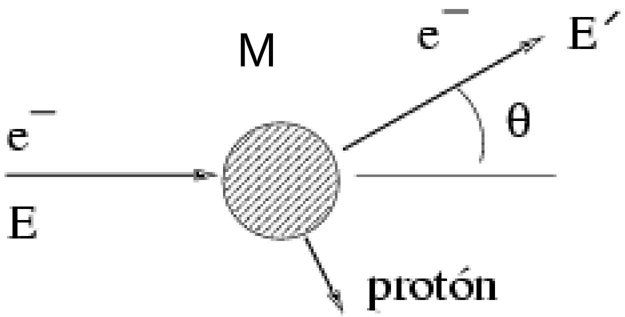
6. Parton Distributions Functions (PDFs)

Outline

- Deep inelastic scattering
- The parton model
- The quark-parton model
- Tests of the quark-parton model
- The pQCD model
- PDF evolution in pQCD
- PDF evolution in QED
- The factorisation theorem

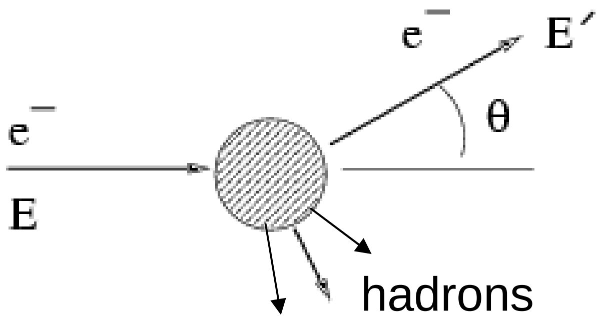
Deep inelastic scattering (1)



Elastic scattering (spin 1/2 target)

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(\frac{E'}{E}\right) \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2}\right)$$

$$\left(\frac{E'}{E}\right) = 1/\left(1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}\right) \text{ and } q^2 = -4EE' \sin^2 \frac{\theta}{2}$$

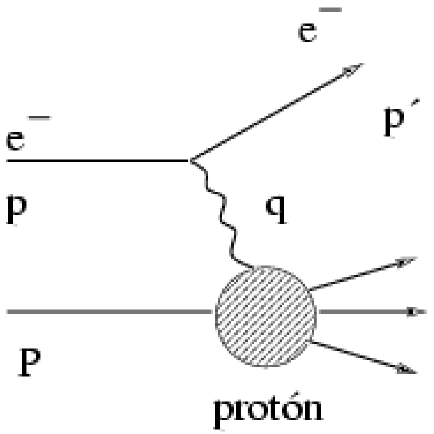


Inelastic scattering (spin 1/2 target)

$$\left(\frac{d\sigma}{d\Omega dE'}\right)_{inel} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left(2W_1 \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2}\right)$$

W_1 and W_2 are form factors

Deep inelastic scattering (2)



Lorentz invariant formulation

$$Q^2 = -q^2 = -(p-p')^2 \approx 4\,E\,E'\sin^2\frac{\theta}{2}$$

$$\nu = \frac{P\,.\,q}{M} = \frac{P.(p-p')}{M} = E-E'$$

Inelastic scattering

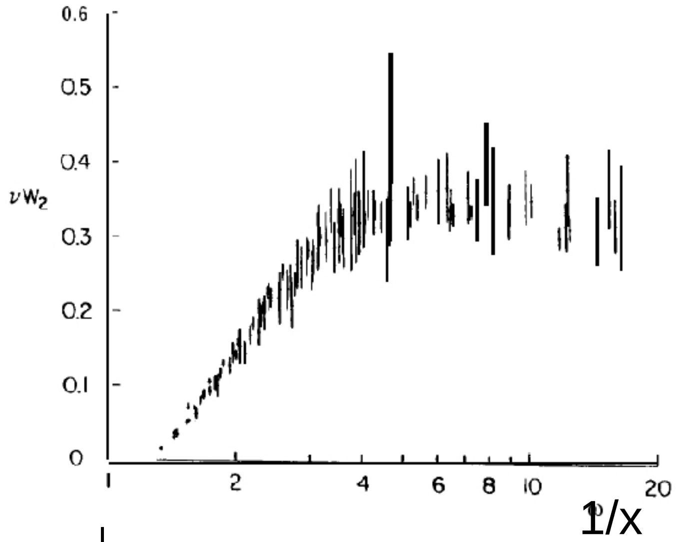
$$\left(\frac{d\sigma}{dQ^2 d\nu}\right)_{inel} = \frac{4\pi\,\alpha^2}{(Q^2)^2}\left[2M^2\frac{Q^2}{s^2}W_1 + \left(1-2M\frac{\nu}{s}-M^2\frac{Q^2}{s^2}\right)W_2\right]$$

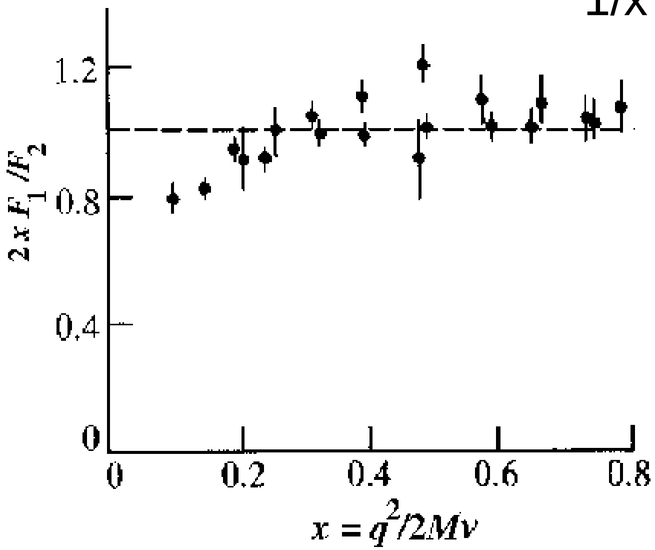
with $s=(p+P)^2$

Elastic scattering

$$W_1 = \frac{Q^2}{4M^2}\delta\left(\frac{Q^2}{2M}-\nu\right) \text{ and } W_2 = \delta\left(\frac{Q^2}{2M}-\nu\right)$$

Deep inelastic scattering (3)





SLAC-MIT Experiment (1967)

$$2MW_1(\nu, Q^2) \to F_1(x)$$
$$\nu W_2(\nu, Q^2) \to F_2(x)$$

Bjorken-Feynman variables

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{2M\nu}{s}$$

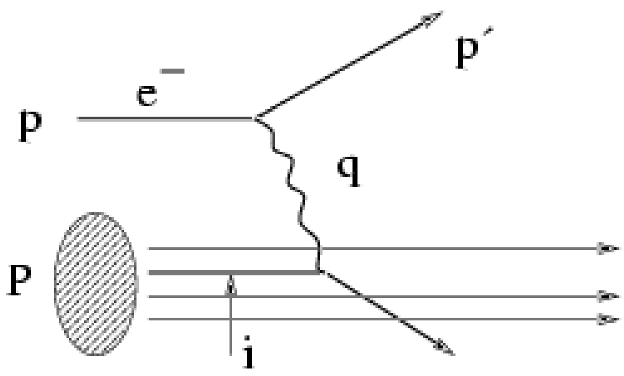
Callan-Gross relation

$$F_2 = 2xF_1$$

The parton model (1)

