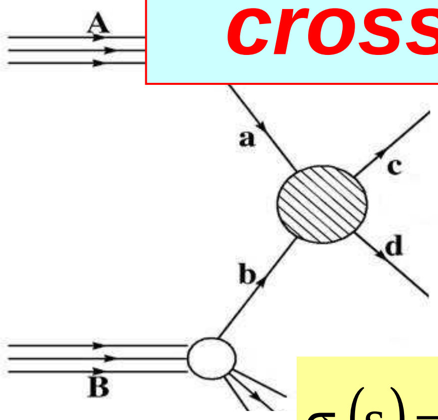
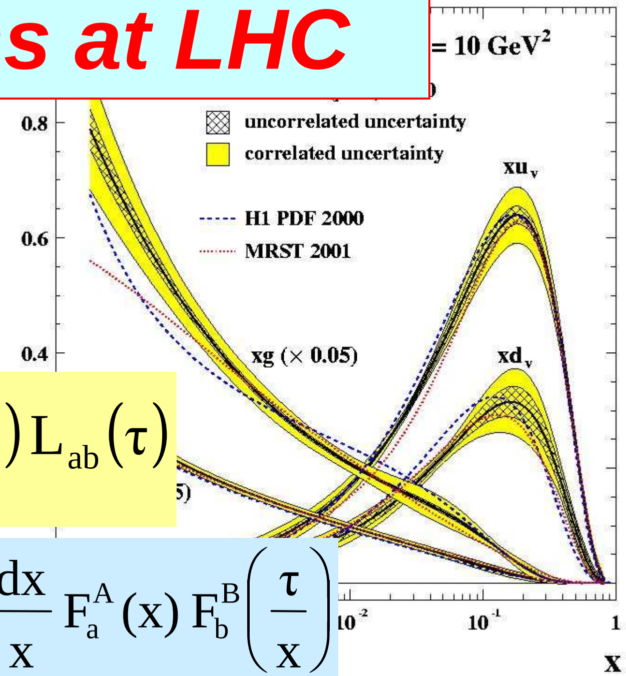
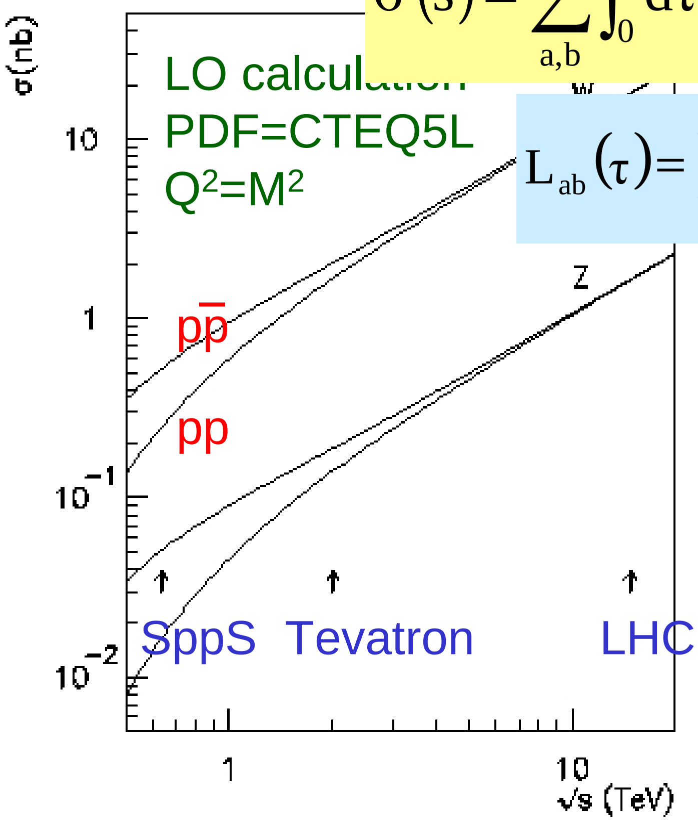
7. How to compute cross-sections at LHC

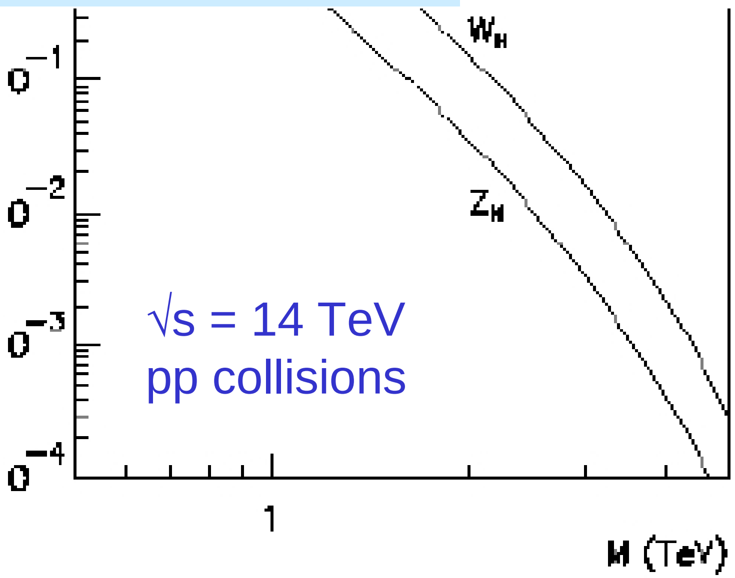




$$\sigma(s) = \sum_{a,b} \int_0^1 d\tau\, \hat{\sigma}_{ab}(\tau s)\, L_{ab}(\tau)$$

$$L_{ab}(\tau) = \sum_{a,b} \frac{1}{\tau} \int_\tau^1 \frac{dx}{x} F_a^A(x)\, F_b^B\left(\frac{\tau}{x}\right)$$

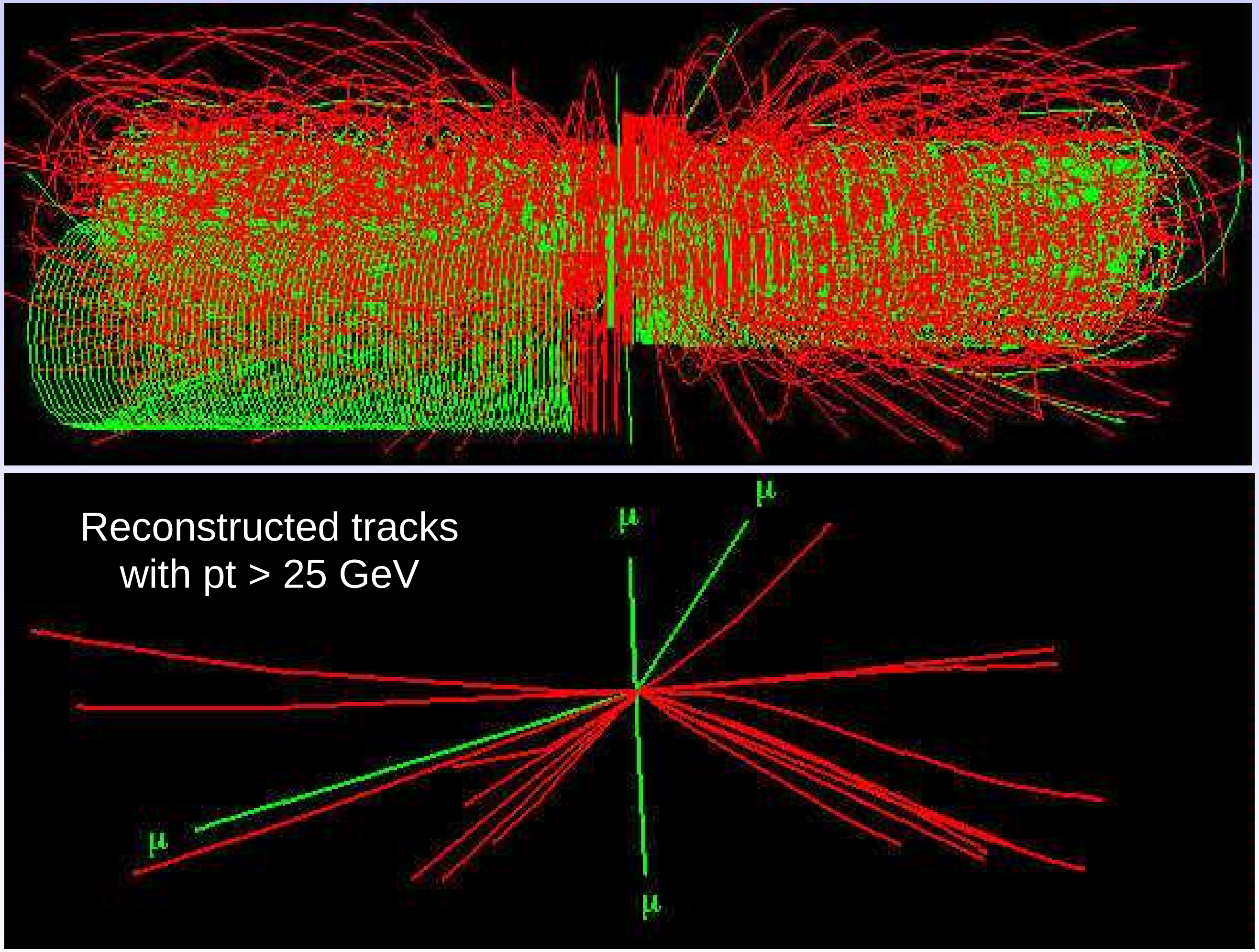




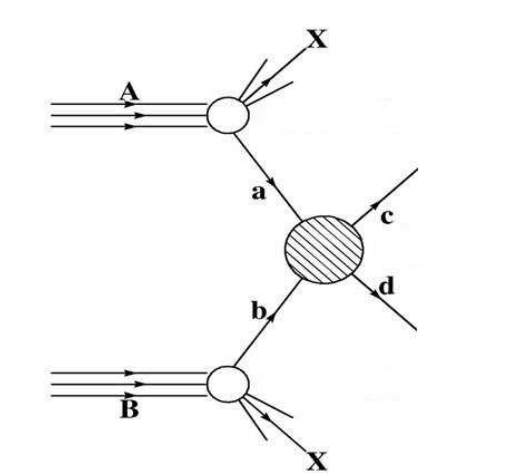
Outline

- Hard processes and the factorisation theorem
- Parton Distribution Functions (PDFs)
- Kinematic variables
- Cross-section uncertainties
- pp $\rightarrow$ Z, W
- pp $\rightarrow$ Z´, W´
- pp $\rightarrow$ single t , double t
- pp $\rightarrow$ single T, double T

