Using Monte Carlo tools for hadron colliders

[Valencia Seminar]



Jan Winter

– Fermilab –



- *Sherpa Monte Carlo – version 1.1.* [a]
- *Dipole shower for hadronic collisions.*
- *Numerical calculation of one-loop gluon amplitudes using unitarity techniques.* [b]

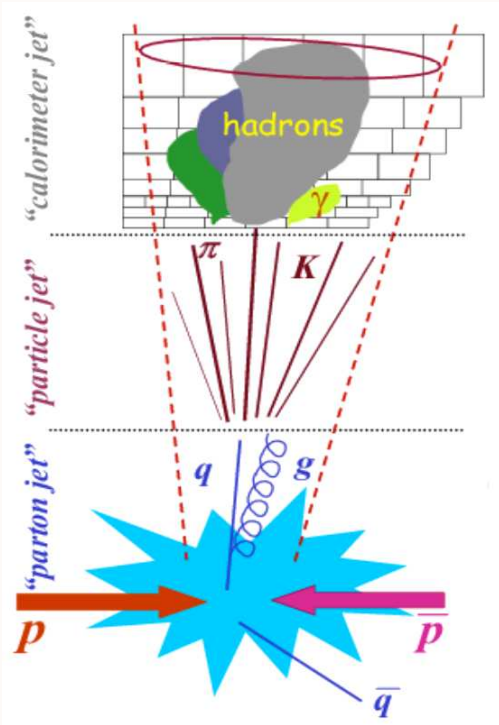
[a] Sherpa authors: T. Gleisberg, S. Höche, F. Krauss, M. Schönherr, S. Schumann, F. Siegert, J. Winter

[b] In collaboration with: W. Giele

http://www.sherpa-mc.de/

Physics at hadron colliders ...

... is largely influenced by jet physics.



- hadronic intial states: available phase space used to emit (additional) strong particles

→ ***Multijets***

- modify signals and backgrounds

Need for tools that model...!!!

Jet production → ***Jet evolution*** → ***Hadronization***

Sherpa project:

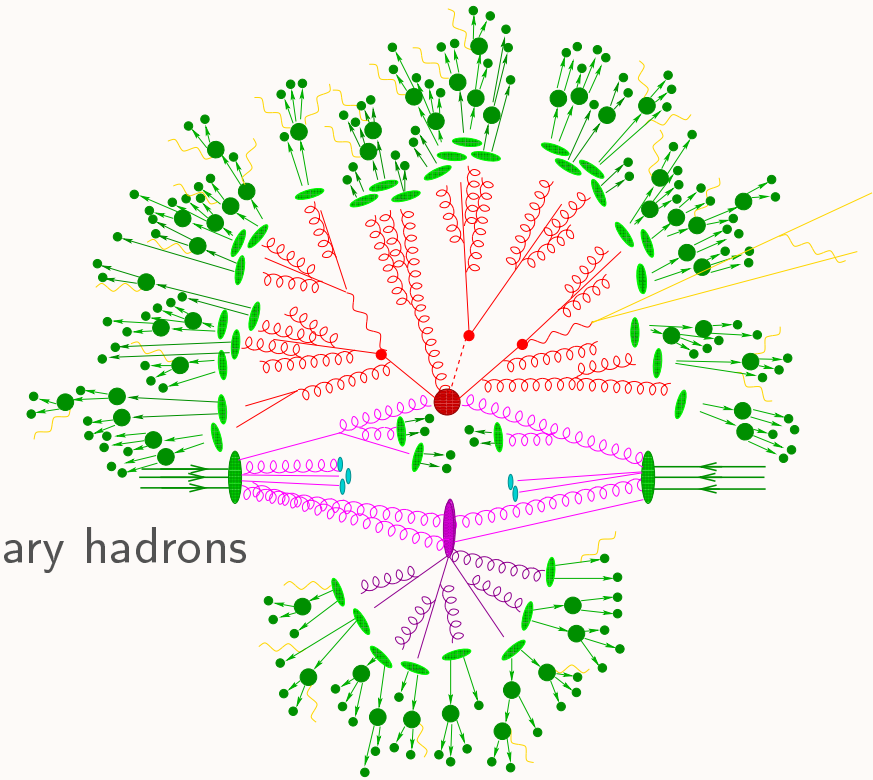
- → *provide a full-fledged, independent Monte Carlo event generator for collider physics – Sherpa 1.1* ✓
- → *with focus on an improved modelling of QCD multijet final states*
- → *provide it as new (C++) code, which is modular, transparent, upgradeable and maintainable*

Monte Carlo picture of jet evolution

➔ ***Factorization approach: divide jet simulation into different phases***

➔ ***Perturbative Phases:*** *[parton jets]*

- ***Hard process/interaction (hard jet production)***
  exact matrix elements $|\mathcal{M}|^2$
- ***QCD bremsstrahlung (soft/coll multiple emissions)***
  initial- and final-state parton showering
- ***Multiple/Secondary interactions***
  modelling the underlying event



➔ ***Non-perturbative Phases:*** *[jet confinement – particle jets]*

- ***Hadronization***
  phenomenological models to convert partons into primary hadrons
- ***Hadron decays***
  phase-space or effective models to decay unstable into stable hadrons as observed in detectors

➔ predictions at hadron level – comparable to experimental data if corrected for detector effects

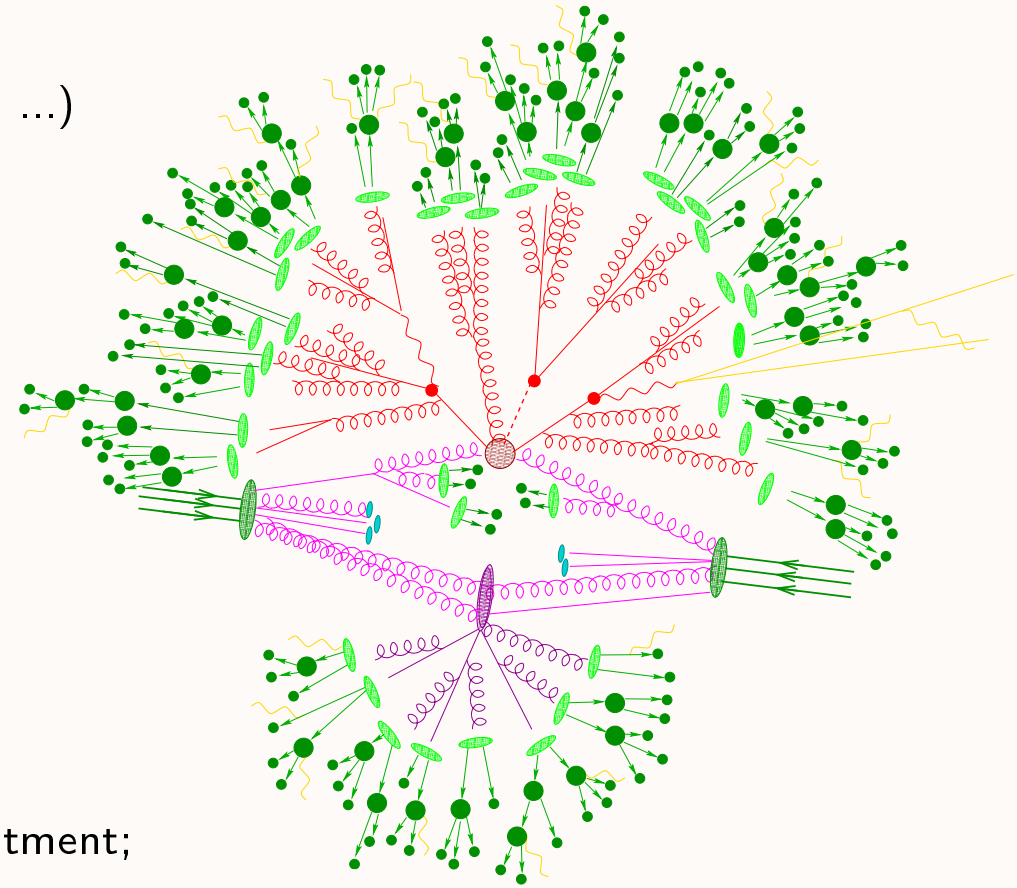
Monte Carlo event generator Sherpa (1.1)

[Gleisberg, Höche, Krauss, Schälicke, Schumann, Winter, JHEP 02 (2004) 056]

[Gleisberg, Höche, Krauss, Schönherr, Schumann, Siegert, Winter, arXiv:0811.4622]

➡ ***Current version: SHERPA 1.1.3 (released Dec/08).***

- ***Tree-level ME generator: AMEGIC***
  (SM+ggH+AGC+4th family, THDM, MSSM, ADD, ...)
- ***IS + FS showers: APACIC***
  virtuality ordered, Pythia-like showers
- ➡ ***ME-PS merging according to CKKW***
- ***Multiple parton interactions: AMISIC***
  based on Sjöstrand–Zijl model
- ***Hadronization: AHADIC***
  cluster-fragmentation model;
  (optional: interface to Pythia's string model)
- ***Hadron decays: HADRONS + PHOTONS***
  [Schönherr, Krauss, arXiv:0810.5071];
  comprehensive hadron & τ decays libs plus YFS treatment;
  (optional: interface to Pythia's decays)



➲ predictions at hadron level – comparable to experimental data if corrected for detector effects

ME+PS merging – CKKW method

[CATANI, KRAUSS, KUHN, WEBBER, JHEP 11 (2001) 063]

[KRAUSS, JHEP 08 (2002) 015]

- *combine parton-shower pros (soft emissions)* +

  ME pros (hard emissions, quantum interferences, correlations)
- *avoid double counting and missing phase space regions*

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Divide multijet phase space into two regimes by k_T jet measure at Q_{jet}.

- tree-level MEs: jet seed (hard parton) production above Q_{jet}
- parton showers: (intra-)jet evolution $Q_{jet} > Q > Q_{\mathrm{cut-off}}$
- MEs regularized by $Q_{ij}^2 = 2\min\left\{p_T^{(i)}, p_T^{(j)}\right\}^2 \frac{\left[\cosh(\eta^{(i)}-\eta^{(j)})-\cos(\phi^{(i)}-\phi^{(j)})\right]}{D^2} > Q_{jet}^2$

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Eliminate/sizeably reduce Q_{jet} dependence.

- identify a possible shower history of MEs via backward clustering
- accordingly reweight MEs by combined α_s and Sudakov weight
- add showers to ME partons and veto emissions above Q_{jet}

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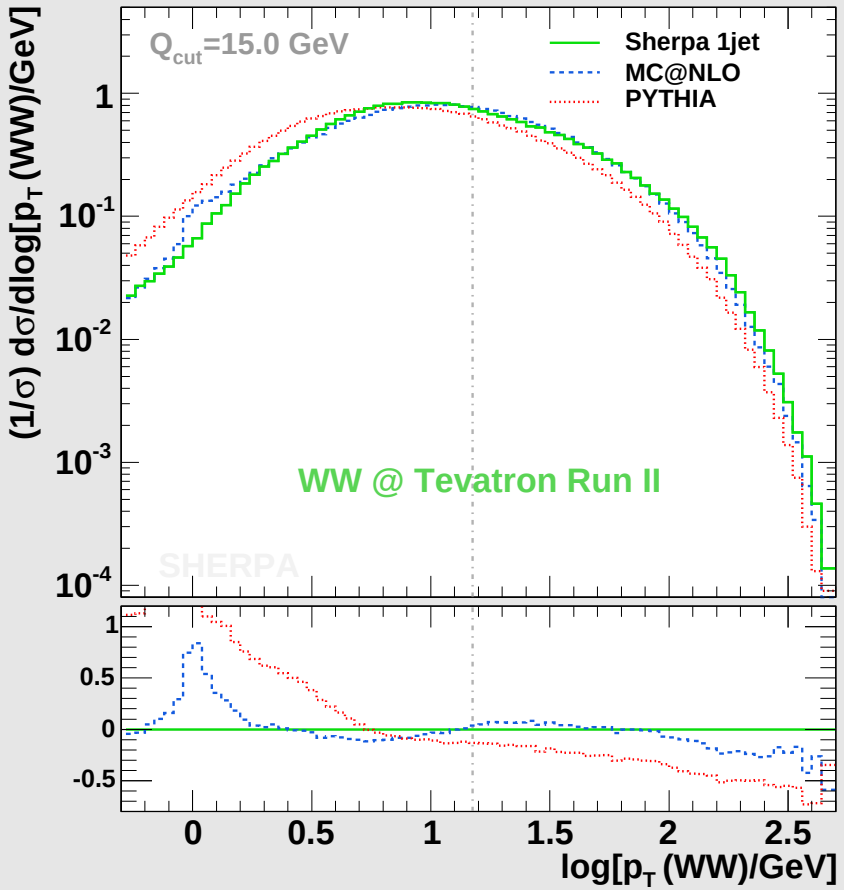
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Eliminate/sizeal

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E.G. DIBOSONS + JETS

GLEISBERG ET AL., PRD72 (2005) 034028



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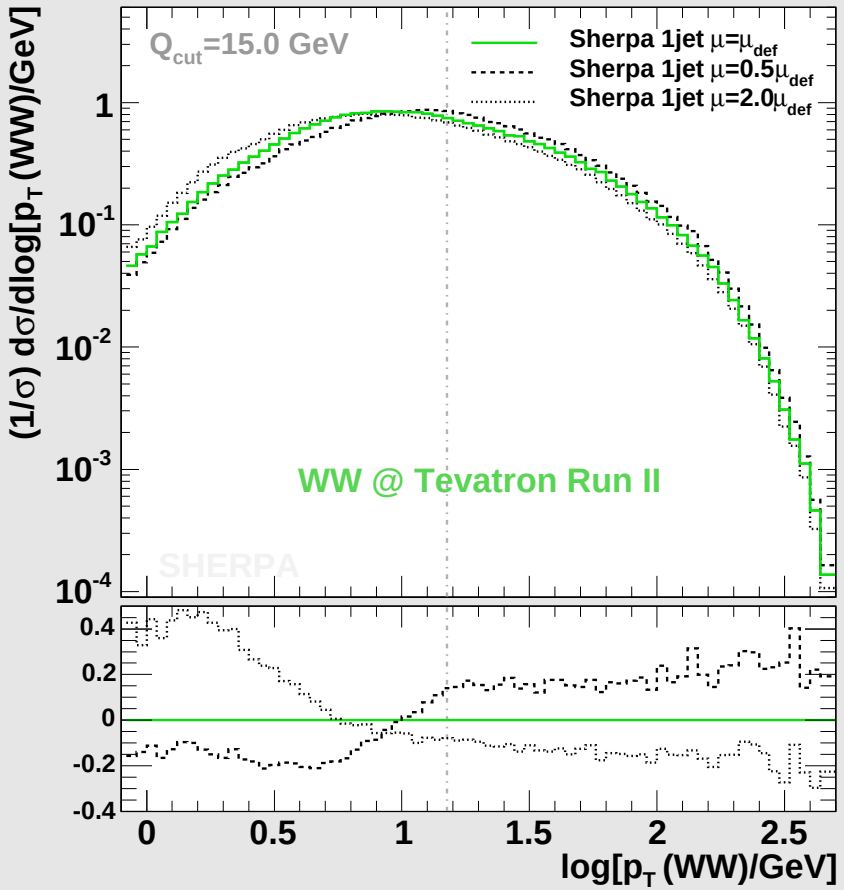
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E.G. DIBOSONS + JETS – SCALE VARIATIONS

