QCD scattering amplitudes beyond Feynman diagrams

Christian Schwinn

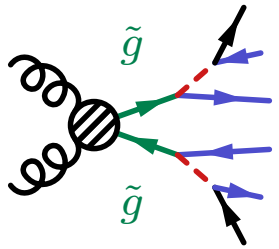
— RWTH Aachen —

03.04.2008

Introduction

Multi-particle final states important for LHC:

- Higgs in Vector-Boson fusion: $VV + jj$
- Higgs + top production: $t\bar{t}H \to t\bar{t}b\bar{b} \quad \Leftrightarrow \quad t\bar{t}jj$
- SUSY signals $\bar{q}q\bar{q}q + \chi^0\chi^0 \Leftrightarrow \quad 4j + Z \to 4j + \nu\bar{\nu}$

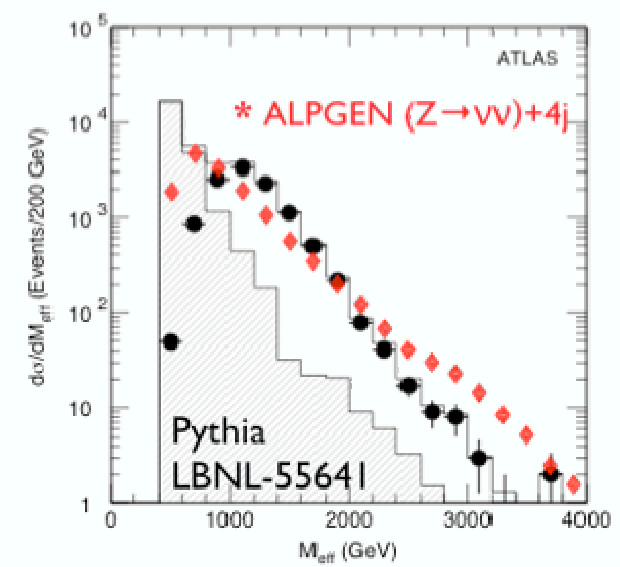


Need accurate predictions to distinguish signals from backgrounds

Les Houches 2005 NLO Wishlist

1. $pp \to VV + j$
2. $pp \to H + jj$
3. $pp \to t\bar{t}b\bar{b}$; $pp \to t\bar{t}jj$
4. $pp \to VVb\bar{b}$, $pp \to VVjj$
5. ...

$\Rightarrow$ need $2 \to 4$ processes at NLO



(Gianotti, Mangano, hep-ph/0504221)

Ingredients of NLO calculation in QCD

LO cross section:

$$\sigma^{(0)}(pp \to n) = \sum_{ab} \int dx_1 dx_2 \;\; f_a(x_1) f_b(x_2) \int d\phi_n |\mathcal{A}_n^{(0)}(x_1 x_2 s)|^2$$

- A_n^0 tree level scattering amplitude $ab \to n$
- $f_{a/b}(x)$: Parton density functions: fitted to experiment
- $\int d\phi_n$: n-body phase space: Monte Carlo methods

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NLO corrections

$$\sigma^{(1)} = \sum_{ab} \int dx_1 dx_2 f_a(x_1) f_b(x_2) \left[\int d\phi_n \; 2\mathrm{Re}\left(\mathcal{A}_n^{(0)} A_n^{(1)}\right) + \int d\phi_{n+1} |\mathcal{A}_{n+1}^{(0)}|^2 \right]$$

- A_n^1 one-loop scattering amplitude $ab \to n$ (virtual corrections)
- A_{n+1}^0 tree level scattering amplitude $ab \to n+g$ (real corrections)
- only sum finite $\Rightarrow$ subtraction formalisms (e.g. Catani, Seymour 96)

Overview of Tree+NLO calculations

Why don't shut up and calculate?

$2 \to 2$ gluon tree amplitude: 4 diagrams

...

$2 \to 6$ gluon tree amplitude: 34300 diagrams

Textbook method: calculate spin averaged amplitude-squared

$$\sum_{s_i} |\mathcal{A}(s_i)|^2 = \sum_{s_i} |\mathcal{M}_1 + \mathcal{M}_2 + \ldots \mathcal{M}_n|^2 = \sum_{s_i} |\mathcal{M}_1|^2 + \mathcal{M}_1 \mathcal{M}_2^* + \cdots + |\mathcal{M}_n|^2$$

N diagrams $\Rightarrow$ N^2 terms: Not useful for $2 \to 5$, even numerically!

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More efficient: calculate helicity amplitudes

$$\mathcal{A}(s_1, s_2, \ldots s_n) = \text{complex number}$$

- evaluate $\mathcal{A}$ numerically (recursive methods): squaring easy
- massless theories: many helicity combinations vanish
- Amplitudes sometimes much simpler then individual diagrams

Overview of Tree+NLO calculations

Since 2003: New methods (initially) for massless QCD amplitudes

- Relation to twistor string theory (Witten 2003)

$\Rightarrow$ New representations of QCD amplitudes

- CSW rules (Cachazo, Svrček, Witten 04)
  - All massless born QCD amplitudes from MHV vertices
- BCFW rules: (Britto, Cachazo, Feng/Witten, 04/05)
  - Construct born amplitudes from on-shell subamplitudes
- Unitarity methods (Britto, Cachazo, Feng 04, Anastasiou et.al 06, Forde 07,...)

Common ideas:

on-shell amplitudes as building blocks, complex kinematics

Plan of the (rest of the) talk

N-Gluon amplitudes

- Color decomposition, Helicity methods
- MHV amplitudes, Berends-Giele recursion

New "Twistor inspired" methods

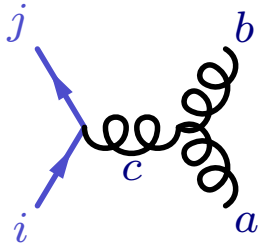
- MHV diagrams
- BCFW recursion
- Extension to massive particles

Impact on phenomenology?