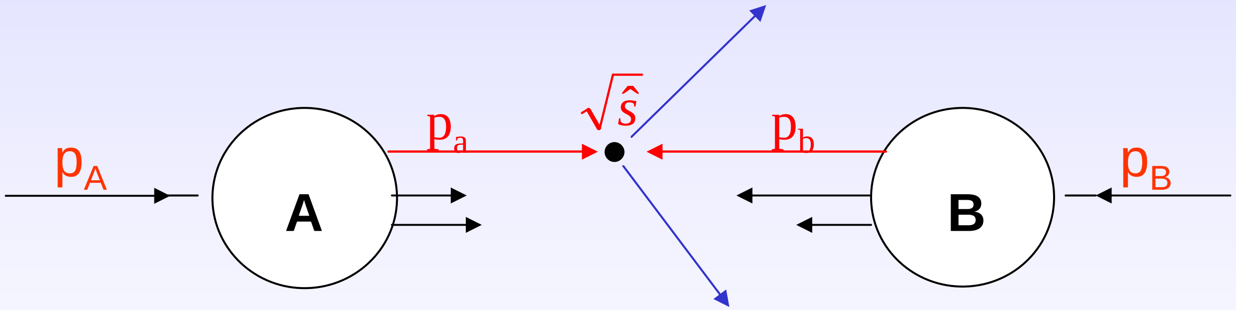
Soft and hard processes in hadron collisions



The Factorisation Theorem



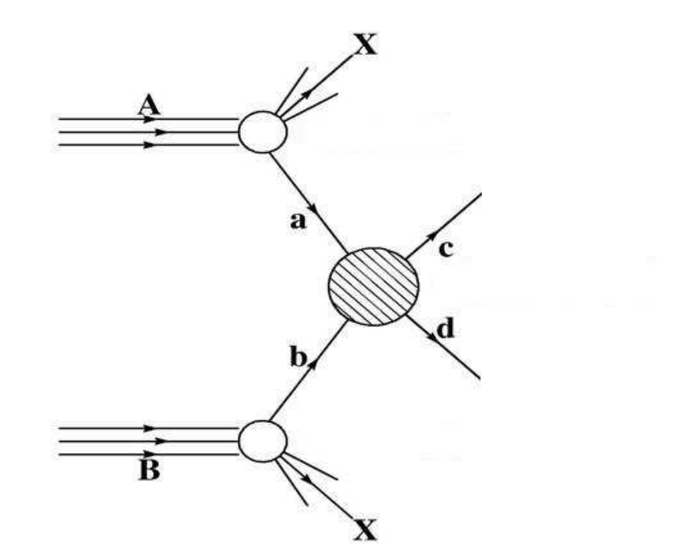
- The collision of two hadrons, A and B, yield a large number of particles in the final state (~1000 particles for √s=14 TeV)
- Most of these particles originate in soft parton processes
- We are interested only in particles originating in a hard parton process (a+b → c+d) where a,b are initial partons (q or g) and c,d any final particles



$$\left.\begin{array}{l} p_a = x_a\, p_A \\ p_b = x_b\, p_B \end{array}\right\} \longrightarrow \quad \hat{s} = (p_a + p_b)^2 \approx 2x_a x_b p_A . p_B \approx x_a x_b s$$

$$\longrightarrow \boxed{\hat{s} \approx x_a x_b s} \longrightarrow \boxed{x < 1 \rightarrow \hat{s} < s}$$

Calculation of cross-sections



The cross-section for A+B→c+d+X is

$$\sigma(s) = \sum_{a,b} \int dx_a \, dx_b \, f_a^A(x_a) \, f_b^B(x_b) \, \hat{\sigma}_{ab}(\hat{s})$$

Sum over initial partonic states a,b

$\hat{\sigma}_{ab}$ = hard scattering cross-section

$f_i^H(x)$ = Parton Density Function

$$\sigma(s) = \sum_{a,b} \int_0^1 d\tau \, \hat{\sigma}_{ab}(\tau s) \, L_{ab}(\tau)$$

L_{ab} is the luminosity function

$$L_{ab}(\tau) = \sum_{a,b} \frac{1}{\tau} \int_\tau^1 \frac{dx}{x} F_a^A(x) \, F_b^B\left(\frac{\tau}{x}\right)$$

and F(x) = x f(x)

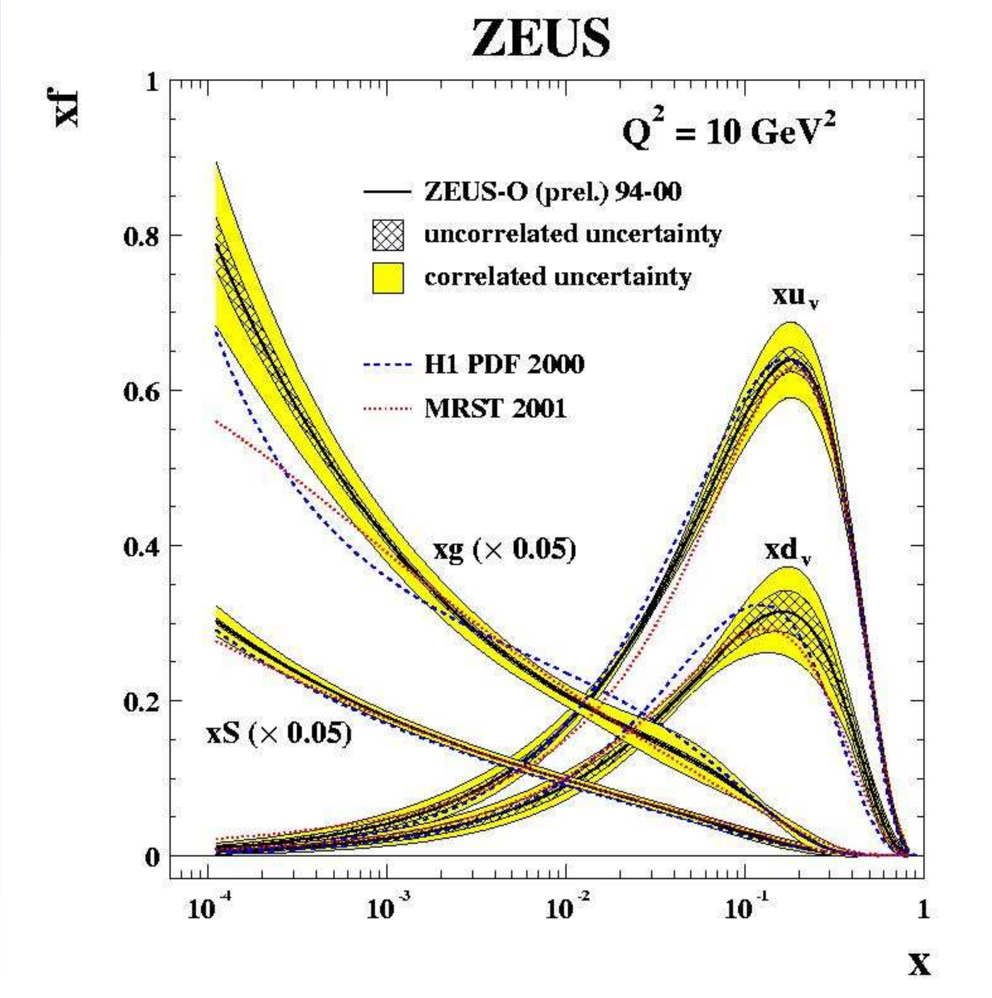
Changing variables x_a, $x_b \leftrightarrow \tau$, y

$$x_{a,b} = \sqrt{\tau} e^{\pm y}$$

$$\tau = \hat{s}/s = x_a x_b \qquad y = \frac{1}{2} \mathrm{Log} \frac{x_a}{x_b}$$

$$0 < \tau < 1 \qquad +\mathrm{Log}\sqrt{\tau} < y < -\mathrm{Log}\sqrt{\tau}$$

The Parton Distribution Functions (PDFs) (1)



u- and d-quarks dominate at large x-values

Gluons dominate at small x

There are uncertainties in the PDFs, in particular on the gluon distribution at small x

PDFs depend also on Q^2
$F(x)=xf(x) \rightarrow F(x,Q^2)$

There are many sets of PDFs (CTEQ, MRST, ZEUS, …)

The Parton Distribution Functions (PDFs) (2)

In principle 11 different parton distributions to consider

$$u, \bar{u}, \quad d, \bar{d}, \quad s, \bar{s}, \quad c, \bar{c}, \quad b, \bar{b}, \quad g$$

$m_c, m_b \gg \Lambda_{\mathrm{QCD}}$ so heavy parton distributions determined perturbatively. Assume $s = \bar{s}$. Leaves 6 independent combinations. Relate s to $1/2(\bar{u} + \bar{d})$ and use

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad \mathrm{sea} = 2 * (\bar{u} + \bar{d} + \bar{s}), \quad \bar{d} - \bar{u}, \quad g.$$

Assuming isospin symmetry $p \to n$ leads to

$$d^p(x) \to u^n(x) \qquad u^p(x) \to d^n(x).$$

Various sum rules constraining parton inputs and conserved order by order in α_S for evolution, i.e. conservation of number of valence quarks.