GLEISBERG ET AL., PRD72 (2005) 034028



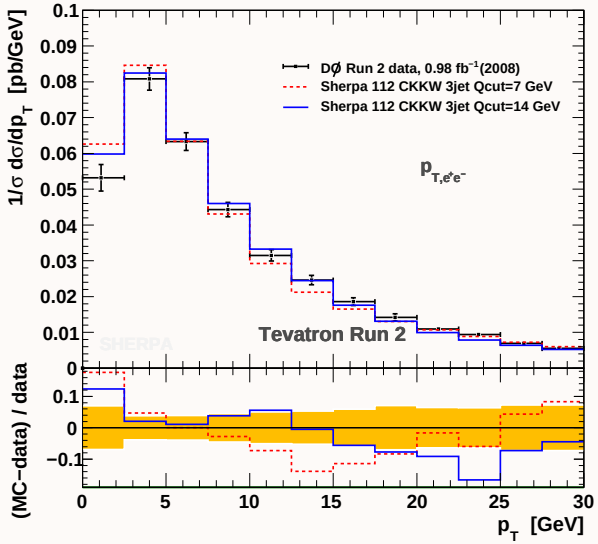
CKKW – key feature of Sherpa

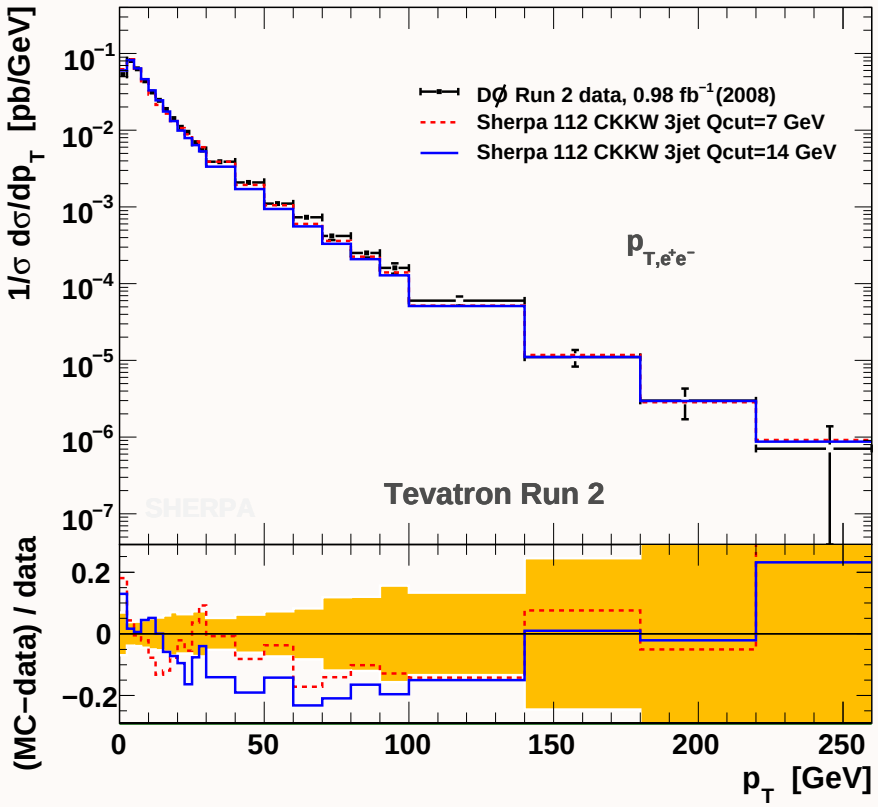
- *Method has been implemented within Sherpa in full generality.*
- *Uses built-in ME generator AMEGIC.*
- *Process-independent implementation.*
- ***Validation and applications***

  $W/Z/WW+$jets, QCD jets, $Zb\bar{b}+X$, VBF, tops, $gg \to H+X$, b-associated Higgs

Recent validation [Abazov et al., PRL **100** (2008) 102002]

- shape comparison
- intrinsic k_T-smearing of order 1 GeV

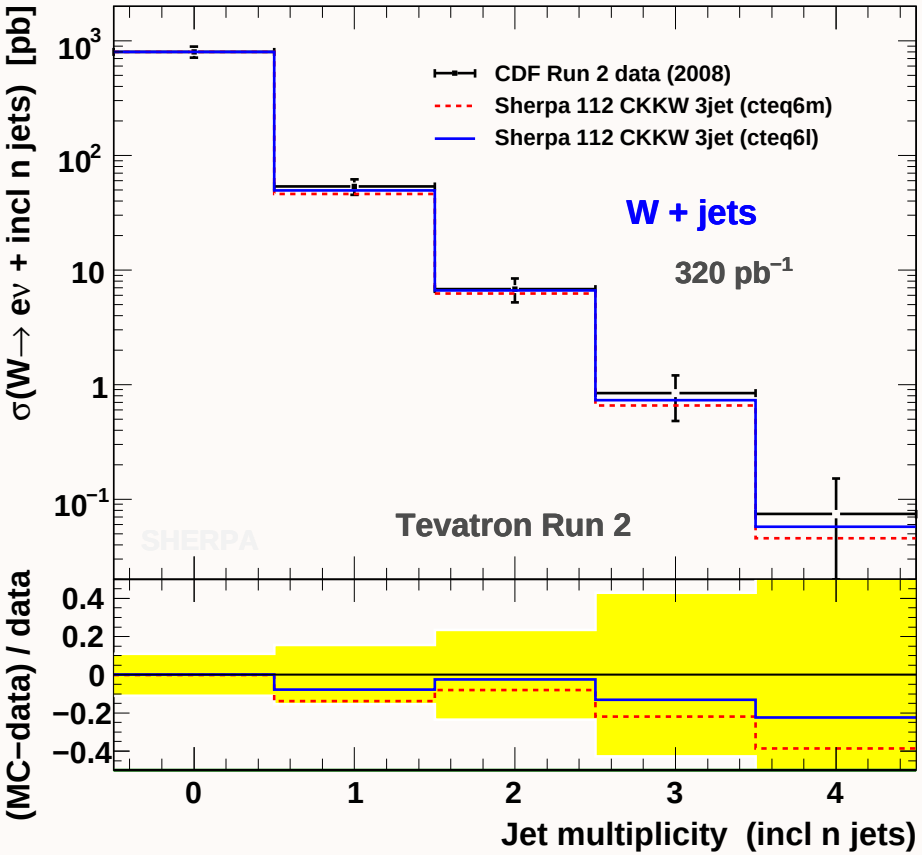


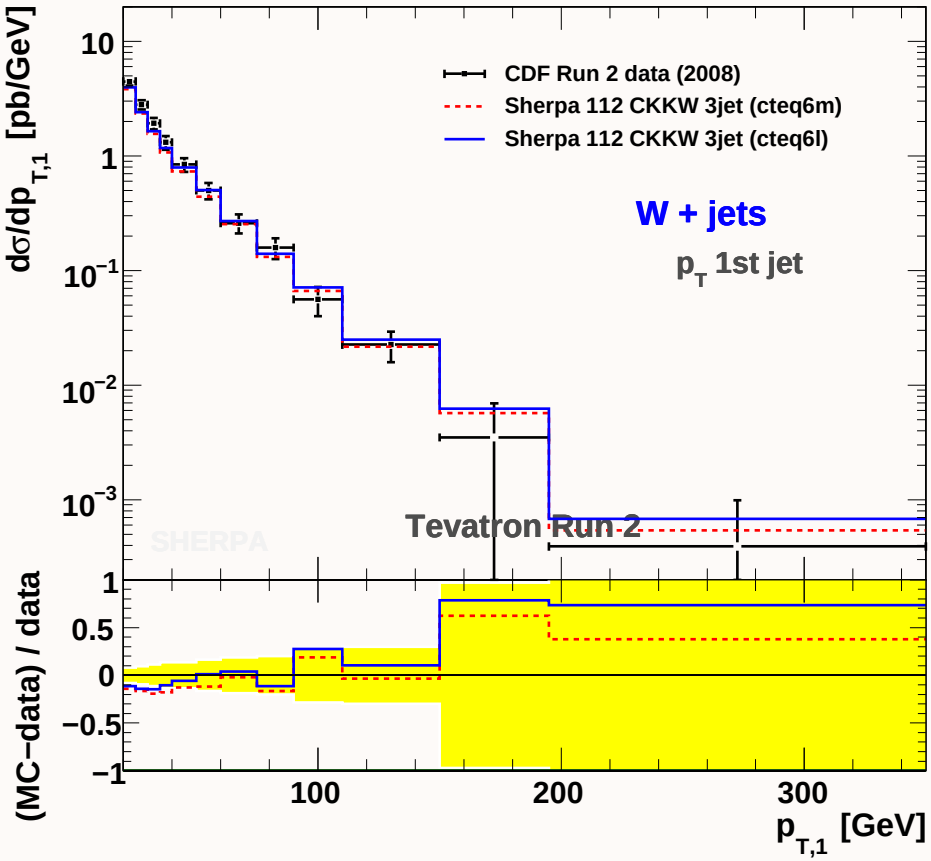


Comparison with recent CDF data: W+jets production

[T. AALTONEN ET AL., PRD **77** (2008) 011108]

- Monte Carlos need to be validated and tuned against most recent Tevatron data.
- Discriminating power increases as more and more data is coming in. Use to refine algorithms.
- Sherpa predictions normalized to total inclusive cross section. Two choices of PDFs.

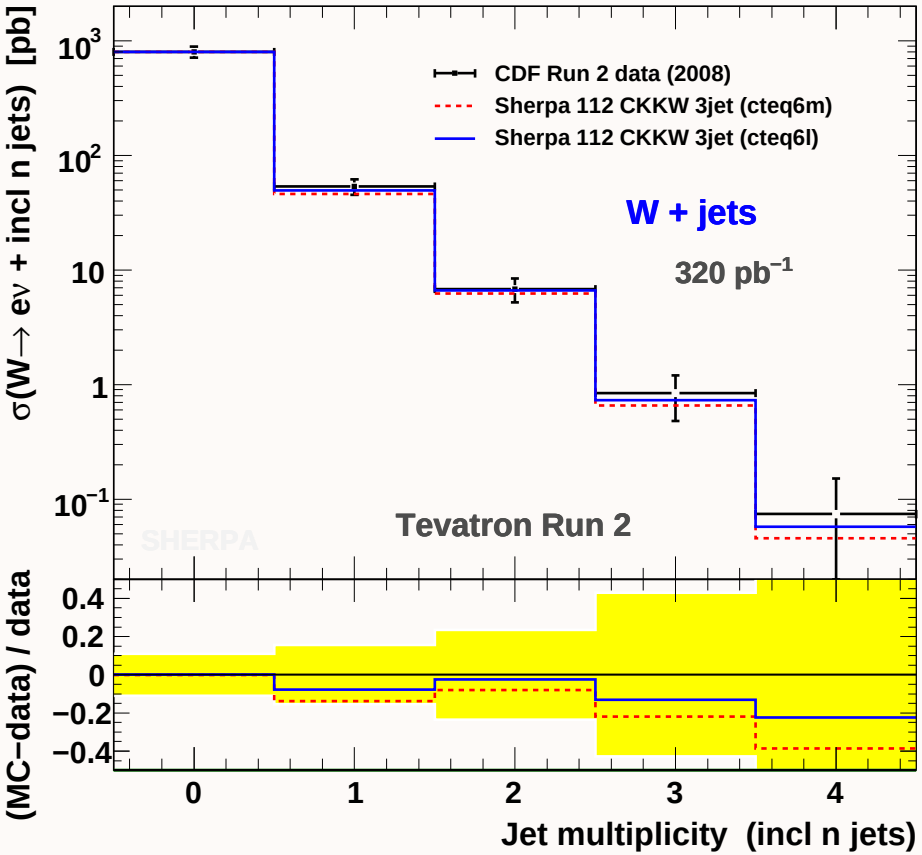


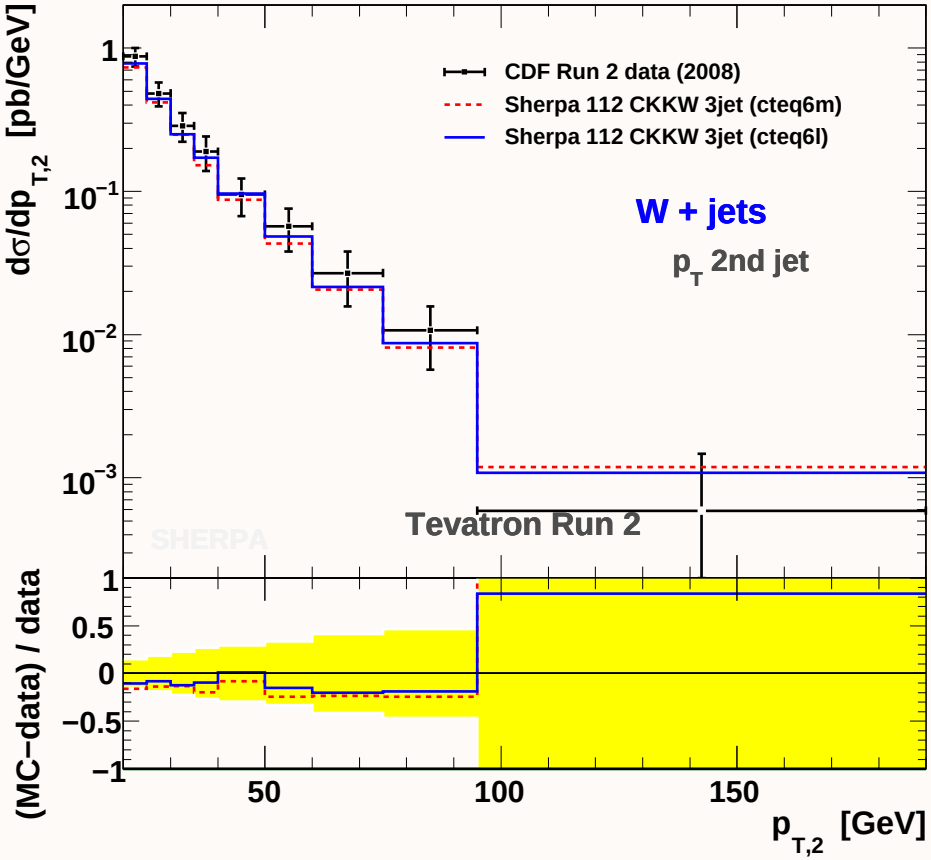


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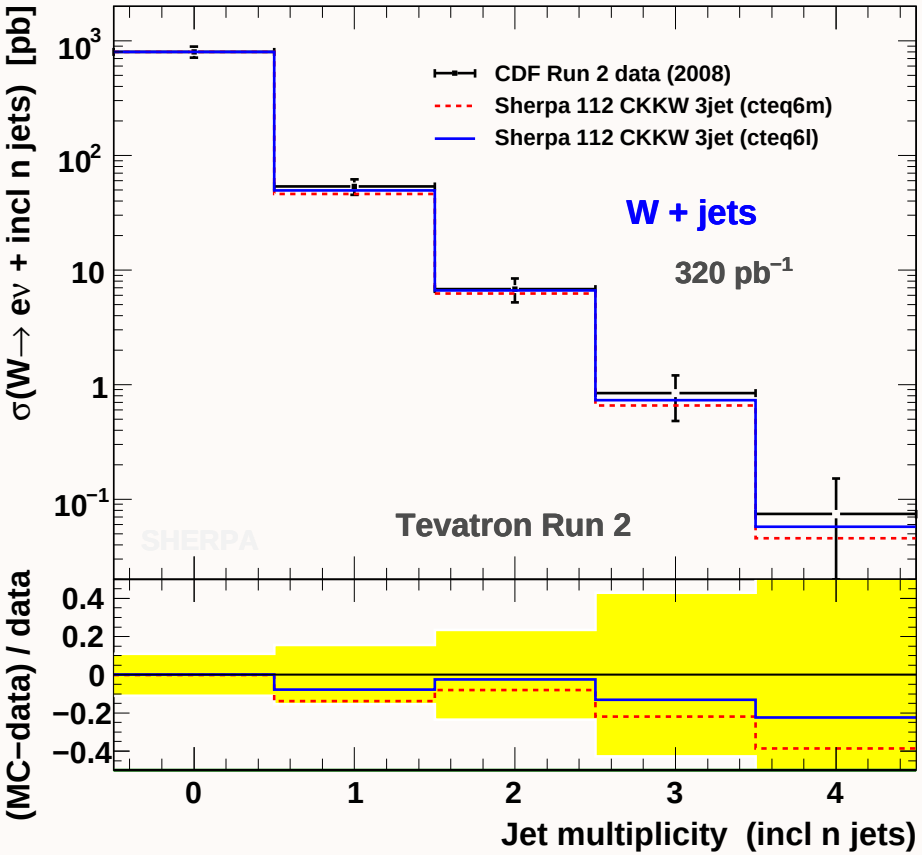