Using Bjorken-Feynman variables
(in the limit $Q^2 >> M^2$):

$$\left(\frac{d\sigma}{dxdy}\right)_{inel} \approx \frac{4\pi\alpha^2}{(Q^2)^2} s\left[xy^2 F_1 + (1-y)F_2\right]$$



Elastic case:

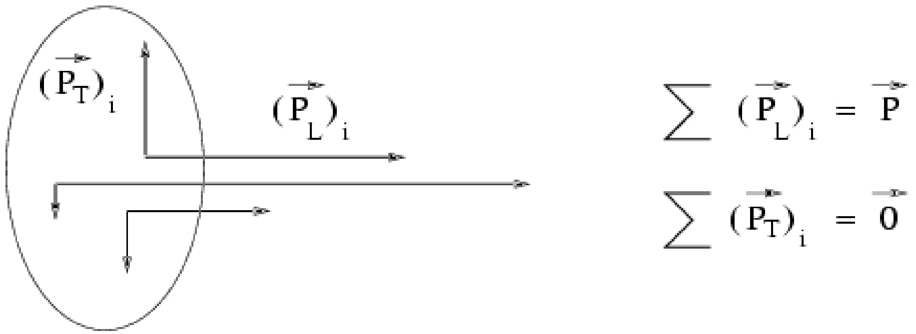
$$F_1 = \frac{x}{2}\delta(1-x) \quad \text{and} \quad F_2 = \delta(1-x)$$

Parton model hypothesis
(Feynman, 1969)

1) The proton is composed of partons. Partons are particles with spin 1/2, charge e_i and negligible mass. The deep inelastic scattering on the proton can be described as an elastic scattering on parton i.

The parton model (2)

2) Each parton carries a fraction of the total proton momentum, such that $P_i = zP$ with $0<z<1$ → Infinite momentum frame satisfying $(P_T)_i=0$ for all i



3) The probability to find parton i with fraction z of the total proton momentum is $f_i(z)$ satisfying:

$$\sigma = \sum_i \int_0^1 dz\, f_i(z)\, \sigma_i(z)$$

$$\int_0^1 dz\, f_i(z) = 1$$

$$\sum_i \int_0^1 dz\, z\, f_i(z) = 1$$

(the functions $f_i(z)$ are called PDF=parton distribution function)

The parton model (3)

1) Elastic scattering on individual partons: where F_1 and F_2 are

$$\left(\frac{d\sigma_i}{dx_i dy_i}\right) \approx \frac{4\pi\alpha^2}{(Q^2)^2} s_i \left[x_i y_i^2 F_1 + (1-y_i)F_2\right]$$

$$F_1 = e_i^2 \frac{x_i}{2} \delta(1-x_i) \quad \text{and} \quad F_2 = e_i^2 \delta(1-x_i)$$

2) Write all variables as function of z (momentum fraction of parton i)

$x_i = x/z \quad y_i = y \quad s_i = zs$ where x,y,s are observable quantities

3) Compute total cross-section using the formula

$$\sigma = \sum_i \int_0^1 dz\, f_i(z)\, \sigma_i(z)$$

and $\delta(1-x/z) = x\delta(z-x)$

The parton model (4)

Form factors according to the parton model:

$$F_1 = \sum_i \frac{e_i^2}{2} f_i(x) \quad \text{and} \quad F_2 = x \sum_i e_i^2 f_i(x)$$

These form factors satisfy the essential relations:

1) F_1 and F_2 depend only on x
2) The Callan-Gross relation $F_2 = 2x\, F_1$

The parton model assumes that partons are spin 1/2 particles but there are no other assumptions, in particular

1) The number of partons i inside the proton is not determined
2) The charge e_i of partons is not determined
3) Nothing is said about the colour of partons

The quark-parton model (1)

1) The simplest hypothesis: partons=constituent quarks
Then there are two PDFs inside the proton: f_u=u and f_d=d, such that

$$\int_0^1 dz\, u(z) = N_u = 2 \qquad \int_0^1 dz\, d(z) = N_d = 1 \qquad \int_0^1 dz\, z\,(u+d) = 1$$

… but it is found experimentally that:

$$\int_0^1 dz\, u(z) \to \infty \qquad \int_0^1 dz\, d(z) \to \infty \qquad \int_0^1 dz\, z\,(u+d) \approx 0.3$$

2) The second simplest hypothesis: partons=constituent quarks+ sea quarks

$$\text{proton} = uud + \underbrace{u\bar{u} + d\bar{d} + s\bar{s} + \ldots}_{\text{sea quarks}}$$

therefore:

$$\int_0^1 dz\,(u-\bar{u}) = 2 \qquad \int_0^1 dz\,(d-\bar{d}) = 1 \qquad \int_0^1 dz\, z\,(u+d+\bar{u}+\bar{d}+\ldots) = 1$$

… and experimentally:

$$\int_0^1 dz\,(u-\bar{u}) = 2 \qquad \int_0^1 dz\,(d-\bar{d}) = 1 \qquad \int_0^1 dz\, z\,(u+d+\bar{u}+\bar{d}+\ldots) \approx 0.5$$

The quark-parton model (2)

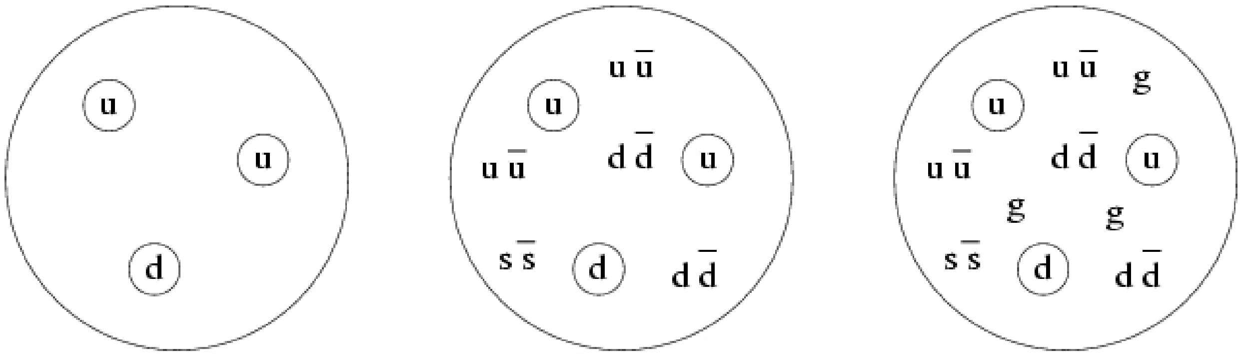
3) The final solution: partons=constituent quarks+ sea quarks+ gluons

$$\text{proton} = uud + \underbrace{u\bar{u} + d\bar{d} + s\bar{s} + \ldots}_{\text{sea quarks}} + g$$

therefore:

$$\int_0^1 dz\,(u - \bar{u}) = 2 \qquad \int_0^1 dz\,(d - \bar{d}) = 1 \qquad \int_0^1 dz\, z\,(u + d + \bar{u} + \bar{d} + \ldots + g) = 1$$

… confirmed experimentally !



constituent quarks

constituent + sea quarks

constituent + sea quarks + gluons

The quark-parton model (3)