$$\int_0^1 u_V(x)\,dx = 2 \qquad \int_0^1 d_V(x)\,dx = 1$$

Also conservation of momentum carried by partons – important constraint on the gluon, which is only probed indirectly.

$$\int_0^1 x\Big(\sum_i (q_i(x) + \bar{q}_i(x)) + g(x)\Big)\,dx = 1.$$

The Parton Distribution Functions (PDFs) (3)

Many sets of PDFs:

CTEQ, MRST, GRV, ZEUS, CMS, …
also LO, NLO, …(CTEQL, CTEQN,…)
and improved sets (CTEQ2, CTEQ4, CTEQ6,…)

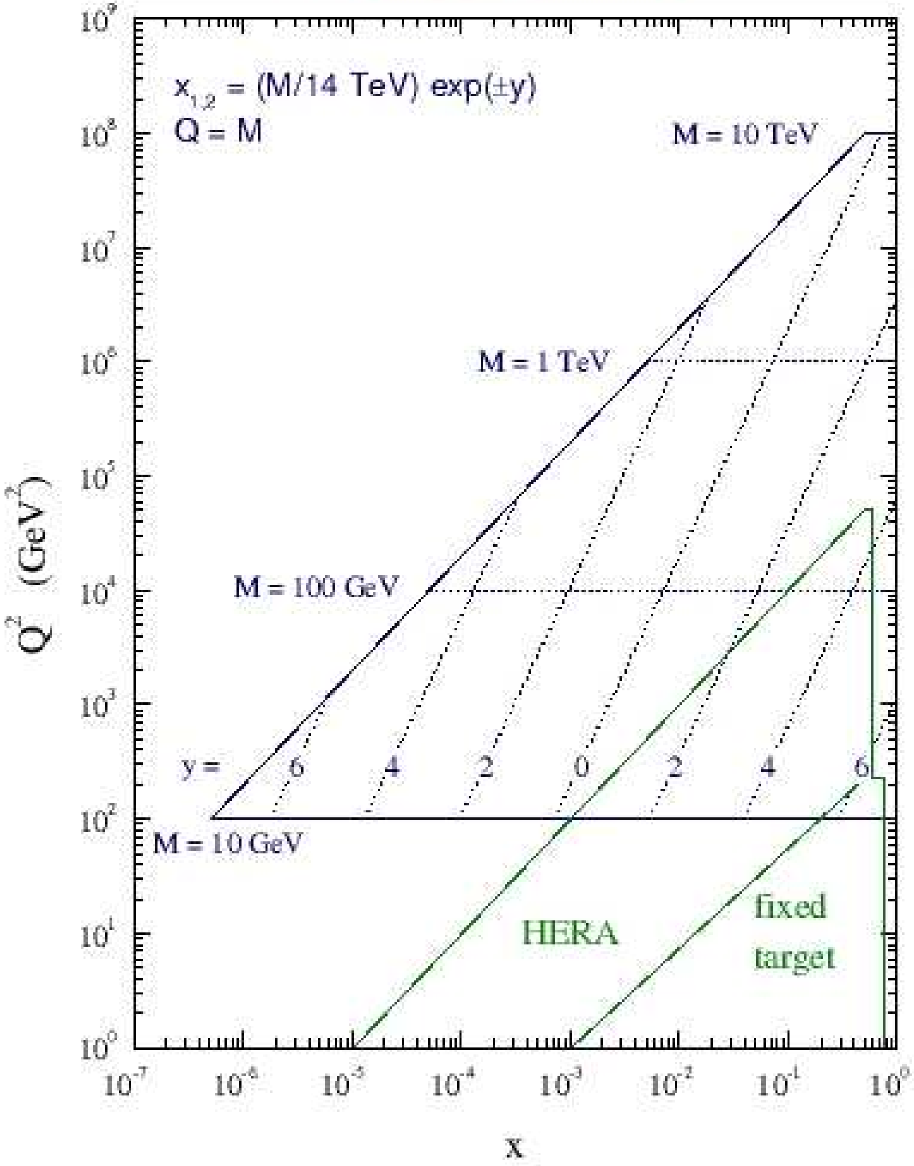
Example of parametrization (ZEUS) at a given Q^2:

$F(x) = x\, f(x) = a_1\,(1-x)^{a2}\,(1+a_3\sqrt{x}+a_4 x)\, x^{a5}$
Parameters $a_1, \ldots, a_5$ fitted to data (HERA and fixed target expts)

Evolution of PDFs:

$F(x,Q_1^2) \rightarrow F(x,Q_2^2)$ using DGLAP equations (perturbative QCD)
$F(x_1,Q^2) \rightarrow F(x_2,Q^2)$ not predicted by perturbative QCD
Lowest measured x values are ~ 10^{-5}

The Parton Distribution Functions (PDFs) (4)

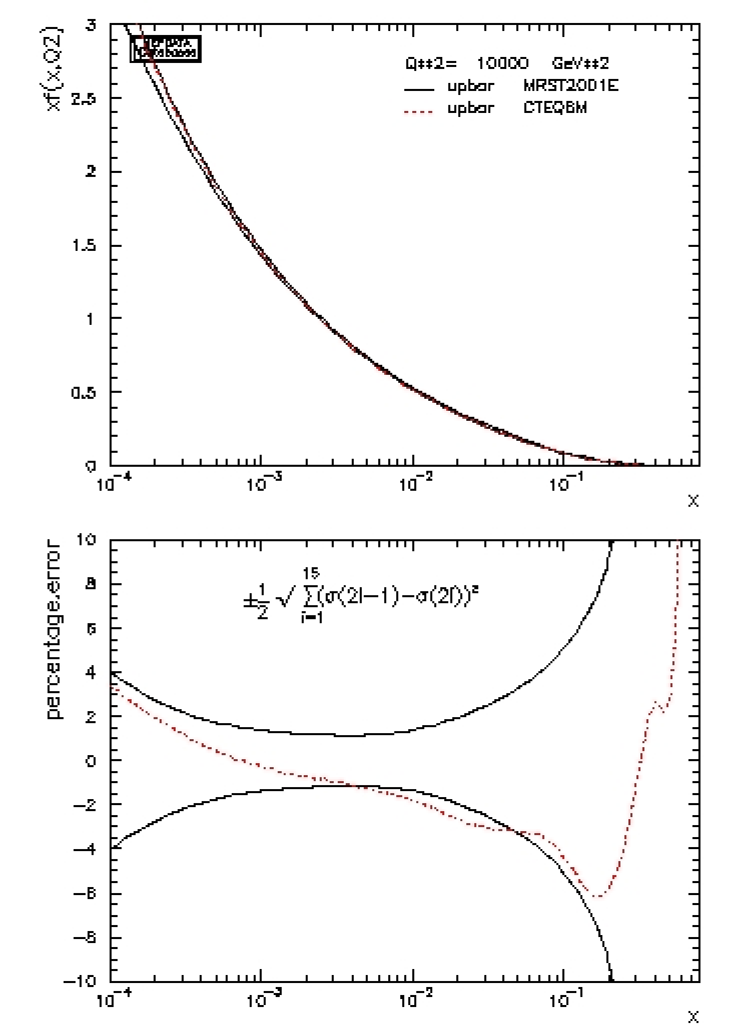


Evolution of PDFs:

$F(x,Q_1^2) \rightarrow F(x,Q_2^2)$
using DGLAP equations
(perturbative QCD)

$F(x_1,Q^2) \rightarrow F(x_2,Q^2)$
not predicted by
perturbative QCD

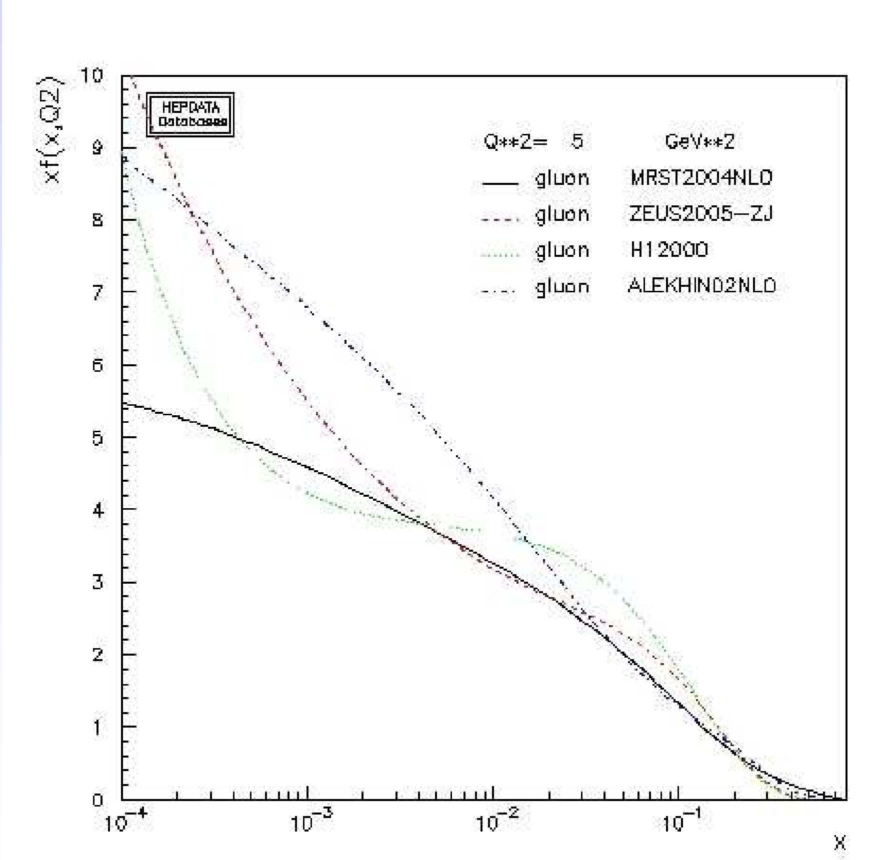
The Parton Distribution Functions (PDFs) (5)