- Numerical comparison
- Outlook to loops

Color decomposition

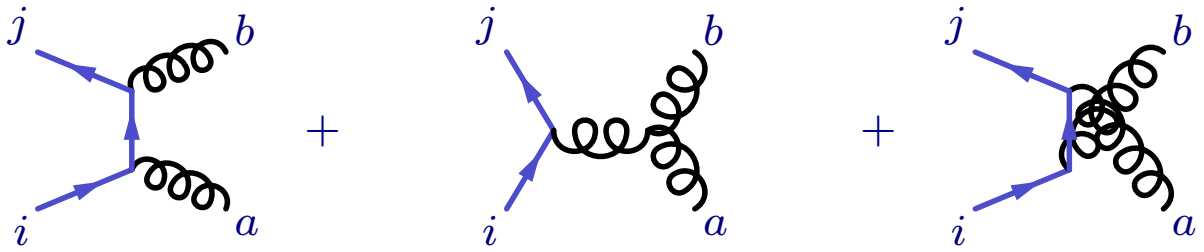
Simplifying color factors using Lie Algebra $[T^a, T^b] = \mathrm{i} f^{abc} T^c$



$$\propto \quad -\mathrm{i} T^c_{ij} f^{abc} V^{\mu_a \mu_b \mu_c}_{ggg}$$

$$= (\mathrm{i})^2 \left[(T^a T^b)_{ij} V^{\mu_a \mu_b \mu_c}_{ggg} + (T^b T^a)_{ij} V^{\mu_b \mu_a \mu_c}_{ggg} \right]$$

Color decomposition: into color ordered partial amplitudes

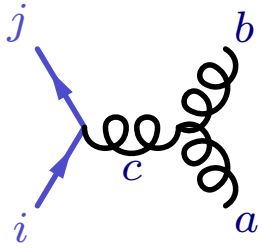


$(T^a T^b)_{ij}$ $\qquad$ $(T^a T^b)_{ij} + (T^b T^a)_{ij}$ $\qquad$ $(T^b T^a)_{ij}$

$$\mathcal{A}_4(Q_i, g_a, g_b, \bar{Q}_j) = (T^a T^b)_{ij} A_4(Q_i, g_a, g_b, Q_j) + (T^b T^a)_{ij} A_4(Q_i, g_b, g_a, Q_j)$$

Color decomposition

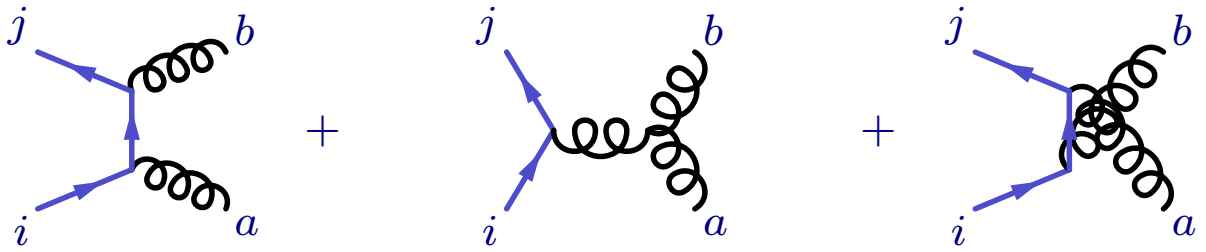
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General decomposition (Berends,Giele, 1987):

$$\mathcal{A}_{n+2}(Q_i, g_1, g_2, ..., g_n, \bar{Q}_j) = g^n \sum_{\sigma \in S_n} (T^{a_{\sigma(1)}} ... T^{a_{\sigma(n)}})_{i,j} \, A_n \left(Q_i, g_{\sigma(1)}, ..., g_{\sigma(n)}, \bar{Q}_j \right)$$

Spinor helicity methods

Two-component Weyl spinors in braket notation

$$|k+\rangle = \lambda_{k,A} = \left(\frac{1+\gamma^5}{2}\right) u(k) \quad , \quad |k-\rangle = \bar{\lambda}_k^{\dot{A}} = \left(\frac{1-\gamma^5}{2}\right) u(k)$$

- Express momenta in terms of spinors: $\langle k+|\gamma^\mu|k+\rangle = 2k^\mu$
- antisymmetric spinor products $\langle pk\rangle = \langle p-|k+\rangle$, $[pk] = \langle p+|k-\rangle$

Polarization vectors of the external gluons

(Kleiss,Sterling; Gunion, Kunszt 1985, Xu, Zhang, Chang 1987)

$$\epsilon_\mu^\pm(k,q) = \pm \frac{\langle q\mp|\gamma_\mu|k\mp\rangle}{\sqrt{2}\,\langle q\mp|k\pm\rangle}$$

with q arbitrary light-like reference momentum

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Simple expression for MHV amplitudes (Parke-Taylor 1986)

$$A_n(g_1^+,\ldots,g_i^-,\ldots,g_j^-,\ldots g_n^+) = \mathrm{i}2^{n/2-1}\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

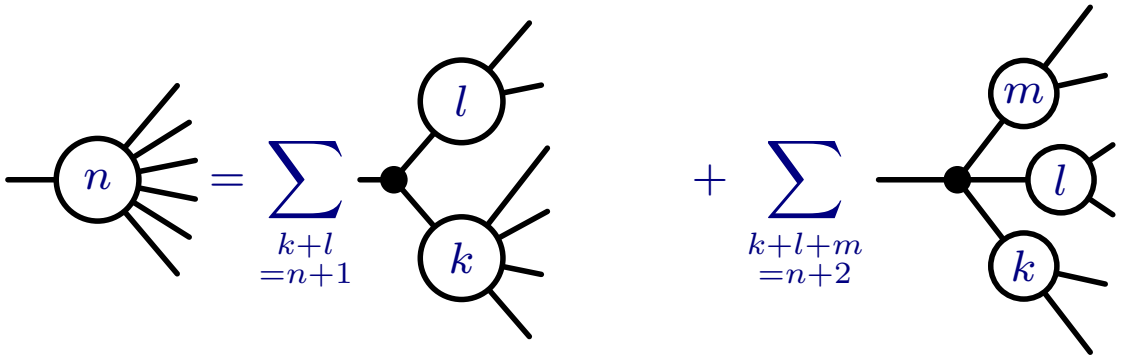
Berends-Giele recursion

Traditional proof of MHV-formula: (Berends, Giele 1988)

