Light-Cone Sum Rules in Soft-Collinear Effective Theory

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based on work with F. De Fazio and T. Hurth,
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Outline

1. Introduction
   - Momentum regions / Factorization / SCET
   - Non-perturbative methods in QCD
2. Light-cone sum rules in SCET
   - Correlation function in SCET_I
   - Derivation of the sum rule
   - Results
   - (Sum rule for factorizable current)
3. Conclusions/Outlook

Introduction

$B \to \pi$ transition form factors:

- 3 independent form factors:

$$\langle \pi(p') | \bar{u}\, \gamma_\mu\, b | B(p) \rangle \quad \longrightarrow \quad F_+(q^2)\,,\; F_0(q^2)$$
$$\langle \pi(p') | \bar{d}\, \sigma_{\mu\nu} q^\nu\, b | B(p) \rangle \quad \longrightarrow \quad F_T(q^2)$$

- $F_+(q^2)$ determines $B \to \pi \ell \nu$ semi-leptonic rate:
  $\Rightarrow$ exclusive determination of $|V_{ub}|$
- $F_+(0)$ enters as hadronic input in $B \to \pi\pi, \pi K$ decays:
  $\Rightarrow$ CKM analysis
- Simplest example of exclusive B decay to energetic hadrons:
  $\Rightarrow$ Theoretical understanding of QCD dynamics

This talk:

Focus on large recoil energy: $E_\pi \sim M_B/2 \;\Leftrightarrow\; q^2 \ll M_B^2$

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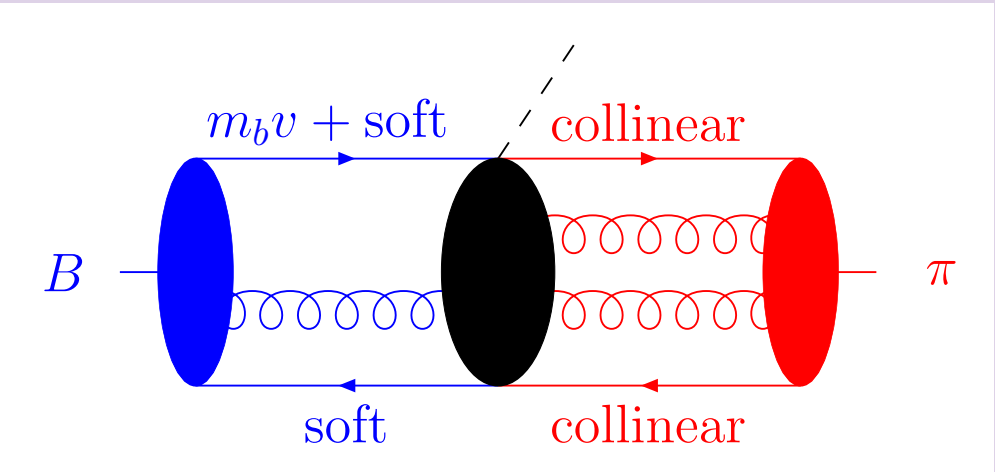
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Momentum regions in $B \to \pi$ transition

→ method of regions [Beneke/Smirnov '97]

Parton picture



B meson at rest:

Pion energetic:

$2E_\pi = \mathcal{O}(m_b)$

long-distance modes

- HQET fields: $\Delta p \sim \Lambda \ [= \mathcal{O}(\Lambda_{QCD})]$
- soft spectators: $p_s^\mu \sim \Lambda$
- collinear quarks and gluons: $E_c \sim m_b, \quad p_c^2 \sim \Lambda^2$