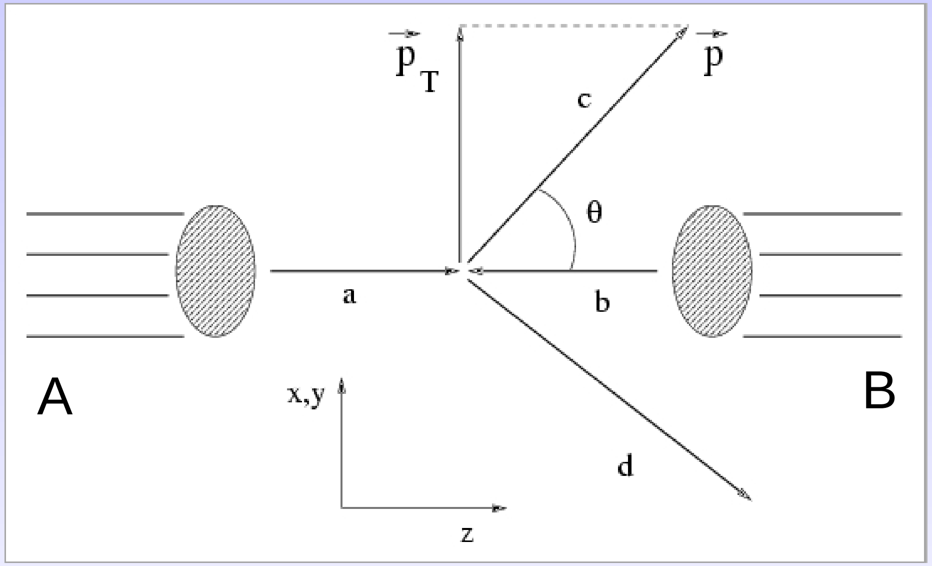
Uncertainties

-From fitting experimental data

-From comparing different sets of pdfs



Kinematic variables used in hadron collisions



Transverse momentum

$$p_T = \sqrt{p_x^2 + p_y^2}$$

Rapidity

$$\eta = \frac{1}{2} \mathrm{Log}\left(\frac{E + p_z}{E - p_z}\right)$$

p_T and η are nearly invariant in a relativistic boost along the z axis

$$p_T \to p'_T = p_T$$

$$\eta \to \eta' = \eta - \mathrm{tgh}^{-1}\beta$$

$$d^2\sigma/dp'_T d\eta' \approx d^2\sigma/dp_T d\eta$$

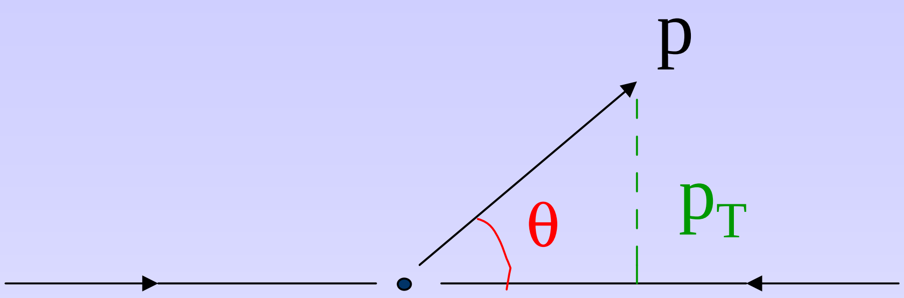
Inverted formulae

$$E = m_T \cosh \eta$$

$$p_z = m_T \sinh \eta$$

$$m_T = \sqrt{m^2 + p_T^2}$$

Kinematic variables for massless particles



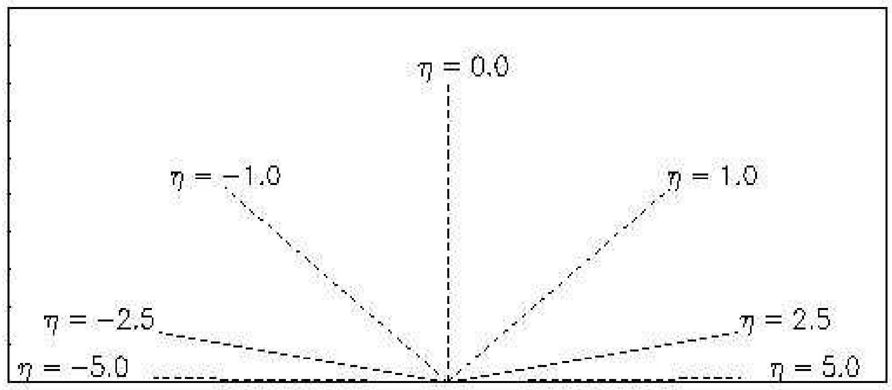
Transverse mass

$$m_T = p_T = p \sin\theta$$

Rapidity = pseudo-rapidity

$$\eta = - \mathrm{Log}\,[\mathrm{tg}(\theta/2)]$$

Pseudo-rapidity versus polar angle

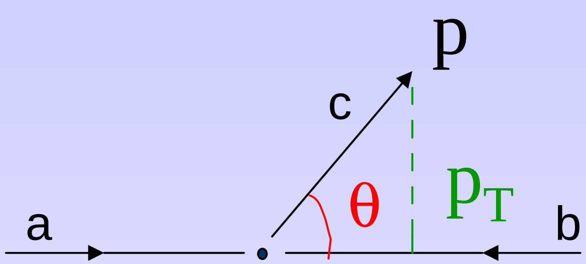


$\theta = 90^o \rightarrow \eta = 0$

$\theta = 10^o \rightarrow \eta \cong 2.4$
$\theta = 170^o \rightarrow \eta \cong -2.4$

$\theta = 1^o \rightarrow \eta \cong 5.0$

Jacobean peak in resonance production

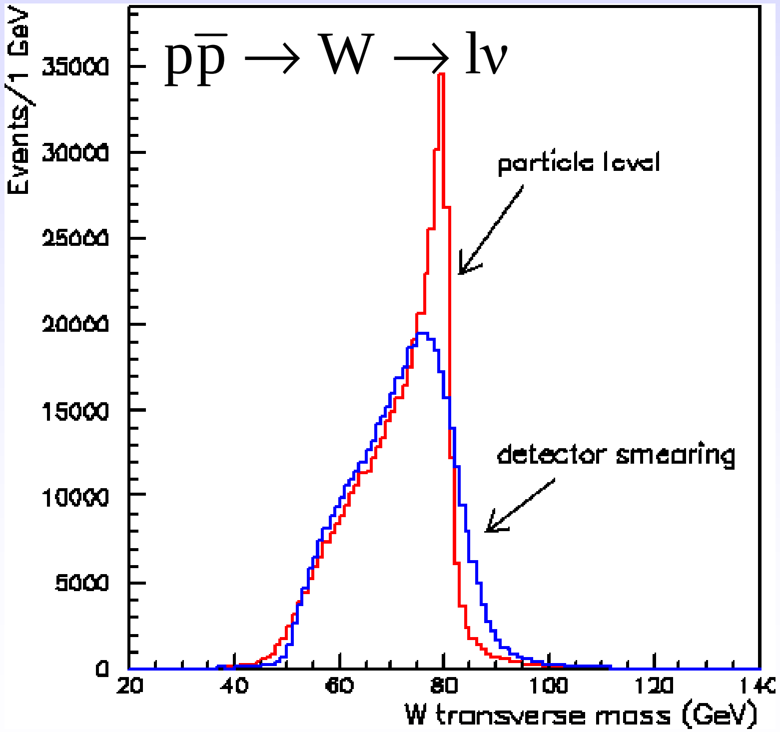


We consider the process ab → R →cd where R is a Resonance with mass M in c.m. frame of partonic process.

$$p_T = p\sin\theta = \frac{M}{2}\sqrt{1-\cos^2\theta}$$

$$\rightarrow \cos\theta = \sqrt{1-4p_T^2/M^2}$$

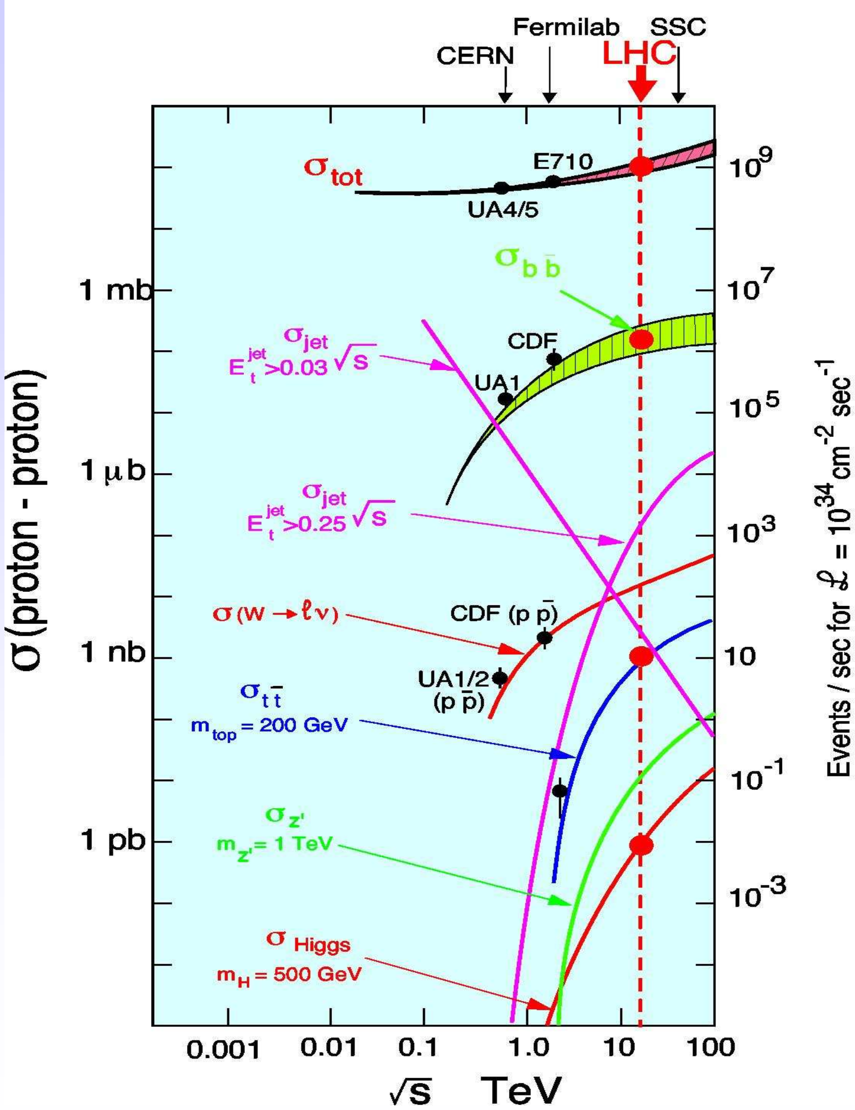
$$\rightarrow \frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\theta}\frac{d\cos\theta}{dp_T} = \frac{f(\cos\theta)}{\sqrt{1-4p_T^2/M^2}}$$