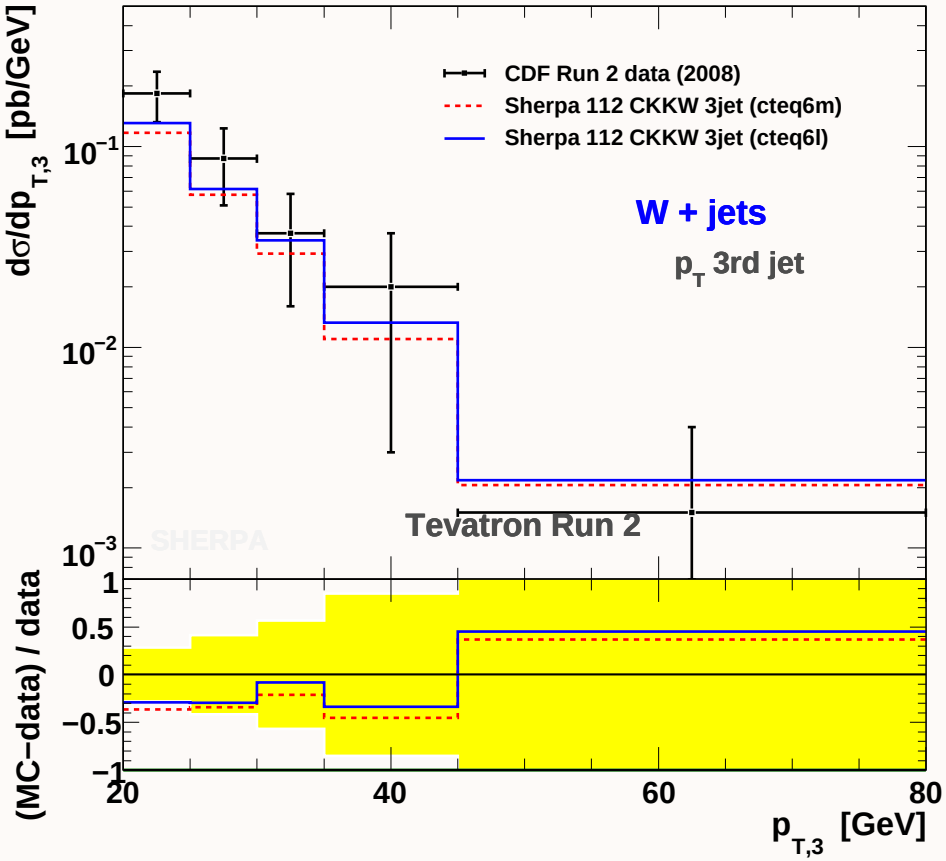


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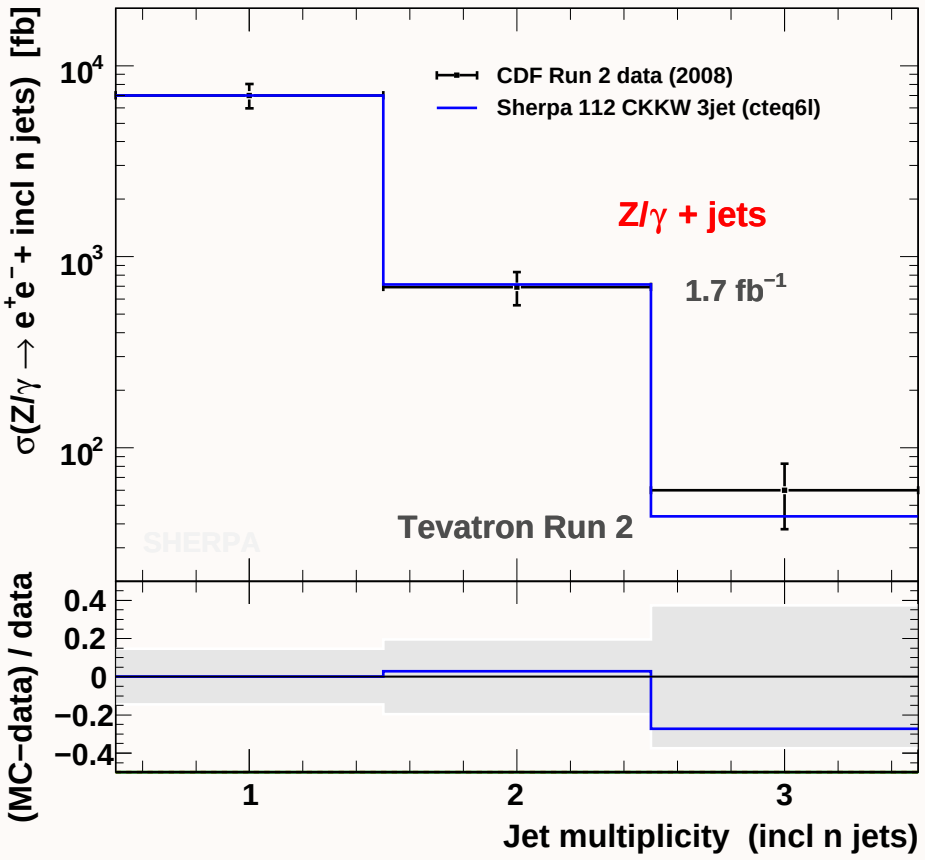


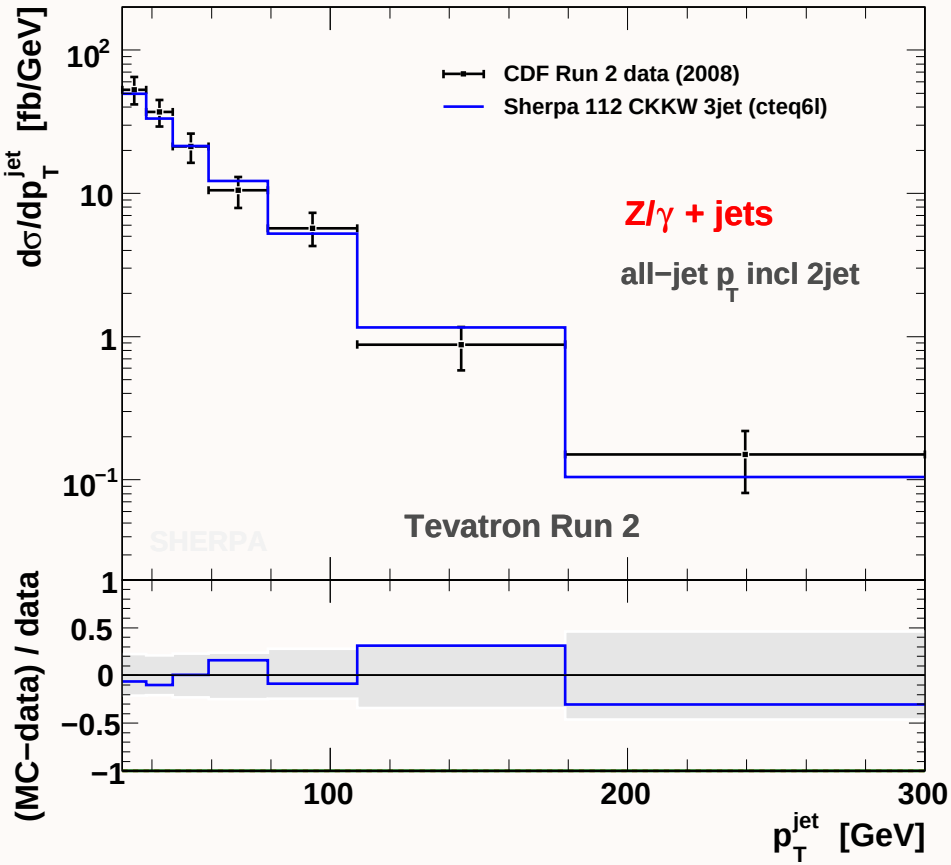


Comparison with recent CDF data: Z+jets production

[T. Aaltonen et al., PRL **100** (2008) 102001]

- Monte Carlos need to be validated and tuned against most recent Tevatron data.
- Discriminating power increases as more and more data is coming in. Use to refine algorithms.
- Sherpa predictions multiplied by constant K factor.





Sherpa projects (beyond 1.1):

new developments ...

- ***COMIX*** *– tree-level SM ME generator based on colour-dressed Berends–Giele recursion relations.* [GLEISBERG, HÖCHE, ARXIV:0808.3675]
- *Automated Catani–Seymour dipole subtraction – dipole & real-correction terms, phase-space integration, NLO PL event generator if interface to one-loop MEs.* [GLEISBERG, KRAUSS, EPJC 53 (2008) 501]
- *Parton shower based on Catani–Seymour dipole factorization – universal dipole terms describe parton splittings, invertible momenta mappings.* [SCHUMANN, KRAUSS, JHEP 03 (2008) 038]
- ➡ *Colour-dipole shower for hadronic collisions.*
- *Multiple-interactions model based on k_T-factorization (BFKL evolution).* [HÖCHE, KRAUSS, TEUBNER, ARXIV:0705.4577].

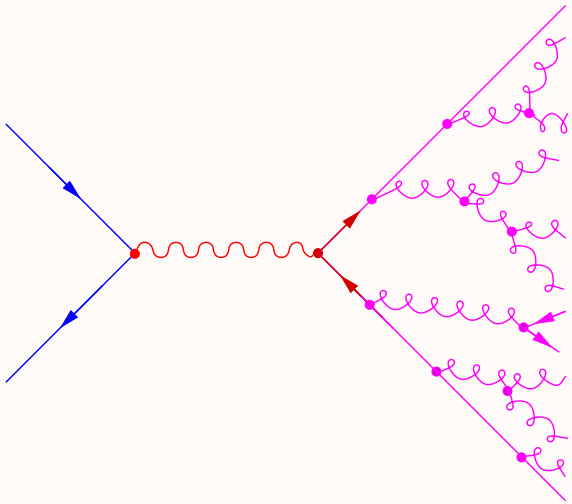
New colour-dipole model for hadronic collisions.

Parton shower concept

Traditional approach: describe additional jet activity by parton showers.

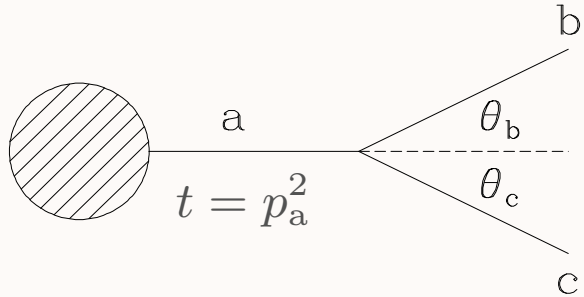
➡ ***QCD emissions preferably populate collinear and soft phase-space regions.***

[Pythia, Herwig, Ariadne]



- QCD amplitudes factorize in the coll/soft limit.
- ➡ Recursive definition of multiple emissions:

$$d\sigma_{n+1} \;=\; d\sigma_n \,\frac{\alpha_s(t)}{2\,\pi}\,\frac{dt}{t}\,dz\,P_{a\to bc}(z) \qquad \text{(here, coll limit)}$$



- coll/soft parton emissions iteratively added to the initial/final states *[LL resummation]*
- good description of bulk of radiation and particle multiplicity growth
- partonic ensemble evolved down to hadronization scale *[ordering variable Q, ϑ, p_T]*
  - ➡ provides suitable input for universal hadronization models *[$\mathcal{O}(1\,\text{GeV})$]*

Lund colour-dipole model

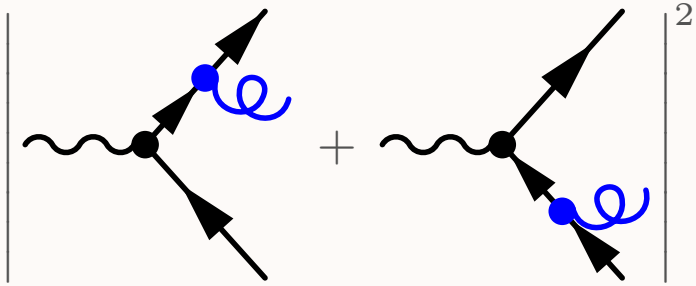
AZIMOV, DOKSHITZER, KHOZE, TROYAN / GUSTAFSON, PETTERSSON, ANDERSSON, LÖNNBLAD

Interesting alternative to conventional parton showers.

Recursive principle applied in limit of soft gluon emissions → soft gluons intrinsically correct; colour coherent radiation pattern

- QCD $q\bar{q}$ antenna (x_i energy fractions)

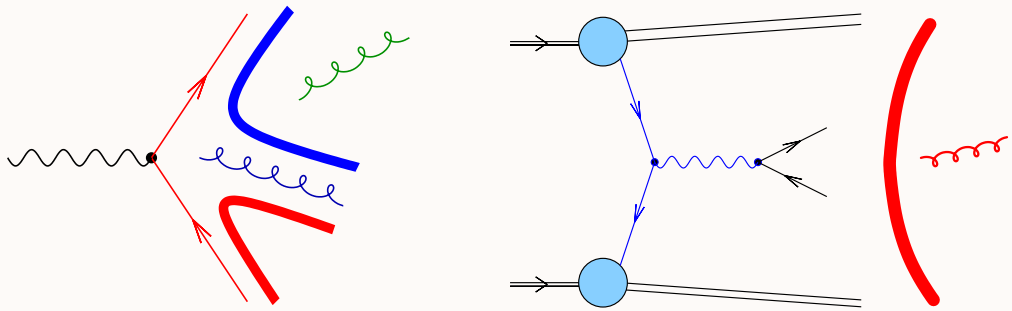
$$d\sigma = d\sigma_0 \frac{2\,\alpha_s}{3\,\pi} \frac{(x_1^2 + x_3^2)\,dx_1 dx_3}{(1-x_1)(1-x_3)} \sim d\sigma_0 \frac{d\omega_g}{\omega_g}\,dy_g$$



Even correct for hard gluons!
Gluon emission is $\mathbf{2 \longrightarrow 3}$ process.

- Semi-classical picture of a dipole cascade

$q\bar{q}$ emits g, in turn $q'g\bar{q}'$ emits softer g',
$q'\bar{q}'$ contribution is neglected $(1/N_C^2)$.



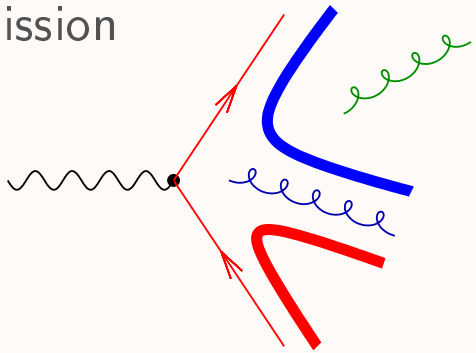
- Local 4-momentum conservation, partons on mass shell! g splitting included: $\hat{q}g \longrightarrow \hat{q}\bar{q}q$.
- ISR mapped onto FSR, parametrize suppression of high-p_T emissions for dipoles containing hadron remnants

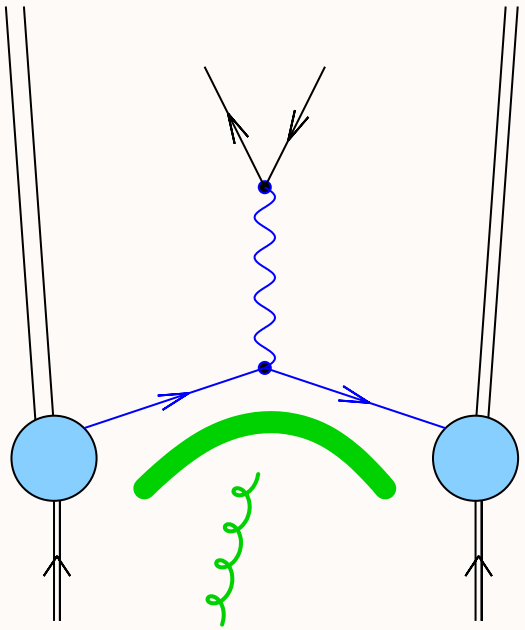
Idea implemented in Ariadne ... very good performance in describing LEP/HERA data

Colour-dipole shower for hadronic collisions

[WINTER, KRAUSS, JHEP 07 (2008),040]

- exploit antenna structure of QCD to describe multiparton emission
- beyond PT: dipoles including beam remnant(s)
- antennae formed by colour-connected partons
- initial, initial–final and final colour dipoles





Our goal: develop fully perturbative CDM:

- incoming parton lines used to form dipoles, hence new types:
  $\bar{q}_\mathrm{i} q_\mathrm{i}$, $g_\mathrm{i} q_\mathrm{i}$, $g_\mathrm{i} g_\mathrm{i}$ and $q_\mathrm{f} q_\mathrm{i}$, $q_\mathrm{f} g_\mathrm{i}$, $g_\mathrm{f} g_\mathrm{i}$
- beam remnants excluded, will be treated during hadronization
- radiation pattern in terms of $2 \to 3$ splittings
  - *dipole ME factorization (re-calculate / crossing symmetry / antenna funcs)*
- generalized $2 \to 3$ onshell kinematics to include new cases
  - *dipole phase-space factorization and invariant evolution variables*
- probabilistic interpretation of Sudakov form factor here based on dipole splitting cross sections
- large N_C limit, all ISR in backward evolution

Gluon emission from colour dipoles

➡ ***Instead of redefining ISR in terms of FSR,***

$$\tilde{k}\,\tilde{\ell} \;\to\; \begin{cases} \tilde{k}\,g_{\rm f}\,\tilde{\ell} & : \quad \text{gluon emission}\,, \\ q\,g_{\rm i}\,\tilde{\ell} & : \quad \text{quark emission, provided that} \quad \tilde{k} = \bar{q}_{\rm i}\,, \\ \tilde{k}\,g_{\rm i}\,\bar{q} & : \quad \text{antiquark emission, provided that} \quad \tilde{\ell} = q_{\rm i}\,. \end{cases}$$

Basic idea: find generalized (and recursive) form for ...