1) The EHQL structure functions of the proton (PDFs)

The first PDFs (1984), now obsolete, but show essential properties.

$$x(u-\bar{u}) = xu_v = 1.78x^{0.5}(1-x^{1.51})^{3.5}$$

$$x(d-\bar{d}) = xd_v = 1.67x^{0.4}(1-x^{1.51})^{4.5}$$

$$x\bar{u} = x\bar{d} = 0.182(1-x)^{8.54}$$

$$xs = x\bar{s} = 0.081(1-x)^{8.54}$$

$$xg = (2.62+9.17x)(1-x)^{5.90}$$

$$\int_0^1 dx\,(u-\bar{u}) = \int_0^1 dx\, u_v = 2$$

$$\int_0^1 dx\,(d-\bar{d}) = \int_0^1 dx\, d_v = 1$$

$$\int_0^1 dx\, x\,(u+d+\bar{u}+\bar{d}+\ldots+g) = 1$$

One can check also that:

$$q(x) \to \infty \quad \text{for} \quad x \to 0$$

$$q(x) \to 0 \quad \text{for} \quad x \to 1$$

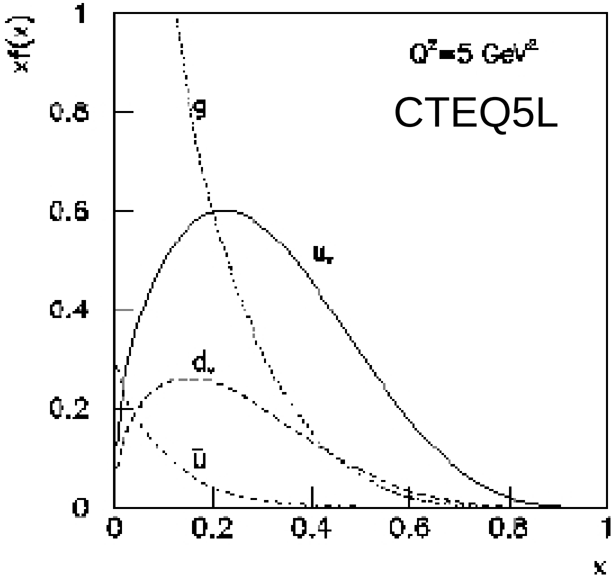
$$\int_0^1 dx\, q(x) \to \infty \quad \text{for all } q$$

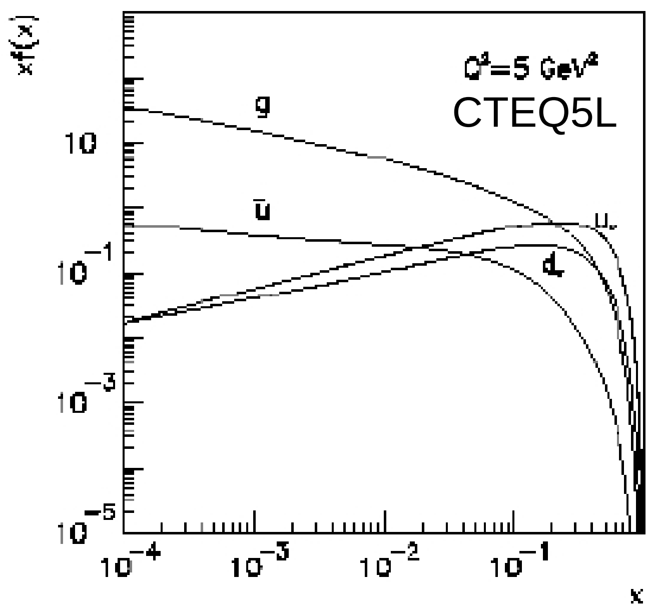
$$d/u \to 0 \quad \text{for} \quad x \to 1$$

The quark-parton model (4)

2) The modern structure functions of the proton (PDFs)

CTEQ, MRST, GRV, ZEUS, …

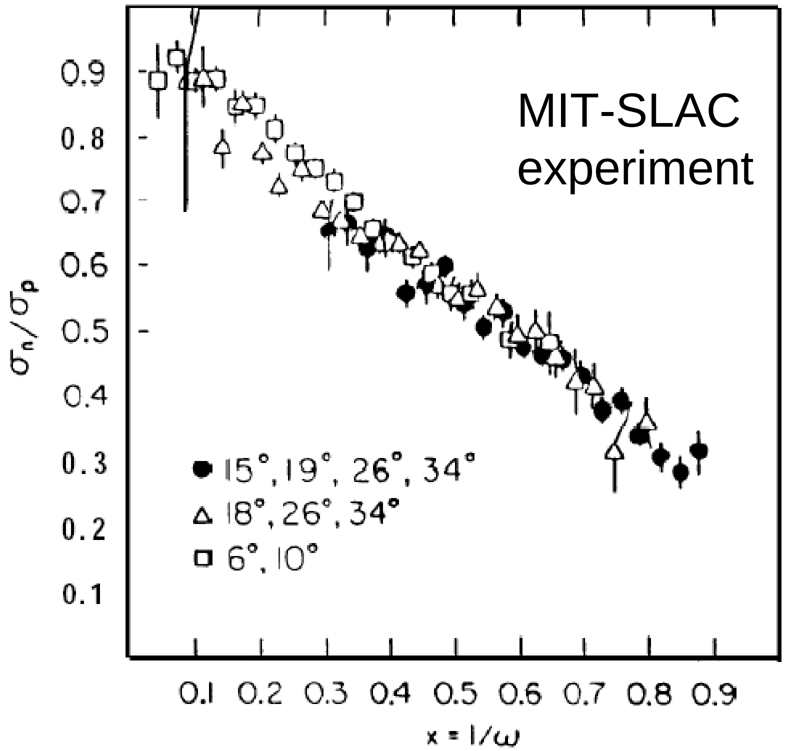




Note that: - PDFs depend also on Q^2
- for $x \to 0$, $xu_v \to 0$, so $xu \approx x\bar{u}$
- gluon dominates at low x

Test of the quark-parton model (1)

The ratio σ(en)/ σ(ep) and the charge of the quarks



$$F_2^{ep} = x\left[e_u^2\left(u+\bar{u}\right)+e_d^2\left(d+\bar{d}\right)+\ldots\right]$$

and using isospin symmetry:

$$F_2^{en} = x\left[e_u^2\left(d+\bar{d}\right)+e_d^2\left(u+\bar{u}\right)+\ldots\right]$$

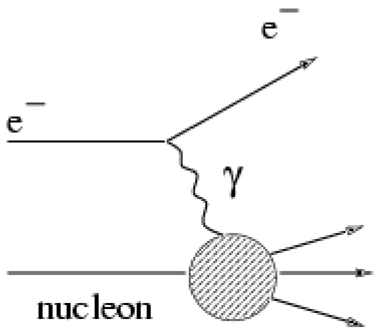
Therefore, for x→1 (d/u→0):

$$\frac{\sigma^{en}}{\sigma^{ep}} \rightarrow \frac{e_u^2 d + e_d^2 u}{e_u^2 u + e_d^2 d} \rightarrow \frac{e_d^2}{e_u^2} = \frac{1}{4}$$

… and deep inelastic scattering is sensitive to quark charges.