The R transverse mass is in the W case $M_T=2p_T$ and therefore the transverse mass distribution is singular for $M_T=M$

$$\frac{d\sigma}{dM_T} \sim \frac{1}{\sqrt{1-M_T^2/M^2}}$$

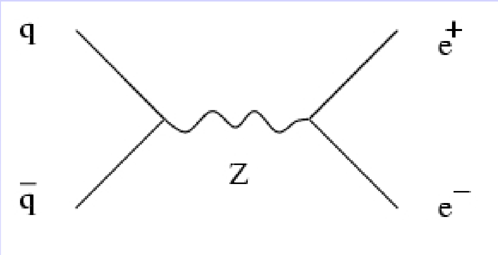
Cross Sections at the LHC

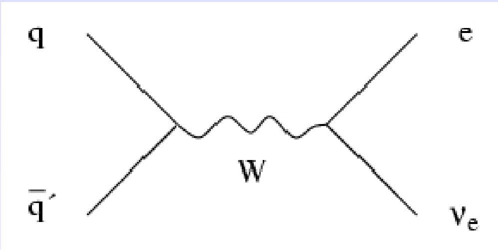


• Inelastic pp reactions:	70 mb
• bb pairs • tt pairs	100 μb 600 pb
• W → e ν • Z → e e	10 nb 1 nb
• **Higgs (500 GeV)** • **Gluino, Squarks (1 TeV)**	**1 pb** **1 pb**

Cross-Sections for pp→W and Z (1)

Partonic process ab→R→cd





Partonic cross-section

$$\hat{\sigma}_{ab}(s) = \frac{\sigma_0 M^2\Gamma^2}{(s-M^2)^2 + M^2\Gamma^2}$$
$$\approx \pi\sigma_0 M\Gamma\,\delta(s-M^2)$$
$$= \hat{\sigma}^0_{ab}\delta(s-M^2)$$

Hadronic process pp→R→cd
In the narrow width approximation

$$\sigma(s) = \sum_{a,b}\int_0^1 d\tau\,\hat{\sigma}_{ab}(\tau s)L_{ab}(\tau)$$

With the luminosity function

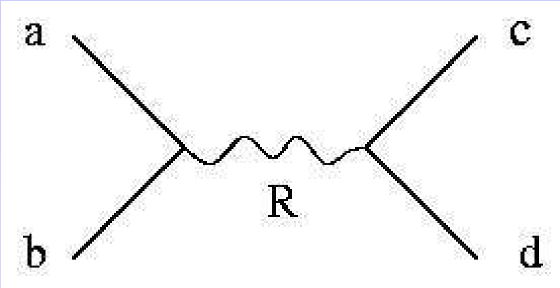
$$\tau L_{ab}(\tau) = \sum_{a,b}\int_\tau^1 \frac{dx}{x} F_a^p(x)\,F_b^p\left(\frac{\tau}{x}\right)$$

Inserting the partonic x-section:

$$\sigma(s) = \sum_{ab}\frac{\hat{\sigma}^0_{ab}}{M_R^2}\tau L_{ab}(\tau)$$

where $\tau = M_R^2/s$

Cross-Sections for pp→W and Z (2)



$$\sigma(s) = \sum_{ab} \frac{\hat{\sigma}^0_{ab}}{M_R^2} \tau L_{ab}(\tau) \quad \text{with} \quad \tau = \frac{M_R^2}{s}$$

$$\hat{\sigma}^0_{ab} = \frac{\pi}{3} g^2 \left(v^2 + a^2\right) BR(R \to cd)$$

$$g_W^2 = 2\sqrt{2} G_F M_W^2 \quad \text{with} \quad v = a = \frac{1}{2} V_{ab} \quad \text{and} \quad V_{ab} \quad \text{is CKM matrix}$$

$$\sigma_W(s) = \sum_{ab} \frac{\pi\sqrt{2}}{3} G_F |V_{ab}|^2 BR\left(W \to l\nu\right) \tau L_{ab}(\tau) \quad \text{with} \quad \tau = \frac{M_W^2}{s}$$

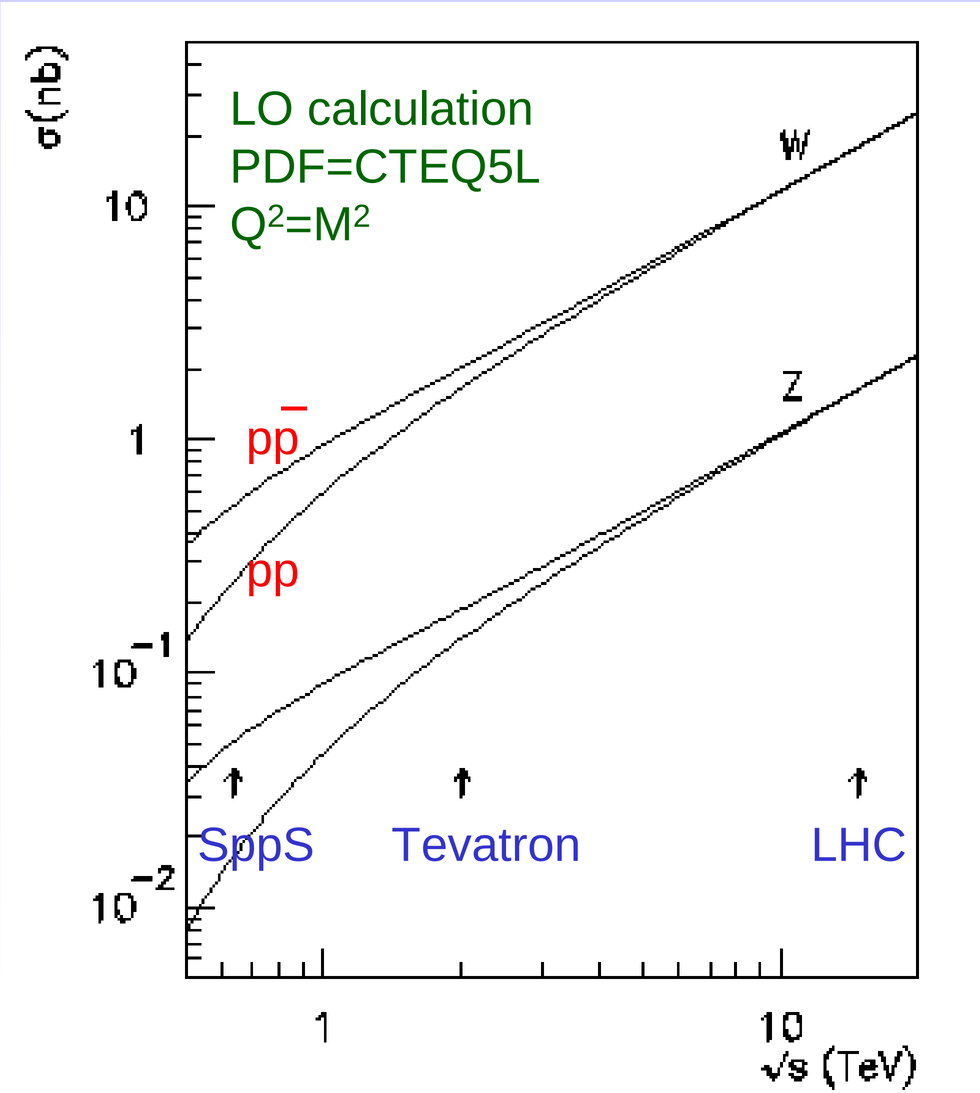
$$\text{For } W^+ \quad ab = u\bar{d}, u\bar{s}, c\bar{d}, c\bar{s} \ldots + a \leftrightarrow b = 2\,(u\bar{d}, u\bar{s}, c\bar{d}, c\bar{s} \ldots)$$

$$g_Z^2 = \sqrt{2} G_F M_Z^2 \quad \text{with} \quad v = T_3 - 2Q\sin^2\theta_W \quad \text{and} \quad a = T_3$$

$$\sigma_Z(s) = \sum_{ab} \frac{\pi\sqrt{2}}{3} G_F \left(v^2 + a^2\right) BR\left(Z \to l^+ l^-\right) \tau L_{ab}(\tau) \quad \text{with} \quad \tau = \frac{M_Z^2}{s}$$

$$\text{For } Z \quad ab = u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c} \ldots + a \leftrightarrow b = 2\,(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c} \ldots)$$

Cross-Sections for pp→W and Z (3)



Remarks

1) $\sigma(W) = \sigma(W^+) + \sigma(W^-)$
   $\sigma(W^+) = \sigma(W^-)$ for $p\bar{p}$
   $\sigma(W^+) > \sigma(W^-)$ for pp