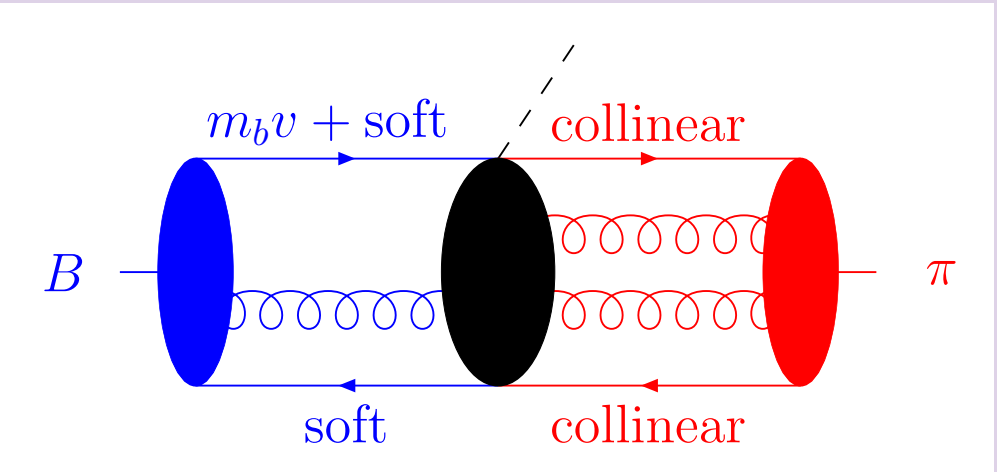
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- hard modes: $(\text{heavy} + \text{collinear})^2 \sim m_b^2$
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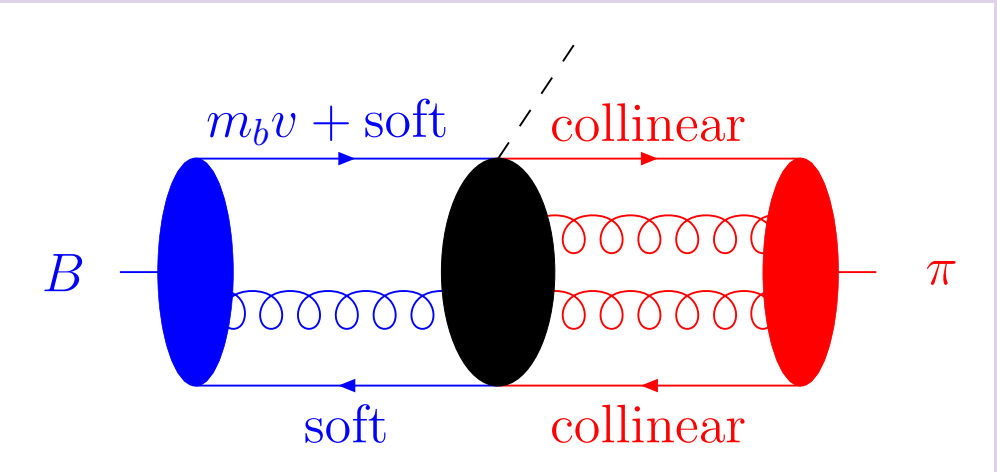
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Factorization of short- and long-distance QCD effects: Why?

- Express form factors in terms of universal (process-independent) hadronic quantities!
- Determine asymptotic dependence on heavy-quark mass! ($B \leftrightarrow D$ power counting, Sudakov logs, ...)
- Renormalization-group improved perturbation theory:
  - Compare: $\alpha_s(m_b) \approx 0.2$ with $\alpha_s(\sqrt{m_b \Lambda}) \sim 0.3 - 0.4$
  - Resum large logarithms $\ln \frac{\sqrt{\Lambda m_b}}{m_b}$ into short-distance coefficients

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Effective theory construction

[Bauer/Fleming/Pirjol/Stewart, Beneke/TF et al., Chay/Kim, Neubert et al., . . . , 2001 – today]

Separate short- and long-distance modes:

$$p_c^2,\, p_s^2 \ll p_{\rm hc}^2 \ll p_{\rm h}^2$$

$\longrightarrow$ Use dimensional regularization: $d^4k \to \mu^{2\epsilon} d^{4-2\epsilon}k$

- modes with $p^2 > \mu^2$ in short-distance coefficients
- modes with $p^2 < \mu^2$ in matrix elements

Two-step matching procedure:

- integrate out hard modes at $\mu^2 \sim m_b^2$ $\longrightarrow$ SCET$_{\rm I}$
- integrate out hard-collinear modes $\longrightarrow$ SCET$_{\rm II}$
- renormalization group in SCET$_{\rm I}$: $\longrightarrow$ evolution to $m_b^2 \to m_b\Lambda$
- RG evolution in SCET$_{\rm II}$: $\longrightarrow$ hadronic scales Λ