Test of the quark-parton model (2)

Neutrino versus lepton scattering

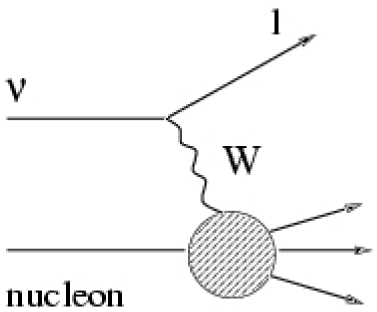


$$N = \frac{1}{2}(p+n)$$

Form factors

F_1, F_2

F_2=2x F_1

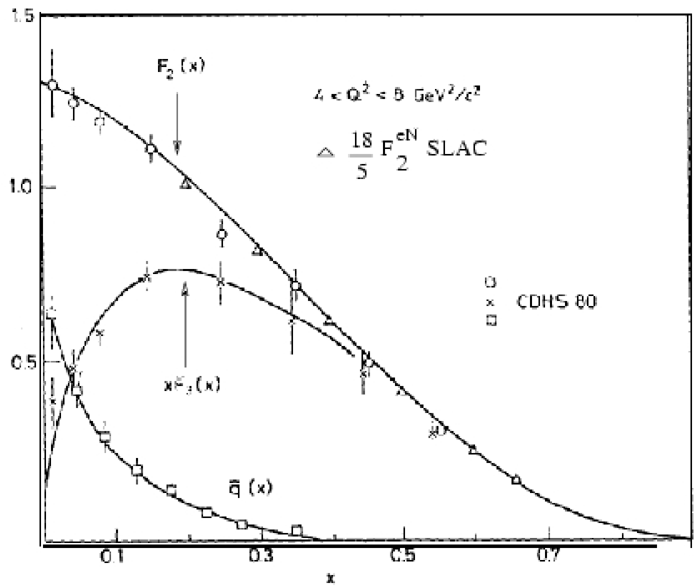


Form factors

F_1, F_2, F_3

F_2=2x F_1

F_3 new



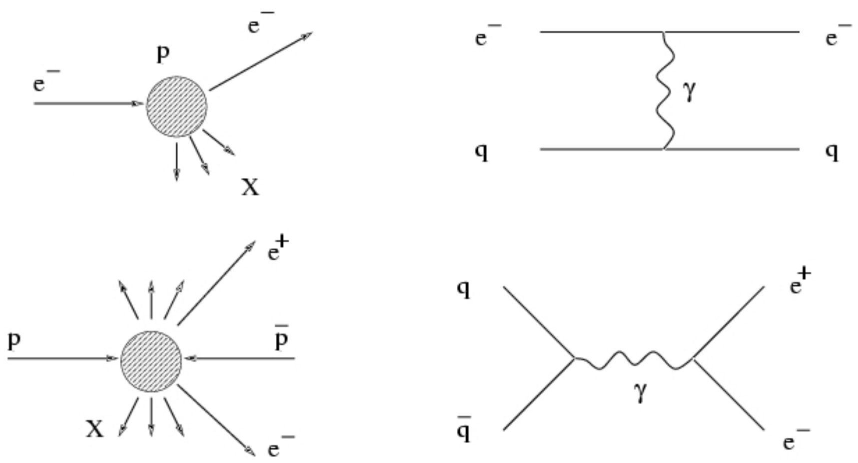
$$\frac{F_2^{eN}}{F_2^{\nu N}} \approx \frac{1}{2}\left(e_u^2 + e_d^2\right) = \frac{5}{18}$$

xF_3= valence quarks → useful to extract PDFs

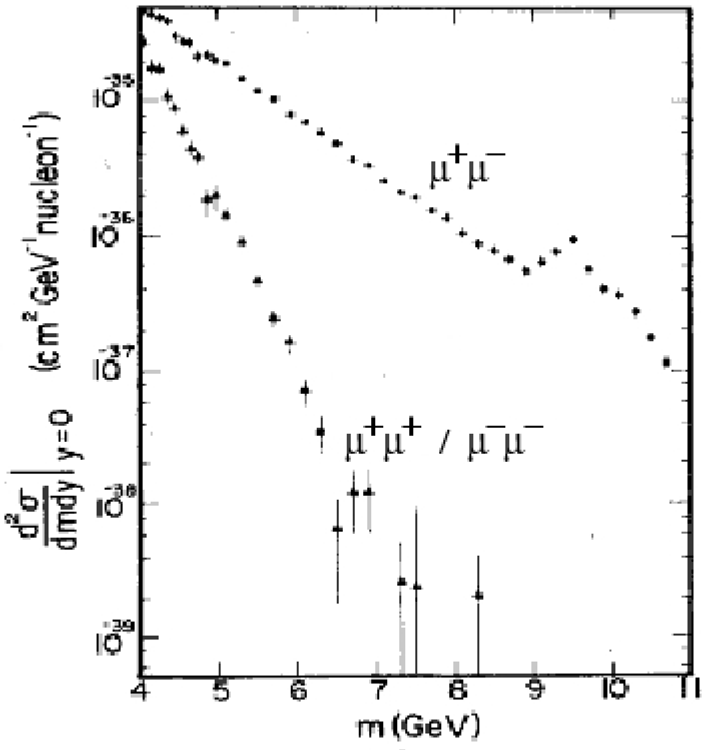
F_2-xF_3= sea quarks

Test of the quark-parton model (3)

Drell-Yan pair production processes



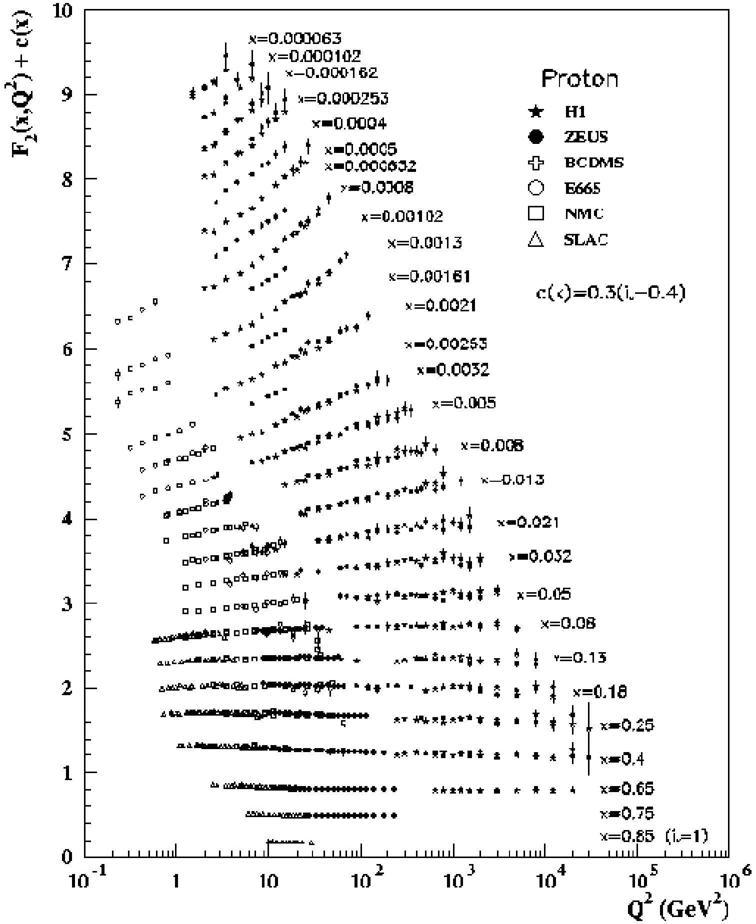
Production of DY muon pairs in fixed target collisions of a 400 GeV proton beam



