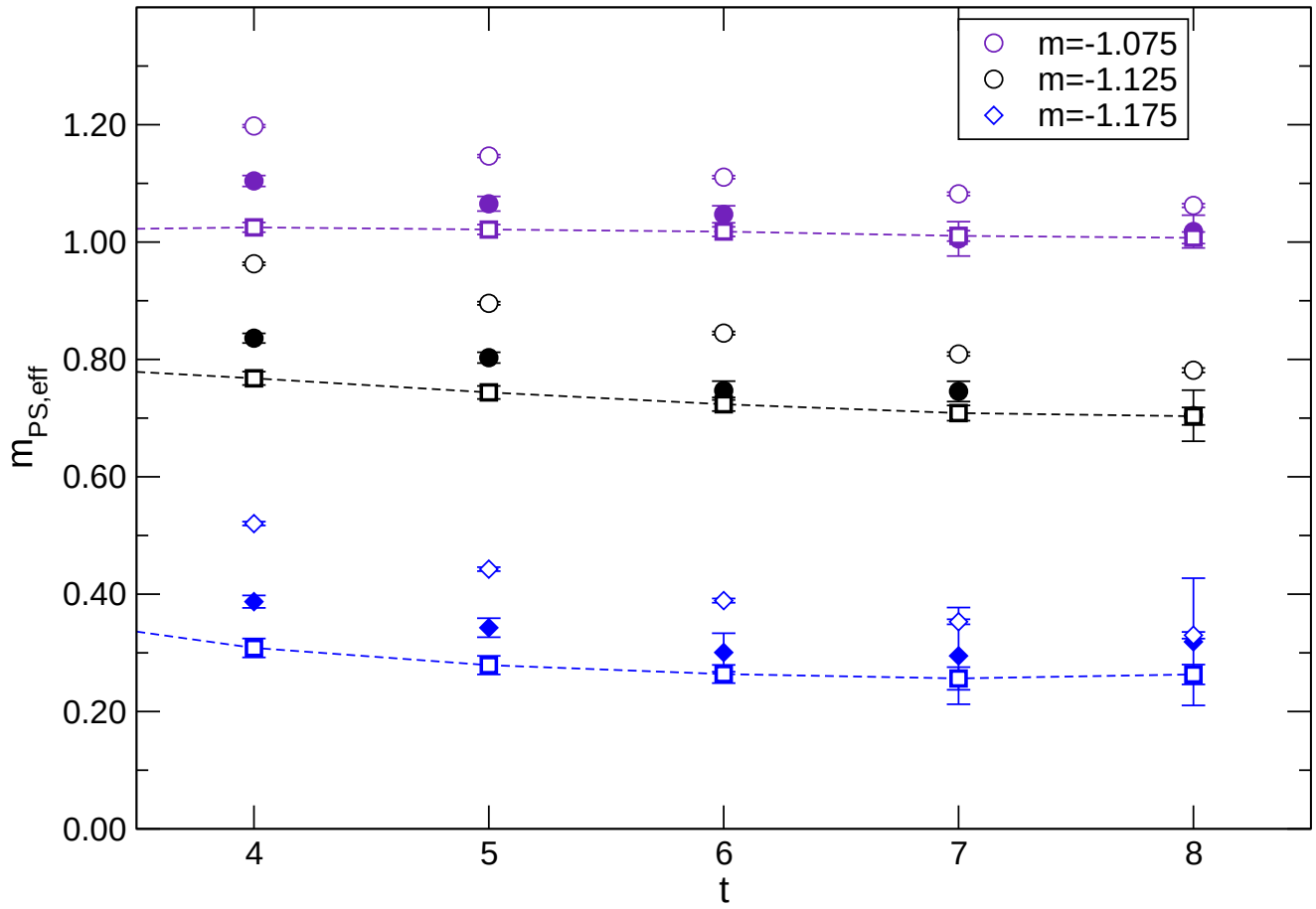
Vector mass (2)