➡ ***differential cross section for a $\tilde{k}\tilde{\ell} \to k g \ell$ dipole splitting:***

$$d\sigma_{0\to k g \ell} \;=\; d\sigma_{0\to \tilde{k}\tilde{\ell}}\;\frac{\alpha_s(\mu_{\rm R})}{2\pi}\;D_{\tilde{k}\tilde{\ell}\to k g \ell}(p_\perp, y)\;\frac{dp_\perp^2}{p_\perp^2}dy$$

➡ ***expressed through dipole splitting functions D and Lorentz invariant evolution variables:***

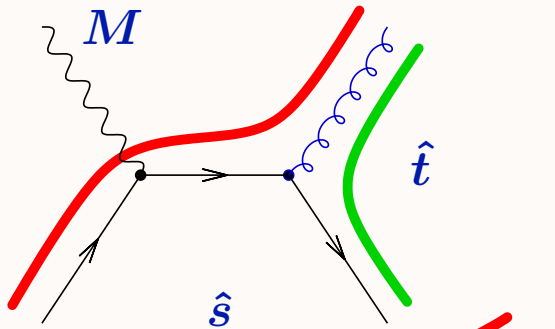
transverse momentum: $p_\perp^2 \;=\; \left|\dfrac{s_{kg}\,s_{g\ell}}{s_{kg\ell}}\right|$, rapidity: $y \;=\; \dfrac{1}{2}\,\ln\left|\dfrac{s_{g\ell}}{s_{kg}}\right|$;

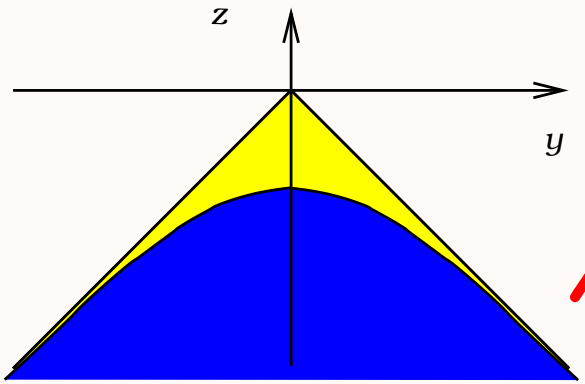
- $s_{mn} \;=\; (\varsigma_m\,p_m + \varsigma_n\,p_n)^2\,, \quad s_{mnr} \;=\; (\varsigma_m\,p_m + \varsigma_n\,p_n + \varsigma_r\,p_r)^2$
- $\varsigma_m = \pm 1$ ($+$ outgoing, $-$ incoming)

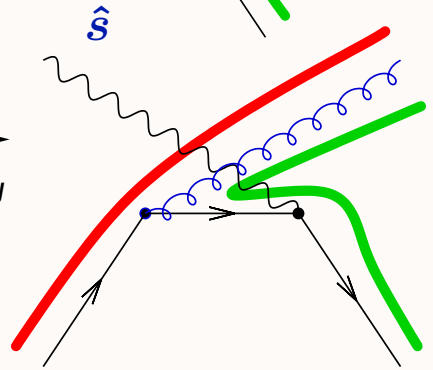
Initial-state dipole branching: $\bar{\imath}' i \longrightarrow \bar{\imath}' g i$

➡ Invariant transverse momentum and rapidity

$$p_\perp^2 \;=\; \left|\frac{s_{\bar{\imath}' g}\, s_{g i}}{s_{\bar{\imath}' g i}}\right| \;=\; \frac{\hat{t}\,\hat{u}}{M^2}\,, \qquad y \;=\; \frac{1}{2}\,\ln\left|\frac{s_{g i}}{s_{\bar{\imath}' g}}\right| \;=\; \frac{1}{2}\,\ln\frac{\hat{u}}{\hat{t}}$$







➡ Phase space, $a = \hat{s}_{\max}/M^2 \le S/M^2$

$$|y| \;\le\; \operatorname{arcosh}\frac{(a-1)M}{2p_\perp} \;\le\; \ln\frac{(a-1)M}{p_\perp} = \ln\frac{1}{z}$$

➡ Dipole splitting function for ***gluon emission*** off II dipoles, $x_{\bar{\imath}',i} \;=\; 1 + \frac{p_\perp}{M}\, e^{\pm y}$

$$D_{\bar{\imath}' i\to\bar{\imath}' g i}(p_\perp, y) \;=\; \frac{f_{\bar{\imath}'}(x_\pm,\mu_{\rm F})\, f_i(x_\mp,\mu_{\rm F})}{f_{\bar{\imath}'}(\tilde{x}_\pm,\tilde{\mu}_{\rm F})\, f_i(\tilde{x}_\mp,\tilde{\mu}_{\rm F})}\, \xi_{\left\{\begin{smallmatrix} F \\ A \end{smallmatrix}\right\}} C_{\left\{\begin{smallmatrix} F \\ A \end{smallmatrix}\right\}}\, \frac{x_{\bar{\imath}'}^{n_{\bar{\imath}'}}(p_\perp, y) + x_i^{n_i}(p_\perp, y)}{\left[x_{\bar{\imath}'}(p_\perp, y) + x_i(p_\perp, y) - 1\right]^2}$$

$$\le\; \mathcal{N}_{\rm PDF}\, \xi_{\left\{\begin{smallmatrix} F \\ A \end{smallmatrix}\right\}} C_{\left\{\begin{smallmatrix} F \\ A \end{smallmatrix}\right\}} \left\{\begin{array}{c} 2 \\ a+1 \end{array}\right\} \;\equiv\; D^{\rm approx}_{\bar{\imath}' i\to\bar{\imath}' g i}(p_\perp, y)$$

$$n_{q,g} = 2, 3; \qquad \left\{\begin{array}{ll} \ldots & \text{for quark dipoles} \\ \ldots & \text{else} \end{array}\right\}$$

Sudakov exponentiation, kinematics and showering

- Rule of thumb: no-branching probability exponentiates

$$\Delta(p^2_{\perp,\mathrm{stt}}, p^2_\perp) \;=\; \exp\left\{-\int_{p^2_\perp}^{p^2_{\perp,\mathrm{stt}}} \frac{dk^2_\perp}{k^2_\perp}\,\mathcal{I}(k^2_\perp)\right\},$$

$$\mathcal{I}(k^2_\perp) \;=\; \frac{\alpha_s[\mu_\mathrm{R}(k_\perp)]}{2\pi} \sum_{\{\tilde{k}\tilde{\ell}\to k g \ell\}} \int_{y_-(k_\perp,\,a)}^{y_+(k_\perp,\,a)} dy\; D_{\tilde{k}\tilde{\ell}\to k g \ell}(k_\perp, y)$$

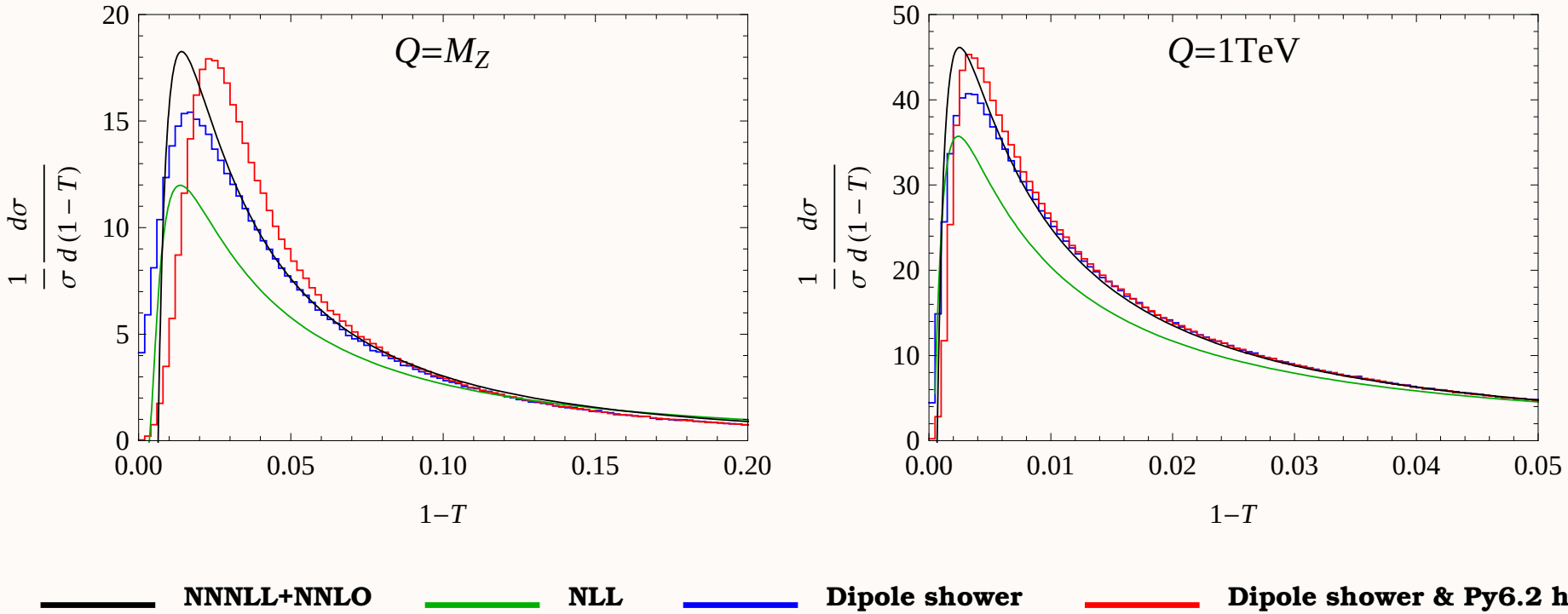
- Differential probability that branching occurs at $p^2_\perp$

$$\frac{dP}{dp^2_\perp} \;=\; \frac{d\Delta(p^2_{\perp,\mathrm{stt}}, p^2_\perp)}{dp^2_\perp} \;=\; \frac{\mathcal{I}(p^2_\perp)}{p^2_\perp}\Delta(p^2_{\perp,\mathrm{stt}}, p^2_\perp)$$

- Monte Carlo method (Veto Algorithm) yields values for evolution variables.
- Set up kinematics: use light-cone variables and recoil strategies.
- Shower algorithm fully defined by fixing:
  - *renormalization and factorization scale choices.*
  - *the initializing scale in dependence on the hard process.*
  - *the maximal phase space of a single emission globally.*
  - *the iteration procedure to generate the cascade and the cut-off(s).*

Results for pure final-state cascading

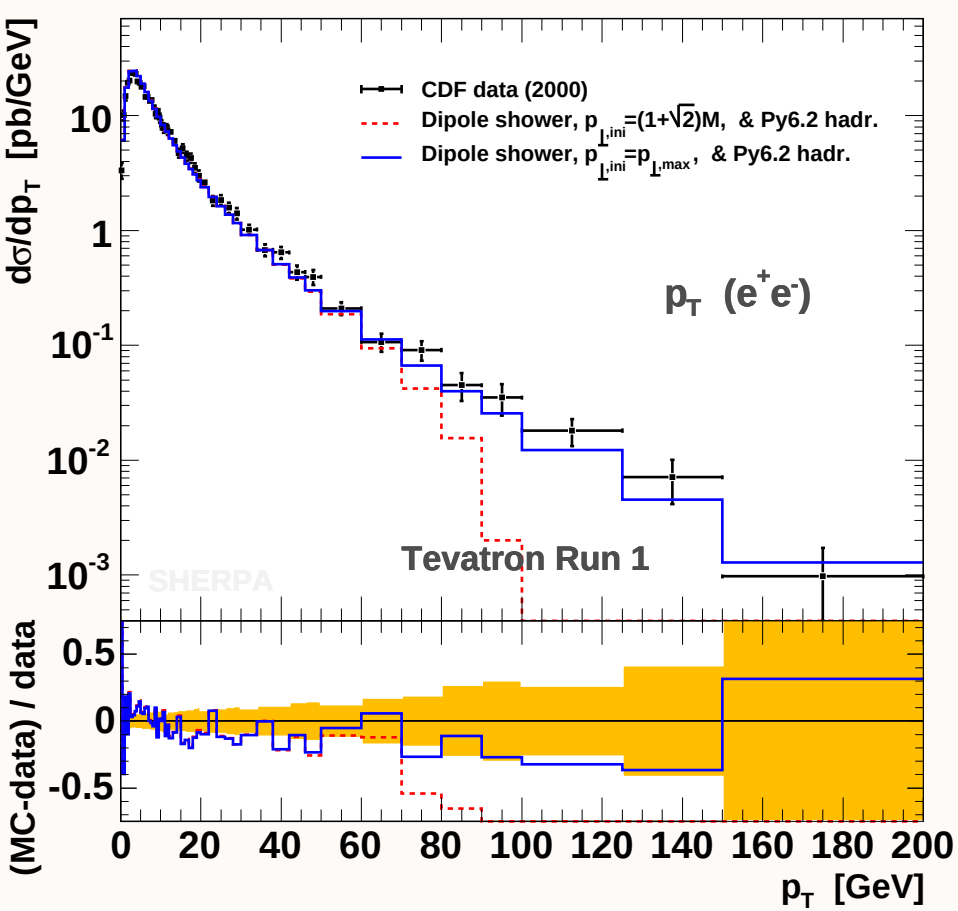
➡ *Testbed: hadron production in electron–positron annihilations*

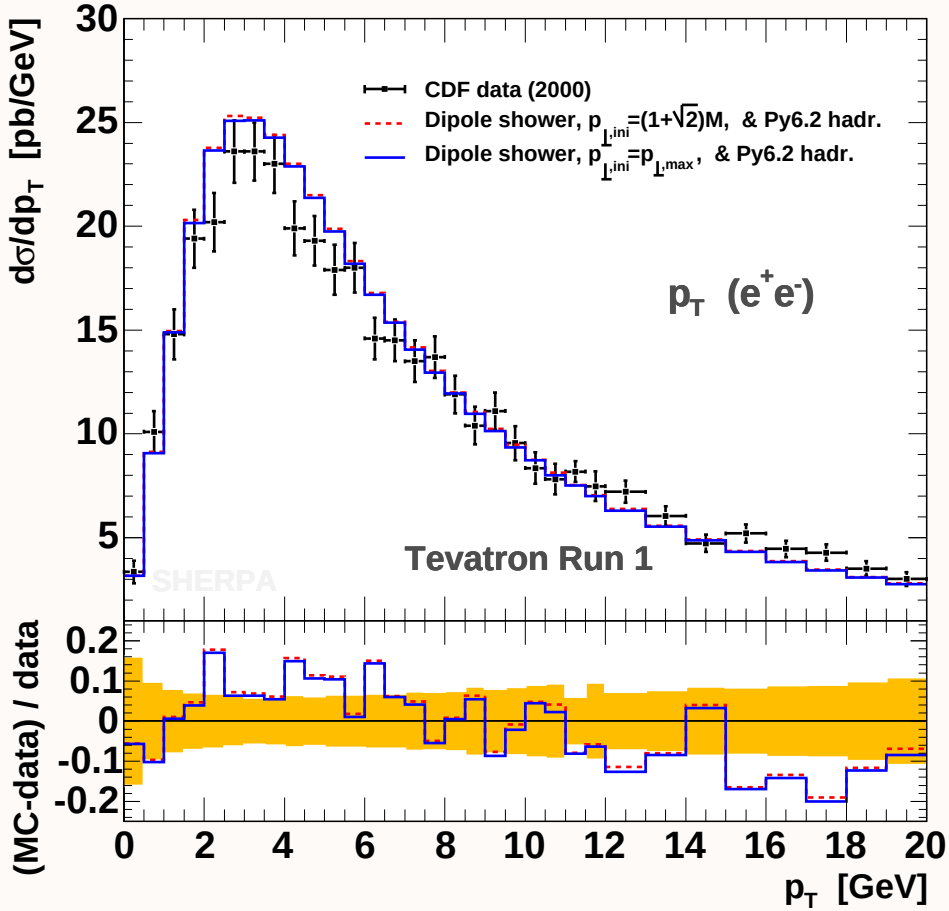


- All-particle 1-thrust event-shape distribution.
- Comparison to theoretical prediction by Becher and Schwartz using SCET. [ARXIV:0803.0342]
- Independent test of resummation encoded in dipole shower.