Drell-Yan x-section in the parton model

$$\sigma(s) = \sum_{a,b} \int dx_a \, dx_b \, f_a^A(x_a) \, f_b^B(x_b) \, \hat{\sigma}_{ab}(\hat{s})$$

The pQCD model (1)



All deep-inelastic data collected to date

Data span over many orders of magnitude in both x and Q^2

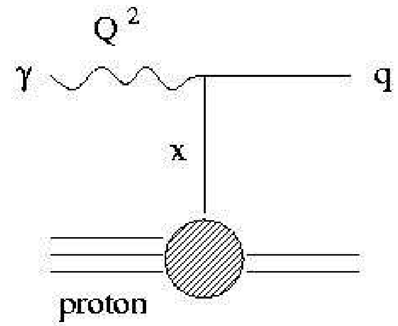
Data show Log Q^2 dependence, especially at low x

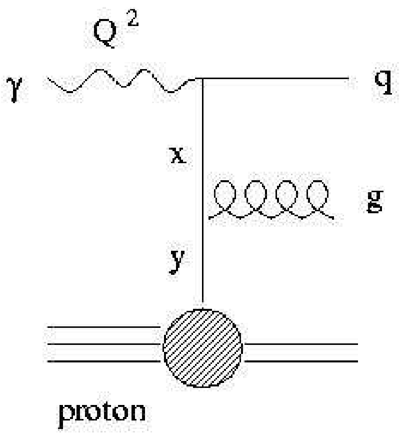
parton model
F is independent of Q^2

pQCD model
$F=F(x,Q^2)$

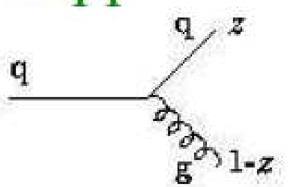
$$\underline{\text{Note}}: \text{remember } F_2 = \sum_i e_i^2 F_i(x)$$

The pQCD model (2)





Pqq



Parton model

q(x)= probability to find q with fraction x

pQCD model

probability is increased by gluon radiation

$$q(x) = q_0(x) + \int_x^1 \frac{dy}{y} q_0(y) F(x/y)$$
$$= q_0(x) + (q_0 \otimes F)(x)$$

where F(z) is the probability of splitting a quark into another quark with fraction z of the energy and a gluon with fraction 1-z

$$F(z) = \frac{\alpha_s}{2\pi} \mathrm{Log}\left(\frac{Q^2}{Q_0^2}\right) P_{qq}(z) \quad \text{with} \quad P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+$$

The pQCD model (3)

The result is therefore:

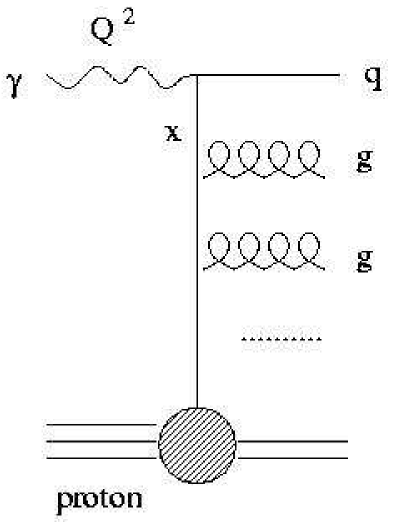
$$q\left(x,Q^2\right)=q_0(x)+\frac{\alpha_s}{2\pi}\mathrm{Log}\left(\frac{Q^2}{Q_0^2}\right)\left(q_0\otimes P_{qq}\right)(x)$$

with $P_{qq}(z)=C_F\left(\frac{1+z^2}{1-z}\right)_+$

showing that q depends now on both x and Q^2

Note also that the QCD correction is singular and needs 2 regulators:

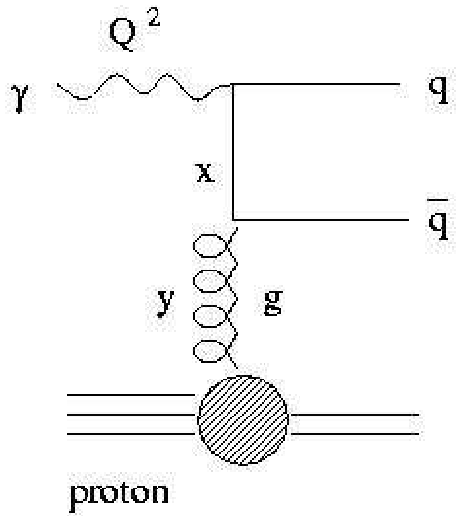
- collinear divergence ($Q_0\rightarrow 0$) is irrelevant (see later)
- Infrared divergence ($z\rightarrow 1$) regulated with + in P_{qq} (see definition later)



Multiple gluon emission has to be included to account for large Log terms in the perturbative expansion. It can be shown that this problem is solved replacing q_0 by q in the expansion:

$$q\left(x,Q^2\right)=q_0(x)+\frac{\alpha_s}{2\pi}\mathrm{Log}\left(\frac{Q^2}{Q_0^2}\right)\left(q\otimes P_{qq}\right)(x)$$

The pQCD model (4)



Additional diagrams contribute to q(x), namely those where a gluon splits into a quark-antiquark pair. After adding them, the result is

$$q(x,Q^2) = q_0(x) + \frac{\alpha_s}{2\pi} \mathrm{Log}(Q^2/Q_0^2)(q \otimes P_{qq})(x) + \frac{\alpha_s}{2\pi} \mathrm{Log}(Q^2/Q_0^2)(g \otimes P_{qg})(x)$$

where P_{qg} is another splitting function

Defining $t = \mathrm{Log}\left(\frac{Q^2}{Q_0^2}\right)$ we obtain: $q(x,t) = q_0(x) + t\frac{\alpha_s}{2\pi}(q \otimes P_{qq} + g \otimes P_{qg})$

Finally deriving by t: $$\frac{dq}{dt} = \frac{\alpha_s}{2\pi}(q \otimes P_{qq} + g \otimes P_{qg})$$

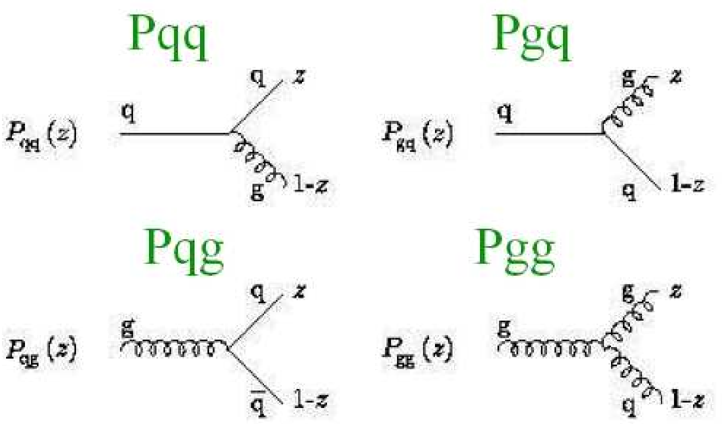
This integro-differential equation provides the evolution of q(x,t)