2) $\sigma(p\bar{p}) > \sigma(pp)$ but
   $\sigma(p\bar{p}) \approx \sigma(pp)$ at large s

3) σ increases at large s
   $\sigma \sim s^d \mathrm{Log}(s)$ with d ~1

4) $R = \sigma(W) / \sigma(Z) \sim 10$

Uncertainties

1) QCD corrections
2) PDFs
3) Scale Q^2

Cross-Sections for pp→W and Z (4)

Remarks

1) $\sigma(W^+) > \sigma(W^-)$ for pp

$$\frac{\sigma(W^+)}{\sigma(W^-)} \approx \frac{u\bar{d}+\ldots}{\bar{u}d+\ldots} \approx \frac{u}{d} > 1$$

2) $\sigma(p\bar{p}) > \sigma(pp)$ but
$\sigma(p\bar{p}) \approx \sigma(pp)$ at large s

$$\frac{\sigma_Z(p\bar{p})}{\sigma_Z(pp)} \approx \frac{uu+dd+\ldots}{u\bar{u}+d\bar{d}+\ldots} \approx \frac{u}{\bar{u}} > 1$$

3) σ increases at large s
$\sigma \sim s^d \mathrm{Log}(s)$ with d ~1

$$F(x) \sim 1/x^d \quad (\text{with } d \sim 1) \text{ at small } x$$

$$\rightarrow \sigma \sim \frac{1}{M^2}\left(\frac{s}{M^2}\right)^d \mathrm{Log}\left(\frac{s}{M^2}\right) \text{ at large } s$$

4) $R = \sigma(W) / \sigma(Z) \sim 10$

$$\frac{\sigma(W)}{\sigma(Z)} \sim \frac{|V_{ab}|^2}{v^2+a^2} \frac{BR(W \rightarrow l\nu)}{BR(Z \rightarrow l^+l^-)} \sim \frac{1}{0.3} 3 \sim 10$$

Cross-Sections for pp→W and Z (5)

Uncertainties

1) QCD corrections → LO, NLO, NNLO, …

2) PDFs → CTEQ, MRST, …..

3) Scale Q^2 → $F(x,Q^2)$ with $Q^2 \sim M^2$

Example: $p\bar{p}$ collisions at $\sqrt{s}$ = 2 TeV

Calculation at LO, using CTEQ5L and $Q^2 \sim M^2$

σ_W = 2020 pb σ_Z = 189 pb R=10.7

Calculation at NNLO

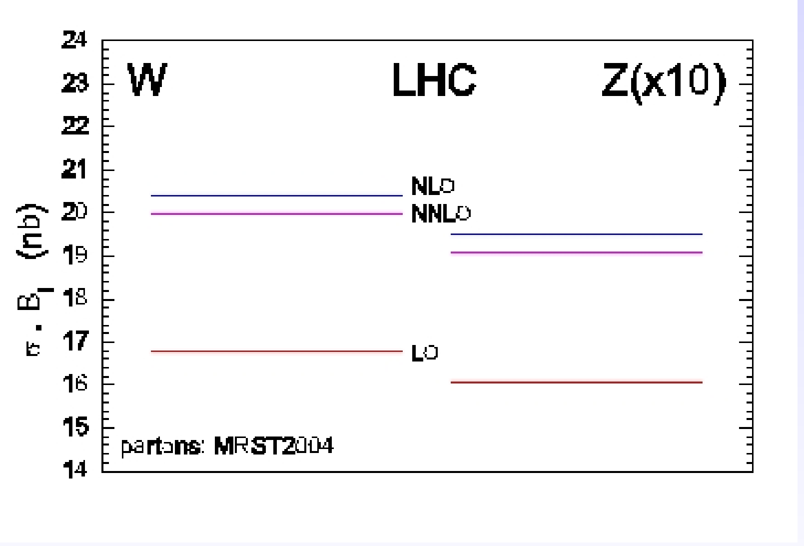
σ_W = 2690 pb σ_Z = 252 pb R=10.7

K factors are large (+33%) but R is very stable

Cross-Sections for pp→W and Z (6)

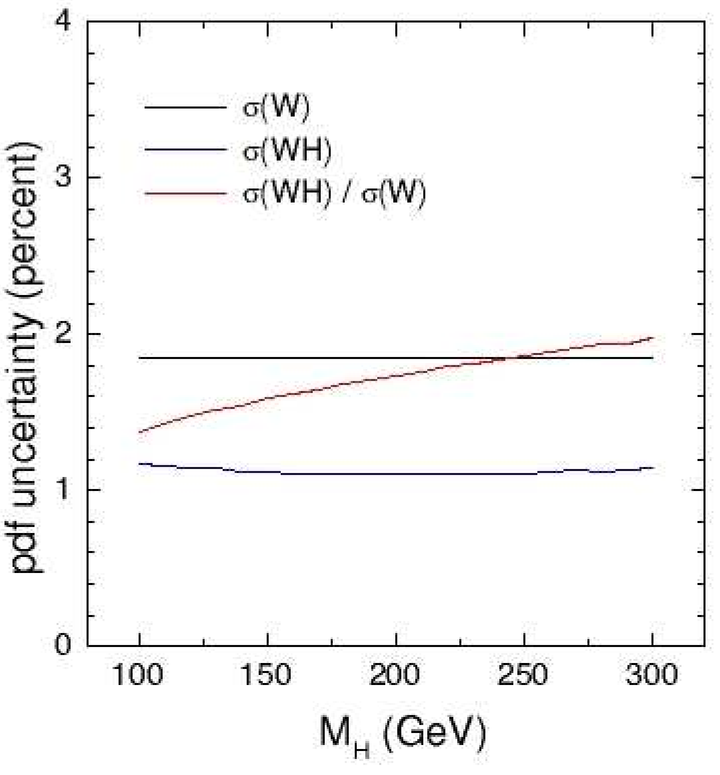
More on uncertainties:

LO, NLO, NNLO, and ….



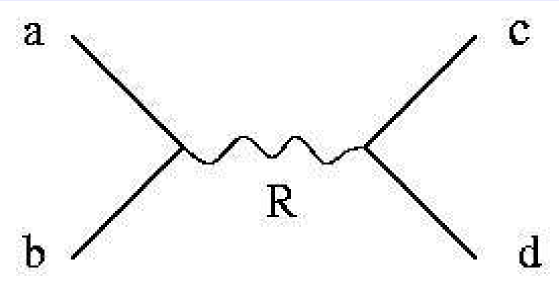
…. smaller Q^2 dependence at larger orders

pdf uncertainties on W, WH cross sections at LHC (MRST2001E)



Cross-Sections for pp→W´ and Z´ (1)

The same formalism is valid for any heavy exotic resonance W´ or Z´
We select as an example the Little Higgs model ($W' = W_H$, $Z' = Z_H$)



$$\sigma(s) = \sum_{ab} \frac{\hat{\sigma}^0_{ab}}{M_R^2} \tau L_{ab}(\tau) \text{ with } \tau = \frac{M_R^2}{s}$$

$$\hat{\sigma}^0_{ab} = \frac{\pi}{3} g^2 \left(v^2 + a^2\right) BR(R \to cd)$$

With the following couplings for $R = W_H$

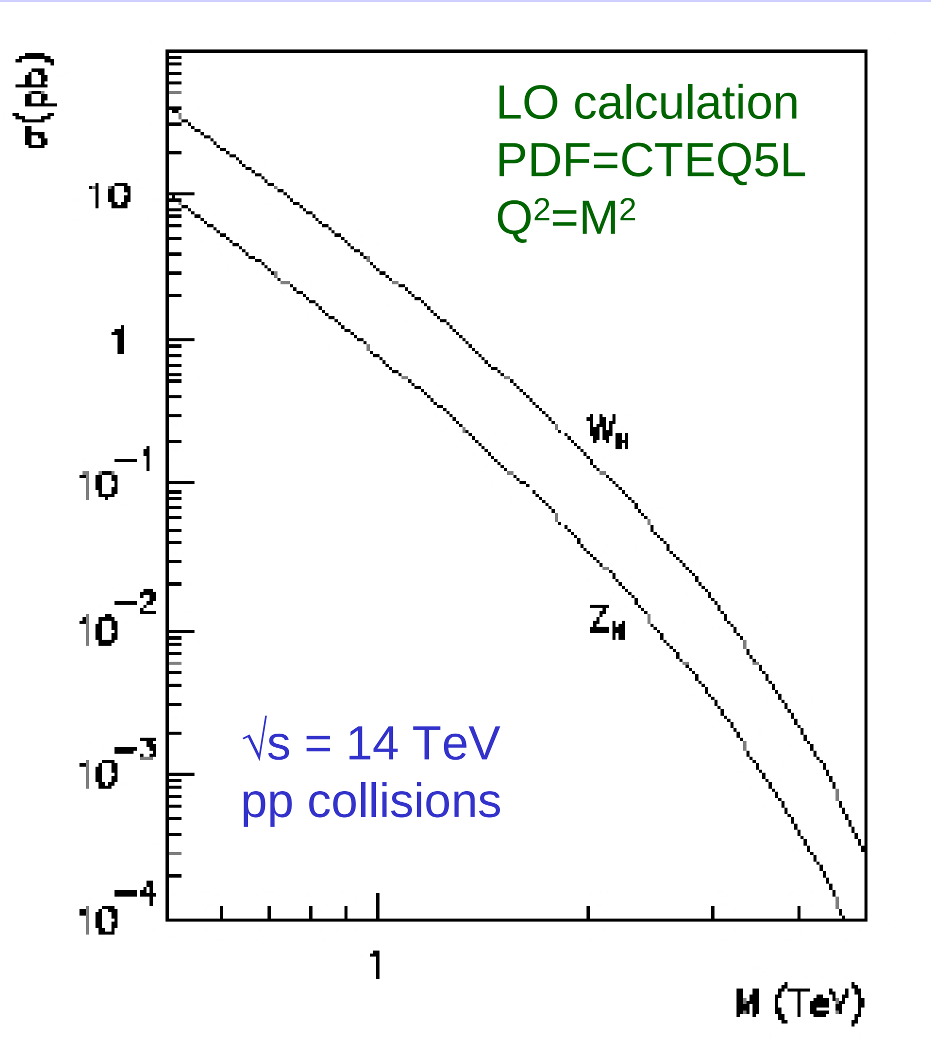
$$g^2 = 2\sqrt{2} G_F M_W^2 \cot^2\theta \text{ and } v = a = \frac{1}{2} V_{ab}$$