Recursion relations for one particle off-shell tree amplitudes

$$A_n(1,\ldots,(n-1),\widehat{n}) = \langle 0|\phi_n(k_n)|k_1,\ldots k_{n-1}\rangle$$

Can be constructed recursively:



- Avoids redundancies ⇒ useful for numerical calculations.
  `Alpha` (Caravaglios, Moretti),
  `Helac` (Kanaki, Papadopoulos), `O'Mega` (Moretti, Ohl, Reuter, CS)
- *But*: off-shell amplitudes required, not ideal for analytical calculations

"Effective Supersymmetry" of QCD: (Parke, Taylor 1985; Kunszt 1986)

Tree level partial amplitudes for massless quarks are the same as for gluinos in a fictitious, unbroken, SUSY QCD.

SUSY transformations of helicity states of gluons and gluinos with Grassmann-valued spinor η:

$$\delta_\eta g^\pm(k) = \langle \eta \pm | k \mp \rangle \, \lambda^\pm(k) \qquad \delta_\eta \lambda^\pm(k) = - \langle \eta \mp | k \pm \rangle \, g^\pm(k)$$

SUSY-relations

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SUSY Ward-Identities (Grisaru, Pendleton 1977)

$$0 = \langle 0 | [Q_{\text{SUSY}}, \psi_1 \dots \psi_n] | 0 \rangle = \sum\nolimits_i A_n(\psi_1 \dots (\delta_\eta \psi_i) \dots \psi_n)$$

Fermionic MHV amplitudes (set $|\eta+\rangle \propto |j+\rangle$) (Parke, Taylor 1985; Kunszt 1986)

$$A_n(\bar{\lambda}_1^-, g_2^+, \dots g_j^-, \dots, \lambda_n^+) = \frac{\langle nj \rangle}{\langle 1j \rangle} A_n(g_1^-, g_2^+, \dots g_j^-, \dots, g_n^+)$$

$$= \mathrm{i} 2^{n/2-1} \frac{\langle 1j \rangle^3 \langle nj \rangle}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

Using MHV amplitudes

So far: All-multiplicity solution for MHV amplitudes

Useful for all amplitudes?

Earlier attempts MHV amplitudes from 2-D field theory (Nair 88),
relation to self-dual Yang-Mills (Bardeen; Cangemi; Chalmers, Siegel 96)

Insights from Twistor space (Witten 2003)

- "Half a Fourier transform":

$$(\lambda_A, \bar{\lambda}^{\dot{A}}) \Rightarrow (\lambda_A, \mu^{\dot{A}}) = (\lambda_A, \frac{\partial}{\partial \bar{\lambda}_{\dot{A}}})$$

- MHV amplitudes nonvanishing on line in twistor space
- Conjectures
  - All QCD amplitudes lie on curves in twistor space, determined by # of negative helicities and loops
  - Can be computed in string theory on twistor space

$\Rightarrow$ New representations of QCD amplitudes

MHV diagrams

MHV diagrams (CSW rules): (Cachazo, Svrček, Witten 2004)

All QCD amplitudes from MHV vertices

$$V_{\text{CSW}}(g_1^+ \dots g_i^- \dots g_j^- \dots g_n^+) = 2^n \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

(with off-shell continuation $|k+\rangle \to \not{k} |\eta-\rangle$)

Scalar propagators $\frac{\mathrm{i}}{k^2}$ connecting $+$ and $-$ labels

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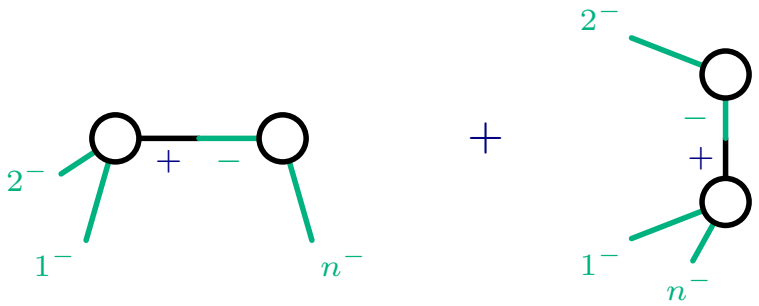
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Example: NMHV amplitudes $A(g_1^-, g_2^-, g_3^+, \dots g_n^-)$:

- Distribute negative helicities over $d = n^- - 1 = 2$ MHV vertices



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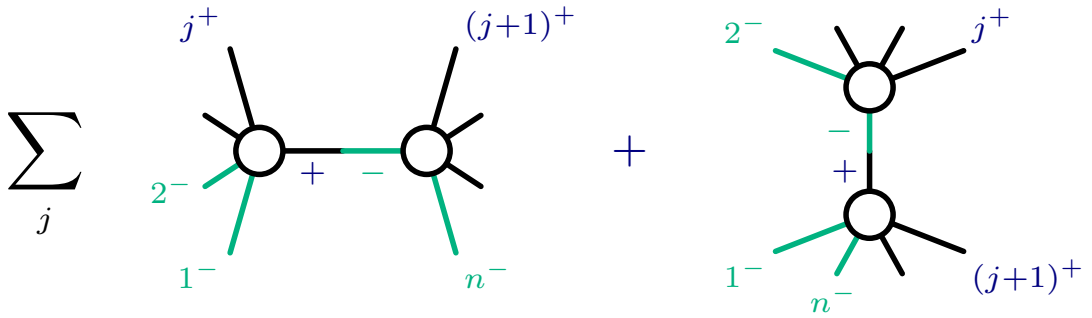
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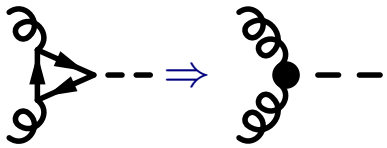