The pQCD model (5)

A similar equation holds for g(x,t), so we get a set of coupled equations:

$$\frac{dq}{dt} = \frac{\alpha_s}{2\pi}\left(q \otimes P_{qq} + g \otimes P_{qg}\right)$$
$$\frac{dg}{dt} = \frac{\alpha_s}{2\pi}\left(q \otimes P_{gq} + g \otimes P_{gg}\right)$$

called DGLAP equations (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi). The various splitting functions are defined below:



$$P_{qq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)_+$$
$$P_{qg}(z) = \frac{T}{N_f}\left[z^2 + (1-z)^2\right]$$
$$P_{gq}(z) = P_{qq}(1-z)$$
$$P_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \frac{11C_A - 4T}{6}\delta(1-z)$$

with the following definitions:

$$\int_0^1 dz\, f(z)F_+(z) = \int_0^1 dz\left[f(z) - f(1)\right]F(z)$$

$$C_F = 4/3 \quad C_A = 3 \quad T = N_f/2$$
$$N_f = \text{number of quark flavours}$$

PDF evolution in pQCD (1)

PDF evolution is obtained by solving the DGLAP equations:

$$\frac{dq}{dt} = \frac{\alpha_s}{2\pi}\left(q \otimes P_{qq} + g \otimes P_{qg}\right)$$

$$\frac{dg}{dt} = \frac{\alpha_s}{2\pi}\left(q \otimes P_{gq} + g \otimes P_{gg}\right)$$

Note that in QCD, the coupling α_s itself depends on Q^2 (hence on t):

$$\alpha_s \equiv \alpha_s(Q^2) = \frac{1}{b\,\mathrm{Log}\left(Q^2/\Lambda^2\right)}$$

$$\text{with} \quad b = \left(11C_A - 4T\right)/\left(12\pi\right)$$

This set of coupled integro-differential equations are solved numerically using the Mellin transform

f(x) → Mellin transform → $\bar{f}(z)$

$$\bar{f}(z) = \int_0^1 dx\, x^{z-1} f(x)$$

analytical solution → Inverse Mellin transform → numerical solution

$$f(x) = \frac{1}{2i\pi}\int_{c-i\infty}^{c+i\infty} dz\, x^{-z}\, \bar{f}(z)$$

Note that analytical solutions exist in Mellin space (also called space of momenta) but not in the direct space.

PDF evolution in pQCD (2)

Analytical solution in Mellin space
for z=n(integer) and the definitions:

$$V = q - \bar{q} \qquad \text{(singlet PDF)}$$
$$\Sigma = \sum_f (q + \bar{q}) \quad \text{(non-singlet PDF)}$$

DGLAP equations are
written in the following way:

$$\frac{dV}{dt} = \frac{\alpha_s}{2\pi}\left(V \otimes P_{qq}\right)$$
$$\frac{d\Sigma}{dt} = \frac{\alpha_s}{2\pi}\left(\Sigma \otimes P_{qq} + 2N_f g \otimes P_{qg}\right)$$
$$\frac{dg}{dt} = \frac{\alpha_s}{2\pi}\left(\Sigma \otimes P_{gq} + g \otimes P_{gg}\right)$$

Using the definition $\bar{f}(n) \equiv f_n$

and the property of Mellin transforms

$$\left(f \otimes g\right)_n = f_n . g_n$$

DGLAP equations are written:

$$\frac{dV_n}{dt} = \frac{\alpha_s}{2\pi}\left(V_n . P^n_{qq}\right)$$
$$\frac{d\Sigma_n}{dt} = \frac{\alpha_s}{2\pi}\left(\Sigma_n . P^n_{qq} + 2N_f g_n . P^n_{qg}\right)$$
$$\frac{dg_n}{dt} = \frac{\alpha_s}{2\pi}\left(\Sigma_n . P^n_{gq} + g_n . P^n_{gg}\right)$$

This set of linear equations can be solved analytically.

PDF evolution in pQCD (3)

As a simple application, we demonstrate that:

$$\frac{dV_1}{dt} = 0 \quad \text{and} \quad \frac{d}{dt}(\Sigma_2 + g_2) = 0$$

$$\text{i.e.} \quad V_1 = \int dx\,(q - \bar{q}) = \text{constant} \ \text{ and}$$

$$\Sigma_2 + g_2 = \int x dx\,\left[\Sigma(q + \bar{q}) + g\right] = \text{constant}$$

that are simply the proton constituent equations and the momentum sum rule.
The derivation makes use of the following integrals:

$$P^1_{qq} = C_F \int_0^1 dz \left(\frac{1+z^2}{1-z}\right)_+ = 0 \qquad P^2_{qq} = C_F \int_0^1 z dz \left(\frac{1+z^2}{1-z}\right)_+ = -\frac{4}{3} C_F$$

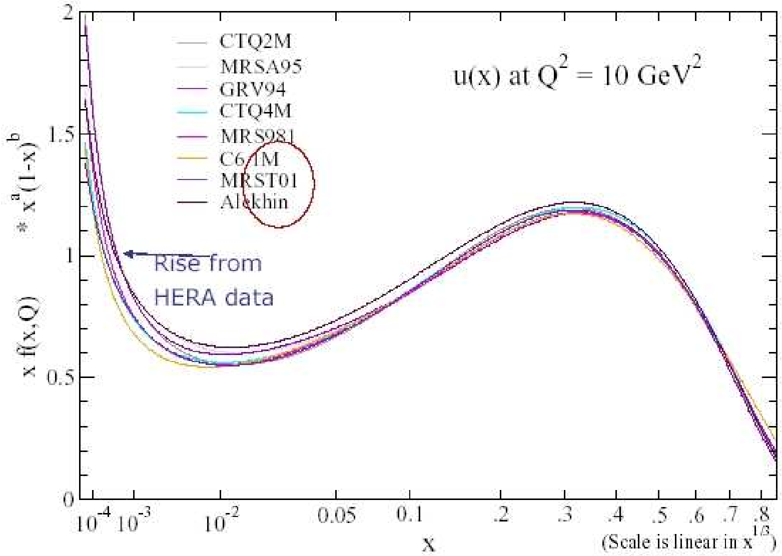
$$2N_f P^2_{qg} = 2N_f \frac{T}{N_f} \int_0^1 z dz \left[z^2 + (1-z)^2\right] = +\frac{2}{3} T$$