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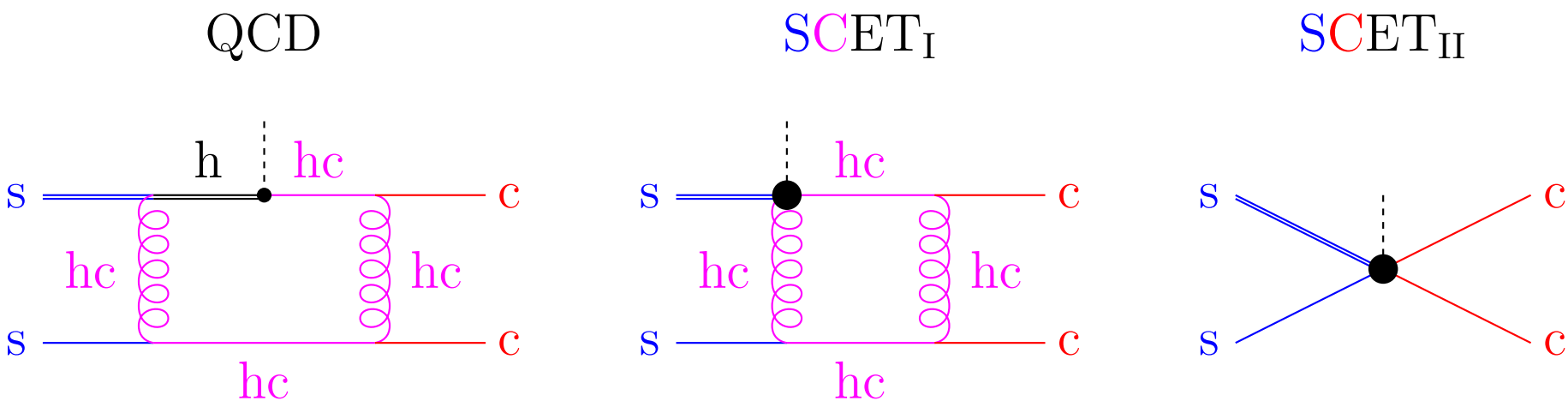
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- renormalization group in $\text{SCET}_{\rm I}$: $\longrightarrow$ evolution to $\mu_2^2 \sim m_b \Lambda$
- integrate out hard-collinear modes $\longrightarrow$ $\boxed{\text{SCET}_{\rm II}}$
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Example



- part of tree-level matching for QCD $\longrightarrow$ SCET$_{\rm I}$
- part of one-loop matching for SCET$_{\rm I}$ $\longrightarrow$ SCET$_{\rm II}$

Hadronic matrix elements in SCET_{II}

Light-cone distribution amplitudes (LCDAs)

- SCET currents are non-local (large energy of collinear particles)
- Matrix elements of non-local currents define LCDAs (analogue of PDFs in DIS)
- Introduce auxiliary light-like vectors: $n_\pm^\mu = (1,0,0,\pm 1),\ \ (v\cdot n_\pm) = 1$

Pion (twist-2)

$$\langle \pi(p')|\bar\Psi_c(sn_+)\,[sn_+,0]_c\,\not{n}_+\gamma_5\Psi_c(0)|0\rangle$$

$$\propto\ f_\pi\,(n_+p')\int_0^1 du\,e^{iu(n_+p')s}\,\phi_\pi(u,\mu)$$

u : quark momentum fraction

B-meson (HQET)

$$\langle 0|\bar\Psi_s(tn_-)\,[tn_-,0]_s\,\not{n}_\mp\gamma_5 b_v(0)|B(v)\rangle$$

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Factorizable and Non-factorizable currents in SCET$_\mathrm{I}$

[Beneke/TF '00, '03, see also Lange/Neubert '03]

Factorization theorem:

$$F_i(q^2) \;=\; T_i^I(q^2)\,\xi_\pi(q^2) \;+\; \phi_+^B \otimes T_i^{II}(q^2) \otimes \phi_\pi \;+\; \Lambda/m_b \text{ corrections}$$

Non-factorizable current

$$\langle \pi | \bar\psi_{\rm hc} [..] h_v | B \rangle \propto \xi_\pi(q^2)$$

[gauge-link (Wilson lines) not shown]

- Soft and collinear fields decouple in SCET$_\mathrm{II}$ Lagrangian, but no simple regularization consistent with soft/collinear factorization.[a]
- ξ_π is universal $\longrightarrow$ symmetry relations [Charles et al. '98]
- Hard dynamics in $T_i^{\rm I} = 1 + \mathcal{O}(\alpha_s)$

[a] Challenged recently by Manohar/Stewart hep-ph/0605001.