- Distribute positive helicities $\Rightarrow 2(n-3)$ diagrams
- Complexity grows with n^-

External massive particles

Effective Higgs-gluon coupling

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{6\pi v} \int dx^4 H \, \text{tr}[F_{\mu\nu} F^{\mu\nu}]$$



Trick: add effective vertex for pseudoscalar

$$\mathcal{L}_{\text{eff},A} = \frac{\alpha_s}{6\pi v} \int dx^4 \, \mathrm{i}A \, \text{tr}[F_{\mu\nu} F_{\rho\sigma}] \epsilon^{\mu\nu\rho\sigma}$$

MHV vertex for complex scalar $\phi = H + \mathrm{i}A$ (Dixon, Glover, Khoze 04)

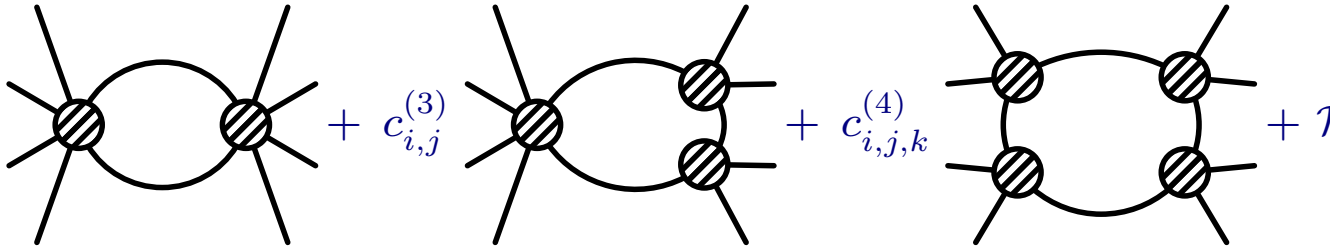
$$V_{\text{CSW}}(\phi, g_1^+ \dots g_i^- \dots g_j^- \dots g_n^+) = \mathrm{i} 2^{n/2-1} \frac{\alpha_s}{6\pi v} \frac{\langle ij \rangle^4}{\langle 12 \rangle \dots \langle (n-1)n \rangle \langle n1 \rangle}$$

External W/Z bosons (Bern, Forde, Kosower, Mastrolia 04)

MHV diagrams at one-loop level

Decomposition of one-loop amplitudes into scalar integrals

$$\mathcal{A} = \sum c_i^{(2)} \; + \; c_{i,j}^{(3)} \; + \; c_{i,j,k}^{(4)} \; + \; \mathcal{R}$$



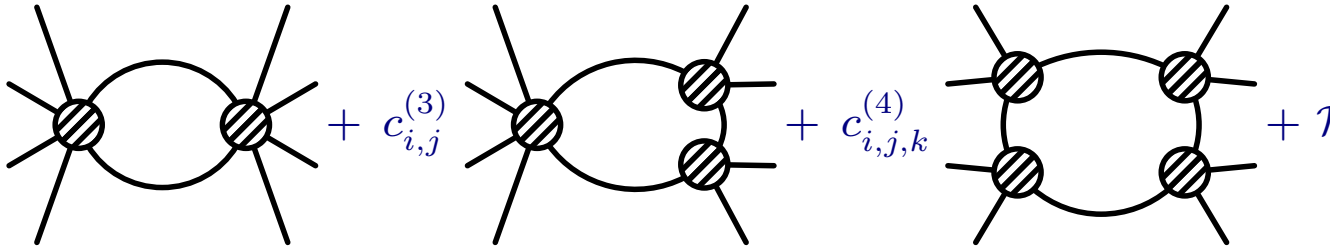
$B_0(k_{1,i}, k_{i,n})$ $C_0(k_{1,i}, k_{i,j}, k_{j,n})$ $D_0(k_{1,i}, k_{i,j}, k_{j,k}, k_{k,n})$

$c^{(n)}$: independent of ϵ; $\mathcal{R}$: "rational part" from $\epsilon \times \frac{1}{\epsilon}$

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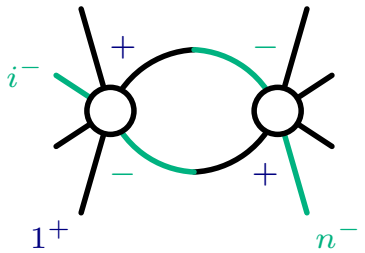
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MHV rules for one-loop amplitudes



- "cut constructable" part of amplitudes

  (Bedford, Brandhuber, Spence, Travaglini; Quigly, Rozali 04)

- Where is all-plus amplitude $A(g_1^+, \ldots g_2^+)$?