Results for hadronic collisions

➡ *Testbed: inclusive production of Drell–Yan lepton pairs @ Tevatron Run I*

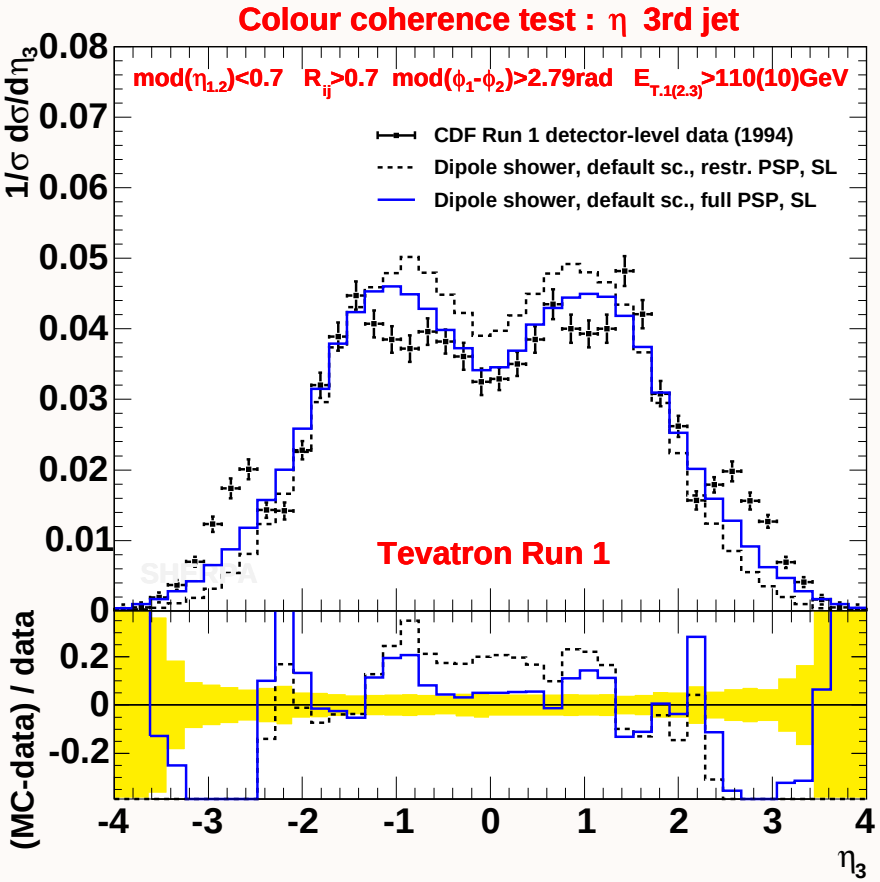


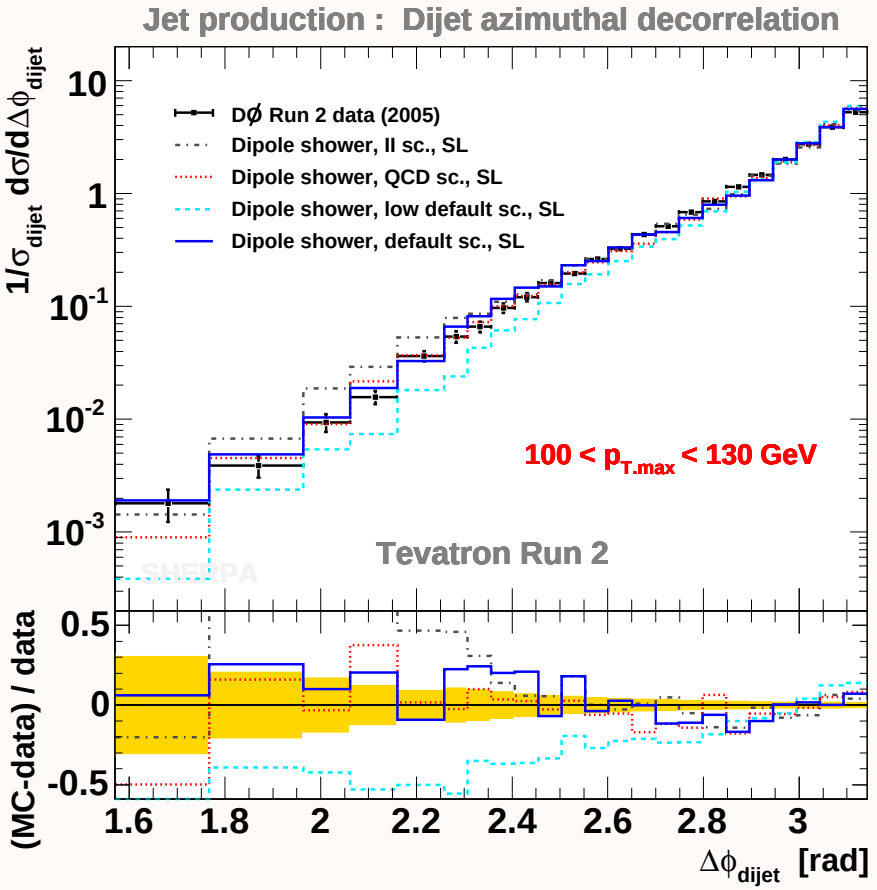


- Boson transverse-momentum distribution and its peak region in $e^+e^- + X$.
- 1st emission of the dipole shower is ME corrected per construction.
- CDF data: Affolder et al. Phys. Rev. Lett. **84** (2000) 845.

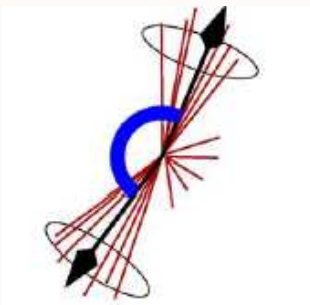
Results for hadronic collisions

➡ *Testbed: inclusive jet production @ Tevatron Run I & II*





- qualitative test for the inclusion of colour coherence – uncorrected data
  CDF [Abe et al., Phys. Rev. **D50** (1994) 5562]
- stringent test of QCD radiation pattern – dijet azimuthal decorrelation
  DØ [Abazov et al., PRL **94** (2005) 221801]



From tree-level to

one-loop gluon amplitudes.

NLO calculations

➡ ***Lessons learned from LEP, HERA, Tevatron:***
LO predictions are fine, yet often give rough estimates only

NLO: 1st real predicition of normalization of many observables
less sensitivity to unphysical input scales (μ_{F} & μ_{R})
more physics (parton merging, jet substructure, ISR, more IS parton species)

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➡ ***Components of NLO calculations***

- tree-level amplitudes (LO & real radiation) + one-loop correction to Born level + subtraction terms to handle and combine singularities + phase-space generator
- Feynman diagram calculations: computational algorithms of exponential complexity
- bottleneck: virtual corrections (tensor-integral reductions generate large # of terms)
- @ tree level: algorithms of polynomial complexity exist ($\tau \sim N^{\#}$)
  recursive methods efficiently re-use recurring groups of offshell Feynman graphs
- @ loop level: unitarity-cut methods factorize one-loop into tree amplitudes
  computing time grows with # of cuts & depends on algorithm employed at tree level

Goal ➡ ***provide algorithm(s) [tools] of polynomial complexity to calculate virtual corrections***

Unitarity techniques for 1-loop amplitudes

active field of research ...

- *Britto et al.*
- *Bern et al.* – *BlackHat project and code.*
- *Ossola et al.* – *CutTools code.*
- *Ellis et al.* – *"Rocket Science".*

- *This work is based on* –

  [Ellis, Giele, Kunszt, arXiv:0708.2398] 4dim method, cut-constructible part

  [Giele, Kunszt, Melnikov, arXiv:0801.2237] Ddim method, rational part

  [Giele, Zanderighi, arXiv:0805.2152] application of Ddim method to pure gluons

Decomposing one-loop amplitudes

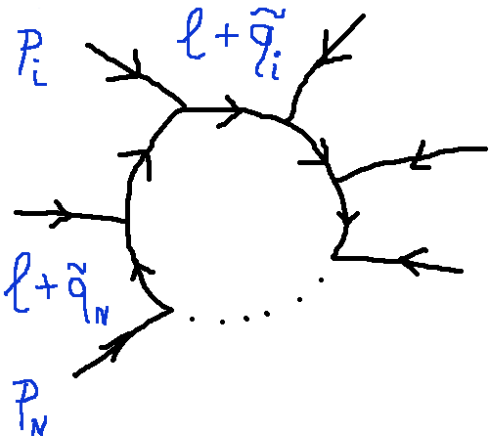
➡ ***into a linear sum of scalar box, triangle, bubble and tadpole master integrals (cut-constructible part) and rational terms***

$$\mathcal{A}_N(\{p_i\}) = \sum_{[i_1|i_4]} d_{i_1 i_2 i_3 i_4} I^{(D)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c_{i_1 i_2 i_3} I^{(D)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b_{i_1 i_2} I^{(D)}_{i_1 i_2} + \sum_{[i_1|i_1]} a_{i_1} I^{(D)}_{i_1} + \mathcal{R}_N$$

- master integrals known in literature
- and implemented in various codes, e.g. QCDLoop [Ellis, Zanderighi] (QCDLoop.fnal.gov)
- to do: determination of the master-integral coefficients ⬅ unitarity techniques
- problem: extraction of lower-point coefficients
  "subtracting terms that are already included in higher-point contributions"
- solution: identify subtraction terms at the integrand level [Ossola, Papadopoulos, Pittau]
  partial fractioning of the integrand: expand over 4,3,2,1 propagator terms
  residues of pole terms contain master-integral coefficient plus finite number of spurious terms
  spurious terms vanish upon integration

note that $[i_1, i_M] = 1 \leq i_1 < i_2 < \ldots < i_M \leq N$ and $I^{(D)}_{i_1 \ldots i_M} = \int d^D\ell \frac{1}{d_{i_1} \ldots d_{i_M}}$

➡ re-expressing the integrand $\quad d_i(\ell) = (\ell + \tilde{q}_i)^2 - m_i^2$



$$\mathcal{A}_N(\{p_i\},\ell) \;=\; \frac{\mathcal{N}(\{p_i\},\ell)}{d_1 d_2 \dots d_N} \;=$$

$$\sum_{[i_1|i_4]} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(\ell)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\bar{c}_{i_1 i_2 i_3}(\ell)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\bar{b}_{i_1 i_2}(\ell)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\bar{a}_{i_1}(\ell)}{d_{i_1}}$$

[ELLIS, GIELE, KUNSZT]

- solve for numerator factors:
  need to find $\ell = \ell_{i_1 \dots i_M}$ such that $d_j(\ell_{i_1 \dots i_M}) = 0$ for $j = i_1, \dots, i_M$
- define $\mathrm{Res}_{i_1 \dots i_M}(\mathcal{A}_N(\ell)) = \{d_{i_1}(\ell) \dots d_{i_M}(\ell) \times \mathcal{A}_N(\ell)\}|_{\ell=\ell_{i_1 \dots i_M}}$ then

$$\bar{d}_{i_1 i_2 i_3 i_4}(\ell) \;=\; \mathrm{Res}_{i_1 i_2 i_3 i_4}(\mathcal{A}_N(\ell))\,, \quad \bar{c}_{i_1 i_2 i_3}(\ell) \;=\; \mathrm{Res}_{i_1 i_2 i_3} \left(\mathcal{A}_N(\ell) - \sum_{[j_1|j_4]} \frac{\bar{d}_{j_1 j_2 j_3 j_4}(\ell)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4}} \right) , \; \dots$$

- find parametric (most general) form of residues, removing spurious terms:
  box coefficient: $\bar{d}_{i_1 i_2 i_3 i_4}(\ell) = d_{i_1 i_2 i_3 i_4} + (\ell n_4)\, \tilde{d}_{i_1 i_2 i_3 i_4}$ ➡ "the Right-Hand-Side"

➡ $$\int d^D\ell \frac{\bar{d}_{i_1 i_2 i_3 i_4}(\ell)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} \;=\; d_{i_1 i_2 i_3 i_4} \int d^D\ell \frac{1}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} \;=\; d_{i_1 i_2 i_3 i_4}\; I_{i_1 i_2 i_3 i_4}$$

Generating the Left-Hand-Side

- What is $\mathrm{Res}_{i_1 \ldots i_M}(\mathcal{A}_N(\ell))$?

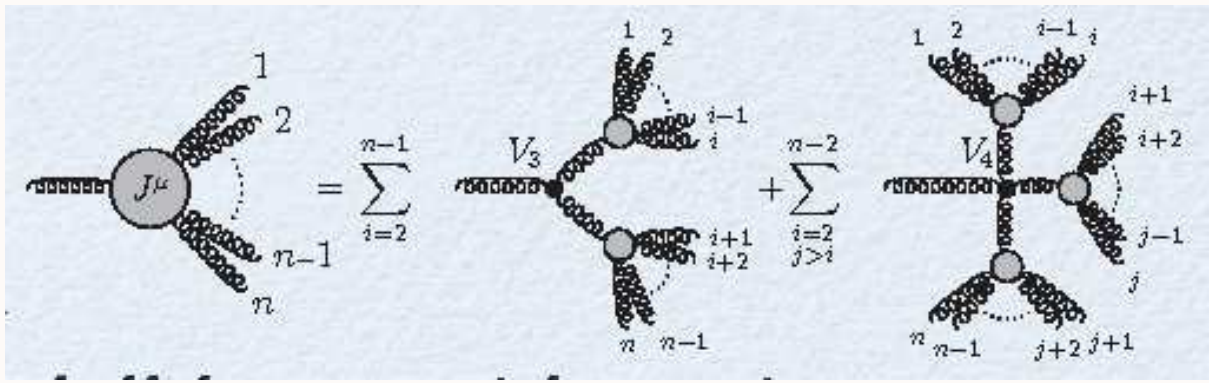
$$= \{ d_{i_1}(\ell) \ldots d_{i_M}(\ell) \times \mathcal{A}_N(\ell) \}|_{d_{i_1}(\ell)=\cdots=d_{i_M}(\ell)=0}$$

- requires calculation of factorized un-integrated one-loop amplitude
- unitarity cuts: M on-shell propagators, amplitude factorizes into M tree-level amplitudes

$$\mathrm{Res}_{i_1 \ldots i_M}(\mathcal{A}_N(\ell)) = \sum_{\{\lambda_1,\ldots,\lambda_M\}=1}^{2} \left(\prod_{k=1}^{M} \mathcal{M}^{(0)} \left(\ell_{i_k}^{(\lambda_k)}; p_{i_k+1}, \ldots, p_{i_{k+1}}; -\ell_{i_{k+1}}^{(\lambda_{k+1})} \right) \right)$$

- two 4-dimensional gluons with complex momenta and 2 polarization states $(\ell_{i_k} = \ell + \tilde{q}_{i_k})$

- Berends–Giele recursion relations to calculate tree-level amplitudes
- very economical scheme
- LHS: take subtractions into account



C++ code

➡ ***Another tool ...*** *Rocket (Rucola) was already launched* [GIELE, ZANDERIGHI]

- independent implementation (from scratch, no translation of Fortran routines)
- allows for independent xchecks of unitarity method and its results
- knowing the tool is knowing the methods, and knowing the details
- C++ ... different philosophy ... modularity, transparency
- allows for combination with other C++ codes ... potentially ... ***COMIX*** ... Gleisberg's automated CS subtraction ... ***Sherpa*** ...