Systematic errors



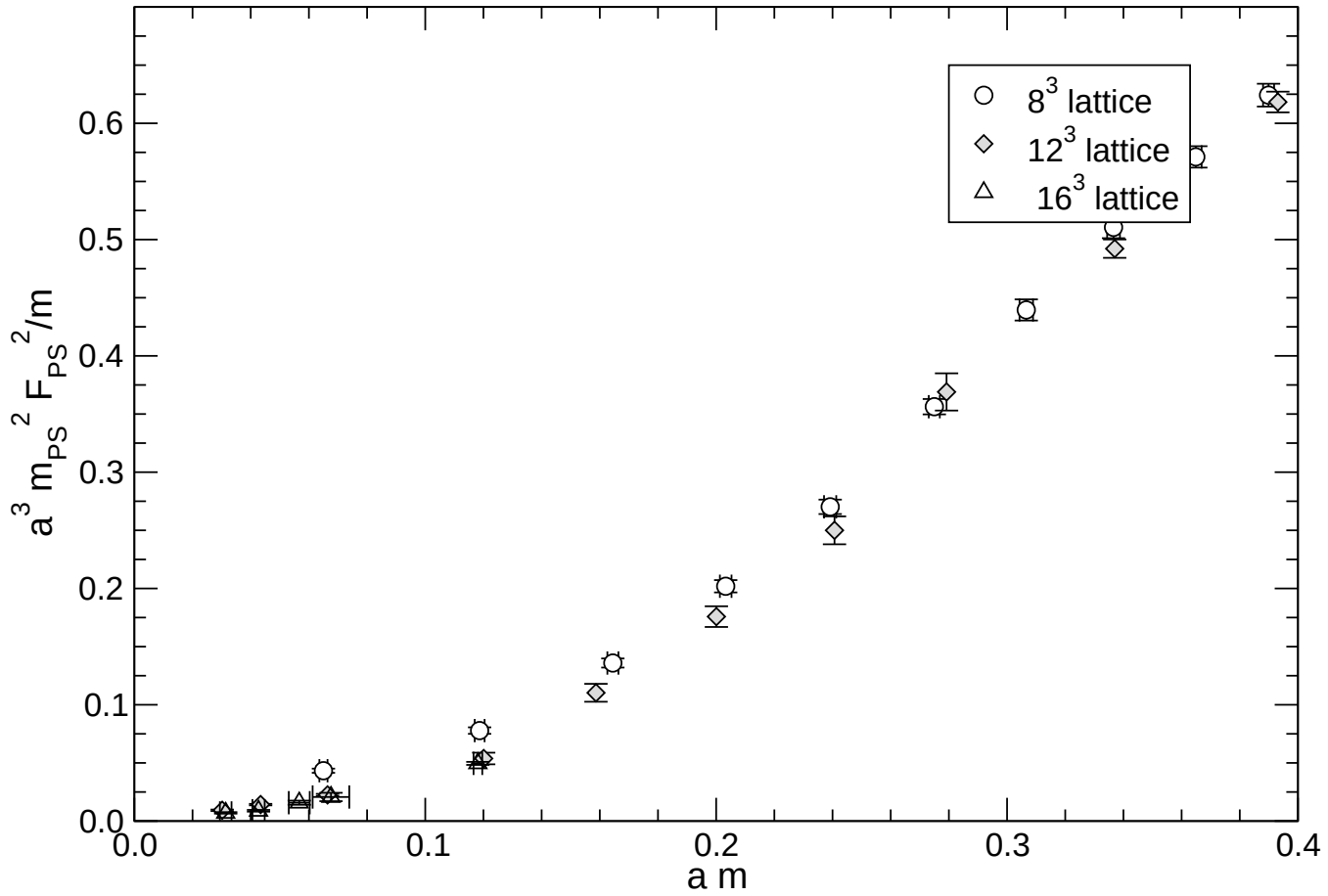
Systematic errors



data from 8^3 lattice – preliminary study

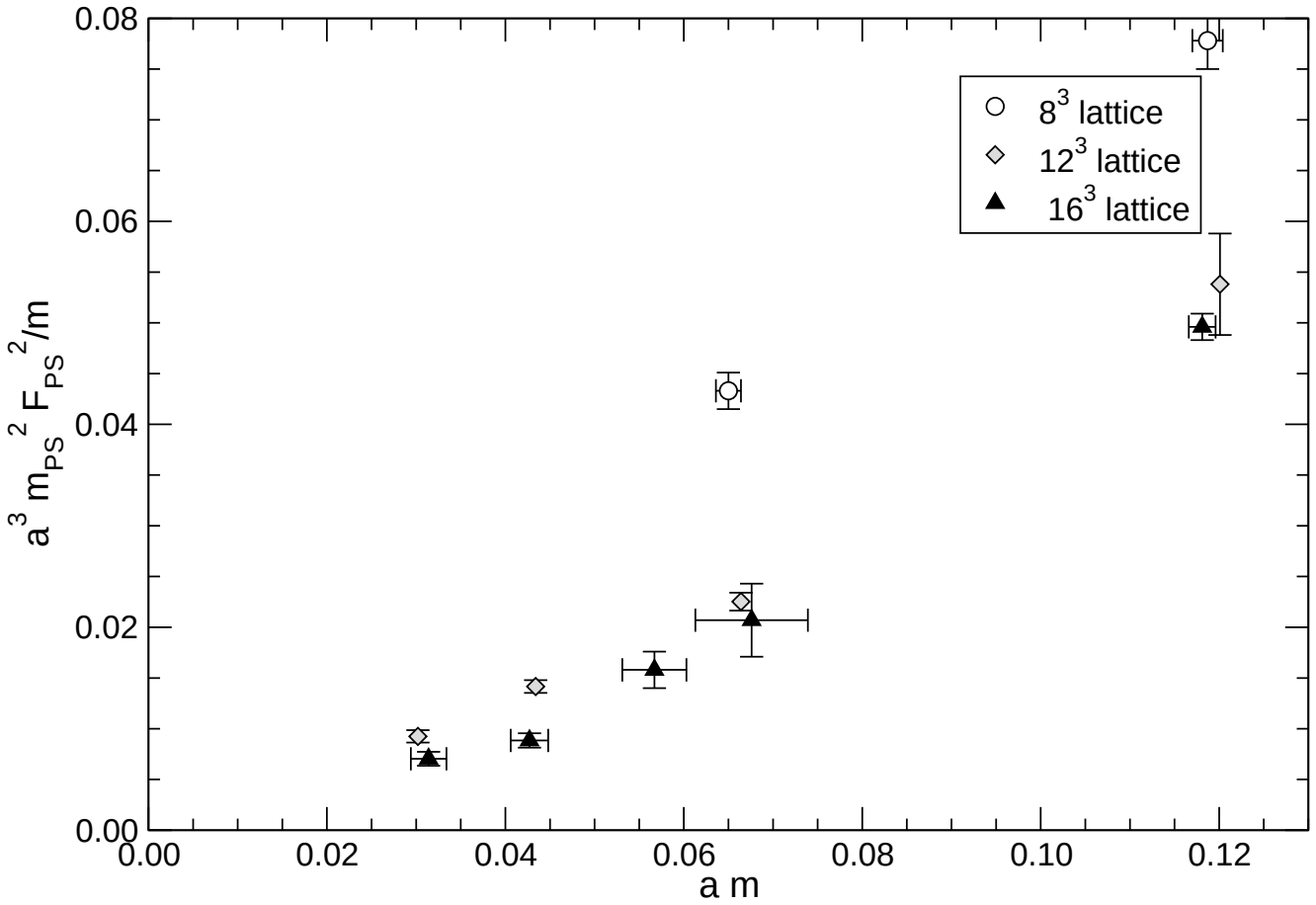
GMOR relation

$$m_\pi^2 f_\pi^2 = Bm$$



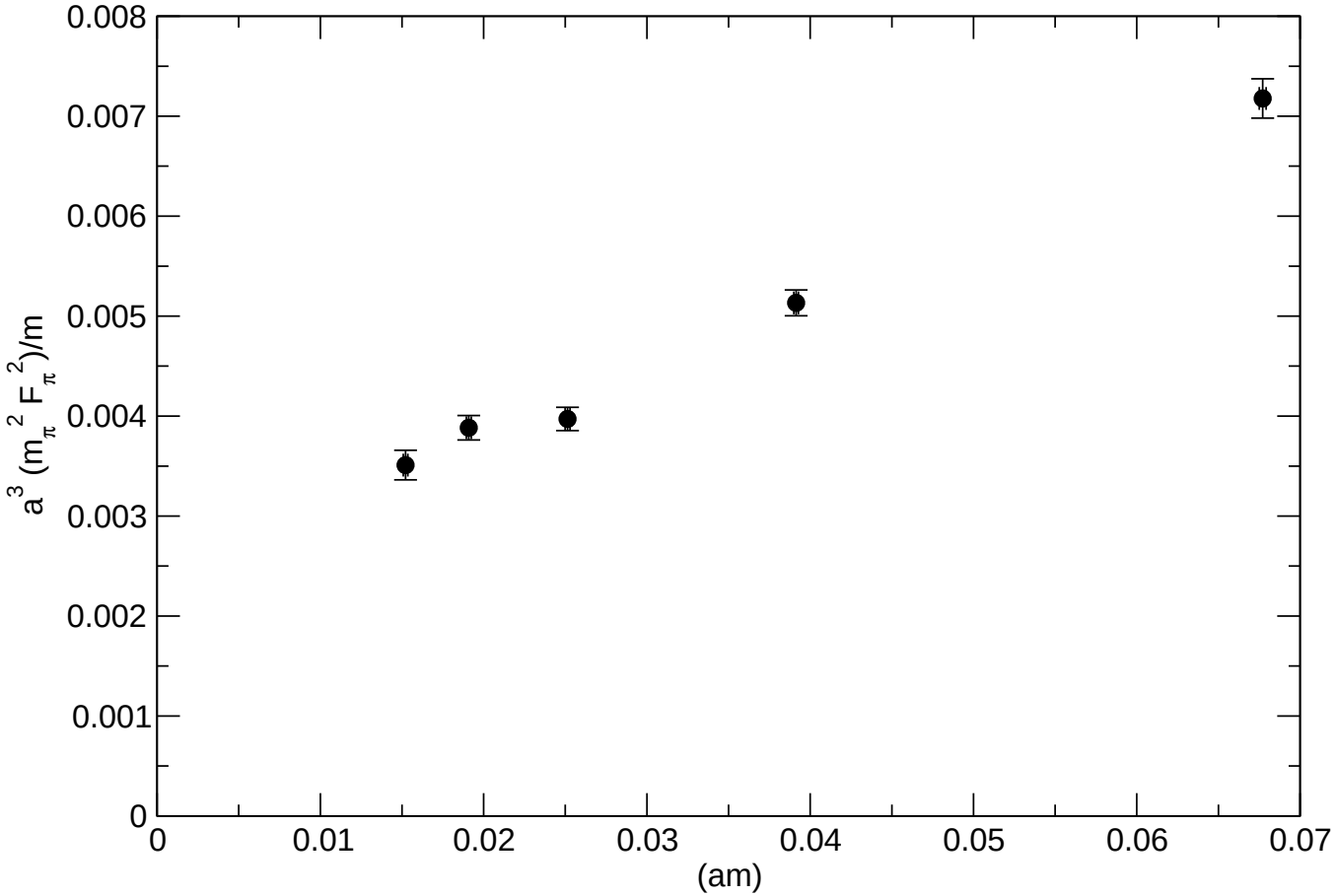
GMOR relation

$$m_\pi^2 f_\pi^2 = Bm$$



GMOR relation

$$m_\pi^2 f_\pi^2 = Bm$$



Conclusions

- generalization of the Dirac operator to higher–dim representations
- quenched spectroscopy
- simulations with dynamical quarks
- even/odd preconditioning, RHMC: arbitrary N and n_f
- importance of reaching the small mass regime!!
- control over the algorithmic performance

- first preliminary results $\longrightarrow$ larger volumes, smaller masses, scaling to control systematics
- improved spectroscopy $\longrightarrow$ smearing, scalar
- NP walking behaviour: Schrödinger functional

  [appelquist et al 07, degrand et al 08, rummukainen et al 09]

  Wilson loop, other schemes [bilgici et al 08]