Several tree-level derivations of CSW rules

- Generalized BCFW recursion (Risager 05)
- Field-redefinition in light-cone QCD (Mansfield 05)
- Gauge theory on twistor space (Boels, Mason, Skinner 06)

Derivations of CSW rules

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$\Rightarrow$ **Proposals for rational part of one-loop amplitudes**

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Extend derivations to CSW rules for massive colored scalar

(R.Boels, CS 07)

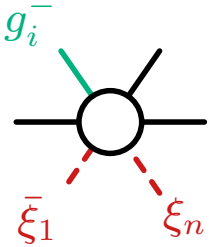
- Toy model for top-quarks
- Rational part of QCD amplitudes $\Leftrightarrow$ Scalar in $D = 4 - 2\epsilon$

CSW rules for massive scalars

Vertices:

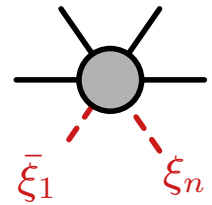
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massless MHV vertices



$$= \mathrm{i}2^{n/2-1} \frac{\langle in\rangle^2 \langle 1i\rangle^2}{\langle 12\rangle \ldots \langle (n-1)n\rangle \langle n1\rangle}$$

new vertex $\sim m^2$



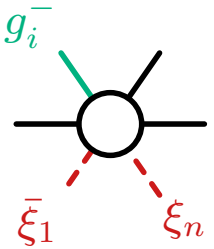
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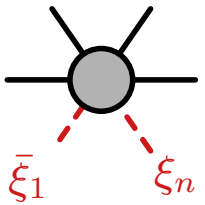
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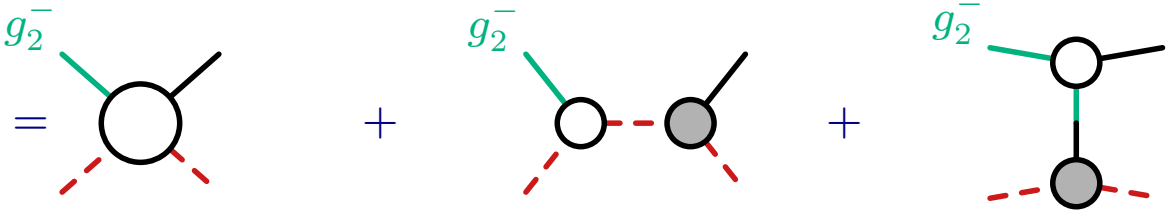
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Example:

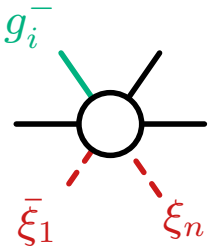
$A_4(\bar{\xi}_1, g_2^-, g_3^+, \xi_4) =$



CSW rules for massive scalars

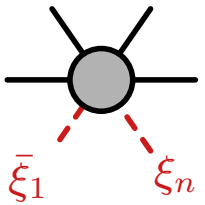
Vertices: (R.Boels, CS, 07)

massless MHV vertices



$$= \mathrm{i}2^{n/2-1} \frac{\langle in\rangle^2 \langle 1i\rangle^2}{\langle 12\rangle \ldots \langle (n-1)n\rangle \langle n1\rangle}$$

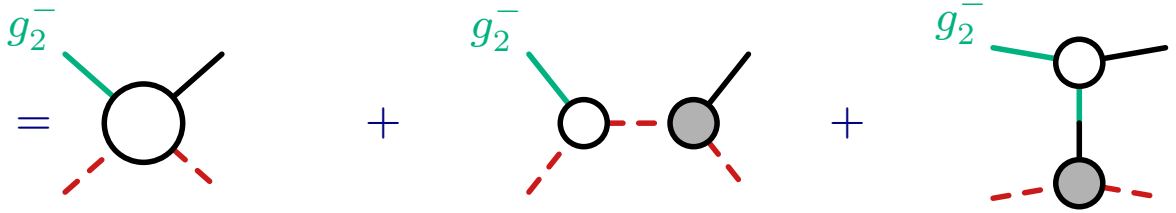
new vertex $\sim m^2$



$$= \mathrm{i}2^{n/2-1} \frac{-m^2 \langle 1n\rangle}{\langle 12\rangle \ldots \langle (n-1)n\rangle}$$

Example:

$A_4(\bar{\xi}_1, g_2^-, g_3^+, \xi_4) =$

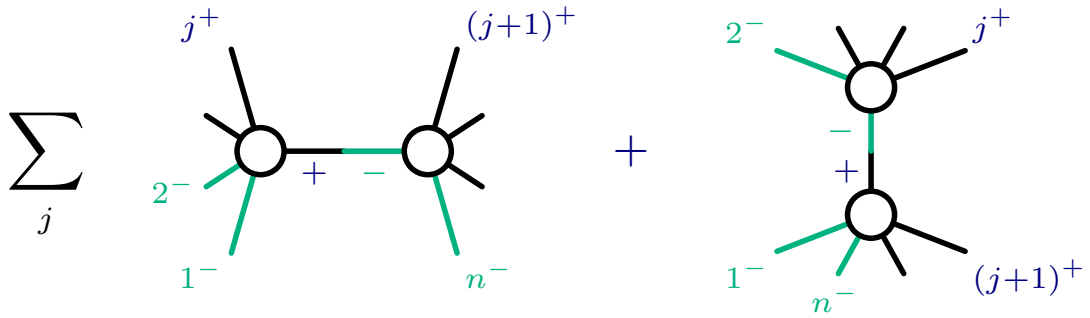


Application: simple prescription for leading piece for $m \to 0$

$$A_n(\bar{\xi}_1, g_2^+ \ldots, , \xi_n) = \mathrm{i}2^{n/2-1} \frac{-m^2 \langle (n-1) + |\not{k}_n \not{k}_1| 2-\rangle}{2(k_1 \cdot k_2) 2(k_n \cdot k_{n-1}) \langle 23\rangle \ldots \langle (n-2)(n-1)\rangle} + \mathcal{O}(m^2)$$

(agrees with Bern, Dixon, Dunbar, Kosower 96)