N external gluons & their polarizations ➡ *(leading-)colour-ordered 1-loop amplitude (FDH)*

- xchecks on numbers
  coefficients itself, poles (known analytically), final numbers (analytic and other calculations)
  gauge invariance, choice of ℓ, dimensionality (D and D_s variation)
- accuracy and numerical stability

$$\varepsilon_{\mathrm{dp,sp}} = \log_{10} \frac{|\mathcal{A}^{(1)(\mathrm{dp,sp})}_{N,\mathrm{C++}} - \mathcal{A}^{(1)(\mathrm{dp,sp})}_{N,\mathrm{anly}}|}{|\mathcal{A}^{(1)(\mathrm{dp,sp})}_{N,\mathrm{anly}}|}, \qquad \varepsilon_{\mathrm{fp}} = \log_{10} \frac{2\,|\mathcal{A}^{(1)(\mathrm{fp})}_{N,\mathrm{C++}}[1] - \mathcal{A}^{(1)(\mathrm{fp})}_{N,\mathrm{C++}}[2]|}{|\mathcal{A}^{(1)(\mathrm{fp})}_{N,\mathrm{C++}}[1]| + |\mathcal{A}^{(1)(\mathrm{fp})}_{N,\mathrm{C++}}[2]|}$$

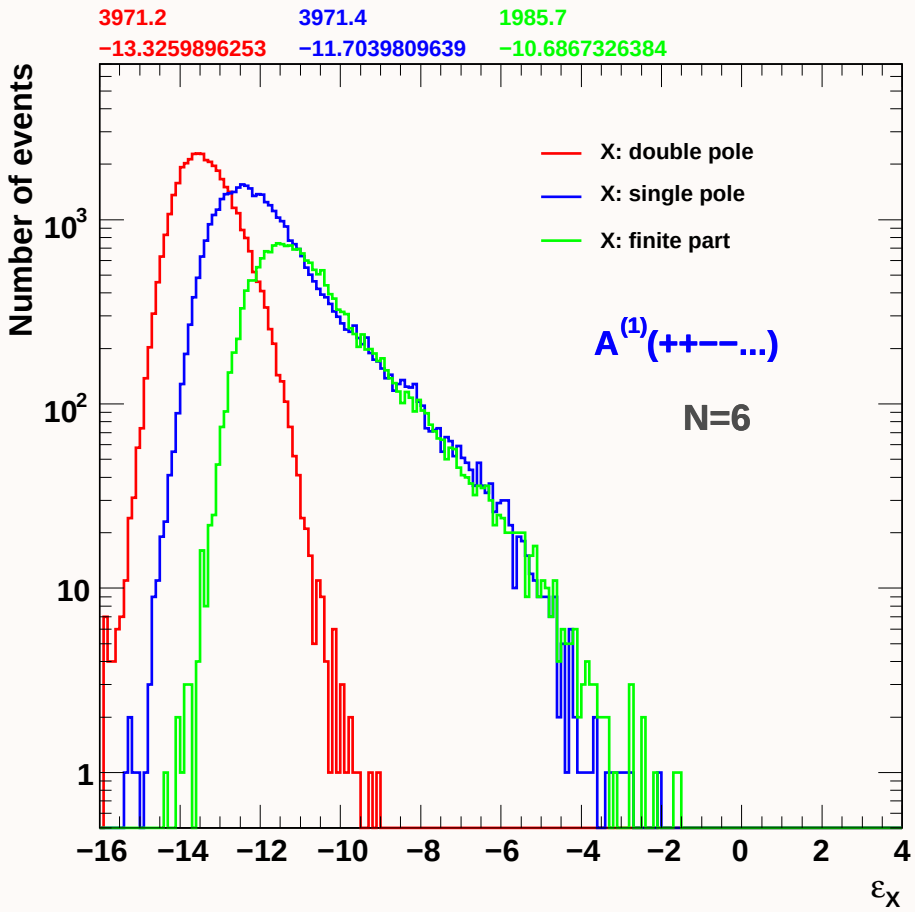
- efficiency – scaling of computing time with # of legs N $\quad\rightarrow\quad$ $\tau \sim N^9$

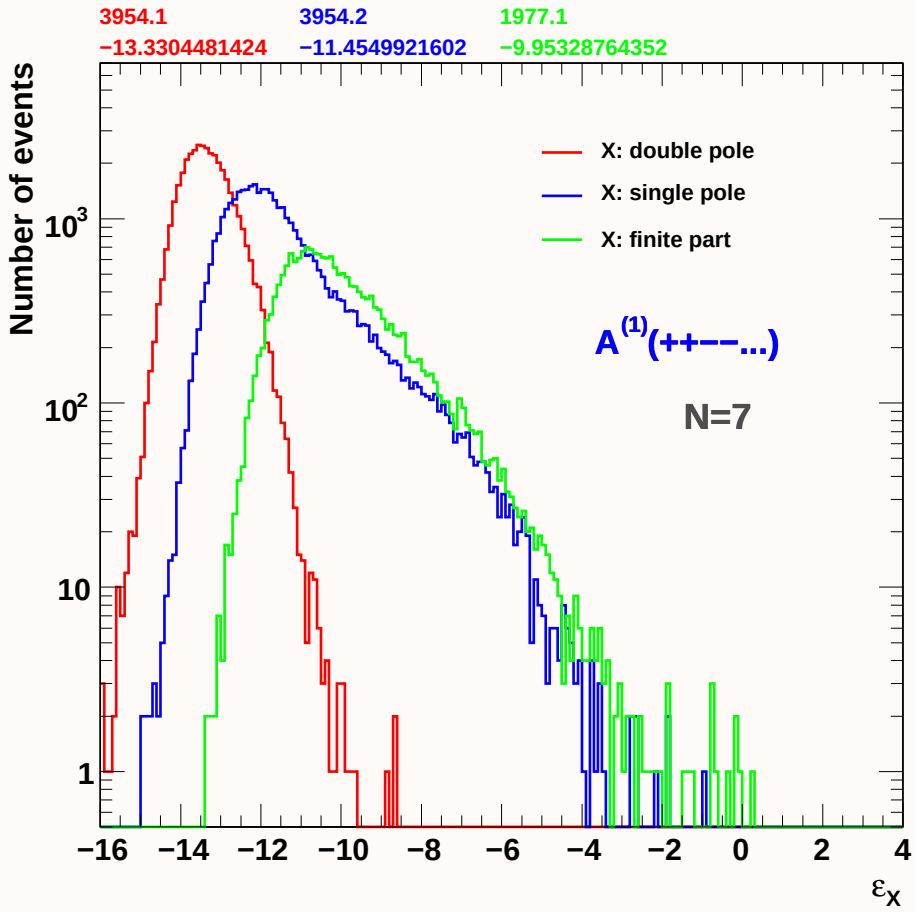
Accuracy

(preliminary) (all calculations in double precision only)

- peak positions are fine, tails seem OK, comparable to Rocket
- losing finite-part precision with $N = 10, 11$, lost for $N = 15$

  (double precision not enough, too many large numbers involved)



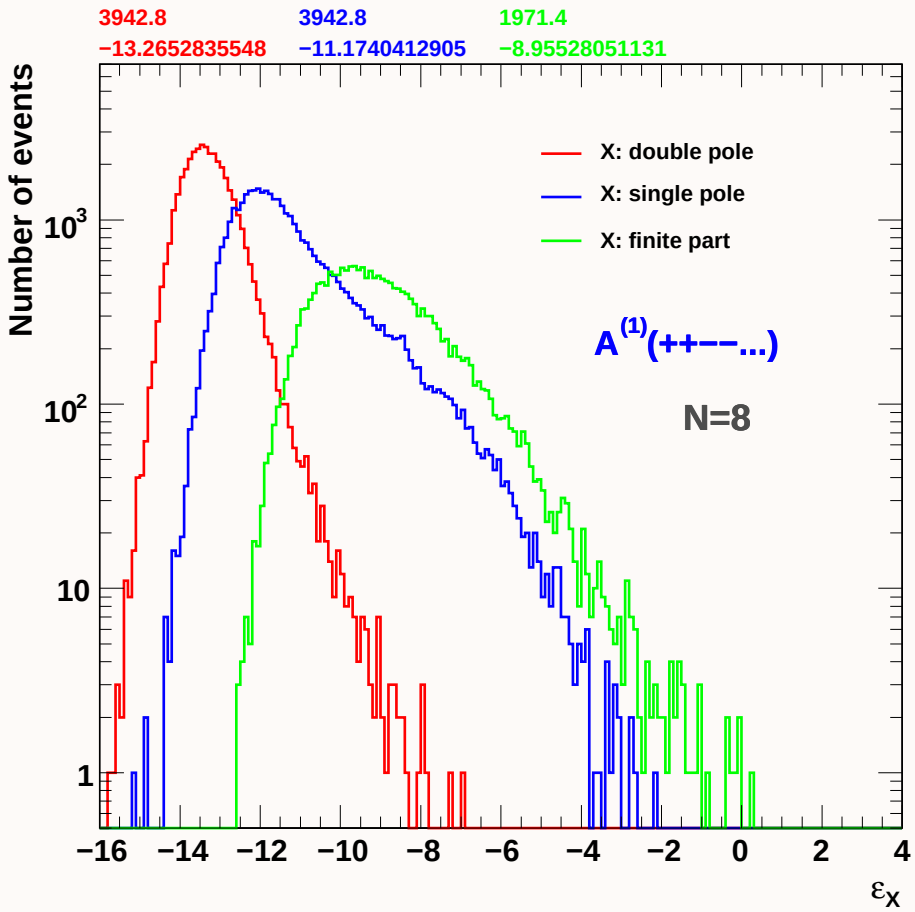


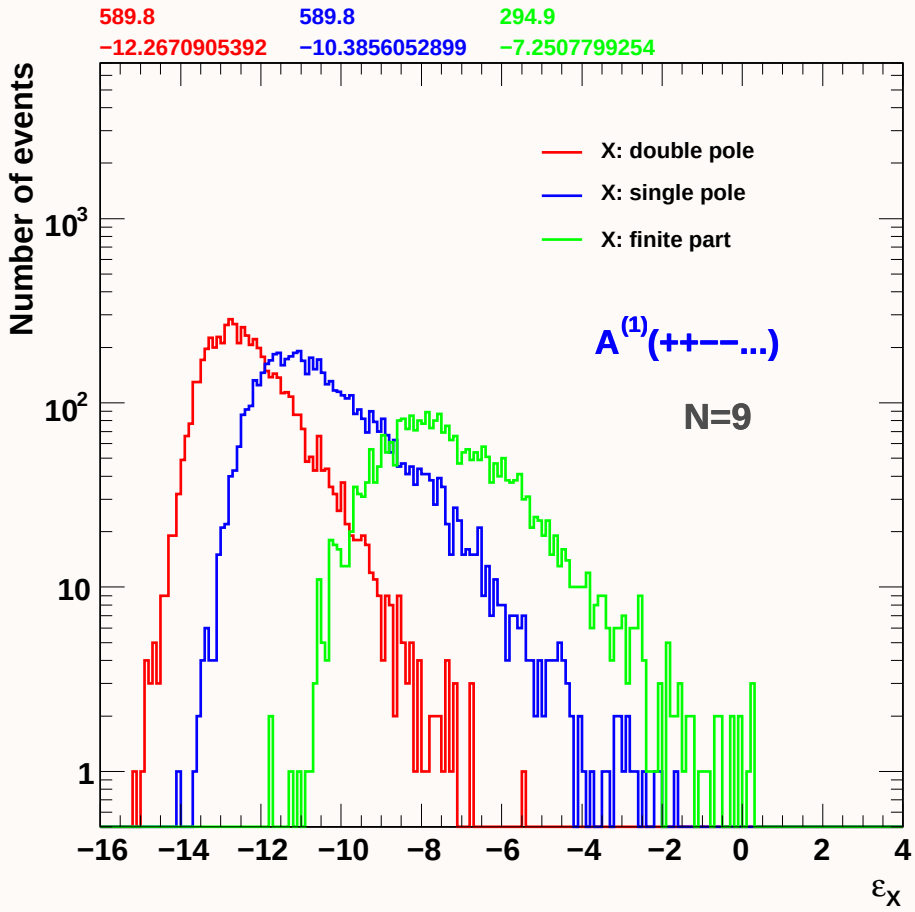
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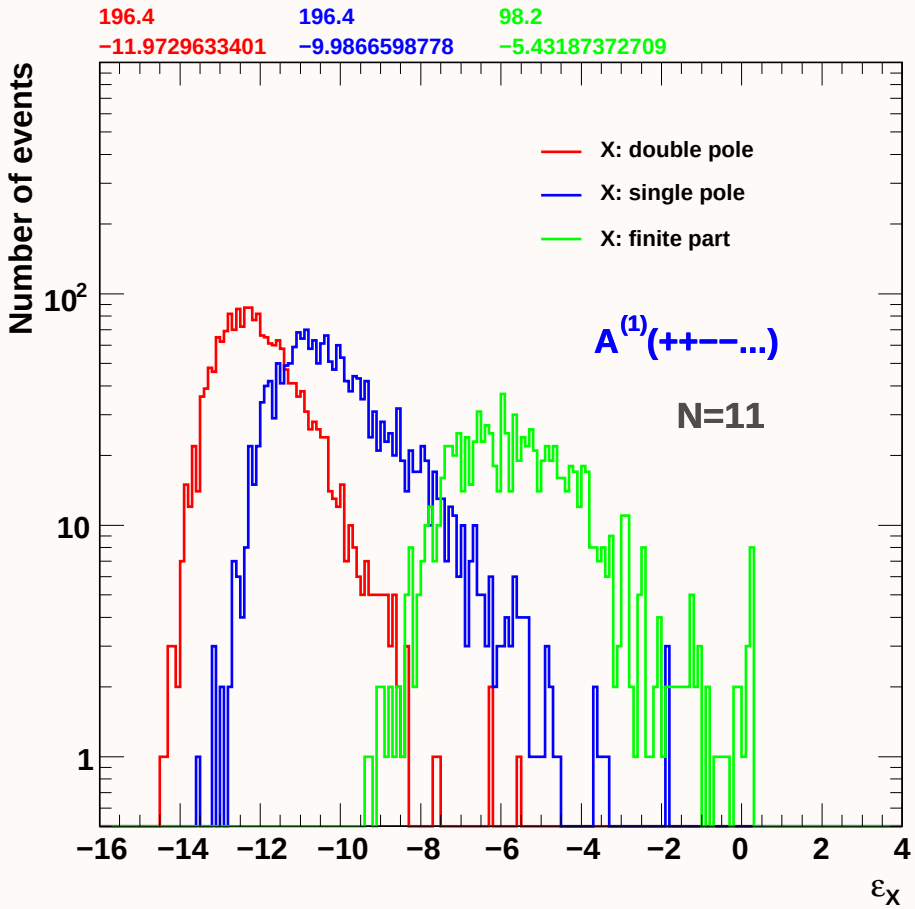


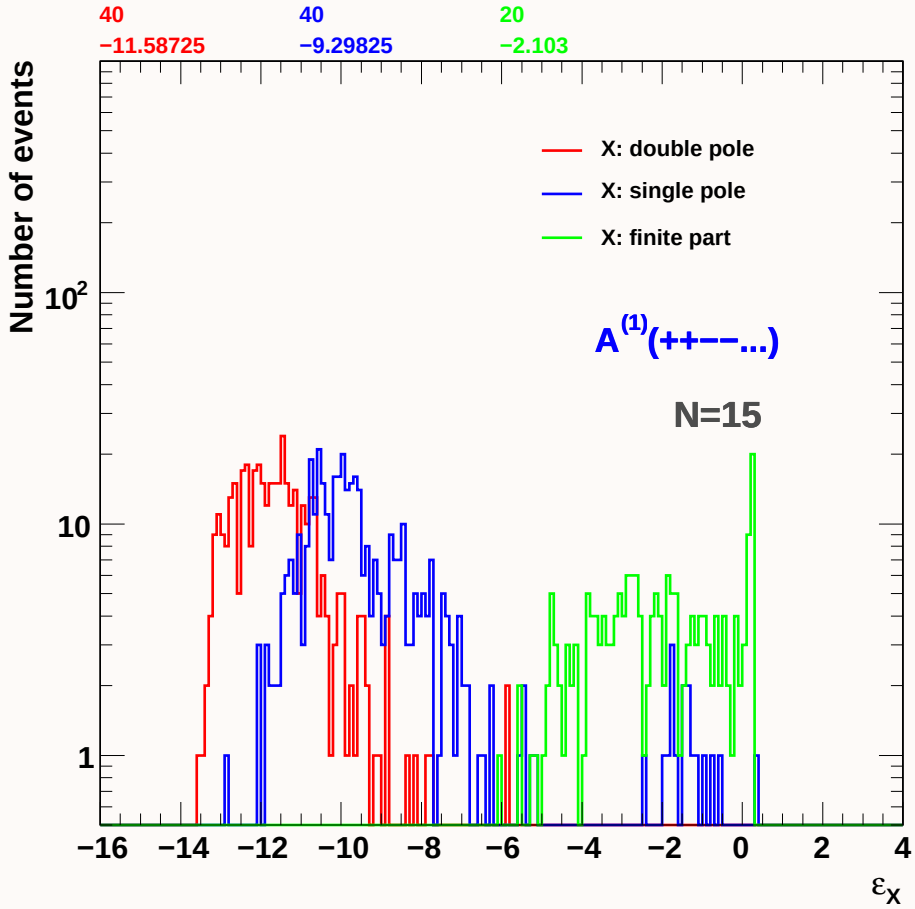
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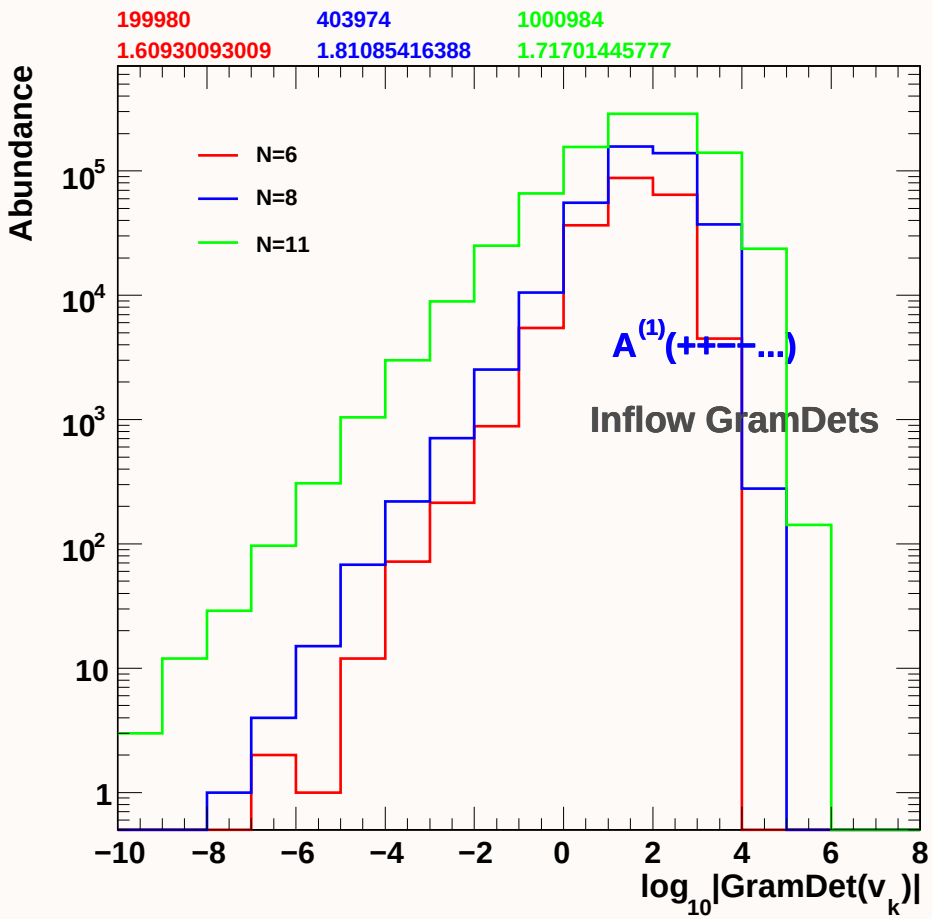