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Non-perturbative methods for $B \to \pi$ decays

Lattice QCD:

- Problem: cannot implement energetic pions!
- Way out: moving NRQCD ?

[Foley/Lepage et al., hep-lat/0209135,0509108]

(traditional) QCD light-cone sum rules:

- Replace heavy meson by *off-shell* interpolating current
- Separate hard and collinear dynamics via light-cone OPE
- Hard and hard-collinear scale not distinguished!
- Heavy-quark limit, $m_b \to \infty$?

[Ball, hep-ph/0308249]
[De Fazio/TF/Hurth, hep-ph/0504088]

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Light-cone sum rules in SCET
[De Fazio/TF/Hurth '05]

Idea:

- First, integrate out hard modes: QCD $\to$ SCET$_{\rm I}$
- Consider correlation function in SCET$_{\rm I}$:
  exclusive final state (e.g. pion) is replaced by interpolating current.
  Separate soft and hard-collinear dynamics in correlator
- Dispersion relation between
  - (unphysical) region of large (hc) space-like momenta
  - physical spectral function, containing the hadronic state

Outcome:

- Sum rule for non-factorizable matrix elements in SCET$_{\rm I}$:
  $\xi_\pi(q^2)$ in terms of light-cone distribution amplitudes of B meson

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Correlation function for ξ_π in SCET$_\mathrm{I}$

Non-factorizable decay current in SCET$_\mathrm{I}$:

$$J_0 = \bar{\psi}_{\mathrm{hc}}\, h_v$$

with

$$\langle \pi(p')|J_0|B(v)\rangle = (n_+p')\,\xi_\pi(n_+p')$$

Interpolating current for pion:

$$\begin{aligned} J_\pi &= -i\bar{\psi}_{\mathrm{hc}}\, \not{n}_+\gamma_5\, \psi_{\mathrm{hc}} \\ &\quad -i\left(\bar{\psi}_{\mathrm{hc}}\, \not{n}_+\gamma_5\, \psi_s + \mathrm{h.c.}\right) \end{aligned}$$

with $\langle 0|J_\pi|\pi(p')\rangle = (n_+p')\, f_\pi$

Define correlation function:

$$\Pi_0(n_-p') = i\int d^4x\, e^{ip'\cdot x}\, \langle 0|\,T\left[J_\pi(x)J_0(0)\right]|B(v)\rangle$$

in Euclidean region, i.e.

$$(n_+p') \simeq 2E_\pi = \mathcal{O}(m_b)\,, \qquad 0 > (n_-p') = \mathcal{O}(\Lambda)\,, \qquad p_\perp^\mu \equiv 0$$

Correlation function for ξ_π in SCET$_\mathrm{I}$

Non-factorizable decay current in SCET$_\mathrm{I}$:

$$J_0 = \bar{\psi}_{\mathrm{hc}}\, h_v$$

with

$$\langle \pi(p')|J_0|B(v)\rangle = (n_+p')\,\xi_\pi(n_+p')$$

Interpolating current for pion:

$$\begin{aligned} J_\pi &= -i\bar{\psi}_{\mathrm{hc}}\, \not{n}_+\gamma_5\, \psi_{\mathrm{hc}} \\ &\quad -i\left(\bar{\psi}_{\mathrm{hc}}\, \not{n}_+\gamma_5\, \psi_s + \text{h.c.}\right) \end{aligned}$$