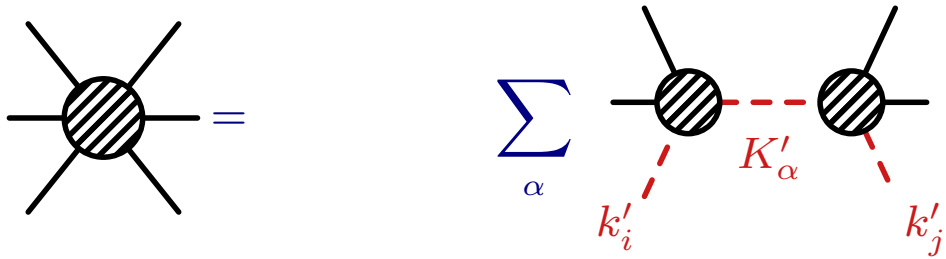
CSW -Summary



- construct all QCD tree-amplitudes from MHV vertices
- Lagrangian methods of derivation
- CSW rules for massive scalars
  - Massless MHV vertices + new vertex from mass term
  - Derivations should extend to massive particles with spin

On-shell recursion relations

Construct amplitudes from on-shell sub-amplitudes



Shifted on-shell momenta:

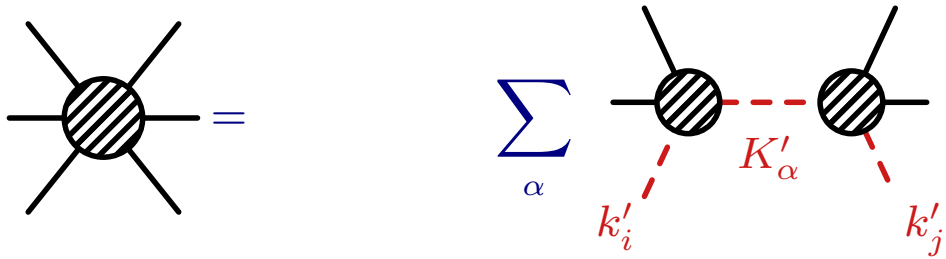
(Britto, Cachazo, Feng/ Witten, 04/05)

$$k'_i = k_i - z_\alpha \eta \qquad k'_j = k_i + z_\alpha \eta$$

with $\eta^2 = 0$ $\qquad k_i \cdot \eta = k_j \cdot \eta = 0$

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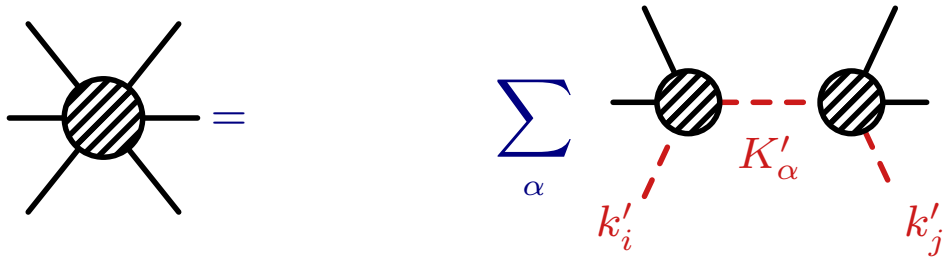
$\Rightarrow \eta^\mu = \frac{1}{2} \langle i+|\gamma^\mu|j+\rangle$ for massless momenta

Corresponds to shifted spinors:

$$|i'+\rangle = |i+\rangle - z\,|j+\rangle \qquad |j'-\rangle = |j-\rangle + z\,|i-\rangle$$

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Choose $z_\alpha = \dfrac{K_\alpha^2}{\langle i+|\not{K}_\alpha|j+\rangle} \quad \Rightarrow \quad K_\alpha'^2 = 0$

Conditions for BCFW recursion:

$A(z)$ has simple poles, $A(z) \to 0$ for $z \to \infty$

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Analysis of $z \to \infty$: Most dangerous diagrams: triple gluon vertices

$$A(z) \sim \underbrace{n \,\text{propagators}}_{z^{-n}} \times \underbrace{(n+1)\,\text{vertices}}_{z^{n+1}} \times \epsilon_i \times \epsilon_j \sim z \times \epsilon_i \times \epsilon_j$$

Scaling of gluon polarization vectors:

$$\epsilon^+(k_i') \sim \frac{1}{z}, \quad \epsilon^-(k_i') \sim z, \quad \epsilon^+(k_j') \sim z, \quad \epsilon^-(k_j') \sim \frac{1}{z}$$

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- (i^+, j^-): $A(z) \sim \frac{1}{z}$ from powercounting; (BCFW 05)
  diagrammatical proof (Draggiotis et.al.; Vaman, Yao; 05)
- (i^+, j^+), (i^-, j^-): from CSW diagrams (BCFW 05)
  auxiliary shift (Badger,Glover,Khoze,Svrček 05)
  Lagrangian analysis (Arkani-Hamed, Kaplan 08)

BCFW recursion for massive scalars (Badger,Glover,Khoze,Svrček 05)

- applied for shifted gluon lines (Forde Kosower 05)
- Simple form of scalar amplitudes with positive helicity gluons:

  (Ferrario, Rodrigo, Talavera 06)

$$A(\bar{\phi}_1^+, g_2^+, \ldots, \phi_n^-) = \frac{i 2^{n/2-1} m^2 \langle 2+|\Pi_{j=3}^{n-2}(y_{1,j} - \not{k}_j \not{k}_{1,j-1})|(n-1)-\rangle}{y_{1,2} \ldots y_{1,n-2} \langle 23\rangle \langle 34\rangle \ldots \langle (n-2)(n-1)\rangle}$$

$$k_{1,j} = \sum_1^j k_j, \qquad y_{1,j} = k_{1,j}^2 - m^2$$

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Recursion for massive fermions (+gauge bosons)

- "stripped" amplitudes: (Badger, Glover, Khoze 05)

$$\sum_{\sigma=\pm} A(\ldots, Q_{K'}^{\sigma}) \frac{\mathrm{i}}{K^2 - m^2} A(\bar{Q}_{K'}^{-\sigma}, \ldots) = A(\ldots, Q_{K'}^{\bullet}) \frac{\mathrm{i}(\not{K}' + m)}{K^2 - m^2} A(\bar{Q}_{K'}^{\bullet}, \ldots)$$