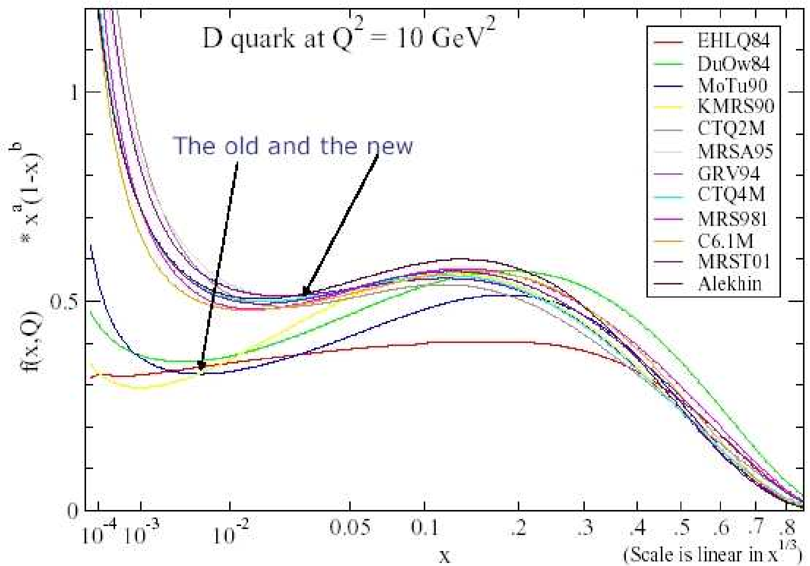
$$P^2_{gq} = C_F \int_0^1 z dz \left(\frac{1+(1-z)^2}{z}\right) = +\frac{4}{3} C_F$$

$$P^2_{gg} = 2C_A \int_0^1 z dz \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \frac{11C_A - 4T}{6} = -\frac{2}{3} T$$

$$\text{and therefore} \quad P^2_{qq} + P^2_{gq} = 2N_f P^2_{qg} + P^2_{gg} = 0$$

PDF evolution in pQCD (4)





PDFs are first parametrized at a given Q^2 value. Then they are evolved and compared to data.

Many different parametrizations are available: EHQL, CTEQ, MRST, GRV, …. and for each type there are periodically improved sets, i.e. CTEQ2, CTEQ4, CTEQ6, …

There are also PDFs at LO and NLO in QCD evolution, i.e. CTEQL and CTEQM.

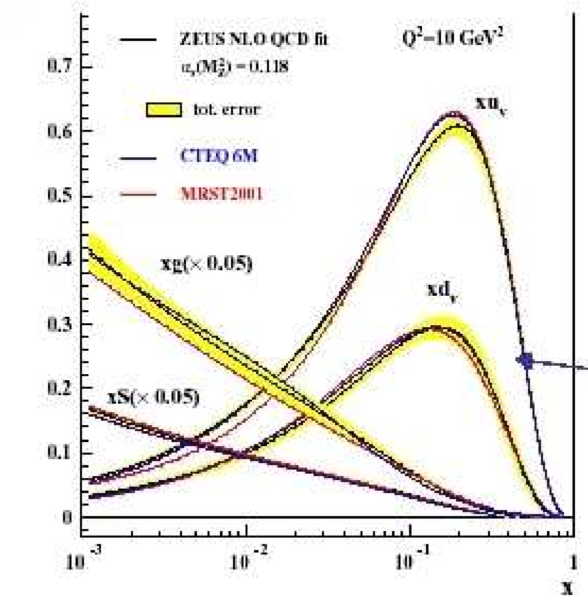
As an example of parametrization, CTEQ PDFs are of the type:

$$xf(x) = Ax^B(1-x)^C e^{Dx}(1+e^E x)^F$$

PDF evolution in pQCD (5)

QCD evolution using DGLAP equations shows that <u>valence quark</u> distributions (u_v and d_v) vary very <u>slowly</u>. The <u>gluon and sea quark</u> distributions grow on the contrary <u>quickly</u> with Q^2.

Q^2=10 GeV^2

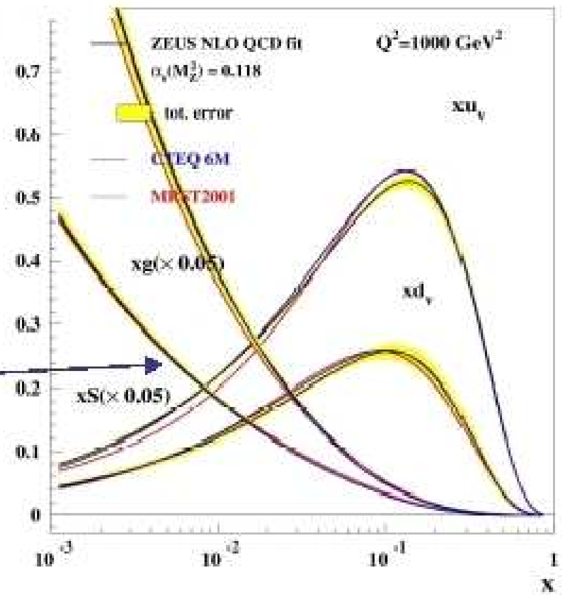


Note that gluon and sea quark PDFs are scaled down.

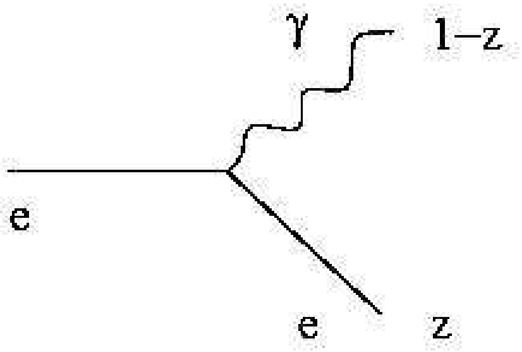
–Valence distributions evolve slowly

Sea/Gluon distributions evolve fast

Q^2=1000 GeV^2



PDF evolution in QED (1)



Splitting of an electron into an electron and a photon

The DGLAP formalism for PDF evolution in QCD can also be applied to QED. One can define a distribution function D_e for the electron, evolving according to:

$$\frac{dD_e}{dt} = \frac{\alpha}{2\pi}\left(D_e \otimes P_{ee}\right)$$

where

$$t = \mathrm{Log}\left(\frac{Q^2}{m_e^2}\right) \qquad P_{ee}(z) = \left(\frac{1+z^2}{1-z}\right)_+$$

This equation is much more simple than in QCD, and can be solved analytically in the limit $z\rightarrow 1$ (infrared limit). Note first that $\alpha \approx$ constant and therefore the evolution equation can be written:

$$\frac{dD_e}{d\beta} = \frac{1}{4}\left(D_e \otimes P_{ee}\right)$$

where

$$\beta = \frac{2\alpha}{\pi}\mathrm{Log}\left(\frac{Q^2}{m_e^2}\right)$$

PDF evolution in QED (2)

An exact solution in the limit z→1 is:

$$D_e(z,\beta) = \frac{\beta}{2}(1-z)^{\frac{\beta}{2}-1} - \frac{\beta}{4}(1+z) + O(\beta^2)$$

It can be easily verified that in the limit β→0,

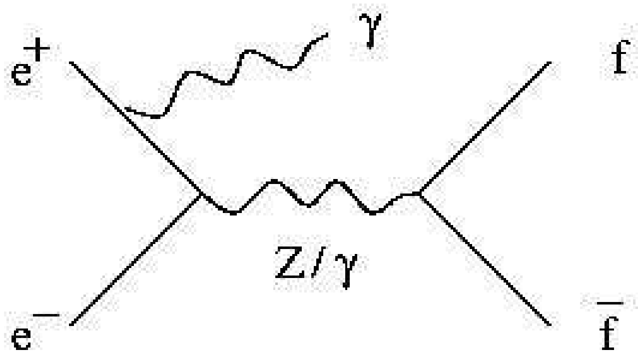
$$D_e(z,\beta) \approx \frac{\beta}{4}\frac{1+z^2}{1-z} + O(\beta^2)$$