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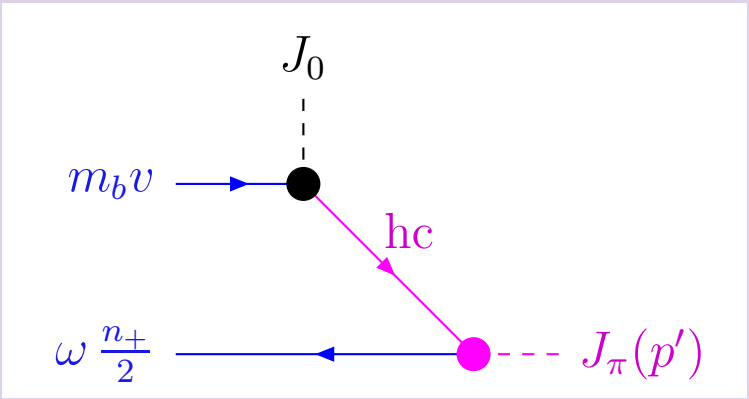
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Calculation of the correlation function

Tree level:

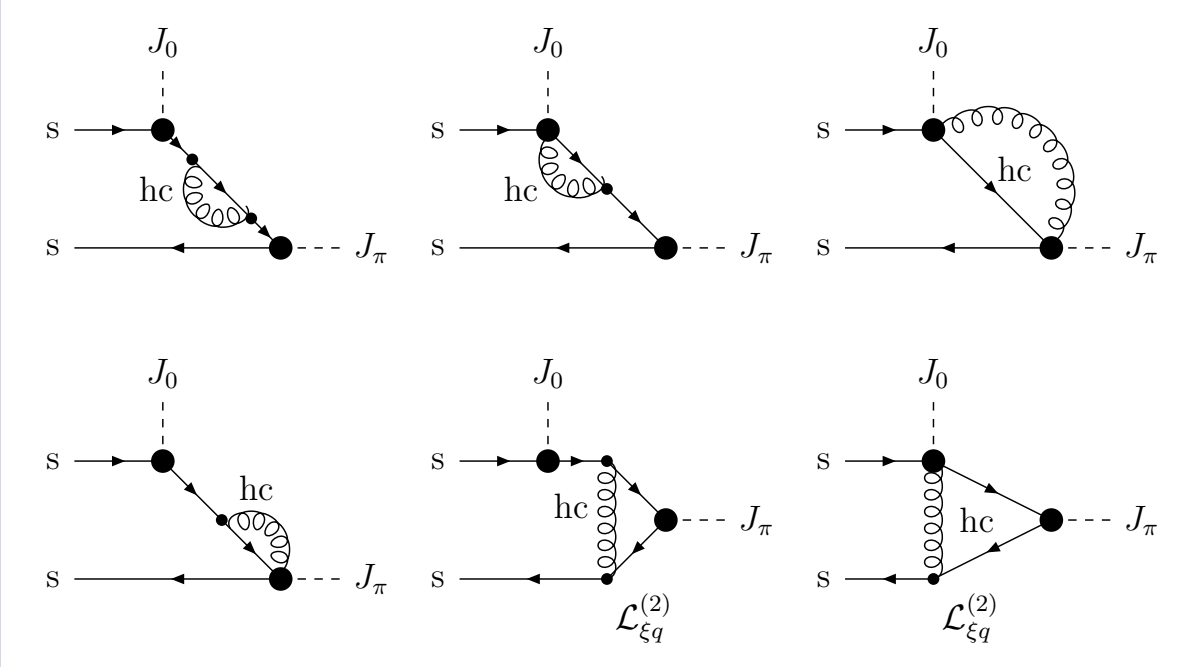
Integrate out hard-collinear quark propagator:



$$\Pi_0(n_- p') = f_B m_B \int_0^\infty d\omega \, \frac{\phi_-^B(\omega)}{\omega - n_- p' - i\eta}$$

- Result in terms of B-meson light-cone wave function ϕ_-^B:

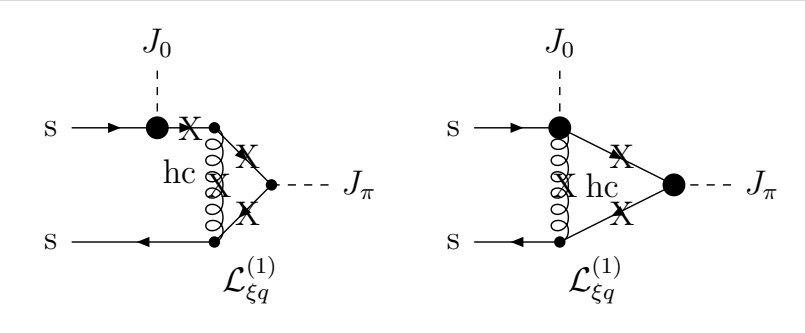
Radiative corrections involving $\phi_-^B(\omega)$:



- hard-collinear effects perturbatively calculable
- soft effects absorbed into *B*-meson LCWF

$\Rightarrow$ Factorization of correlation function in SCET_I

Radiative Corrections involving 3-particle LCDA:



$X =$ insertion of one soft gluon

(not calculated yet)

Renormalization-scale dependence

- Wilson coefficient in front of J_0 from SCET_I matching/running; resums Sudakov logs between m_b and μ.
  [Bauer/Fleming/Pirjol/Stewart 2000]

- Scale-dependence induced by hard-collinear loops. (see above)

- RG evolution of soft LCWF:
  - Universal effect from *cusp anomalous dimension*.
  - Single logs only known for $\phi_+^B(\omega,\mu)$ [Lange/Neubert 2002]

Leading scale dependence $\sim \ln^2 \mu$ drops out $\surd$

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Derivation of the sum rule ("DUALITY")

Dispersion relation:

$$\Pi_0(n_- p') = \frac{1}{\pi} \int_0^\infty d\omega' \, \frac{\mathrm{Im}\left[\Pi_0(\omega')\right]}{\omega' - n_- p' - i\eta}$$

For example, perturbative spectrum at tree level:

$$\mathrm{Im}\left[\Pi_0(\omega')\right] = \int d\omega \, f_B m_B \, \phi_-^B(\omega) \, \pi\delta(\omega - \omega') + \mathcal{O}(\alpha_s)$$

Continuum model:

For $\omega' > \omega_s \approx m^2_{[3\pi]}/(n_+ p')$ use:

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Pion contribution:

For $\omega' < \omega_s$:

spectrum saturated by pion pole,

$$\Pi_0\Big|_{\text{hadr.}} = -\frac{(n_+ p')}{(n_- p')} \, \xi_\pi(n_+ p') \, f_\pi$$

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  Choose Borel parameter ω_M of order $\Lambda^2_{\text{QCD}}/(n_+p')$:
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Systematic uncertainties reflected by ω_s and ω_M

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Numerical result for ξ_π at $q^2 = 0$

Numerical estimate (incl. NLO radiative corrections, excl. 3-particle LCDA)

$$\frac{T_i^{\rm I}(\mu)}{T_i^{\rm I}(m_b)} \cdot \xi_\pi(m_B,\mu) = 0.27 \pm 0.02\big|_{\mu} \pm 0.07\big|_{f_B\phi_-} {}^{+0.05}_{-0.08}\big|_{\rm s.r.} {}^{+0.00}_{-0.04}\big|_{\rm sys.}$$

Dependence on ω_M

Dependence on μ

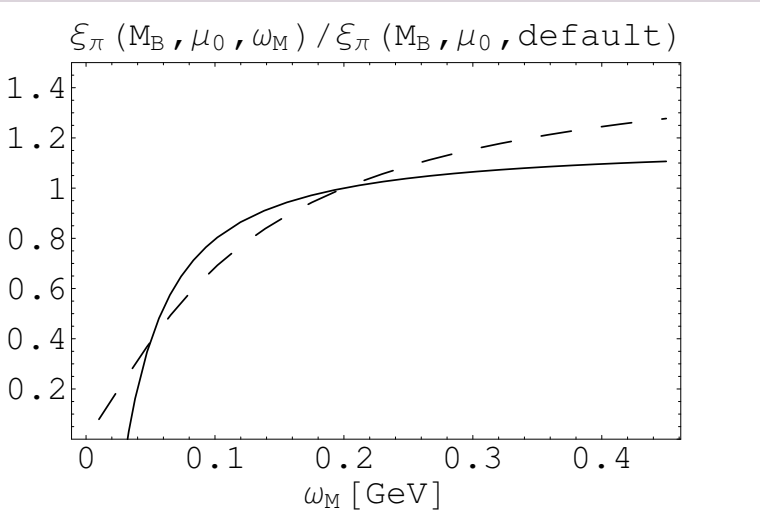
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(for $\omega_s = 200$ MeV, $\mu_0 = 1$ GeV)