- 5-6 point $t\bar{t} + g$ amplitudes calculated (Ozeren,Stirling 06; Hall 07)

SUSY-relations for massive quarks

Spinors for massive quarks (Kleiss, Stirling 85;. . . ; CS, S.Weinzierl 05)

$$u(\pm, q) = \frac{1}{\langle p^\flat \pm | q \mp \rangle} (\not{p} + m) | q \mp \rangle$$

with light cone projection $p^\flat = p - \frac{p^2}{2p \cdot q} q$

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Embed QCD with massive quark $Q = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$ in $N = 1$ SQCD

$\Rightarrow$ two complex scalars $\phi_\pm$ as superpartners

SUSY transformation of helicity states (CS, S.Weinzierl 06)

$$\delta_\eta \phi_k^+ = \langle \eta k^\flat \rangle \, Q_k^+ + m \frac{\langle q\eta \rangle}{\langle q k^\flat \rangle} Q_k^- \qquad \delta_\eta Q^- = \langle \eta k^\flat \rangle \, \phi_k^- - m \frac{[q\eta]}{[q k^\flat]} \phi_k^+$$

Simplify for $|\eta \pm\rangle \propto |q \pm\rangle \Rightarrow$ similar to massless case!

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SUSY relations to scalar amplitudes:

$$\langle q 1^\flat \rangle A(\bar{Q}_1^+, 2^+, \ldots, Q_n^-) = \langle n^\flat q \rangle A(\bar{\phi}_1^+, 2^+, \ldots, \phi_n^-)$$

(Verified explicitly in Ferrario, Rodrigo, Talavera 06)

Shift of massive quark lines (CS, S.Weinzierl 07)

Decompose general momenta into light-like $l_{i/j}$: (del Aguila, Pittau 04)

$$p_i = l_i + \alpha_j l_j \quad , \quad p_j = \alpha_i l_i + l_j$$

with $$\alpha_\ell = \frac{2p_ip_j \mp \sqrt{\Delta}}{2p_\ell^2}, \qquad \Delta = (2p_ip_j)^2 - 4p_i^2p_j^2$$

On-shell recursion for massive quarks

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Define shifted spinors:

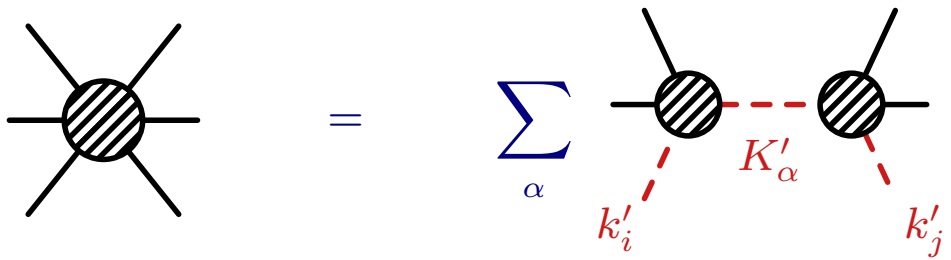
$$u_i{}'(-) = u_i(-) - z\,|l_j+\rangle \quad , \quad \bar{u}_j'(+) = \bar{u}_j(+) + z\,\langle l_i+|$$

with reference spinors $|q_i\pm\rangle = |l_j\pm\rangle$, $|q_j\pm\rangle = |l_i\pm\rangle$

Corresponds to shifted momenta

$$p_i'^{\mu} = p_i^\mu - \frac{z}{2}\,\langle l_i+|\gamma^\mu|l_j+\rangle \ , \quad p_j'^{\mu} = p_j^\mu + \frac{z}{2}\,\langle l_i+|\gamma^\mu|l_j+\rangle$$

BCFW-Summary



- Recursive construction of scattering amplitudes from on-shell sub-amplitudes
- compact expressions in analytical calculations
- Extension to massive quarks, shift of massive quark lines

Impact on phenomenology?

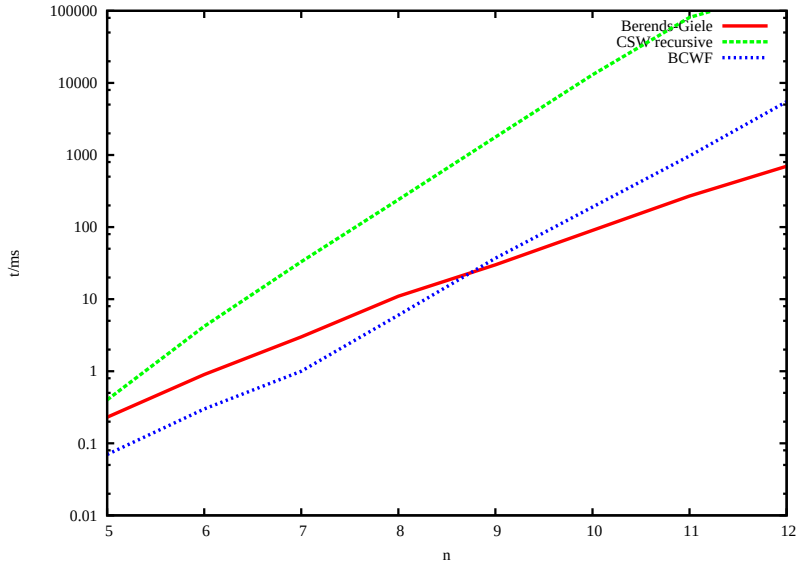
Numerical comparison of n-gluon amplitudes

Color ordered amplitudes

(Dindsdale, Ternik, Weinzierl, 06)

(CSW: recursive reformulation

(Bena, Bern, Kosower, 04))