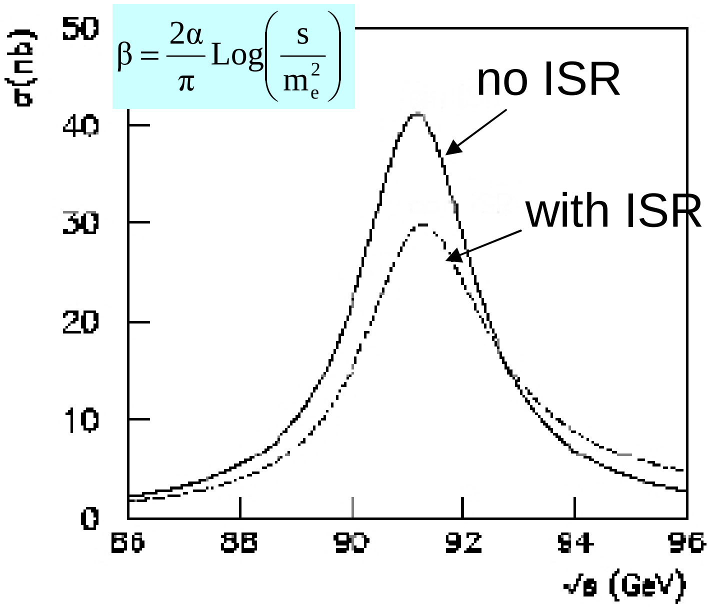
But this formula is not valid for z→1. The correct expansion is:

$$D_e(z,\beta) \approx \delta(1-z) + \frac{\beta}{4}\left(\frac{1+z^2}{1-z}\right)_+ + \ldots = \delta(1-z) + \frac{\beta}{4}P_{ee}(z) + \ldots$$

An example of application to ISR (Initial State Radiation) is given next.

PDF evolution in QED (3)





$$\beta = \frac{2\alpha}{\pi}\,\mathrm{Log}\left(\frac{s}{m_e^2}\right)$$

The formula of ISR for the process

$$e^+e^- \to Z/\gamma \to f\,\bar{f}$$

where f is any final fermion is:

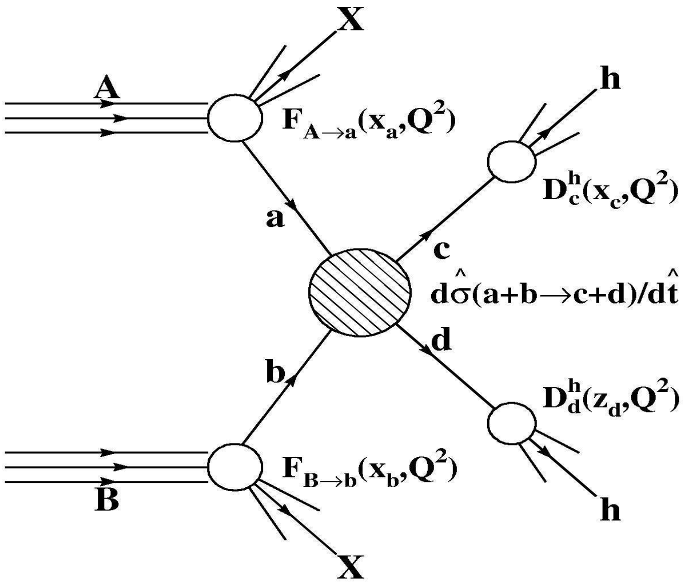
$$\sigma(s) = \int dz_1\, dz_2\, D_e(z_1)\, D_e(z_2)\, \hat{\sigma}(z_1 z_2 s)$$
$$= \int dz\, (D_e \otimes D_e)(z)\, \hat{\sigma}(zs)$$
$$= \int dz\, G(z,\beta)\, \hat{\sigma}(zs)$$

with the following G function:

$$G(z,\beta) = (D_e \otimes D_e)(z,\beta)$$
$$= \beta(1-z)^{\beta-1} - \frac{\beta}{2}(1+z) + \ldots$$

The effect of ISR for hadron production on the Z peak is shown in the figure.

The factorisation theorem (1)



In the parton model, the cross-section for hard processes in hadronic collisions is computed as follows:

$$\sigma(s) = \sum_{a,b} \int dx_a \, dx_b \, f_a^A(x_a) \, f_b^B(x_b) \, \hat{\sigma}_{ab}(\hat{s})$$

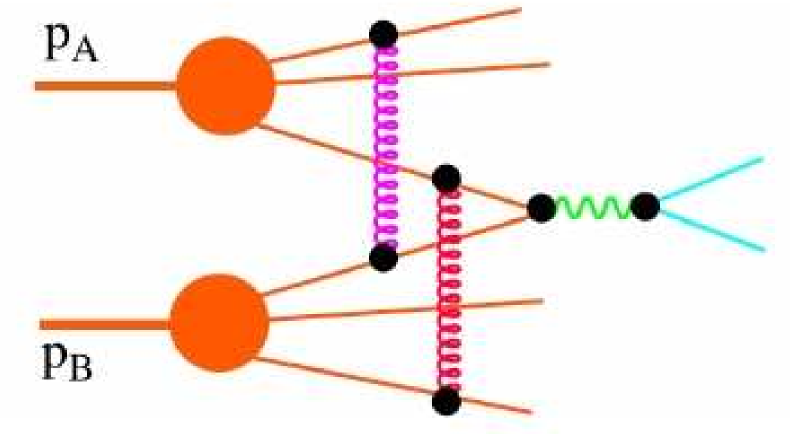
In pQCD, PDFs depend on Q^2 therefore the cross-section is:

$$\sigma(s) = \sum_{a,b} \int dx_a \, dx_b \, f_a^A(x_a, Q_F^2) \, f_b^B(x_b, Q_F^2) \, \hat{\sigma}_{ab}(\hat{s}, Q_F^2, Q_R^2)$$

It seems therefore that σ(s) depends also on an arbitrary scale Q^2. Morover, it could depend on two different scales: the factorisation scale (Q_F) for PDFs and the renormalisation scale (Q_R) for the hard prosess calculated perturbatively in QCD.

The factorisation theorem (2)

It can be proved however that σ(s) is independent of both scales Q_F and Q_R if the complete perturbative series of QCD is computed. In practice this is impossible, hence a scale uncertainty is always present in the computation of cross-sections for hard processes.



QCD introduces additional corrections, since spectators quarks may also affect the hard process via gluon exchange.

The factorisation theorem states that this effect can be neglected. In fact, this result is only exact in the limit $Q^2 \to \infty$ (due to power corrections in $1/Q^2$) and is only proved for a few simple cases.

Summary of the lecture

- Deep inelastic scattering data suggest that the proton is composed of partons, charged particles with spin 1/2 and negligible mass. The data are reproduced by the so-called Parton Model of Bjorken-Feynman, in particular the property of scale invariance.

- The Parton Model introduces the Proton Distribution Functions (PDFs), also called Proton Structure Functions. They give the probability to find a parton inside the proton with a certain fraction of the total momentum. In the Parton Model, these PDFs are independent of the scale Q^2

- QCD introduces gluon radiation in addition to quarks. As a result, PDFs depend also on the scale Q^2. The evolution of PDFs with Q^2 can be computed in perturbative QCD using the so-called DGLAP equations