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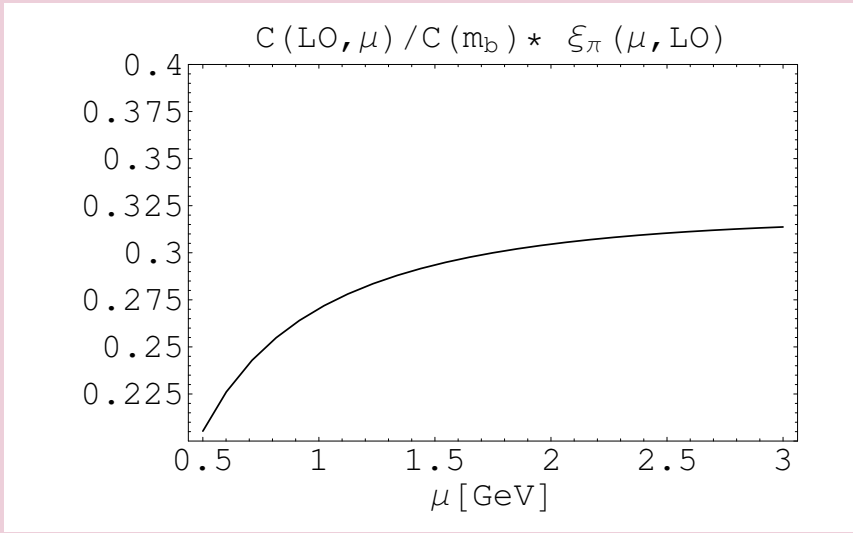
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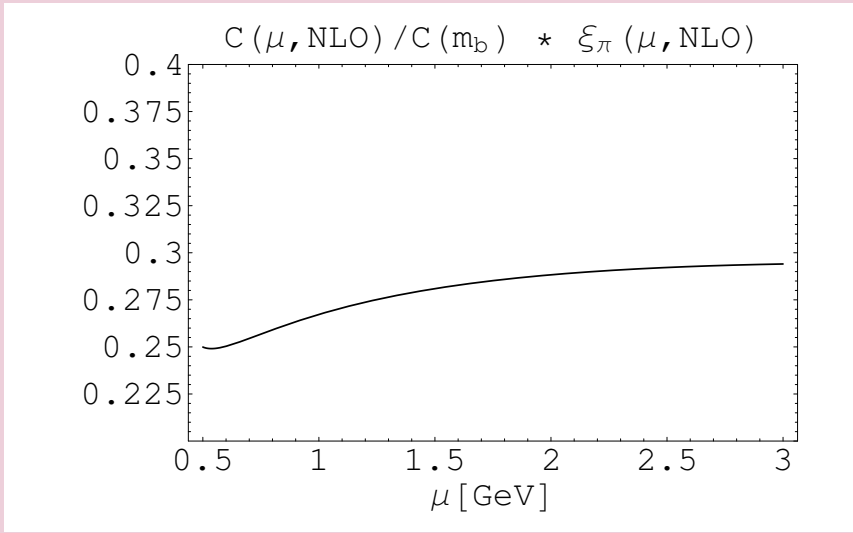
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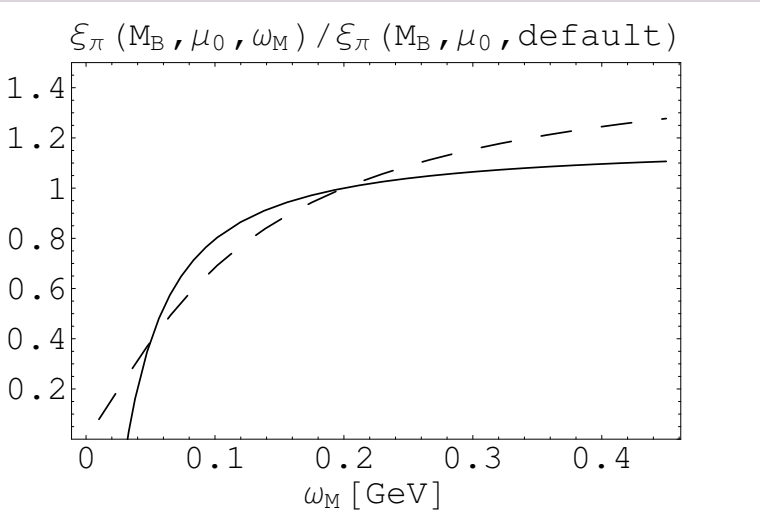
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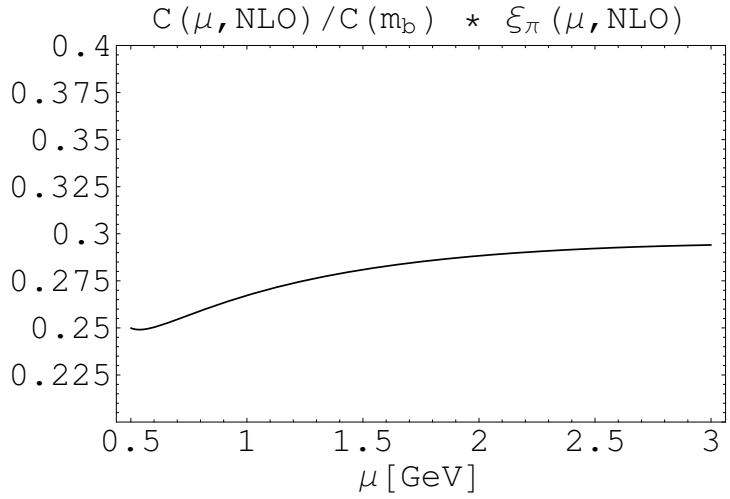