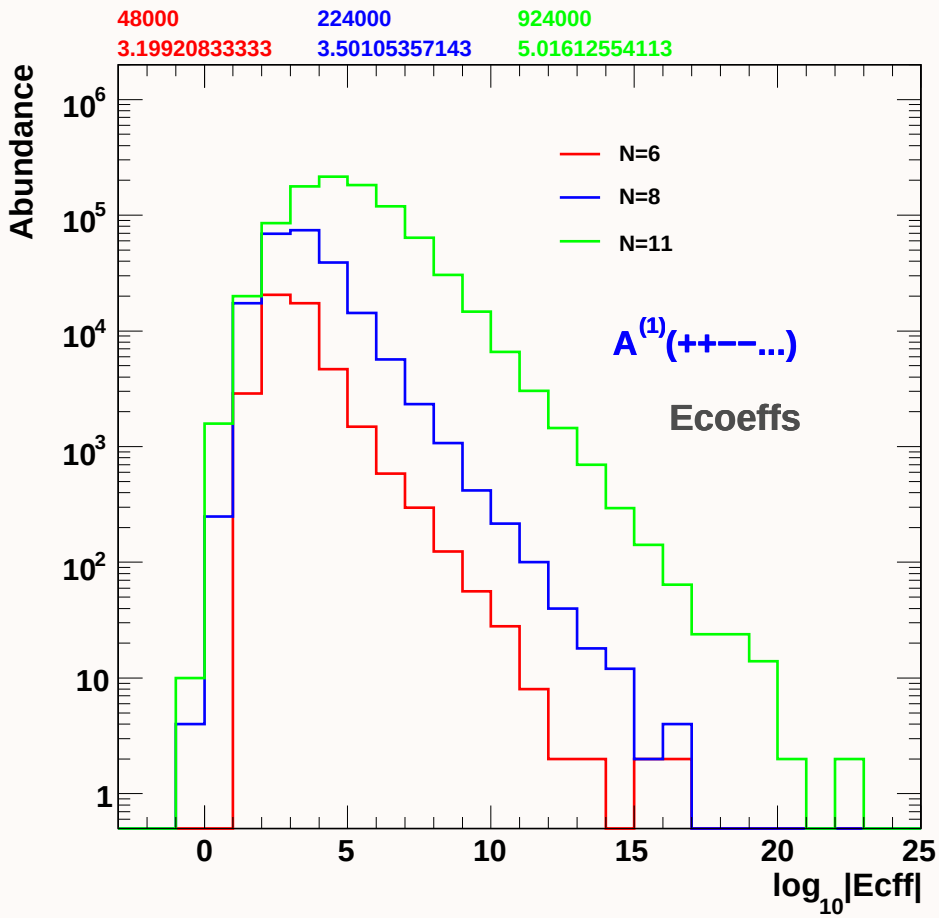


Accuracy

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- range of numbers increases with N – Gram dets of external gluons and $e^{(0)}_{ijklm}$ coefficients may become small and large, respectively

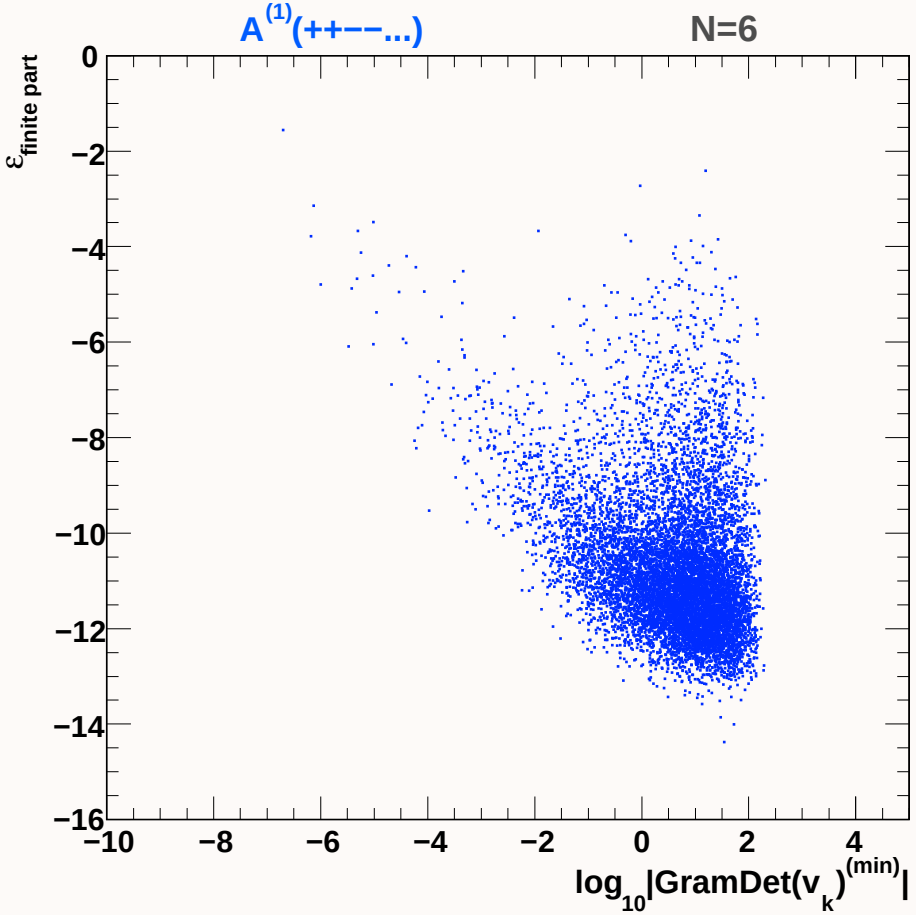


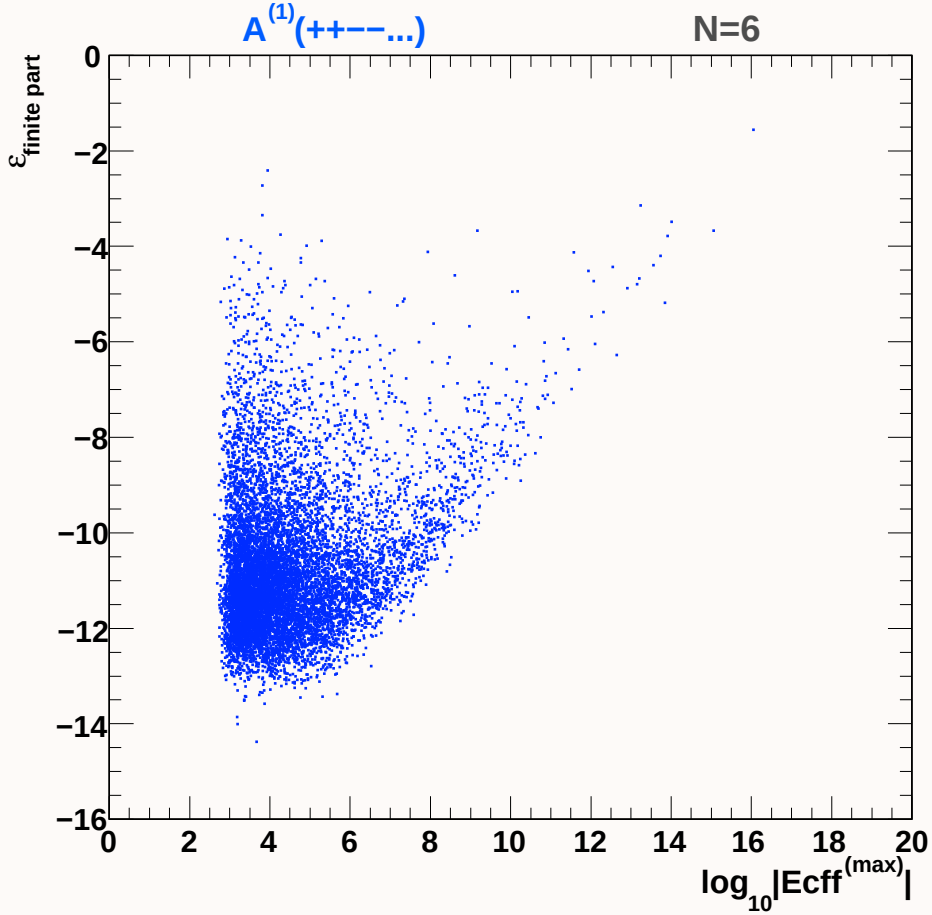


Correlations

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- precision of finite term partly correlated with smallness/largeness of Gram dets/coefficients
- still other denominators that can become small
- e.g. the leftover d_j in the subtraction terms (even when coefficients are not large)

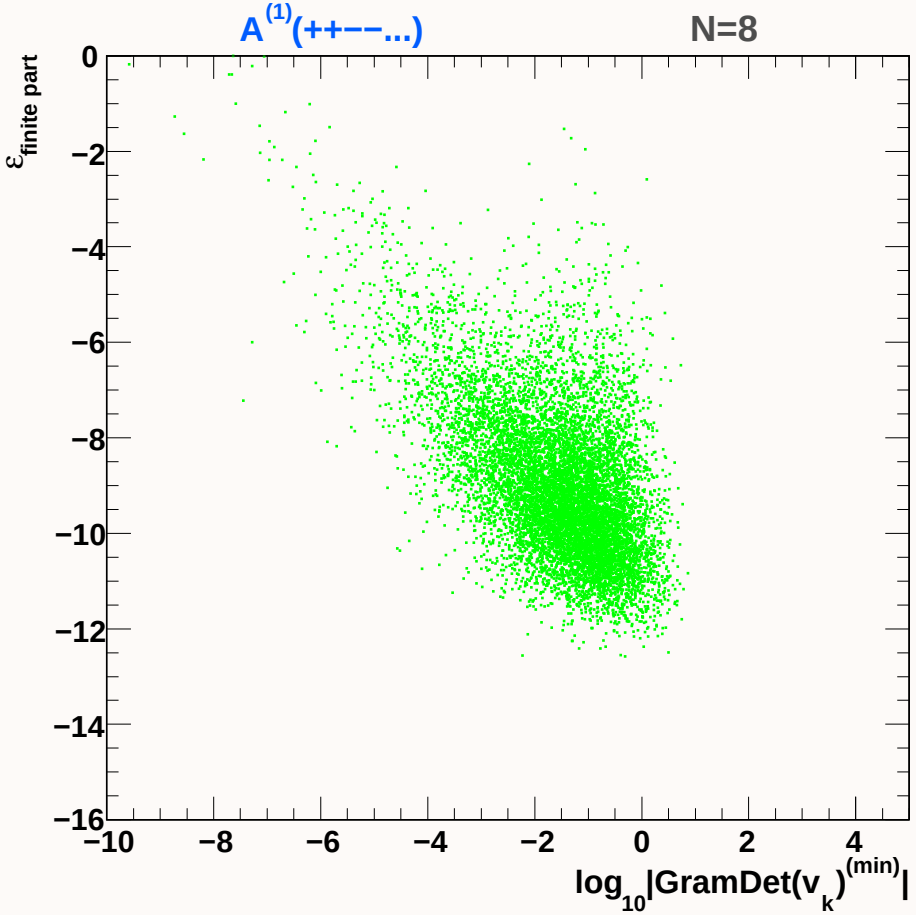


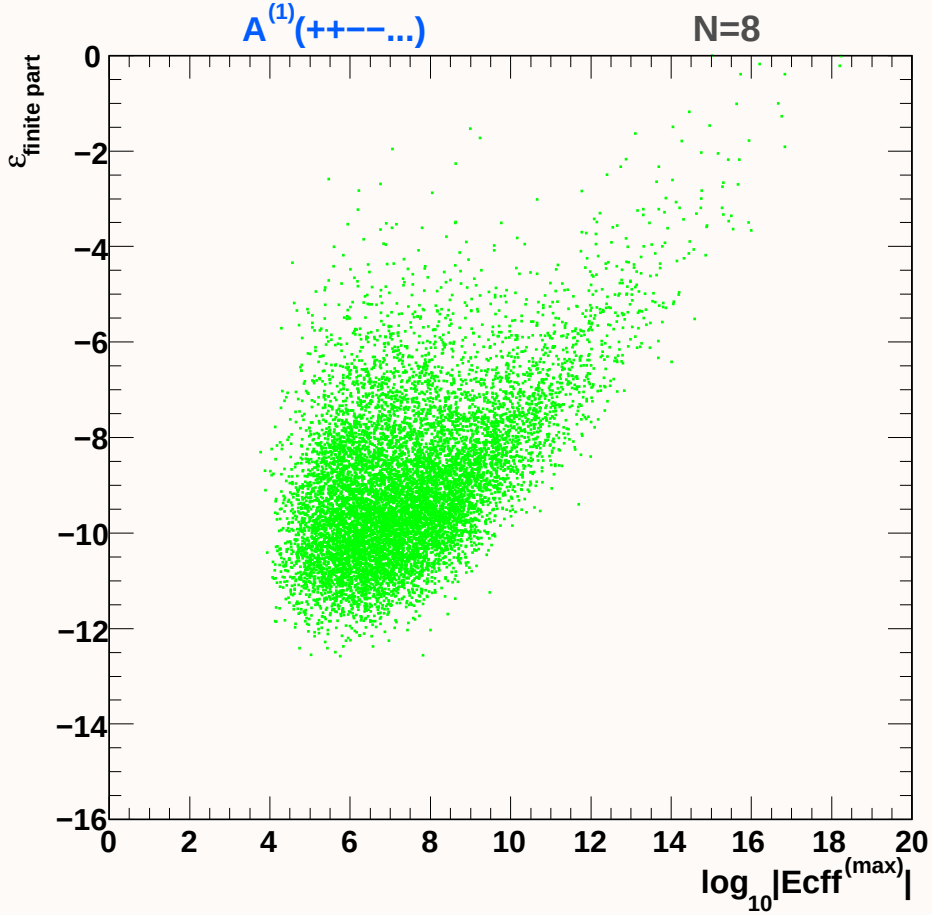


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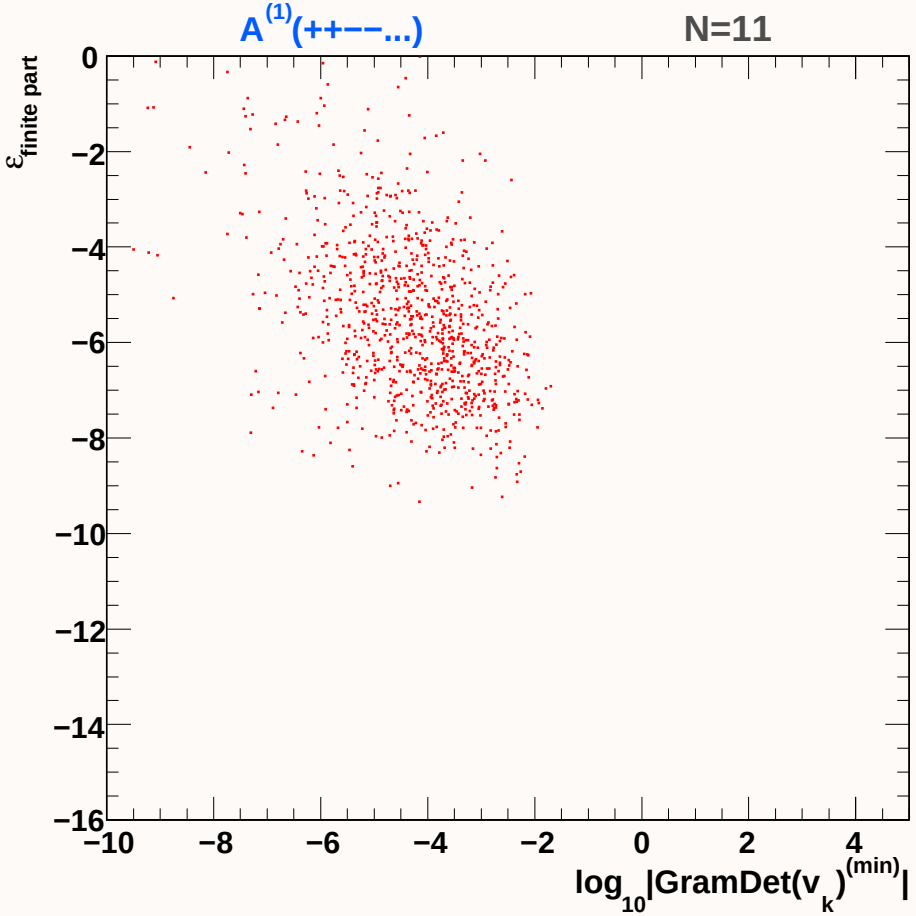