Therefore the result is

$$\sigma_R(s) = \sum_{ab} \frac{\pi\sqrt{2}}{3} G_F |V_{ab}|^2 \left(\frac{M_W}{M_R}\right)^2 \cot^2\theta \, BR(R \to cd) \, \tau L_{ab}(\tau) \text{ with } \tau = \frac{M_R^2}{s}$$

And the same for $R = Z_H$ replacing $|V_{ab}|^2$ by 1/2

Cross-Sections for pp→W´ and Z´ (2)



Branching ratios

BR ($W_H \to l\nu$) = 8.3 %

BR ($Z_H \to l^+l^-$) = 4.2 %

Ratio of cross-ssections

$$\frac{\sigma(W_H)}{\sigma(Z_H)} \approx 2 \frac{BR\left(W_H \to l\nu\right)}{BR\left(Z_H \to l^+l^-\right)} \approx 4$$

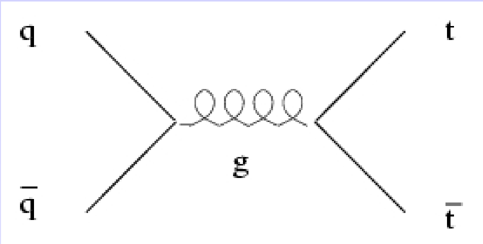
Mass dependence

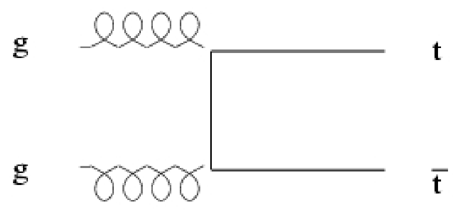
$$\sigma \sim \frac{1}{M^2}\left(\frac{s}{M^2}\right)^d \mathrm{Log}\left(\frac{s}{M^2}\right)$$

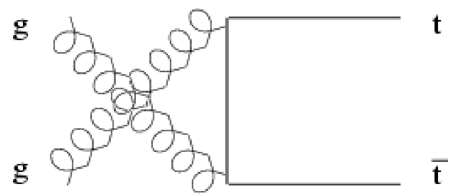
$$d \sim 1 \to \sigma \sim \frac{1}{M^4} \text{ for } M \sim 1\,\mathrm{TeV}$$

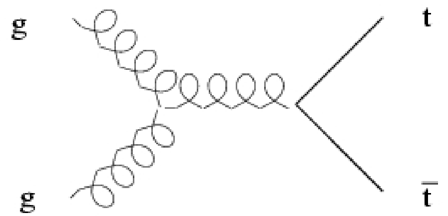
Cross-Section for double-top production (1)

Partonic processes









Partonic cross-sections

$$\hat{\sigma}_{qq}(s) = \frac{8}{27}\frac{\pi\alpha_s^2}{s}\beta\left(1+\frac{\rho}{2}\right)$$

$$\hat{\sigma}_{gg}(s) = \frac{1}{3}\frac{\pi\alpha_s^2}{s}\left[a\,\mathrm{Log}\left(\frac{1+\beta}{1-\beta}\right) - b\,\beta\right]$$

with $\rho = 4m_t^2/s$ and $\beta = \sqrt{1-\rho}$

$$a = 1+\rho+\frac{\rho^2}{16} \quad \text{and} \quad b = \frac{7}{4}+\frac{31}{16}\rho$$

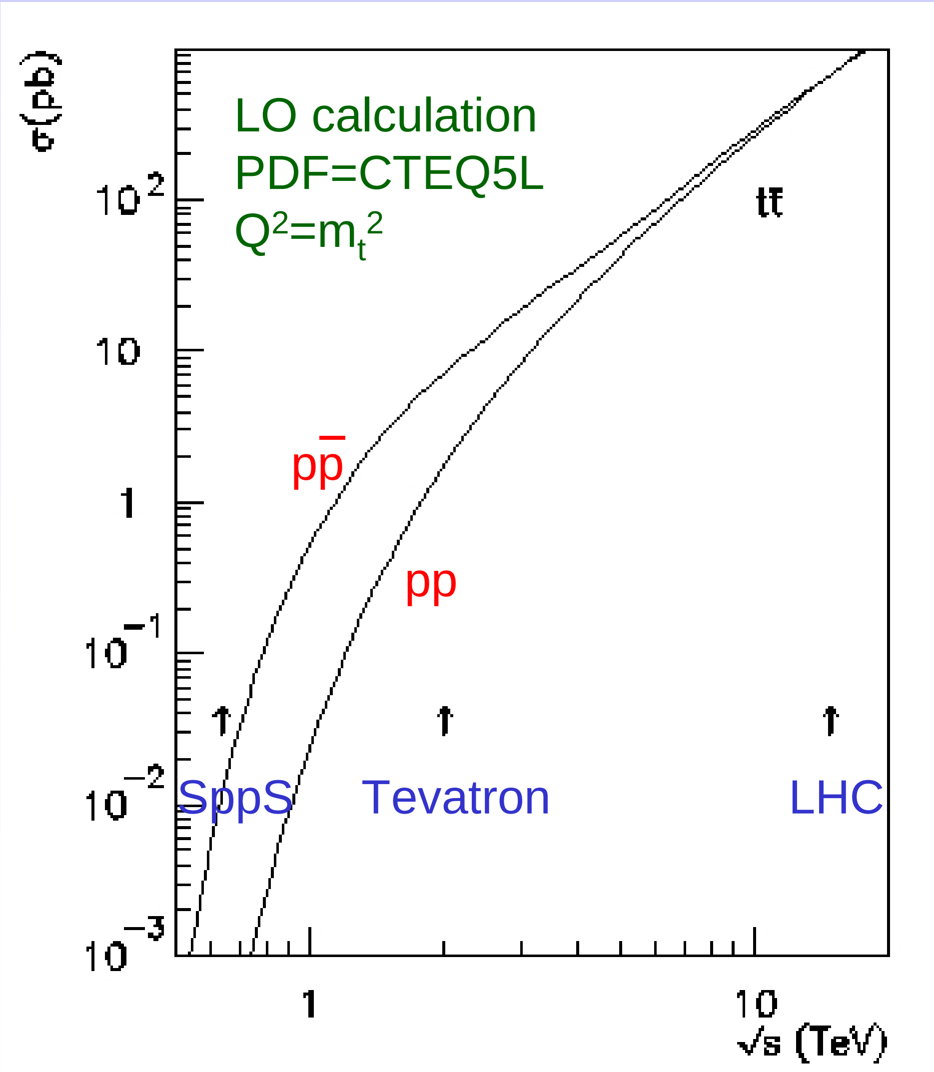
Threshold

$\sqrt{s} > 2m_t \rightarrow \rho<1 \rightarrow \beta>0$

Limit at large s

$$\hat{\sigma}_{qq}(s) \rightarrow \frac{1}{s} \text{ and } \hat{\sigma}_{gg}(s) \rightarrow \frac{\mathrm{Log}(s)}{s}$$

Cross-Section for double-top production (2)



Hadronic cross-sections

$$\sigma(s) = \sum_{a,b} \int_0^1 d\tau\, \hat{\sigma}_{ab}(\tau s)\, L_{ab}(\tau)$$

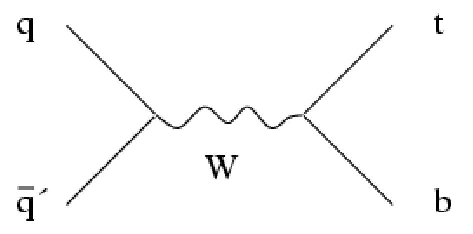
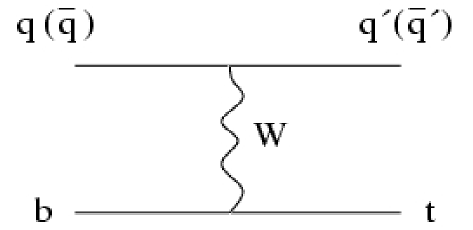
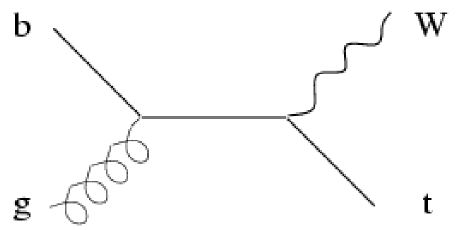
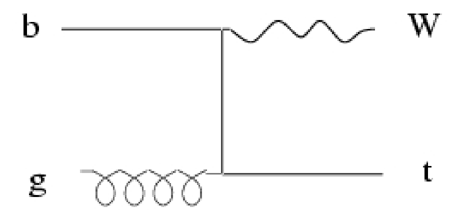
$$= \sum_{a,b} \int_\varepsilon^1 \frac{d\rho}{\rho}\, \hat{\sigma}_{ab}(\rho) \frac{\varepsilon}{\rho} L_{ab}\left(\frac{\varepsilon}{\rho}\right)$$

with $\varepsilon = 4m^2/s$

√s (TeV)	σ (pb) (LO)	σ(pb) (NLO)	
1.8 ($p\bar{p}$)	5.2	4.9	- 6 %
14 (pp)	580	803	+38 %

$$\sigma_{gg} \approx \begin{cases} \text{10 \% at Tevatron} \\ \text{85 \% at LHC} \end{cases}$$