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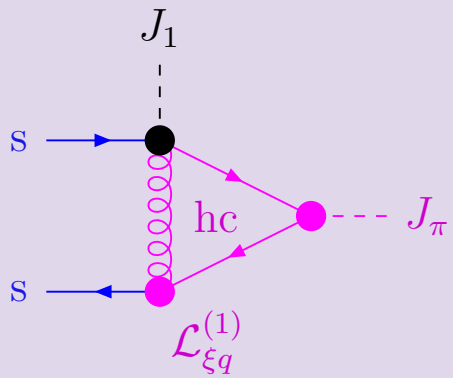


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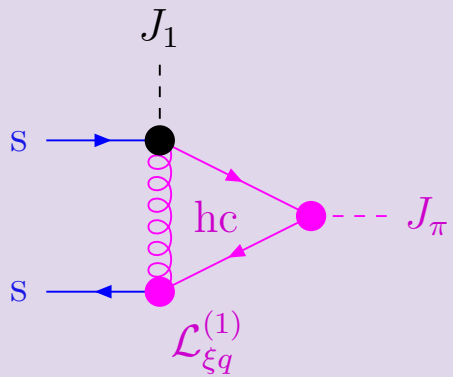


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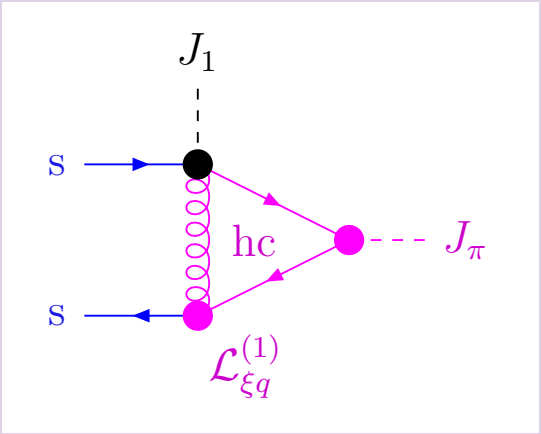


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Factorizable current at one-loop:

- After Borel transform correlator can be written as

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- Collinear dynamics modelled by threshold/Borel parameters.
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- Apply to other B-decays
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