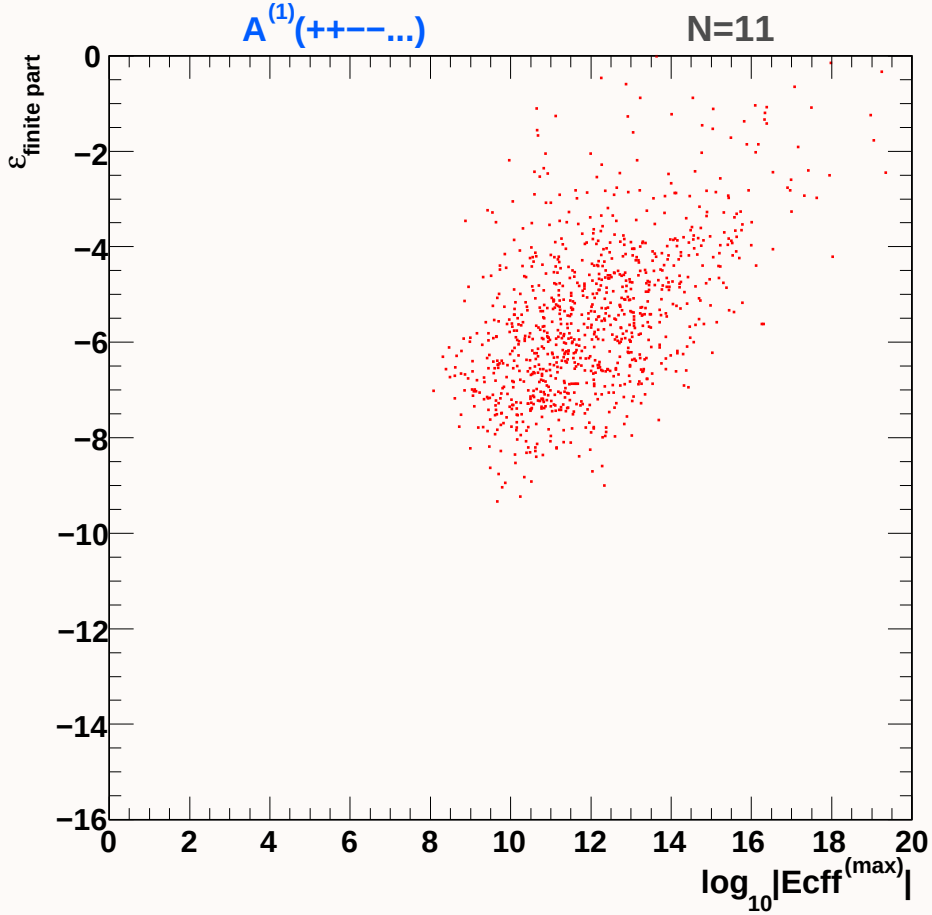


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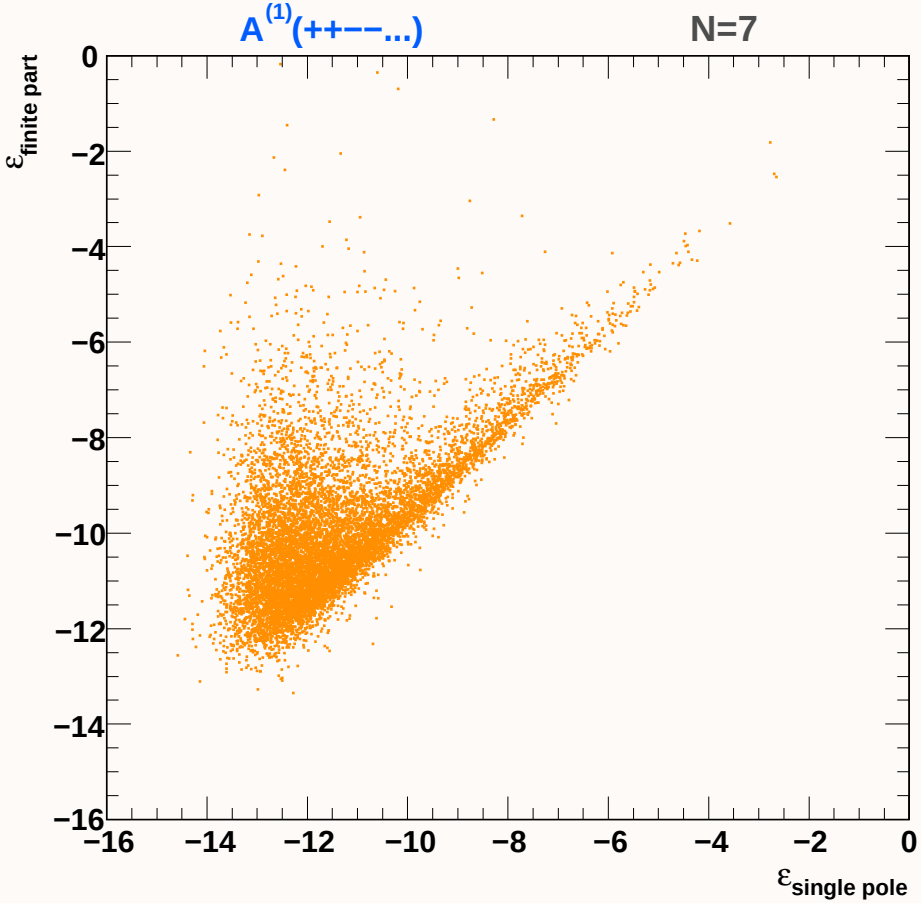


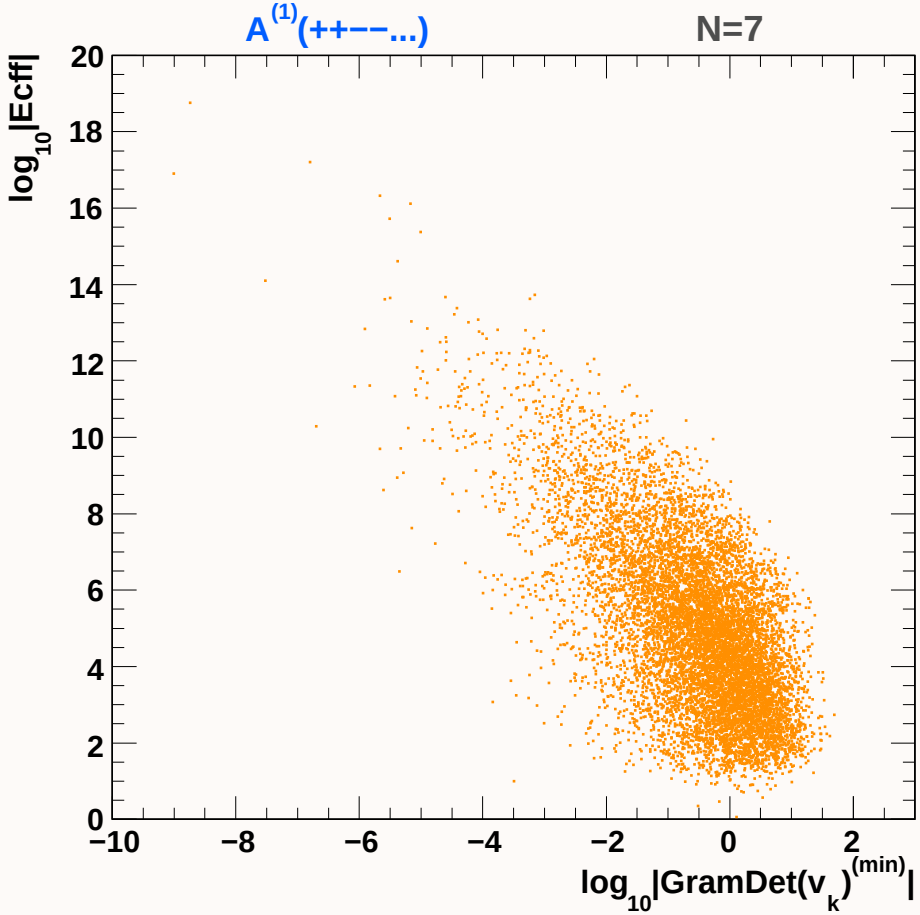


Correlations

(preliminary) (all calculations in double precision only)

- left: correlation between single-pole and finite-part accuracy
- right: which $e^{(0)}_{ijklm}$ coefficient occurs when external-gluon Gram det is minimal?

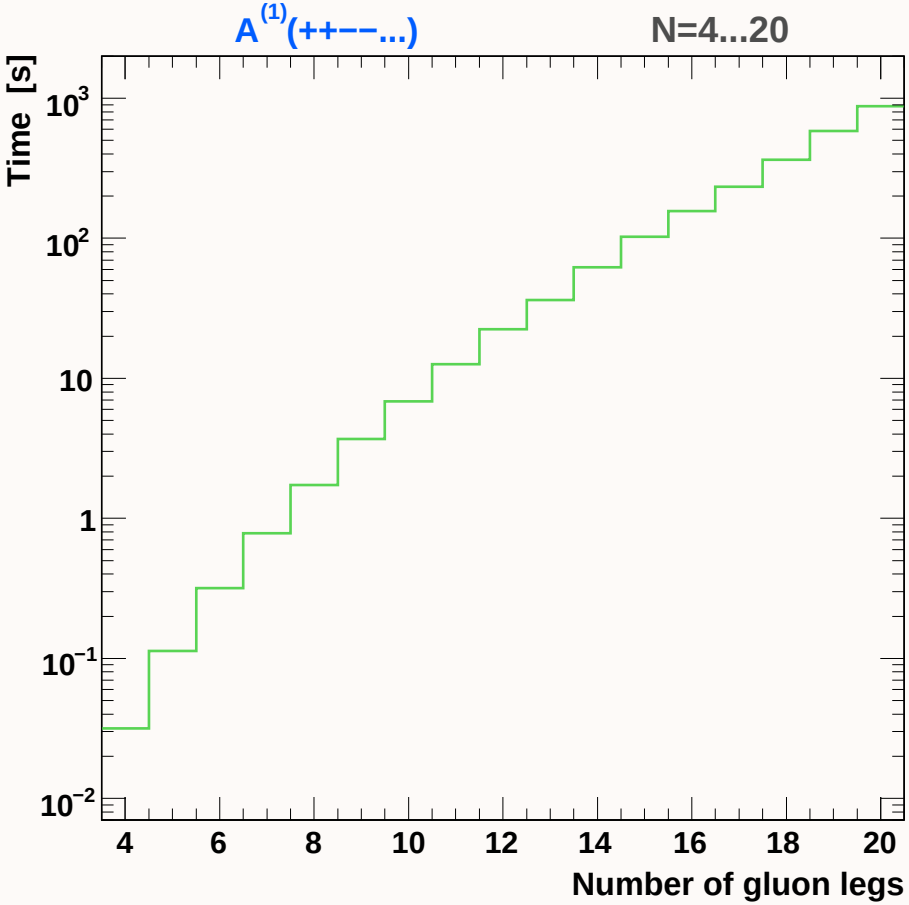


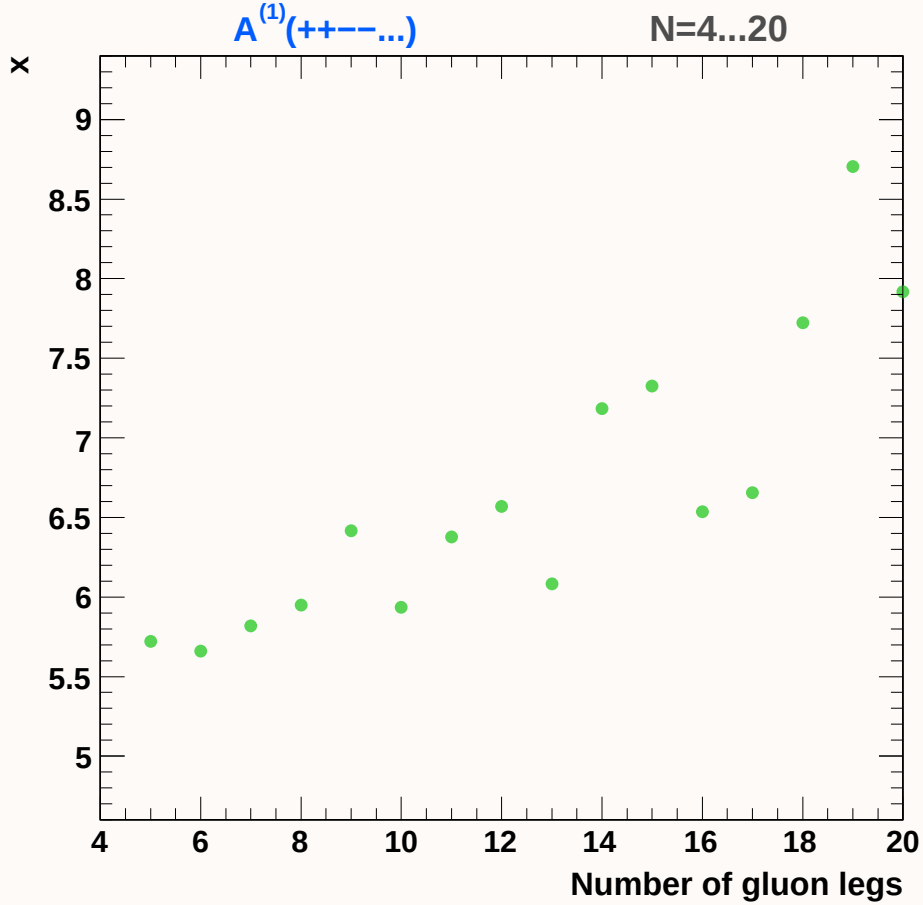


Speed of the calculation

(preliminary) (all calculations in double precision only)

- check for algorithm of polynomial complexity ($\tau \sim N^x$)
- check fractions: $x = \ln \frac{\tau_{N+1}}{\tau_N} / \ln \frac{N+1}{N}$





Summary

- The LHC physics program requires a detailed understanding of QCD.
- New major (physics) release Sherpa 1.1 ➡ fully independent, complete MC event generator.

  Improved description of hard multijet configurations together with jet fragmentation !

  CKKW is implemented for SM processes in Sherpa.

  ⇒ tool for jet physics. ⇒ full simulation of hadron-level events.
- Dipole shower for hadronic collisions ➡ valid & working approach, no non-PT assumptions.
- Predictions show good agreement with hadron-collider data.

  ⇒ interesting alternative to parton-shower models.

  ⇒ to do: some gluon-splitting channels, mass effects, merge/match with matrix elements.
- C++ code that implements Ellis–Giele–Kunszt–Melnikov method of calculating one-loop amplitudes using unitarity cuts. Basic tool is working.

  ⇒ good double-precision results for gluon case.

  ⇒ to do: fitting coefficients, higher precision, fully include quarks, squared amplitudes.