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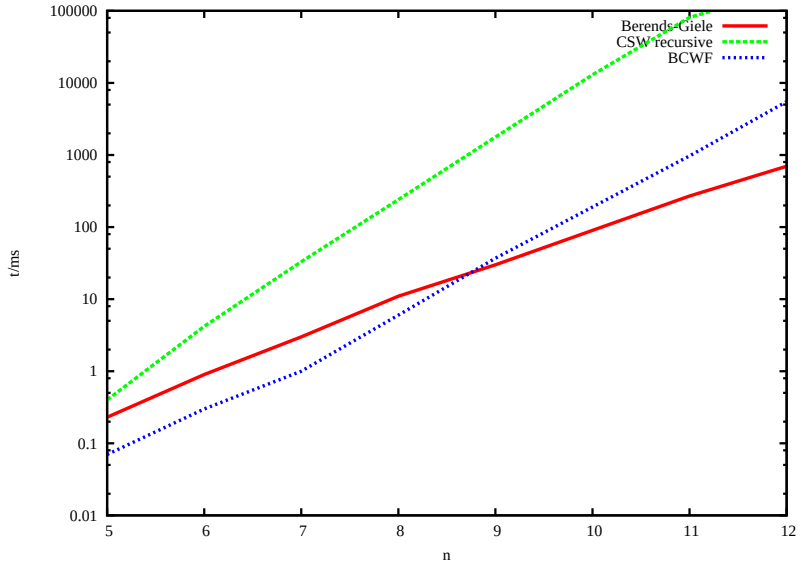
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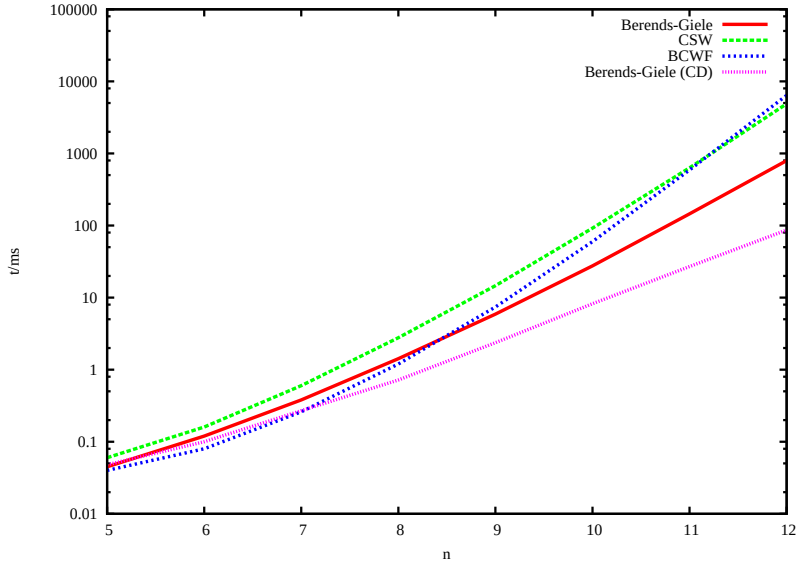
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Averaged color ordered/dressed amplitudes (Duhr, Höche, Maltoni, 06)

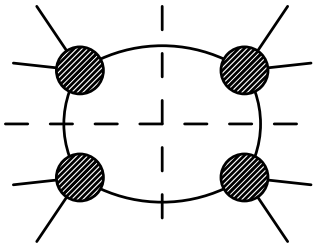
(CSW: reformulation with cubic vertices)





New unitarity methods

- Box coefficients from quadruple cuts (Britto, Cachazo, Feng 04)



$$\Rightarrow c_{ijk}^{(4)} \sim \sum_{k_{1/2}} A_{1,i}\, A_{i,j}\, A_{j,k}\, A_{k,n}$$

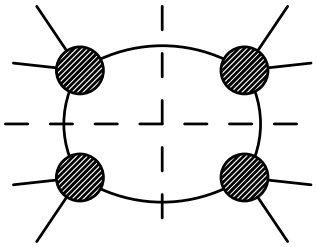
  complex momenta to solve constraints

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On-shell recursion for one-loop amplitudes?

- Proof requires $A(z) \xrightarrow{z\to\infty} 0$, single poles in z
  $\Rightarrow$violated by one loop amplitudes
- works for rational part R (Bern, Dixon, Kosower 05)
- More complicated than tree level, need auxiliary shifts,...

Impact on phenomenology?

Analytical results from combination of methods:

- all one-loop six-gluon helicity amplitudes

  (Unitarity+MHV+BCFW+dedicated PV)

  (Numerical calculation: Ellis, Giele, Zanderighi 06)

- one-loop Higgs +gluon amplitudes

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Automated implementations

- `CutTools:` (Ossola, Papadopoulos, Pittau 06-08)
  Numerical variant of generalized unitarity
  Results for 6γ amplitudes, VVV production

- `BlackHat:` (Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre 08)
  generalized unitarity +BCFW for $n \leq 8$ gluon amplitudes

Summary

Methods for QCD calculations:

- spinor helicity
- color ordering
- closed expression for MHV amplitudes
- Berends-Giele recursion

New methods:

- CSW diagrams
- on-shell recursion relations
- new unitarity methods

Extension to massive particles

- new CSW vertex for massive scalars
- On-shell recursion for massive quarks

Summary

Impact on phenomenology?

- new methods: compact analytical expressions
- numerical tree-level calculations:
  Berends-Giele asymptotically more efficient
- NLO: unitarity methods suitable for automatization,
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Impact on phenomenology?

- new methods: compact analytical expressions
- numerical tree-level calculations:
  Berends-Giele asymptotically more efficient
- NLO: unitarity methods suitable for automatization,
  numerical stability remains to be checked
- "Conventional methods" practioners:

  "Unitarity inspired methods interesting but to late for LHC"
- "New methods" proponents:

  "Changes the computational approach to NLO corrections, generic NLO calculator seems feasible"

Stay tuned for more surprises