Cross-Section for single-top production (1)

Partonic processes		Partonic cross-sections
	s-channel $\longrightarrow$	$\dfrac{d\hat{\sigma}}{dt} = -\dfrac{g^4}{16\pi}\left\|V_{qq'}\right\|^2\left\|V_{tb}\right\|^2 \dfrac{u}{s^2}\dfrac{u-m_t^2}{\left(s-m_W^2\right)^2}$
	t-channel $\longrightarrow$	$\dfrac{d\hat{\sigma}}{dt} = \dfrac{g^4}{16\pi}\left\|V_{tb}\right\|^2 \dfrac{1}{s}\dfrac{s-m_t^2}{\left(t-m_W^2\right)^2}$ (q) $\dfrac{d\hat{\sigma}}{dt} = \dfrac{g^4}{16\pi}\left\|V_{tb}\right\|^2 \dfrac{u}{s^2}\dfrac{u-m_t^2}{\left(t-m_W^2\right)^2}$ ($\bar{q}$)
 	gb-fusion $\longrightarrow$	$\dfrac{d\hat{\sigma}}{dt} \approx -\dfrac{g^2\alpha_s}{24\,s^2}\left\|V_{tb}\right\|^2\left(\dfrac{s}{u}+\dfrac{u}{s}+\dfrac{1}{2}\dfrac{m_t^2}{m_W^2}\dfrac{t^2}{us}\right)$

Cross-Section for single-top production (2)

Hadronic cross-sections

$$\sigma(s) = \sum_{a,b} \int_{\tau_{min}}^{1} d\tau \, \hat{\sigma}_{ab}(\tau s) \, L_{ab}(\tau)$$

$$= \sum_{a,b} \int_{0}^{z_{max}} dz \, \hat{\sigma}_{ab}(\tau s) \, \tau \, L_{ab}(\tau)$$

with $z = -\mathrm{Log}(\tau)$ and $\tau_{min} = m_0^2/s$

LO, CTEQ5L, $Q_2 = m_t^2$

√s (TeV)	σ (pb) s-chan	σ(pb) t-chan	σ(pb) gb-fus
2.0 ($p\bar{p}$)	0.6	2.0	0.2
14 (pp)	7.3	214	55

Thresholds

$m_0 \approx m_t$ for s and t channels
$m_0 \approx m_t + m_W$ for gb fusion

Asymptotic limits $(\hat{s} \to \infty)$

$\hat{\sigma} \to 1/\hat{s}$ for s – channel

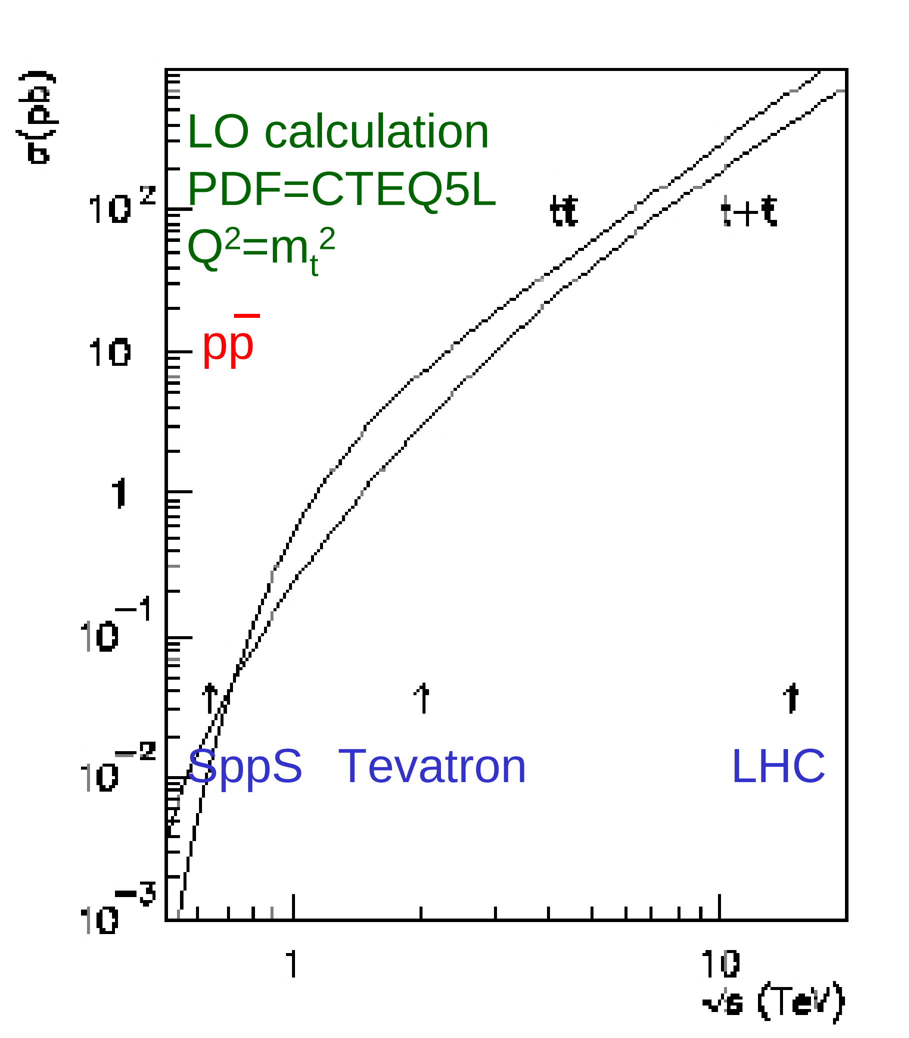
$\hat{\sigma} \to \dfrac{g^4 |V_{tb}|^2}{16\pi \, m_W^2}$ for t – channel

$\hat{\sigma} \to \mathrm{Log}(\hat{s})/\hat{s}$ for gb fusion

→ t-channel dominant

$\sigma(\text{t-channel}) \approx \begin{cases} \text{71 \% at Tevatron} \\ \text{78 \% at LHC} \end{cases}$

Single-top vs. double-top production

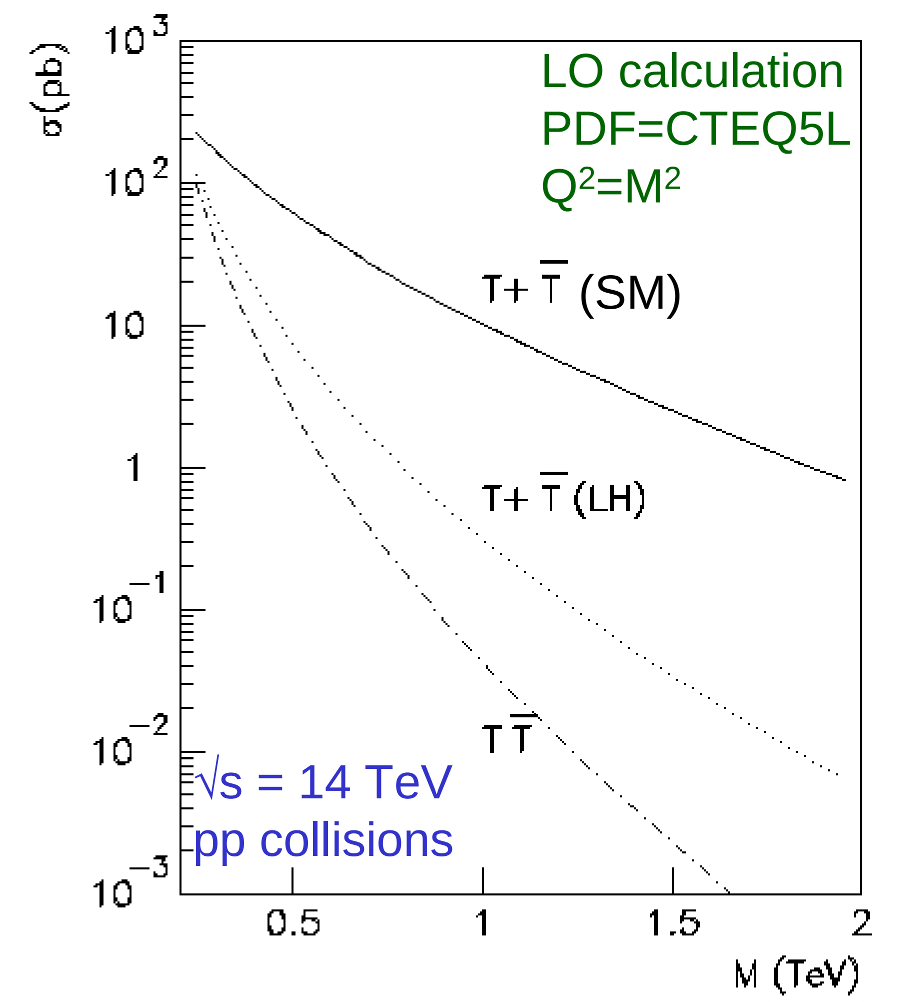


Hadronic cross-sections

√s (TeV)	σ (pb) (2 t)	σ(pb) (1 t)
2.0 ($p\bar{p}$)	6.0	2.8
14 (pp)	580	275

Single top cross-section is very large but top is produced with low p_T

Cross-Sections for pp→T and TT



Assume T=heavy top quark
(i.e. top quark with mass $M > m_t$)

Then single T production is in general the dominant process

We consider 2 models:

1) T has the same couplings as t (SM model)

2) The WTb vertex is suppressed by a factor $(m_t/M)^2$ (Little Higgs model)

Summary of the Lecture

Hadronic pp collisions involve soft and hard processes

The hard process can be easily calculated at LO using the factorisation theorem and proton PDFs

The uncertainties are however large. They include:
- NLO QCD corrections
- PDF uncertainties
- Scale depedence

Examples of calculations:
pp → Z, W, Z´, W´, t, tt, T, TT
Also interesting but not discussed here
pp → hard photons, jets, H, SUSY particles, …